

Primary Constrains of Newer General Relativity

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Outlook

- **The geometrical arena for symmetric teleparallel gravity;**
- **Irreducible decomposition formulation and ADM variables;**
- **Canonical Momenta;**
- **Kernel Elements;**
- **Primary Constraints;**
- **Review of the possible models;**
- **Covariant Primary Constraints;**
- **Conclusions & Perspectives.**

The geometric arena for symmetric teleparallel gravity

We consider a spacetime manifold equipped with two geometric, independent entities, the metric $g_{\mu\nu}$ and an affine connection $\Gamma_{\mu\nu}^\lambda$. In a general metric-affine framework, we

the **curvature tensor** $R_{\beta\mu\nu}^\alpha = \partial_\mu \Gamma_{\beta\nu}^\alpha - \partial_\nu \Gamma_{\beta\mu}^\alpha + \Gamma_{\sigma\mu}^\alpha \Gamma_{\beta\nu}^\sigma - \Gamma_{\sigma\nu}^\alpha \Gamma_{\beta\mu}^\sigma$;

the **torsion tensor** $T_{\mu\nu}^\alpha = \Gamma_{\nu\mu}^\alpha - \Gamma_{\mu\nu}^\alpha$;

the **nonmetricity tensor** $Q_{\alpha\mu\nu} = \nabla_\alpha g_{\mu\nu} = \partial_\alpha g_{\mu\nu} - \Gamma_{\mu\alpha}^\sigma g_{\sigma\nu} - \Gamma_{\nu\alpha}^\sigma g_{\mu\sigma}$.

The geometric arena for symmetric teleparallel gravity

If $T^{\alpha}_{\mu\nu} = 0$, the independent connection is symmetric, and if additionally $R^{\alpha}_{\beta\mu\nu} = 0$, it is determined as $\Gamma^{\rho}_{\mu\nu} = \frac{\partial x^{\rho}}{\partial \xi^a} \frac{\partial^2 \xi^a}{\partial x^{\mu} \partial x^{\nu}}$, with $\frac{\partial \xi^a}{\partial x^{\mu}}$ the (invertible) coordinate transformation matrix. At lowest order in derivatives, and restricting to parity-even terms, the most general gravitational action built from all quadratic scalars in the nonmetricity tensor is:

$$\begin{aligned} S(g_{\mu\nu}, \xi^a) &= \int d^4x \mathcal{L} = \frac{1}{2} \int d^4x \sqrt{-g} \mathbb{Q} \\ &= \frac{1}{2} \int d^4x \sqrt{-g} [c_1 Q_{\rho\mu\nu} Q^{\rho\mu\nu} + c_2 Q_{\nu\mu\rho} Q^{\rho\mu\nu} + c_3 Q^{\mu} Q_{\mu} + c_4 \widetilde{Q}^{\mu} \widetilde{Q}_{\mu} + c_5 Q^{\mu} \widetilde{Q}_{\mu}], \end{aligned}$$

where the nonmetricity scalar \mathbb{Q} is implicitly defined, and the coefficients C_i control the five independent quadratic nonmetricity invariants. This theory is called **Newer GR**. The Symmetric Teleparallel Equivalent of General Relativity (STTEGR) is recovered when $c_1 = -1/4$, $c_2 = 1/2$, $c_3 = 1/4$, $c_4 = 0$, $c_5 = -1/2$.

The geometric arena for symmetric teleparallel gravity

For the nonmetricity scalar it is commonly used in the literature in the form $\mathbb{Q} = \mathcal{P}^{\rho\mu\nu} Q_{\rho\mu\nu}$, where the nonmetricity conjugate or superpotential

$$\mathcal{P}^{\rho\mu\nu} = c_1 Q^{\rho\mu\nu} + c_2 Q^{(\nu\mu)\rho} + c_3 Q^\rho g^{\mu\nu} + c_4 g^{\rho(\nu} \tilde{Q}^{\mu)} + \frac{c_5}{2} \left(g^{\rho(\nu} Q^{\mu)} + \widetilde{Q}^\rho g^{\mu\nu} \right).$$

