Space-time structure and phenomenology of Cartan Khronon theory

Lucy Zheng

Relevant publications "khronon theory" "Lorentz gauge theory" "Spin(4) gauge theory", ...etc.

2018 **Spacetime and dark matter from spontaneous breaking of Lorentz symmetry**
Złośnik, Urban, Marzola, Koivisto e-Print: 1807.01100 [gr-qc] DOI: 10.1088/1361-6382/aaea96

2019 **The General Linear Cartan Khronon**
Koivisto, Hohmann, Złośnik e-Print: 1905.02967 [gr-qc] DOI: 10.3390/universe5070168

2021 **The Λ and the CDM as Integration Constants**
Gallagher, Koivisto e-Print: 2103.05435 [gr-qc] DOI: 10.3390/sym13112076

2022 **Pregeometric first order Yang-Mills theory**
Gallagher, Koivisto, Marzola e-Print: 2202.05657 [hep-th] DOI: 10.1103/PhysRevD.105.125010

2023 **Paths to gravitation via the gauging of parametrized field theories**
Koivisto, Tom Zlosnik e-Print: 2212.04562 [gr-qc] DOI: 10.1103/PhysRevD.107.124013

Cosmology in the Lorentz gauge theory
Koivisto e-Print: 2306.00963 [gr-qc] DOI: 10.1142/S0219887824500403

Hamiltonian formulation of gravity as a spontaneously-broken gauge theory of the Lorentz group
Nikjoo, Zlosnik e-Print: 2308.01108 [gr-qc] DOI: 10.1088/1361-6382/ad1c84

Consistent first-order action functional for gauge theories e-Print: 2311.07464 [hep-th]
Gallagher, Koivisto, Marzola, Varrin, Zlosnik DOI: 10.1103/PhysRevD.109.L061503

2024 **Canonical aspects of pregeometric vector-based first order gauge theory**
Gallagher e-Print: 2403.02578 [hep-th] DOI: 10.1103/PhysRevD.110.085010

Black holes in Lorentz gauge theory
Koivisto, Zheng e-Print: 2408.10100 [gr-qc] DOI: 10.1103/PhysRevD.111.064008

2025 **Spin(4) gauge theory of space, time, gravitation, matter, and dark matter**
Koivisto, Zheng, Zlosnik e-Print: 2507.00968 [gr-qc] DOI: 10.1103/xx3x-s5wj

Relevant theses (2026)

(At the Corner of Quantum and Gravity)
Ludovic Varrin [arXiv: 2603.21941]

On the internal gauge theory analogy to the Cartan Khronon theory of gravity
Ernest Michael Priidik Gallagher [University of Tartu Press]

The Gravitating Universe
Mehraveh Nikjoo [TBA]

Cartan Khronon – $\mathbb{R}$eal space-time structure –
Lucy Zheng [TBA]

$$(+,+,+,+)$$

“But we live in a Lorentzian spacetime”...?

Lorentzian	Euclidean
Gravity	Quantum field theory
Cosmology	Black hole thermodynamics
⋮	Statistical field theory
Causal dynamics	

luxi.zheng@ut.ee Lucy Zheng

The necessity of Euclidean time

Path integral in quantum theory

$$\langle g_1, \phi_1, S_1 | g_2, \phi_2, S_2 | g_1, \phi_1, S_1 | g_2, \phi_2, S_2 \rangle = \int \mathrm{D}[g, \phi] \exp(iI[g, \phi]) \, ,$$

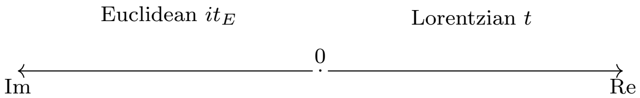
Black hole thermodynamics

$$Z = \mathrm{Tr}\{\exp(-\beta H)\} = \int \mathrm{D}[g, \phi] \exp(-I_E[g, \phi]) \, ,$$

(Hawking temperature $T = 1/\beta$, intrinsic entropy $S = \beta \frac{\partial I_E}{\partial \beta} - I_E$)

Is Euclidean spacetime the fundamental framework to define physics?

