Geometric Foundations in and beyond Gravity: The Perspective of Particle Detector Models

Sebastian Schuster with
(1) Finnian Gray (Macquarie), and Christian Pfeifer (ZARM, UBremen)
(2) Sam Dolan (Sheffield), Magdalena Zych (SU), Navdeep Arya (SU), ...TBA...?

Fysikum
Stockholms universitet

30th June 2026

Stockholm University

Geometric Foundations in and beyond Gravity: The Perspective of Particle Detector Models

Sebastian Schuster with
(1) Finnian Gray (Macquarie), and Christian Pfeifer (ZARM, UBremen)
(2) Sam Dolan (Sheffield), Magdalena Zych (SU), Navdeep Arya (SU), …TBA…?

Fysikum
Stockholms universitet

30th June 2026

Stockholm University

Outline

Introduction: Analogues and Geometric Foundations

Caveats & Conventions

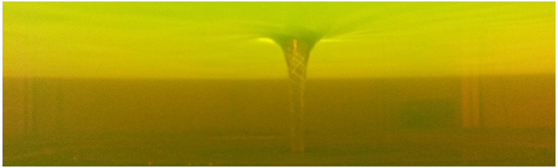
- Signature: $-+++$
- $G = c = \hbar = 1$
- Space-time indices: $abcd\ldots$
- Boys–Post constitutive relations:

$$\begin{pmatrix}\mathbf{D}\\ \mathbf{H}\end{pmatrix} = \begin{pmatrix}\epsilon & \zeta \\ \zeta^\dagger & \mu^{-1}\end{pmatrix}\begin{pmatrix}\mathbf{E}\\ \mathbf{B}\end{pmatrix} \qquad \Longleftrightarrow \qquad G^{ab} = Z^{abcd}F_{cd}.$$

- PSA: EM terminology *not* gravitoelectromagnetism
- PSA: ‘Analogue Space-Times’ $\gg$ ‘Analogue Gravity’[1]
- arXiv numbers for convenience, not to imply ‘unpublished’

[1] But see, *e.g.*, Erkul & Leonhardt (2025) arXiv:2508.11300, Volovik – *The Universe in a Helium Droplet*, ...

What Are Analogues?



Black hole

White hole

Image sources: (L) Jessica Santiago (2017),

(R) G. Rousseaux, DOI:10.1007/978-3-319-00266-8_5 in Faccio *et al.* — Analogue Gravity Phenomenology (2013), p.99

Why Analogues Matter for Geometric Foundations of Gravity?

- Alternate/modified theories of gravity influence(d by) semi-classical and quantum gravity

Why Analogues Matter for Geometric Foundations of Gravity?

- Alternate/modified theories of gravity influence(d by) semi-classical and quantum gravity
- Operational methods to identify what is ‘physical’

Why Analogues Matter for Geometric Foundations of Gravity?

- Alternate/modified theories of gravity influence(d by) semi-classical and quantum gravity
- Operational methods to identify what is ‘physical’
- In semi-classical gravity, the Unruh–DeWitt (UDW) detector is such a method

Why Analogues Matter for Geometric Foundations of Gravity?

- Alternate/modified theories of gravity influence(d by) semi-classical and quantum gravity
- Operational methods to identify what is ‘physical’
- In semi-classical gravity, the Unruh–DeWitt (UDW) detector is such a method
- UDW detectors sensitive to modifications of the underlying physics/geometry

Why Analogues Matter for Geometric Foundations of Gravity?

- Alternate/modified theories of gravity influence(d by) semi-classical and quantum gravity
- Operational methods to identify what is ‘physical’
- In semi-classical gravity, the Unruh–DeWitt (UDW) detector is such a method
- UDW detectors sensitive to modifications of the underlying physics/geometry
- $\Longrightarrow$ Analogues can teach and learn from geometry, too!

Why Analogues Matter for Geometric Foundations of Gravity?

- Alternate/modified theories of gravity influence(d by) semi-classical and quantum gravity
- Operational methods to identify what is ‘physical’
- In semi-classical gravity, the Unruh–DeWitt (UDW) detector is such a method
- UDW detectors sensitive to modifications of the underlying physics/geometry
- $\implies$ Analogues can teach and learn from geometry, too!
- Let’s study semi-classical effects of geometry in analogues!

1-Slide Introduction: Unruh–DeWitt Detectors

- Operational definition of 'particle'
- 'A particle makes a detector click'
- Introduces a (usually) non-relativistic detector model on the space-time
- Example for a scalar field ϕ:

$$H_{\text{int}}(\tau) = g_{\text{coupling}} \chi(\tau) \phi(x^\mu(\tau)) \otimes \mu(\tau)$$

- Detector response function

$$\mathcal{F}(E) := \int \mathrm{d}\tau' \, \mathrm{d}\tau'' \, \chi' \chi'' e^{-iE(\tau' - \tau'')} \mathscr{W}(\tau', \tau''),$$

$\mathscr{W}$: pull-back of Wightman function to detector's worldline.

Electromagnetism and the Earliest Analogue

The Gordon Analogue and Its Issue

- In macroscopic EM, the Gordon analogue goes as: Find constitutive tensor Z such that

$$S \stackrel{!}{=} -\frac{1}{8}\int \mathrm{d}^4x\sqrt{-\det g_{\text{eff}}}\left(\left[g_{\text{eff}}^{-1}\right]^{ac}\left[g_{\text{eff}}^{-1}\right]^{bd}-\left[g_{\text{eff}}^{-1}\right]^{ad}\left[g_{\text{eff}}^{-1}\right]^{bc}\right)F_{ab}F_{cd},$$

$$\textit{i.e.,}\quad Z^{abcd}=\frac{1}{2}\frac{\sqrt{\det g_{\text{eff}}}}{\sqrt{\det g}}\left(\left[g_{\text{eff}}^{-1}\right]^{ac}\left[g_{\text{eff}}^{-1}\right]^{bd}-\left[g_{\text{eff}}^{-1}\right]^{ad}\left[g_{\text{eff}}^{-1}\right]^{bc}\right).$$

- The consistency condition (for isotropic media $\epsilon=\mu$) follows from $\mathrm{d.o.f.}(g^{\text{eff}}_{ab})\neq\mathrm{d.o.f.}(Z^{abcd})$.[2]
- Warning! $\nabla_{g_{\text{lab}}}$ is not metric-compatible with $\nabla_{g_{\text{eff}}}$ and *vice versa*

[2]See, for example, SeSc, Visser arXiv:1706.06280 for the general case.