Then we collect the quadratic nonmetricity terms into $\mathbb{Q} = \widetilde{P}^{\alpha\beta\gamma\rho\mu\nu} Q_{\alpha\beta\gamma} Q_{\rho\mu\nu}$, so that

$$\widetilde{P}^{\alpha\beta\gamma\rho\mu\nu} = c_1 g^{\alpha\rho} g^{\beta\mu} g^{\gamma\nu} + c_2 g^{\alpha\nu} g^{\beta\mu} g^{\gamma\rho} + c_3 g^{\alpha\rho} g^{\beta\gamma} g^{\mu\nu} + c_4 g^{\alpha\gamma} g^{\beta\mu} g^{\rho\nu} + c_5 g^{\alpha\mu} g^{\beta\gamma} g^{\rho\nu}.$$

Since the nonmetricity tensor is symmetric in the last two indices we also define a fully symmetrized object:

$$\begin{aligned} & \widetilde{P}^{\alpha(\beta\gamma)\rho(\mu\nu)} \\ &= \frac{c_1}{2} g^{\alpha\rho} (g^{\beta\mu} g^{\gamma\nu} + g^{\beta\nu} g^{\gamma\mu}) + \frac{c_2}{4} (g^{\alpha\nu} g^{\beta\mu} g^{\gamma\rho} + g^{\alpha\nu} g^{\gamma\mu} g^{\beta\rho} + g^{\alpha\mu} g^{\beta\nu} g^{\gamma\rho} + g^{\alpha\mu} g^{\gamma\nu} g^{\beta\rho}) \\ &+ c_3 g^{\alpha\rho} g^{\beta\gamma} g^{\mu\nu} + \frac{c_4}{4} (g^{\alpha\gamma} g^{\beta\mu} g^{\rho\nu} + g^{\alpha\beta} g^{\gamma\mu} g^{\rho\nu} + g^{\alpha\gamma} g^{\beta\nu} g^{\rho\mu} + g^{\alpha\beta} g^{\gamma\nu} g^{\rho\mu}) \\ &+ \frac{c_5}{2} g^{\beta\gamma} (g^{\alpha\mu} g^{\rho\nu} + g^{\alpha\nu} g^{\rho\mu}). \end{aligned}$$

The geometric arena for symmetric teleparallel gravity

The metric equations of motion of Newer GR are

$$\frac{2}{\sqrt{-g}} \nabla_{\rho} (\sqrt{-g} \mathcal{P}^{\rho\mu\nu}) + \mathcal{P}^{\mu\rho\lambda} Q_{\rho\lambda}^{\nu} + 2\mathcal{P}^{\rho\lambda\mu} Q_{\rho\lambda}^{\nu} - \mathbb{Q} g^{\mu\nu} = 0$$

while the connection field equations:

$$\nabla_{\rho} \nabla_{\mu} (\sqrt{-g} \mathcal{P}_{\nu}^{\rho\mu}) = 0,$$

the metric equations are 3rd order in derivatives in ξ^a , while in the connection equations they appear with derivatives of 4th order, while being up to derivatives of 3rd order in the metric tensor.

Irreducible decomposition formulation and ADM variables

We present an alternative formulation to the spectral analysis, using an irreducible decomposition of the most general kernel element.

We compute expressions for the canonical momenta separating explicitly the velocity components on the metric from the nondynamical parts. For this, we write the general expression for the Hessian of Newer GR defined as $K^{\alpha\beta\mu\nu} = P^{0(\alpha\beta)0(\mu\nu)} + P^{0(\mu\nu)0(\alpha\beta)}$, in terms of metric components

$$\begin{aligned} K^{\alpha\beta\mu\nu} &= c_1 g^{00} (g^{\alpha\mu} g^{\beta\nu} + g^{\alpha\nu} g^{\beta\mu}) + 2c_3 g^{00} g^{\alpha\beta} g^{\mu\nu} \\ &+ \frac{c_2 + c_4}{2} (g^{0\beta} g^{0\nu} g^{\alpha\mu} + g^{0\alpha} g^{0\nu} g^{\beta\mu} + g^{0\beta} g^{0\mu} g^{\alpha\nu} + g^{0\alpha} g^{0\mu} g^{\beta\nu}) + c_5 g^{\alpha\beta} g^{0\mu} \end{aligned}$$

Canonical momenta

Thus, canonical momenta are defined as

$$\pi^{\alpha\beta} = \frac{1}{2} \sqrt{-g} (K^{\alpha\beta\mu\nu} Q_{0\mu\nu} + B^{\alpha\beta})$$

where

$$B^{\alpha\beta} = (P^{0(\alpha\beta)i(\mu\nu)} + P^{i(\mu\nu)0(\alpha\beta)}) Q_{i\mu\nu}$$

encode terms independent of the velocities.