Standard Wick rotation



- does not apply to certain spacetime metrics
- index rotation of the gamma matrices with $\gamma^4 = -i\gamma^0$ fails to
  - establish Euclidean symmetry in Dirac action
  - maintain hermiticity $\psi_\pm^\dagger = \psi_\mp$
  - maintain self-conjugacy of Majorana spinor
    (components become complex)

Cartan Khronon framework

Khronon field
ϕ^a - a clock field
symmetry breaking into
SO(3) $\phi^a = (\phi, 0, 0, 0)$

gauge field
$\boldsymbol{A}^{ab}$ - connection
SO(1,3)?
SO(4)? Spin(4)?

Chiral gravity
$\boldsymbol{A} = {}^{+}\boldsymbol{A} + {}^{-}\boldsymbol{A}$
$\boldsymbol{F} = {}^{+}\boldsymbol{F} + {}^{-}\boldsymbol{F}$

Euclidean background
$(+, +, +, +)$

Minimal tetrad formalism
$\mathbf{e}^a = \mathbf{D}\phi^a$

Spinors
Clifford algebra or
inherent in Spin(4)
=SU(2)×SU(2)

Effective DM
$$\frac{\delta I_{tot}}{\delta \phi^a} = \mathbf{D}M^a$$

Lorentzian-Euclidean correspondence
$-1/m_P^2 \leftrightarrow \kappa$
$it \leftrightarrow \tau$

Canonical source currents
$$\boldsymbol{t}_a = \frac{\delta I_m}{\delta \mathbf{e}^a}$$
$$\boldsymbol{O}_{ab} = \frac{\delta I_m}{\delta \boldsymbol{A}^{ab}}$$

Khronon + Spin(4) as a solution

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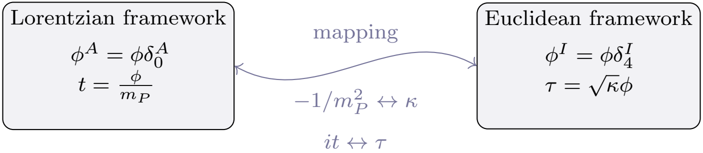
Works for Kerr, Cosmological background & perturbations, etc.

Euclidean-Lorentzian correspondence – the dimension

For a dimensionless symmetry breaking scalar field ϕ^a, $\mathbf{e}^a = \mathbf{D}\phi^a \sim [E]$.
Introducing dimensionful $[E]$ parameters m_P and $1/\sqrt{\kappa}$ in respective framework,

$$t = \phi/m_P \text{ and } \tau = \sqrt{\kappa}\phi$$

$\mathbf{e}^0 = \mathbf{d}t$ and $\mathbf{e}^4 = \mathbf{d}\tau$ both become dimensionless.



All the dimensionful parameters require mapping, e.g., spin parameter $a = J/M$

Euclidean-Lorentzian correspondence – spinors

A unitary rotation $U = \exp\left(\gamma^4\gamma^5\,\frac{\theta}{2}\right)$ by $\theta = \pi/2$ maps spinors between the two frameworks without violating each group symmetry

$$\gamma_E^i = \gamma_L^i\,, \quad \gamma_E^4 = \gamma_L^5\,, \quad \gamma_E^5 = \gamma_E^1\gamma_E^2\gamma_E^3\gamma_E^4 = -\gamma^4\,,$$

which satisfy the corresponding hermiticity relations.

$$\left\{\gamma_E^I, \gamma_E^J\right\} = 2\delta^{IJ}\mathbb{1}, \qquad (\gamma_E^I)^\dagger = -\gamma_E^5\gamma_E^I\gamma_E^5 = \gamma_E^I, \qquad (\gamma_E^5)^2 = \mathbb{1}.$$