The Gordon Analogue and Its Issue

- In macroscopic EM, the Gordon analogue goes as: Find constitutive tensor Z such that

$$S \overset{!}{=} -\tfrac{1}{8} \int \mathrm{d}^4x \sqrt{-\det g_{\text{eff}}} \left(\left[g_{\text{eff}}^{-1}\right]^{ac} \left[g_{\text{eff}}^{-1}\right]^{bd} - \left[g_{\text{eff}}^{-1}\right]^{ad} \left[g_{\text{eff}}^{-1}\right]^{bc} \right) F_{ab} F_{cd},$$

$$\textit{i.e.,} \quad Z^{abcd} = \tfrac{1}{2} \frac{\sqrt{\det g_{\text{eff}}}}{\sqrt{\det g}} \left(\left[g_{\text{eff}}^{-1}\right]^{ac} \left[g_{\text{eff}}^{-1}\right]^{bd} - \left[g_{\text{eff}}^{-1}\right]^{ad} \left[g_{\text{eff}}^{-1}\right]^{bc} \right).$$

- The consistency condition (for isotropic media $\epsilon = \mu$) follows from $\mathrm{d.o.f.}(g^{\text{eff}}_{ab}) \neq \mathrm{d.o.f.}(Z^{abcd})$.[2]
- Warning! $\nabla_{g_{\text{lab}}}$ is not metric-compatible with $\nabla_{g_{\text{eff}}}$ and *vice versa*
- $\implies$ **Cartographic distortions, non-metricity**

$$q^{\text{lab/eff}}_{bca} := \nabla^{\text{lab/eff}}_{a} g^{\text{eff/lab}}_{bc},$$

- This has often been (implicitly) observed, *e.g.*, Cummer, Fathi, Frauendiener, Thompson (arXiv:1006.3364, arXiv:1602.08341, arXiv:1705.11108); Sawicki, Trenkler, Vikman (arXiv:2412.21169); SeSc, Visser (arXiv:1808.07987)

[2] See, for example, SeSc, Visser arXiv:1706.06280 for the general case.

Issues of Coordinates

- Algebraic EM analogue of Schwarzschild:

$$\epsilon^{ab} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r(r-2M)} & 0 \\ 0 & 0 & \frac{1}{r(r-2M)\sin^2\theta} \end{pmatrix},$$

$$\mu^{ab} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r(r-2M)} & 0 \\ 0 & 0 & \frac{1}{r(r-2M)\sin^2\theta} \end{pmatrix},$$

$$\zeta^a{}_b = 0.$$

See SeSc, Visser arXiv:1808.07987

3+1D Cartography in Action

Issues of Coordinates

- Algebraic EM analogue of Schwarzschild:

$$\epsilon^{ab} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r(r-2M)} & 0 \\ 0 & 0 & \frac{1}{r(r-2M)\sin^2\theta} \end{pmatrix},$$

$$\mu^{ab} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r(r-2M)} & 0 \\ 0 & 0 & \frac{1}{r(r-2M)\sin^2\theta} \end{pmatrix},$$

$$\zeta^{a}{}_{b} = 0.$$

See SeSc, Visser arXiv:1808.07987

3+1D Cartography in Action

Issues of Coordinates

- Algebraic EM analogue of Schwarzschild:

$$\epsilon^{ab} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r(r-2M)} & 0 \\ 0 & 0 & \frac{1}{r(r-2M)\sin^2\theta} \end{pmatrix},$$

$$\mu^{ab} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r(r-2M)} & 0 \\ 0 & 0 & \frac{1}{r(r-2M)\sin^2\theta} \end{pmatrix},$$

$$\zeta^a{}_b = 0.$$

- Coordinates for $(M, g_{\text{lab}}) \neq$ Coordinates for (M, g_{eff})!

See SeSc, Visser arXiv:1808.07987

3+1D Cartography in Action

Issues of Coordinates

- Algebraic EM analogue of Schwarzschild:

$$\epsilon^{ab} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r(r-2M)} & 0 \\ 0 & 0 & \frac{1}{r(r-2M)\sin^2\theta} \end{pmatrix},$$

$$\mu^{ab} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r(r-2M)} & 0 \\ 0 & 0 & \frac{1}{r(r-2M)\sin^2\theta} \end{pmatrix},$$

$$\zeta^a{}_b = 0.$$

- Coordinates for $(M, g_{\text{lab}}) \neq$ Coordinates for (M, g_{eff})!

Example: Scaled lab coordinates

Hawking temperature of a Schwarzschild BH

- Hawking temperature can be rephrased in terms of either M or $\epsilon, \left[\mu^{-1}\right], \zeta$

- $T_{\text{H,eff}} = \left(\frac{\hbar}{4\pi\sqrt{\det g_{\text{lab}} \ \det \epsilon}} \right)_{,r_{\text{eff}}} \Big|_{r_{\text{eff}}=r_+}$

See SeSc, Visser arXiv:1808.07987

3+1D Cartography in Action

Issues of Coordinates

- Algebraic EM analogue of Schwarzschild:

$$\epsilon^{ab} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r(r-2M)} & 0 \\ 0 & 0 & \frac{1}{r(r-2M)\sin^2\theta} \end{pmatrix},$$

$$\mu^{ab} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r(r-2M)} & 0 \\ 0 & 0 & \frac{1}{r(r-2M)\sin^2\theta} \end{pmatrix},$$

$$\zeta^{a}{}_{b} = 0.$$

- Coordinates for $(M, g_{\text{lab}}) \neq$ Coordinates for (M, g_{eff})!

See SeSc, Visser arXiv:1808.07987

Example: Scaled lab coordinates

Hawking temperature of a Schwarzschild BH

- Hawking temperature can be rephrased in terms of either M or $\epsilon, [\mu^{-1}], \zeta$

- $T_{\text{H,eff}} = \left(\frac{\hbar}{4\pi\sqrt{\det g_{\text{lab}} \ \det\epsilon}} \right)_{,r_{\text{eff}}} \Bigg|_{r_{\text{eff}}=r_+}$

- Assume $r_{\text{eff}} = a\, r_{\text{lab}}$

3+1D Cartography in Action

Issues of Coordinates

- Algebraic EM analogue of Schwarzschild:

$$\epsilon^{ab} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r(r-2M)} & 0 \\ 0 & 0 & \frac{1}{r(r-2M)\sin^2\theta} \end{pmatrix},$$

$$\mu^{ab} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r(r-2M)} & 0 \\ 0 & 0 & \frac{1}{r(r-2M)\sin^2\theta} \end{pmatrix},$$

$$\zeta^{a}{}_{b} = 0.$$

- Coordinates for $(M, g_{\text{lab}}) \neq$ Coordinates for (M, g_{eff})!