After the 3+1 split of the the spacetime components, $\mu = 0, i$, of the Hessian, the canonical momenta are

$$\begin{aligned} \frac{2\pi^{00}}{\sqrt{-g}} &= K^{0000} g_{00} + 2K^{000i} g_{0i} + K^{00ij} g_{ij} + C^{00}, & g_{00} &= -\alpha^2 + \beta_i \beta^i, g_{0i} = \beta_i, g_{ij} = \gamma_{ij}, \\ \frac{2\pi^{0i}}{\sqrt{-g}} &= K^{0i00} g_{00} + 2K^{0i0j} g_{0j} + K^{0ikl} g_{kl} + C^{0i}, & n_\mu &= (-\alpha, 0), n^\mu = (1/\alpha, -\beta^i/\alpha) \\ \frac{2\pi^{ij}}{\sqrt{-g}} &= K^{ij00} g_{00} + 2K^{ij0k} g_{0k} + K^{ijkl} g_{kl} + C^{ij}, & g_{00} &= -2\alpha\dot{\alpha} + 2\beta^i \dot{\beta}_i - \beta^i \beta^j \gamma_{ij}, g_{0i} = \dot{\beta}_i, g_{ij} = \gamma_{ij}. \end{aligned}$$

$$C^{\alpha\beta} = B^{\alpha\beta} - K^{\alpha\beta\mu\nu} q_{0\mu\nu}.$$

Canonical momenta

The conjugate to the ADM variables

$$\begin{aligned}
 \pi^\alpha &= -\sqrt{\gamma} \left[\frac{4\hat{c}}{\alpha^3} \dot{\alpha} + \left(\frac{c_{35}}{\alpha^2} \gamma^{ij} - \frac{2\hat{c}}{\alpha^4} \beta^i \beta^j \right) \dot{\gamma}_{ij} + \alpha^2 C^{00} \right] \\
 \pi^{\beta^k} &= \sqrt{\gamma} \left[\frac{2c_{124}}{\alpha^3} \gamma^{ij} \dot{\beta}_j - \left(\frac{c_{124}}{\alpha^3} (\beta^j \gamma^{ik} + \beta^k \gamma^{ij}) + \frac{2\hat{c}}{\alpha^5} \beta^i \beta^j \beta^k \right) \dot{\gamma}_{ij} + \alpha (\beta^i C^{00} + C^{0i}) \right] \\
 \pi^{\gamma^{ij}} &= \sqrt{\gamma} \left[-\frac{c_{35}}{\alpha^2} \gamma^{ij} \dot{\alpha} - \frac{c_{124}}{\alpha^3} (\beta^i \gamma^{jk} + \beta^j \gamma^{ik}) \dot{\beta}_k \right. \\
 &\quad + \left(\frac{\hat{c}}{\alpha^5} \beta^i \beta^j \beta^k \beta^l + \frac{c_{124}}{2\alpha^3} (\gamma^{ik} \beta^j \beta^l + \gamma^{il} \beta^j \beta^k + \gamma^{jk} \beta^i \beta^l + \gamma^{jl} \beta^i \beta^k) - \frac{c_1}{2\alpha} (\gamma^{ik} \gamma^{jl} + \gamma^{il} \gamma^{jk}) - \frac{c_3}{\alpha} \gamma^{ij} \gamma^{kl} \right) \dot{\gamma}_{kl} \\
 &\quad \left. + \frac{\alpha}{2} (C^{ij} - \beta^i \beta^j C^{00}) \right].
 \end{aligned}$$

In comparison to the ADM formulation of GR, the momenta conjugate to the lapse and shift do not vanish identically. All dependence on nondynamical pieces of the connection is contained in the velocity-independent terms $C^{\mu\nu}$, which shift the canonical momenta but do not change the rank of the Hessian.