This new system can be understood as a reinterpretation of γ^4 and γ^5.

luxi.zheng@ut.ee Lucy Zheng

Phenomenology in the purely right-(left-)handed limit

The full ambidextrous action

$$I = \int \mathbf{D}\phi^I \wedge \mathbf{D}\phi^J \wedge \left(g_+ {}^+\boldsymbol{F}_{IJ} - g_- {}^-\boldsymbol{F}_{IJ}\right) - \int \star\lambda - \int \mathbf{D}\phi^I \wedge \boldsymbol{t}_I - \int \boldsymbol{A}^{IJ} \wedge \boldsymbol{O}_{IJ} \,,$$

In the $(g_+, g_-) = (1,0),(0,1)$ limit, when $\boldsymbol{O}^{IJ} = 0$, we recover...

- FLRW background + $\Lambda(= -m_P^{-2}\lambda)$ + effective ideal dust $\boldsymbol{M}^I$
  (vanishing pressure and sound speed for $\boldsymbol{M}^I$)
- Standard GR black hole solutions
- Gravitational wave propagating at the speed of light

luxi.zheng@ut.ee Lucy Zheng

Phenomenology in the general chiral configuration

The full ambidextrous action

$$I = \int \mathbf{D}\phi^I \wedge \mathbf{D}\phi^J \wedge \left(g_+ {}^+\boldsymbol{F}_{IJ} - g_- {}^-\boldsymbol{F}_{IJ}\right) - \int \star\lambda - \int \mathbf{D}\phi^I \wedge \boldsymbol{t}_I - \int \boldsymbol{A}^{IJ} \wedge \boldsymbol{O}_{IJ} \,,$$

For the general $g_+ g_- \neq 0$, when $\boldsymbol{O}^{IJ} = 0$, we obtain...

- FLRW background with $\boldsymbol{M}^I$-effect included in the vacuum + $\Lambda(= -m_P^{-2}\lambda)$
  (vanishing sound speed for $\boldsymbol{M}^I$)
- No black hole solutions have been found
- Gravitational wave propagating at the speed of $|\gamma| \equiv |\frac{g_+ + g_-}{g_+ - g_-}|$

multi-messenger data from the neutron star merger GW170817 :
$|g_\mp / g_\pm| \lesssim 10^{-15}$ at present

Phenomenology in the chirally symmetric limit

The full ambidextrous action

$$I = \int \mathbf{D}\phi^I \wedge \mathbf{D}\phi^J \wedge \left(g_+ {}^+\boldsymbol{F}_{IJ} - g_- {}^-\boldsymbol{F}_{IJ}\right) - \int \star\lambda - \int \mathbf{D}\phi^I \wedge \boldsymbol{t}_I - \int \boldsymbol{A}^{IJ} \wedge \boldsymbol{O}_{IJ}\,,$$

For the chirally symmetric $g_+ = -g_-$ case, when $\boldsymbol{O}^{IJ} = 0$, we obtain...

- khronon field acting as scale factor $a \sim \phi$
- radiation-dominated universe $H \sim t^{-1}$
- $\boldsymbol{M}^I$: candidate for dark radiation...?

Likely scenario:

chirally symmetric gravity $g_+ = -g_-$ in early universe (before BBN)
transitions into the (almost) purely right-(left-)handed gravity

Cartan Khronon Cosmic Timeline

$t = 0$		Symmetric Phase $\phi^a = \vec{0}, \quad g_+ = -g_-$	
		Inflationary era — Phase transition	
?		• Phase transition towards $g_+ \sim 1$, $g_- \sim 0$	
		Radiation-Dominated Era	
~ 1 sec	$\sim 10^{10}$ eV	• Big Bang Nucleosynthesis	
$\sim 10^4$ yr	~ 1 eV	Start of structure formation	$a \sim 3.3 \times 10^{-4}$
		Matter-Dominated Era	
$\sim 3.8 \times 10^5$ yr	~ 0.4 eV	• Photon decoupling produces CMB	$a \sim 9 \times 10^{-4}$
$\sim 9.8 \times 10^9$ yr	$\sim 10^{-3}$ eV	Freezing of structure formation	$a \sim 0.73$
		Dark Energy-Dominated Era	
$\sim 10^{10}$ yr	$\sim 10^{-4}$ eV	• Present	$a = 1$