See SeSc, Visser arXiv:1808.07987

Example: Scaled lab coordinates

Hawking temperature of a Schwarzschild BH

- Hawking temperature can be rephrased in terms of either M or ϵ, $\left[\mu^{-1}\right]$, ζ
- $T_{\text{H,eff}} = \left(\frac{\hbar}{4\pi\sqrt{\det g_{\text{lab}} \;\det \epsilon}} \right)_{,r_{\text{eff}}} \Bigg|_{r_{\text{eff}}=r_+}$
- Assume $r_{\text{eff}} = a\, r_{\text{lab}}$
- $T_{\text{H, lab}} = \frac{\hbar}{8\pi a^{9/2} M} = a^{-9/2}\, T_{\text{H, eff}}$

Issues of Coordinates

- Algebraic EM analogue of Schwarzschild:

$$\epsilon^{ab} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r(r-2M)} & 0 \\ 0 & 0 & \frac{1}{r(r-2M)\sin^2\theta} \end{pmatrix},$$

$$\mu^{ab} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r(r-2M)} & 0 \\ 0 & 0 & \frac{1}{r(r-2M)\sin^2\theta} \end{pmatrix},$$

$$\zeta^a{}_b = 0.$$

- Coordinates for $(M, g_{\text{lab}}) \neq$ Coordinates for (M, g_{eff})!

Example: Scaled lab coordinates

Hawking temperature of a Schwarzschild BH

- Hawking temperature can be rephrased in terms of either M or ϵ, $[\mu^{-1}]$, ζ
- $T_{\text{H,eff}} = \left(\frac{\hbar}{4\pi\sqrt{\det g_{\text{lab}} \ \det \epsilon}} \right)_{,r_{\text{eff}}} \Big|_{r_{\text{eff}}=r_+}$
- Assume $r_{\text{eff}} = a\, r_{\text{lab}}$
- $T_{\text{H, lab}} = \frac{\hbar}{8\pi a^{9/2} M} = a^{-9/2}\, T_{\text{H, eff}}$
- $T_{\text{H,eff}}$ conformally invariant under conformal *space-time* transformations of (M, g_{eff}); not under *space* transformations!

See SeSc, Visser arXiv:1808.07987

Area Metrics, Finsler Metrics, and Dispersion Relations

- For QG or EM—'area metrics'; reinterpreting Z as mapping area-forms[3]

$$\text{EM:} \quad S_{\text{EM}} = -\tfrac{1}{8} \int_M \mathrm{d}^4x \sqrt[6]{\det G^{6\times 6}_{\text{Petrov}}}\, F_{ab} Z^{abcd} F_{cd},$$

$$\text{GR+:} \quad S_{\text{EH}} = \int_M \mathrm{d}^4x \sqrt[6]{\det G^{6\times 6}_{\text{Petrov}}}\, `R'.$$

$\implies$ 'Area metric geometry'

[3] For details: Punzi, Schuller, Witte, Wohlfarth (arXiv:hep-th/0508170, arXiv:hep-th/0612141, arXiv:0908.1016).

Area Metrics, Finsler Metrics, and Dispersion Relations

- For QG or EM—'area metrics'; reinterpreting Z as mapping area-forms[3]

$$\text{EM:} \quad S_{\text{EM}} = -\frac{1}{8}\int_M \mathrm{d}^4x \sqrt[6]{\det G^{6\times 6}_{\text{Petrov}}}\, F_{ab} Z^{abcd} F_{cd},$$

$$\text{GR+:} \quad S_{\text{EH}} = \int_M \mathrm{d}^4x \sqrt[6]{\det G^{6\times 6}_{\text{Petrov}}}\, \text{`}R\text{'}.$$

$\Longrightarrow$ 'Area metric geometry'

- Z also tied to Finsler geometry:[4]

$$\mathrm{d}s = F(x,y), \qquad x \in M, y \in T_x M, \quad \text{homogeneous of degree 1 in } y$$

[3] For details: Punzi, Schuller, Witte, Wohlfarth (arXiv:hep-th/0508170, arXiv:hep-th/0612141, arXiv:0908.1016).

[4] For details: Pfeifer (arXiv:1903.10185); for application to EM: Perlick – *Ray Optics, Fermat's Principle, and Applications to General Relativity*.

Area Metrics, Finsler Metrics, and Dispersion Relations

- For QG or EM—'area metrics'; reinterpreting Z as mapping area-forms[3]

$$\text{EM:} \quad S_{\text{EM}} = -\frac{1}{8}\int_M \mathrm{d}^4x \sqrt[6]{\det G^{6\times 6}_{\text{Petrov}}} F_{ab} Z^{abcd} F_{cd},$$

$$\text{GR+:} \quad S_{\text{EH}} = \int_M \mathrm{d}^4x \sqrt[6]{\det G^{6\times 6}_{\text{Petrov}}}\, `R'.$$

$\implies$ 'Area metric geometry'

- Z also tied to Finsler geometry:[4]

$$\mathrm{d}s = F(x,y), \qquad x \in M, y \in T_x M, \quad \text{homogeneous of degree 1 in } y$$

- Lastly, also tied to general/modified/...dispersion relations $\omega(\mathbf{p})$ or $P_x(q) = m^{\deg P}$, $P_x : T^*_x M \to \mathbb{R}$.[5]

[3] For details: Punzi, Schuller, Witte, Wohlfarth (arXiv:hep-th/0508170, arXiv:hep-th/0612141, arXiv:0908.1016).

[4] For details: Pfeifer (arXiv:1903.10185); for application to EM: Perlick – *Ray Optics, Fermat's Principle, and Applications to General Relativity*.

[5] See Rätzel, Rivera, Schuller (arXiv:1010.1369). Warning! Technical & physical requirements on P omitted.

The Detector Model

- Ingredients for uniaxial crystal:
  - Flat background η,
  - Crystal rest frame's worldline U,
  - $Z^{abcd} = \eta^{a[c}\eta^{d]b} + U^{[a}X^{b]}U^{[c}X^{d]}$

The Detector Model

- Ingredients for uniaxial crystal:
  - Flat background[6] η,
  - Crystal rest frame's worldline U,
  - $Z^{abcd} = \eta^{a[c}\eta^{d]b} + U^{[a}X^{b]}U^{[c}X^{d]}$

[6] Well, unless the constitutive tensor is assumed to *define* a more general (birefringent) space-time geometry.