Kernel elements

In order to obtain the primary constraints for Newer GR, we must obtain the kernel $X_{\alpha\beta} = X_{\beta\alpha}$ of the Hessian $K^{\alpha\beta\mu\nu}$:

$$K^{\alpha\beta\mu\nu} X_{\mu\nu} = 0, \forall \mu, \nu.$$

If $X_{\alpha\beta}$ is an element of the kernel, then multiplying it by the left of $\frac{2\pi^{\alpha\beta}}{\sqrt{-g}} - C^{\alpha\beta} = K^{\alpha\beta\mu\nu} g_{\mu\nu}$, we obtain the primary constraints $X_{\alpha\beta} \left(\frac{2\pi^{\alpha\beta}}{\sqrt{-g}} - C^{\alpha\beta} \right) = X_{\alpha\beta} K^{\alpha\beta\mu\nu} g_{\mu\nu} = 0$.

Thus, the Hessian does not have a kernel for arbitrary values of the c_i , and different cases will give a different kernel and consequently different number of primary constraints.

Any arbitrary component of the kernel can be decomposed as

$$X_{\mu\nu} = a n_{\mu} n_{\nu} + b \gamma_{\mu\nu} + 2n_{(\mu} V_{\nu)} + S_{\mu\nu},$$

V_{μ} a purely spatial vector: $n^{\mu} V_{\mu} = 0$;

$S_{\mu\nu}$ a symmetric trace-free tensor: $n^{\mu} S_{\mu\nu} = 0, g^{\mu\nu} S_{\mu\nu} = \gamma^{\mu\nu} S_{\mu\nu} = 0$.

Kernel elements

The action of the generic kernel element $X_{\alpha\beta}$ in the Hessian in three components is

$$K^{\alpha\beta\mu\nu}X_{\mu\nu} = K_{scalar}^{\alpha\beta} + K_{vector}^{\alpha\beta} + K_{tensor}^{\alpha\beta}.$$

$$K_{scalar}^{\alpha\beta} = M = \begin{pmatrix} 2g^{00}(c_1 + c_2 + c_3 + c_4 + c_5) & -3g^{00}(2c_3 + c_5) \\ -g^{00}(2c_3 + c_5) & 2g^{00}(c_1 + 3c_3) \end{pmatrix} \begin{pmatrix} (a) \\ (b) \end{pmatrix},$$

$$K_{vector}^{\alpha\beta} = (2c_1 + c_2 + c_4)g^{00}(V^\alpha n^\beta + V^\beta n^\alpha)$$

$$K_{tensor}^{\alpha\beta} = 2c_1g^{00}S^{\alpha\beta}.$$

Moreover, the three degeneracy conditions

$$c_1^2 + c_1(c_2 + 4c_3 + c_4 + c_5) + 3c_3(c_2 + c_4) - \frac{3}{4}c_5^2 = 0$$

$$2c_1 + c_2 + c_4 = 0,$$

$c_1 = 0$ represents the factorization of the Hessian determinant into the three kind of modes. The results of this method fully coincide with the spectral analysis

Primary constraints

- The tensor sector is controlled by the degeneracy condition $c_1 = 0$. If this condition is achieved, then every spatial symmetric trace-free tensor $S_{\mu\nu}$ belongs to the kernel of the Hessian. In three spatial dimensions, $S_{\mu\nu}$ has 5 independent components, hence there are 5 primary constraints.
- In the vector sector, every spatial vector V_μ belongs to the kernel if the condition $2c_1 + c_2 + c_4 = 0$ is satisfied. The three independent components of this arbitrary spatial vector give rise to 3 primary constraints
- The scalar sector is two-dimensional, spanned by $n_\mu n_\nu$ and $\gamma_{\mu\nu}$, becoming degenerate when

$$c_1^2 + c_1(c_2 + 4c_3 + c_4 + c_5) + 3c_3(c_2 + c_4) - \frac{3}{4}c_5^2 = 0$$

If the scalar block has rank 1, then the kernel is one-dimensional, and there is exactly 1 scalar primary constraint.

Primary constraints

If $(a, b) \neq (0,0)$ is a kernel vector of the scalar matrix

$$\begin{pmatrix} 2(c_1 + c_2 + c_3 + c_4 + c_5) & -3(2c_3 + c_5) \\ -(2c_3 + c_5) & 2(c_1 + 3c_3) \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix},$$

Then, the scalar primary constraint is $an_\mu n_\nu Y^{\mu\nu} + b\gamma_{\mu\nu} Y^{\mu\nu} \approx 0$. It could be chosen a convenient generic kernel vector such as $a = 2(c_1 + 3c_3)$, $b = 2c_3 + c_5$.