The Detector Model

- Ingredients for uniaxial crystal:
  - Flat background[6] η,
  - Crystal rest frame's worldline U,
  - $Z^{abcd} = \eta^{a[c}\eta^{d]b} + U^{[a}X^{b]}U^{[c}X^{d]}$
- Ingredients for detector
  - Detector worldline $\dot{x}$
  - Dipole M^a

[6]Well, unless the constitutive tensor is assumed to *define* a more general (birefringent) space-time geometry.

The Detector Model

- Ingredients for uniaxial crystal:
  - Flat background[6] η,
  - Crystal rest frame's worldline U,
  - $Z^{abcd} = \eta^{a[c}\eta^{d]b} + U^{[a}X^{b]}U^{[c}X^{d]}$
- Ingredients for detector
  - Detector worldline $\dot{x}$
  - Dipole M^a
- $\eta(\dot{x}, \dot{x}) = \eta(U, U) = -1$, $\eta(M, \dot{x}) = \eta(U, X) = 0$

[6] Well, unless the constitutive tensor is assumed to *define* a more general (birefringent) space-time geometry.

The Detector Model

- Ingredients for uniaxial crystal:
  - Flat background[6] η,
  - Crystal rest frame's worldline U,
  - $Z^{abcd} = \eta^{a[c}\eta^{d]b} + U^{[a}X^{b]}U^{[c}X^{d]}$
- Ingredients for detector
  - Detector worldline $\dot{x}$
  - Dipole M^a
- $\eta(\dot{x}, \dot{x}) = \eta(U, U) = -1$, $\eta(M, \dot{x}) = \eta(U, X) = 0$
- Fresnel equation[7] determines wave front momenta: $\eta^{-1}(k,k)\zeta^{-1}(k,k) = 0$ with

$$\zeta_{ab} = \eta_{ab} + \frac{\xi^2}{1+\xi^2}U_aU_b - \frac{1}{1+\xi^2}X_aX_b$$

[6] Well, unless the constitutive tensor is assumed to *define* a more general (birefringent) space-time geometry.
[7] See, for example, Hehl, Obukhov – *Foundations of Classical Electrodynamics*

The Detector Model

- Ingredients for uniaxial crystal:
  - Flat background[6] η,
  - Crystal rest frame's worldline U,
  - $Z^{abcd} = \eta^{a[c}\eta^{d]b} + U^{[a}X^{b]}U^{[c}X^{d]}$
- Ingredients for detector
  - Detector worldline $\dot{x}$
  - Dipole M^a
- $\eta(\dot{x},\dot{x}) = \eta(U,U) = -1$, $\eta(M,\dot{x}) = \eta(U,X) = 0$
- Fresnel equation[7] determines wave front momenta: $\eta^{-1}(k,k)\zeta^{-1}(k,k) = 0$ with

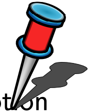
$$\zeta_{ab} = \eta_{ab} + \frac{\xi^2}{1+\xi^2}U_a U_b - \frac{1}{1+\xi^2}X_a X_b$$

- Early results: depending on $\angle(U,\dot{x})$, detector response even for inertial motion

[6] Well, unless the constitutive tensor is assumed to *define* a more general (birefringent) space-time geometry.
[7] See, for example, Hehl, Obukhov – *Foundations of Classical Electrodynamics*

- Ingredients for uniaxial crystal:
  - Flat background[6] η,
  - Crystal rest frame's worldline U,
  - $Z^{abcd} = \eta^{a[c}\eta^{d]b} + U^{[a}X^{b]}U^{[c}X^{d]}$
- Ingredients for detector
  - Detector worldline $\dot{x}$
  - Dipole M^a
- $\eta(\dot{x},\dot{x}) = \eta(U,U) = -1$, $\eta(M,\dot{x}) = \eta(U,X) = 0$
- Fresnel equation[7] determines wave front momenta: $\eta^{-1}(k,k)\zeta^{-1}(k,k) = 0$ with

$$\zeta_{ab} = \eta_{ab} + \frac{\xi^2}{1+\xi^2}U_aU_b - \frac{1}{1+\xi^2}X_aX_b$$



- Early results: depending on $\angle(U,\dot{x})$, detector response even for inertial motion

[6] Well, unless the constitutive tensor is assumed to *define* a more general (birefringent) space-time geometry.
[7] See, for example, Hehl, Obukhov – *Foundations of Classical Electrodynamics*

Dimension-Dependence: Exciton-Polaritons as an Analogue

The Goal

- Detector response function $\mathscr{F}$ both mass- and dimension-dependent:

	even space-time dim.	odd space-time dim.
$\mathscr{F}^{\text{fermions}}_{m=0}$	$\sim p_{\text{Fermi–Dirac}}$	$\sim p_{\text{Bose–Einstein}}$
$\mathscr{F}^{\text{bosons}}_{m=0}$	$\sim p_{\text{Bose–Einstein}}$	$\sim p_{\text{Bose–Einstein}}$
$\mathscr{F}^{\text{either}}_{m\neq 0}$	$\sim \textbf{a mess}[\{K_\nu(\cdot), \cos(\mathrm{h}), \sin(\mathrm{h}), \ln, \dots\}]$	

- See: Takagi ('86) doi:10.1143/PTP.88.1 & Ooguri ('85) doi:10.1103/PhysRevD.33.3573 & Terashima ('99) arXiv:hep-th/9903062, Louko & Toussaint ('16) arXiv:1608.01002
- Many analogues are $2+1$-dimensional, not fully $3+1$ as the previous example.

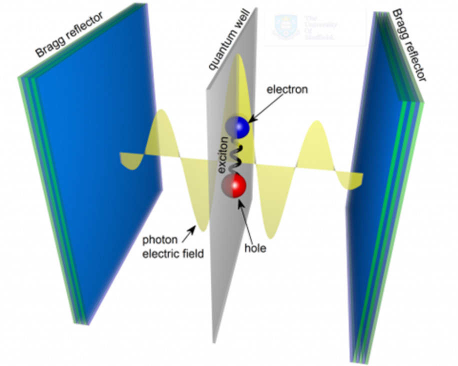
The Goals

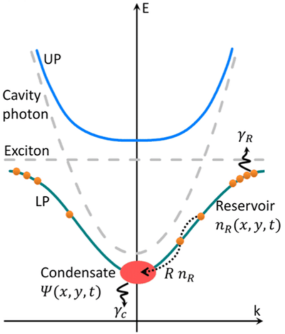
- Detector response function $\mathscr{F}$ both mass- and dimension-dependent:

	even space-time dim.	odd space-time dim.
$\mathscr{F}^{\text{fermions}}_{m=0}$	$\sim p_{\text{Fermi–Dirac}}$	$\sim p_{\text{Bose–Einstein}}$
$\mathscr{F}^{\text{bosons}}_{m=0}$	$\sim p_{\text{Bose–Einstein}}$	$\sim p_{\text{Bose–Einstein}}$
$\mathscr{F}^{\text{either}}_{m\neq 0}$	$\sim$ **a mess**$[\{K_\nu(\cdot), \cos(\text{h}), \sin(\text{h}), \ln, \dots\}]$	

- See: Takagi ('86) doi:10.1143/PTP.88.1 & Ooguri ('85) doi:10.1103/PhysRevD.33.3573 & Terashima ('99) arXiv:hep-th/9903062, Louko & Toussaint ('16) arXiv:1608.01002
- Many analogues are $2+1$-dimensional, not fully $3+1$ as the previous example.
- *Yet* another WIP: Potential for an analogue Penrose effect... (Solnyshkov *et al.*, arXiv:1809.05386)

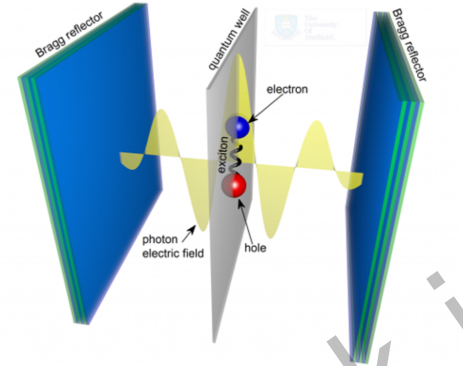
Proposed Set-Up for Checking Takagi Statistics 'Inversion'

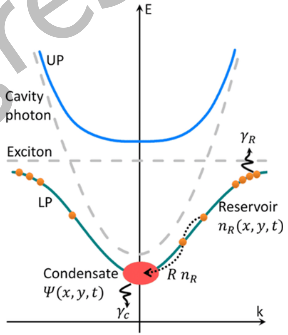




- Interaction strength can be tuned to get $m_{\text{eff}} = 0$ and $m_{\text{eff}} \neq 0$.
  $\implies$ Not just conformal geometry, but full metric structure available!
- Use a second laser as UDW Detector, as in Gooding *et al.* arXiv:2308.07892.

Image sources: Sich *et al.* ('16), doi:10.1016/j.crhy.2016.05.002, Gargoubi *et al.* ('16) doi:10.1103/PhysRevE.94.043310





- Interaction strength can be tuned to get $m_{\text{eff}} = 0$ and $m_{\text{eff}} \neq 0$.
  $\Longrightarrow$ Not just conformal geometry, but full metric structure available!
- Use a second laser as UDW Detector, as in Gooding *et al.* arXiv:2308.07892.
- Project details: WIP ☹

Image sources: Sich *et al.* ('16), doi:10.1016/j.crhy.2016.05.002, Gargoubi *et al.* ('16) doi:10.1103/PhysRevE.94.043310

Summary

Summary and Outlook

Summary:

- Analogues fit naturally into geometries beyond Lorentzian metrics
- One can find covariant dispersion relations
- Many application to modified theories of gravity

Outlook

- Inertial UdW detector response in a uniaxial crystal
- Hopefully also accelerated UdW detector response...

Summary and Outlook

Summary:

- Analogues fit naturally into geometries beyond Lorentzian metrics
- One can find covariant dispersion relations
- Many application to modified theories of gravity

Outlook

- Inertial UdW detector response in a uniaxial crystal
- Hopefully also accelerated UdW detector response...
- Any *FuSe-icists* in the room?

Summary and Outlook

Summary:

- Analogues fit naturally into geometries beyond Lorentzian metrics
- One can find covariant dispersion relations
- Many application to modified theories of gravity

Outlook

- Inertial UdW detector response in a uniaxial crystal
- Hopefully also accelerated UdW detector response...
- Any *FuSe-icists* in the room? I have *ideas* about seismic analogue space-times and need people with knowledge of elasticity and seismology!

Thanks! Aitäh! Tack! Danke!

References I

EL25 E. E. Erkul & U. Leonhardt. *Einstein's Equations in Electromagnetic Media*. 2025. arXiv: 2508.11300 [gr-qc].

FPS18 C. J. Fewster, C. Pfeifer & D. Siemssen. Quantum energy inequalities in premetric electrodynamics. *Physical Review D* **97**, 025019. doi:10.1103/PhysRevD.97.025019. arXiv: 1709.01760 (Jan. 2018).

FT16 M. Fathi & R. T. Thompson. Cartographic distortions make dielectric spacetime analog models imperfect mimickers. *Physical Review D* **93**, 124026. doi:10.1103/PhysRevD.93.124026. arXiv: 1602.08341 (June 2016).

Gar+16 H. Gargoubi, T. Guillet, S. Jaziri, J. Balti & B. Guizal. Polariton condensation threshold investigation through the numerical resolution of the generalized Gross-Pitaevskii equation. *Physical Review E* **94**, 043310. doi:10.1103/PhysRevE.94.043310 (Oct. 2016).

Goo+24 C. Gooding, A. Sachs, R. B. Mann & S. Weinfurtner. Vacuum entanglement probes for ultra-cold atom systems. *New Journal of Physics* **26,** 105001. doi:10.1088/1367-2630/ad8675. eprint: 2308.07892 (Oct. 2024).

HO03 F. W. Hehl & Y. N. Obukhov. *Foundations of Classical Electrodynamics.* ISBN: 3-7643-4222-6 (Birkhäuser, 2003).

HPV22 M. Hohmann, C. Pfeifer & N. Voicu. Mathematical foundations for field theories on Finsler spacetimes. *Journal of Mathematical Physics* **63,** 032503. doi:10.1063/5.0065944. arXiv: 2106.14965 [math-ph] (2022).

LP09 U. Leonhardt & T. G. Philbin. Transformation Optics and the Geometry of Light. *Progress in Optics* **53,** 69–152. doi:10.1016/S0079-6638(08)00202-3. arXiv: 0805.4778 (2009).