The scalar sector gives two constraints only if the full 2×2 scalar block vanishes.

Thus, simultaneously, $2c_3 + c_5 = 0$, $c_1 + 3c_3 = 0$, $(c_1 + c_2 + c_3 + c_4 + c_5) = 0$. In this case, both scalar directions belong to the kernel and the two scalar primary constraints are $n_\mu n_\nu Y^{\mu\nu} \approx 0$, $\gamma_{\mu\nu} Y^{\mu\nu} \approx 0$, which in components, are written as

$$\frac{2\pi^{00}}{\sqrt{-g}} - C^{00} \approx 0$$

$$\gamma_{ij} \left(\frac{2\pi^{ij}}{\sqrt{-g}} - C^{ij} \right) + 2\beta_i \left(\frac{2\pi^{0i}}{\sqrt{-g}} - C^{0i} \right) + \beta_i \beta^i \left(\frac{2\pi^{00}}{\sqrt{-g}} - C^{00} \right) \approx 0$$

Review of the possible models

The Hessian of Newer GR decomposes into irreducible tensor, vector and scalar blocks of dimensions 5, 3 and 2, respectively. The corresponding primary constraints are therefore naturally classified according to the degeneracies of these three sectors.

Case	Tensor	Vector	Scalar	NPC	Relation to [41]
0	no	no	no	0	Sector 0
1	no	no	1 scalar	1	Part of Sector I / scalar-deg. sector
2	no	no	2 scalars	2	Not present
3	no	yes	no	3	Sector II
4	no	yes	1 scalar	4	Sector V (includes GR)
5	yes	no	no	5	Sector III
6	yes	no	1 scalar	6	Sector IV
7	yes	yes	no	8	Sector VI
8	yes	yes	1 scalar	9	Sector VII
9	yes	yes	2 scalars	10	Sector VIII / fully deg. case

Possible combinations of primary constraints from the tensor, vector and scalar sectors of the Hessian. NPC denotes number of primary constraints. The equivalence is not full.

Covariant primary constraints

One may try to define canonical momenta directly for the scalar fields ξ^a determining the symmetric teleparallel connection. Since the connection depends on second derivatives of ξ^a , this would require an Ostrogradsky procedure. Introducing the auxiliary variable $\Xi = \dot{\xi}^a$, one would define two momenta

$$\begin{aligned}\Pi_a &= \frac{\partial \mathcal{L}}{\partial \dot{\xi}^a} - \partial_t \left(\frac{\partial \mathcal{L}}{\partial \ddot{\xi}^a} \right) = \frac{\partial \mathcal{L}}{\partial \Xi^a} - \partial_t \left(\frac{\partial \mathcal{L}}{\partial \dot{\Xi}^a} \right), \\ P_a &= \frac{\partial \mathcal{L}}{\partial \dot{\xi}^a} = \frac{\partial \mathcal{L}}{\partial \dot{\Xi}^a}.\end{aligned}$$

Problems:

- ξ^a play the role of coordinate fields as well as dynamical variables.
- the Ostrogradsky construction in suggests that the momenta will appear linearly in the Hamiltonian, which as a result will give a Hamiltonian unbounded from below and be prone to ghosts.

Covariant primary constraints

To reduce the order of the derivatives in the connection, a tetrad-like approach to the symmetric teleparallel connection was proposed, by defining

$$e_{\mu}^a = \frac{\partial \xi^a}{\partial x^{\mu}}$$

Let E_a^{μ} denote the inverse matrix. The symmetric teleparallel connection is then written as

$$\Gamma_{\mu\nu}^{\rho} = E_a^{\rho} \partial_{\nu} e_{\mu}^a.$$

The integrability condition $\partial_{\mu} e_{\nu}^a - \partial_{\nu} e_{\mu}^a = 0$ ensures that e_{μ}^a is locally of the form $\partial_{\mu} \xi^a$, and also enforces the torsionless condition for the connection above.

Covariant primary constraints

The symmetric teleparallel connection can be written as $\Gamma_{\rho\mu\nu} = g_{\rho\sigma} \frac{\partial x^\sigma}{\partial \xi^a} \frac{\partial^2 \xi^a}{\partial x^\mu \partial x^\nu} = E_{a\rho} \partial_\nu e_\mu^a$. As a consequence, the nonmetricity tensor can be written as

$$Q_{\rho\mu\nu} = \partial_\rho g_{\mu\nu} - E_{a\nu} \partial_\rho e_\mu^a - E_{a\mu} \partial_\rho e_\nu^a.$$

and the momenta conjugate

$$P^{a\sigma} = \frac{\partial \mathcal{L}}{\partial \partial_0 e_\sigma^a} = \frac{\partial \mathcal{L}}{\partial Q_{\rho\mu\nu}} \frac{\partial Q_{\rho\mu\nu}}{\partial (\partial_0 e_\sigma^a)}$$

We have that $\frac{\partial \mathcal{L}}{\partial Q_{\rho\mu\nu}} = \sqrt{-g} \mathcal{P}^{\rho\mu\nu}$, and the variation of the nonmetricity tensor in terms of the tetrads gives the following

$$\frac{\partial Q_{\rho\mu\nu}}{\partial (\partial_0 e_\sigma^a)} = -\delta_\rho^0 (E_{a\nu} \delta_\mu^\sigma + E_{a\mu} \delta_\nu^\sigma)$$

The new momenta are then

$$P_a^\sigma = -2E_{a\nu} \pi^{\sigma\nu} + \lambda_a^{0\sigma} - \lambda_a^{\sigma 0} \rightarrow \chi_a^\sigma = P_a^\sigma + 2E_{a\nu} \pi^{\sigma\nu} - 2\lambda_a^{0\sigma} \approx 0.$$

Conclusions & Perspective

- Using the irreducible decomposition, we found tensor, vector, and scalar sectors, each with its own degeneracy condition and corresponding primary constraints;
- The tensor sector contributes five primary constraints;
- The vector sector contributes three primary constraints;
- The most general condition for constraints in the scalar sector, in any spacetime dimension, is vanishing of matrix determinant 2×2 scalar block;
- Two independent scalar primary constraints;
- The introduction of first-order variables $e_\mu^a = \partial_\mu \xi^a$ avoids Ostrogradsky instabilities caused by the second derivatives of the ξ^a ;
- In this approach, we were able to find the primary constraints from the covariant formulation. These constraints provide the natural starting point for a fully covariant Hamiltonian analysis of symmetric teleparallel theories beyond the coincident gauge;

Conclusions & Perspective

- They express the fact that, once the connection is written in first-order pure-gauge form, the momenta conjugate to the connection variables are not independent from the metric momenta;
- This primary-constraint analysis depends only on the kinetic Hessian of the quadratic nonmetricity action. In this sense, this methods can also be read as a classification of the degeneracies of a globally Lorentz-covariant quadratic kinetic structure for a symmetric rank-two tensor;
- This work does not yet determine the final number of propagating degrees of freedom. For that, one must continue the Dirac-Bergmann algorithm: construct the primary Hamiltonian, impose time preservation of primary constraints, determine the secondary constraints, and classify the full constraint set into first- and second-class;
- In future work, we would like to determine the Hamiltonian and prove that these constraints commute with the remaining primary and secondary constraints;
- Determine the Hamiltonian for Metric-Affine gravity for a theory with general Ricci scalar and quadratic nonmetricity.



THANK YOU!

References

- [1] Ferrara, C., Golovnev, A., and Guzmán, M. J., “Primary Constraints of Newer General Relativity arXiv:2605.30221, 2026. doi:10.48550/arXiv.2605.30221 (accepted on IJGMMP).
- [2] D'Ambrosio, F. and Heisenberg, L., “Classification of primary constraints of quadratic non-metricity theories of gravity”, Journal of High Energy Physics, vol. 2021, no. 2, Art. no. 170, Springer, 2021. doi:10.1007/JHEP02(2021)170.
- [3] Bajardi, F. and Blixt, D., “Primary constraints in general teleparallel quadratic gravity”, Physical Review D, vol. 109, no. 8, Art. no. 084078, APS, 2024. doi:10.1103/PhysRevD.109.084078
- [4] Capozziello, S., De Falco, V., and Ferrara, C., “Comparing equivalent gravities: common features and differences”, European Physical Journal C, vol. 82, no. 10, Art. no. 865, Springer, 2022. doi:10.1140/epjc/s10052-022-10823-x.