LT16 J. Louko & V. Toussaint. Unruh-DeWitt detector's response to fermions in flat spacetimes. *Physical Review D* **94,** 064027. doi:10.1103/PhysRevD.94.064027. arXiv: 1608.01002 [gr-qc] (Sept. 2016).

References III

ODe70 T. H. O'Dell. *The Electrodynamics of Magneto-Electric Media.* ISBN: 0-7204-1461-X (North-Holland, 1970).

Oog85 H. Ooguri. Spectrum of Hawking Radiation and the Huygens Principle. *Physical Review D* **33,** 3573–3580. doi:10.1103/PhysRevD.33.3573 (Oct. 1985).

Per00 V. Perlick. *Ray Optics, Fermat's Principle, and Applications to General Relativity.* ISBN: 3-540-66898-5 (Springer, 2000).

Pfe19 C. Pfeifer. Finsler spacetime geometry in Physics. *International Journal of Geometric Methods in Modern Physics* **16,** 1941004. doi:10.1142/S0219887819410044. arXiv: 1903.10185 [gr-qc] (2019).

Pos62 E. J. Post. *Formal Structure of Electromagnetics.* (North-Holland, 1962). Reprinted by Dover Publications, ISBN: 978-0-486-65427-0.

PS16 C. Pfeifer & D. Siemssen. Electromagnetic potential in pre-metric electrodynamics: Causal structure, propagators and quantization. *Physical Review D* **93,** 105046. doi:10.1103/PhysRevD.93.105046. arXiv: 1602.00946 (May 2016).

References IV

PSW07 R. Punzi, F. P. Schuller & M. N. R. Wohlfarth. Area metric gravity and accelerating cosmology. *Journal of High Energy Physics* **02,** 030. doi:10.1088/1126-6708/2007/02/030. arXiv: hep-th/0612141 (2007).

RRS11 D. Rätzel, S. Rivera & F. P. Schuller. Geometry of physical dispersion relations. *Physical Review D* **83,** 044047. doi:10.1103/PhysRevD.83.044047. arXiv: 1010.1369 [hep-th] (2011).

Sol+19 D. D. Solnyshkov, C. Leblanc, S. V. Koniakhin, O. Bleu & G. Malpuech. Quantum analogue of a Kerr black hole and the Penrose effect in a Bose-Einstein condensate. *Physical Review B* **99,** 214511. doi:10.1103/PhysRevB.99.214511. arXiv: 1809.05386 [cond-mat.quant-gas] (June 2019).

SSK16 M. Sich, D. V. Skryabin & D. N. Krizhanovskii. Soliton physics with semiconductor exciton–polaritons in confined systems. *Comptes Rendus Physique* **17,** 908–919. doi:10.1016/j.crhy.2016.05.002 (Oct. 2016).

STV25 I. Sawicki, G. Trenkler & A. Vikman. Causality and stability from acoustic geometry. *Journal of High Energy Physics* **10,** 227. doi:10.1007/JHEP10(2025)227. arXiv: 2412.21169 [gr-qc] (2025).

SV17 S. Schuster & M. Visser. Effective metrics and a fully covariant description of constitutive tensors in electrodynamics. *Physical Review D* **96,** 124019. doi:10.1103/PhysRevD.96.124019. arXiv: 1706.06280 (Dec. 2017).

SV18 S. Schuster & M. Visser. Electromagnetic analogue space-times, analytically and algebraically. *Classical and Quantum Gravity* **36,** 134004. doi:10.1088/1361-6382/ab2159. arXiv: 1808.07987 [gr-qc] (June 2018). Focus Issue on Gravity in the Lab.

SW06 F. P. Schuller & M. N. R. Wohlfarth. Geometry of manifolds with area metric: multi-metric backgrounds. *Nuclear Physics B* **747,** 398–422. doi:10.1016/j.nuclphysb.2006.04.019. arXiv: hep-th/0508170 (2006).

References VI

SWW10 F. P. Schuller, C. Witte & M. N. R. Wohlfarth. Causal structure and algebraic classification of area metric spacetimes in four dimensions. *Annals of Physics* **325,** 1853–1883. doi:10.1016/j.aop.2010.04.008. arXiv: 0908.1016 (Sept. 2010).

Tak86 S. Takagi. Vacuum Noise and Stress Induced by Uniform Acceleration: Hawking–Unruh Effect in Rindler Manifold of Arbitrary Dimension. *Progress of Theoretical Physics Supplement* **88,** 1–142. doi:10.1143/PTP.88.1 (Mar. 1986).

TCF11a R. T. Thompson, S. A. Cummer & J. Frauendiener. A Completely Covariant Approach to Transformation Optics. *Journal of Optics* **13,** 024008. doi:10.1088/2040-8978/13/2/024008. arXiv: 1006.3118 (Nov. 2011).

TCF11b R. T. Thompson, S. A. Cummer & J. Frauendiener. Generalized Transformation Optics of Linear Materials. *Journal of Optics* **13,** 055105. doi:10.1088/2040-8978/13/5/055105. arXiv: 1006.3364 [physics.optics] (Apr. 2011).

References VII

Ter99 H. Terashima. Fluctuation dissipation theorem and the Unruh effect of scalar and Dirac fields. *Physical Review D* **60,** 084001. doi:10.1103/PhysRevD.60.084001. arXiv: hep-th/9903062 (1999).

TF10 R. T. Thompson & J. Frauendiener. Dielectric Analog Space-Times. *Physical Review D* **82,** 124021. doi:10.1103/PhysRevD.82.124021. arXiv: 1010.1587 (Dec. 2010).

TF17 R. T. Thompson & M. Fathi. Covariant kinematics of light in media and a generalized Raychaudhuri equation. *Physical Review D* **96,** 105006. doi:10.1103/PhysRevD.96.105006. arXiv: 1705.11108 [gr-qc] (Nov. 2017).

Tho17 R. T. Thompson. Covariant electrodynamics in linear media: Optical metric. *Physical Review D* **97,** 065001. doi:10.1103/PhysRevD.97.065001. arXiv: 1712.06872 (Mar. 2017).

References VIII

Vol09 G. E. Volovik. *The Universe in a Helium Droplet.* *International Series of Monographs on Physics* **117.** ISBN: 9780199564842. doi:10.1093/acprof:oso/9780199564842.001.0001 (Oxford University Press, 2009).

Microscopic Maxwell equations

Macroscopic Maxwell equations

$$\nabla_{[a}F_{bc]} = 0, \qquad \nabla_a F^{ab} = J^b$$

non-trivial constitutive relations →

$$\nabla_{[a}F_{bc]} = 0, \qquad \nabla_a \underbrace{\left(Z^{abcd}F_{cd}\right)}_{G^{ab}} = J^b,$$

$$\mathbf{D} = \epsilon\,\mathbf{E} + \zeta\,\mathbf{B},$$
$$\mathbf{H} = \zeta^{\dagger}\mathbf{E} + \mu^{-1}\mathbf{B}.$$

Reminder: Orthogonal Decomposition

For any two-form F_{ab} in four dimensional space-time and any four-velocity V^a, there exist two vector fields E^a and B^a, both orthogonal to V^a, such that

$$F_{ab} = V_a E_b - V_b E_a + \varepsilon_{abcd} V^c B^d.$$

- Similar statements for any possible tensor
- View our constitutive equation as tensorial map of field strength $F_{ab} \xrightarrow{Z} G$, excitation tensor
- Z has 21 d.o.f.[8] if compatible with an action $\mathscr{L} \propto F_{ab} Z^{abcd} F_{cd}$
- $$\epsilon_V^{ab} = -2 Z^{dacb} V_d V_c, \qquad [\mu_V^{-1}]^{ab} = 2(*Z*)^{dacb} V_d V_c,$$
  $$\zeta_V^{ab} = 2(*Z)^{dacb} V_d V_c, \qquad \left[\zeta_V^{\dagger}\right]^{ab} = 2(Z*)^{dacb} V_d V_c.$$

[8] Often the axion-part $\propto \varepsilon_{abcd}$ is omitted, but see, for example, Hehl *et al.* (2009), arXiv:0903.1261.

Relativity and Moving Media

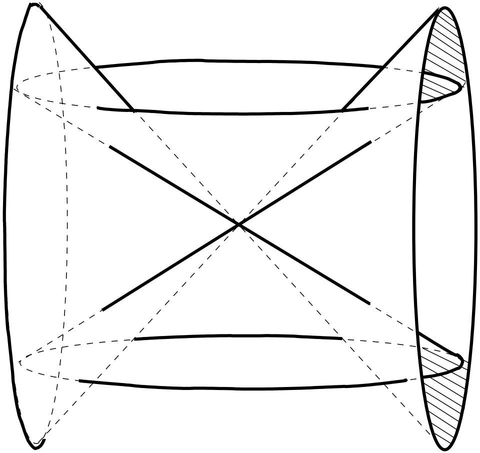
- This directly translates to the physics of moving media![9]
- An isotropic medium in its rest frame V viewed by W:

$$\begin{aligned}\epsilon_W^{bd} =& -2Z^{abcd}W_aW_c,\\ =&\mu^{-1}(g^{bd}+W^bW^d)+(\epsilon-\mu^{-1})\left(g^{bd}(V\cdot W)^2\right.\\ &\left.-(W^bV^d+V^bW^d)(V\cdot W)-V^bV^d\right),\\ [\mu_W^{-1}]^{bd} =&\frac{h_W^{bd}}{\mu}+(\mu^{-1}-\epsilon)\left((V\cdot W)^2h_W^{bd}-h_W^{be}h_{ef}h_W^{fd}\right),\\ \zeta_W^{ac} =&(\epsilon-\mu^{-1})(V\cdot W)\left(\epsilon^{acef}W_eV_f\right).\end{aligned}$$

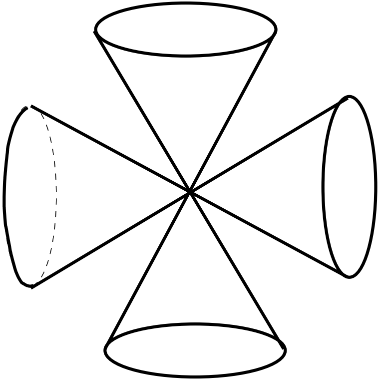
- Reminders: $(V\cdot W)^2=\gamma_{V,W}$, $\qquad 1-1/(\epsilon\mu)=1-1/n^2$
- Isotropy is lost in the moving medium!

[9] A.k.a. the Fresnel–Fizeau effect; see, *e.g.*, Post – *Formal Structure of Electromagnetics* or O'Dell - *The Electrodynamics of Magneto-Electric Media*.

What It Looks Like



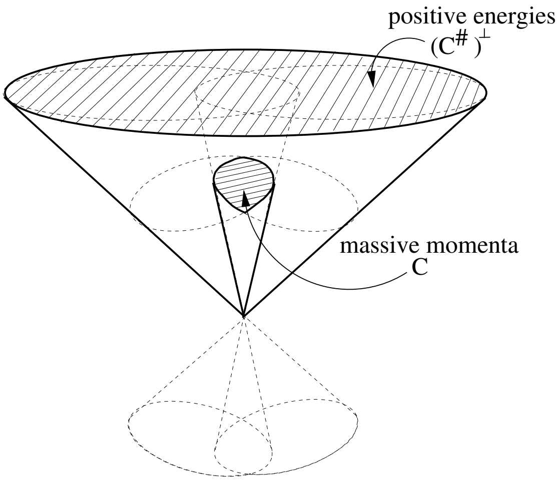
cotangent space

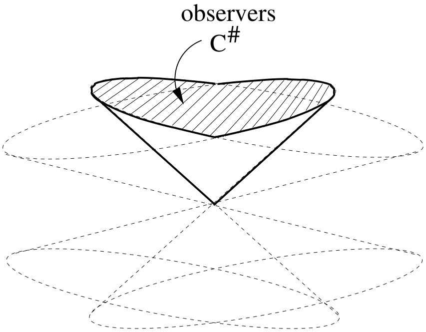


tangent space

Images sourced from: Rätzel, Rivera, Schuller (arXiv:1010.1369)

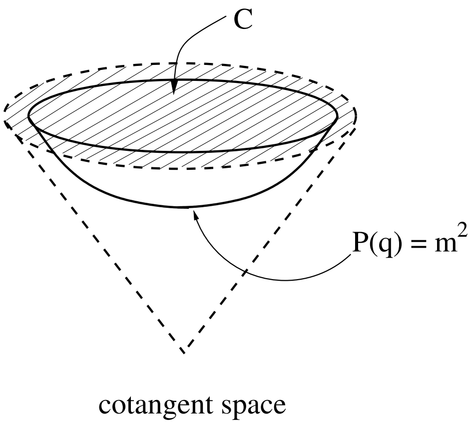
What It Looks Like

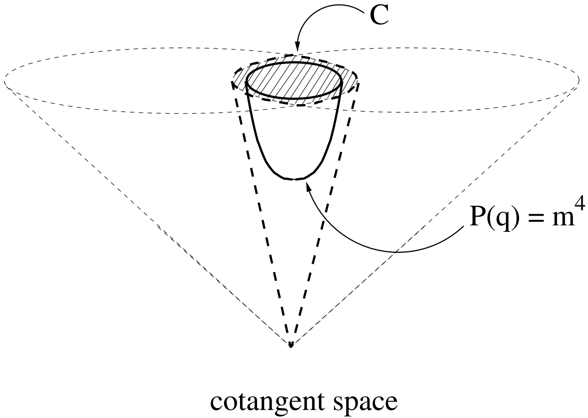




Images sourced from: Rätzel, Rivera, Schuller (arXiv:1010.1369)

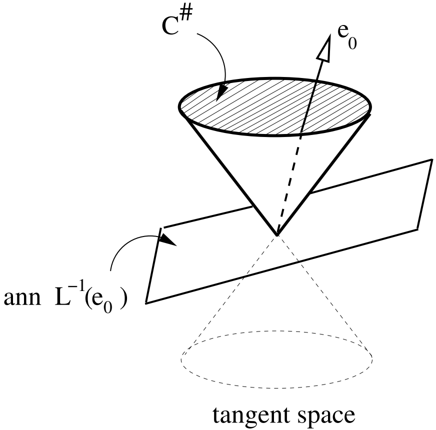
What It Looks Like





Images sourced from: Rätzel, Rivera, Schuller (arXiv:1010.1369)

What It Looks Like—Addendum ‘Observers’



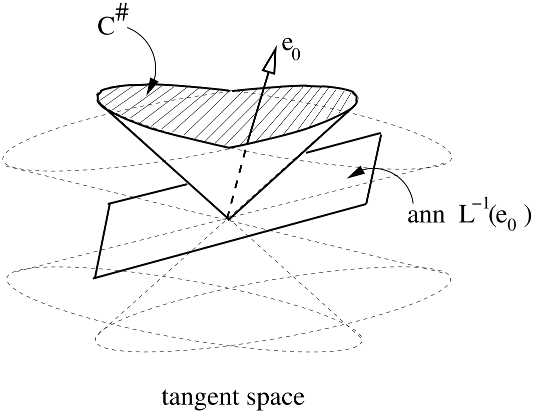


Image sourced from: Rätzel, Rivera, Schuller (arXiv:1010.1369)

What It Looks Like—Addendum ‘Stability’

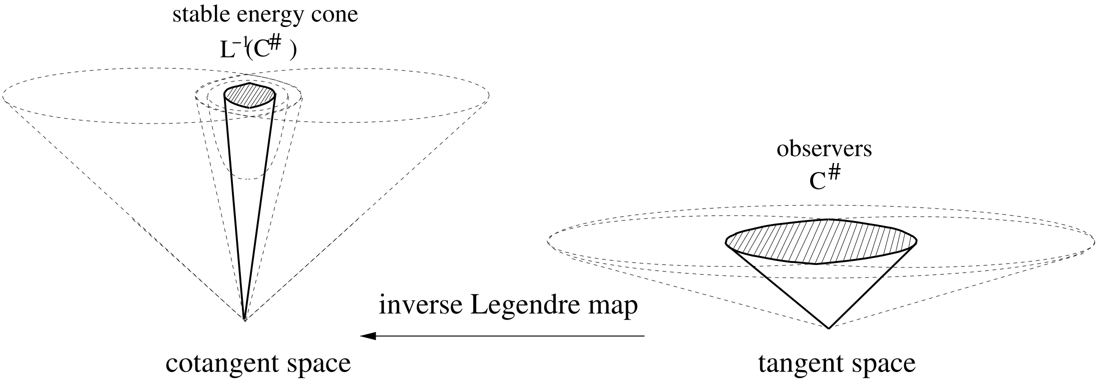


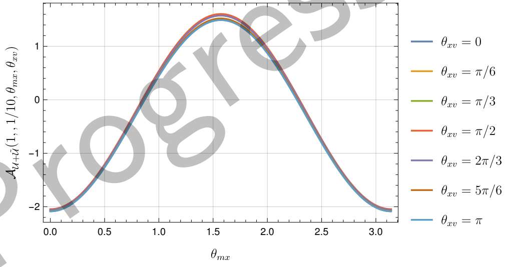
Image sourced from: Rätzel, Rivera, Schuller (arXiv:1010.1369)

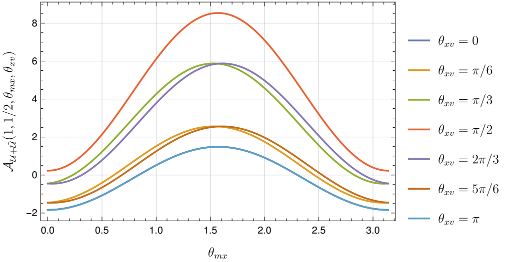
Early Results: Inertial Motion

- Building on results of Fewster, Pfeifer, Siemssen (arXiv:1709.01760, arXiv:1602.00946)
- Currently, only (partially) done for inertial $\dot{x}$
- Calculate $\mathscr{M}^a \mathscr{M}^c \dot{x}^b \dot{x}^d \langle 0 | F_{cd}(x(\tau')) F_{ab}(x(\tau)) | 0 \rangle \big|_{\xi=0}$
- Using clever variable choices[10]

$$
\begin{aligned}
U^a &= (1, \vec{0})^a, \hat{X}^a = (0, \hat{x})^a, \\
\dot{x}^a &= \gamma(1, v)^a, \\
\eta(U, \dot{x}) &= -\gamma \\
\mathscr{M}^a &= |\mathscr{M}|(\gamma v \cos\theta_{mv}, \hat{m} + (\gamma - 1)\cos\theta_{mv} \hat{v})^a, \\
\cos\theta_{mv} &= \hat{m} \cdot \hat{v}, \\
\eta(\hat{X}, \dot{x}) &= v\gamma \cos\theta_{vx},
\end{aligned}
$$

isolate factors $A_{U+\tilde{U}}(\xi, v, \theta_{mx}, \theta_{mv})$





[10] Thank you, Finn!