luxi.zheng@ut.ee Lucy Zheng

Phenomenology with spin current

The full ambidextrous action

$$I = \int \mathbf{D}\phi^I \wedge \mathbf{D}\phi^J \wedge \left(g_+ {}^+\boldsymbol{F}_{IJ} - g_- {}^-\boldsymbol{F}_{IJ}\right) - \int \star\lambda - \int \mathbf{D}\phi^I \wedge \boldsymbol{t}_I - \int \boldsymbol{A}^{IJ} \wedge \boldsymbol{O}_{IJ} \,,$$

When ${}^\pm\boldsymbol{O}^i = \sqrt{\kappa}{}^\pm O \star \mathbf{e}^i \neq 0$, we obtain...

- Coupling between effective DM and material spin current

$$\dot{\hat{\rho}} + 3H\left(\hat{\rho} + \frac{2\beta}{\kappa\tau^2}\right) = \frac{6\beta}{\kappa\tau^3} - \frac{{}^+O + {}^-O}{2\gamma\tau^2}$$

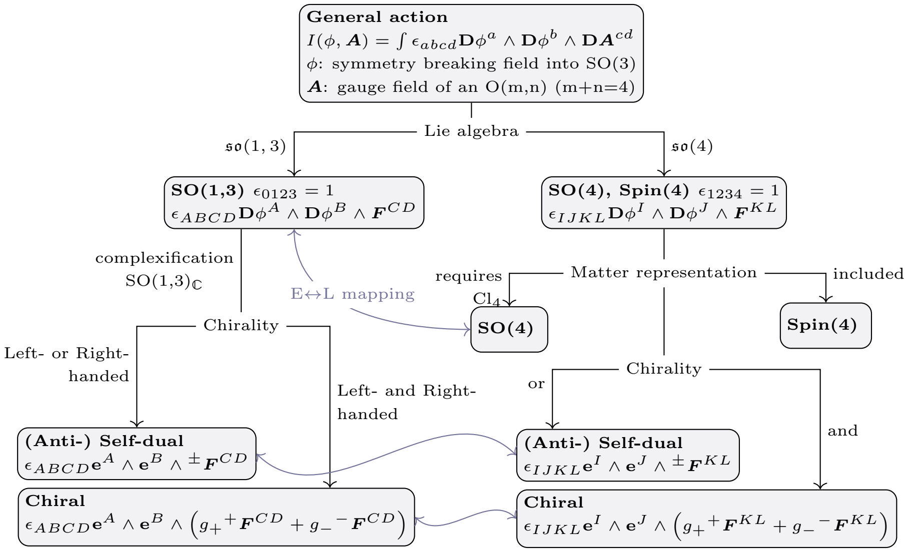
- New effect in the Friedmann equations
  $\Omega \equiv \frac{\kappa}{2\gamma}\left({}^+O + {}^-O\right)$, $\Sigma \equiv \frac{\kappa^{3/2}}{\alpha\gamma^2\phi}\left(g_+ {}^-O - g_- {}^+O\right)$

$$3\alpha H^2 + 3H\Omega + \frac{3}{4}\frac{\Omega^2}{\alpha} = \kappa\left(\rho + \hat{\rho}\right) + 3\beta\tau^{-2} \,,$$

$$3\alpha H^2 + 2\alpha\dot{H} + 2H\Omega + \frac{\Omega^2}{4\alpha} + \dot{\Omega} + \frac{\Sigma}{\sqrt{\kappa}} = -\kappa p + \beta\tau^{-2} \,.$$

Is Lorentzian spacetime – where we *observe* physics –
the optimal framework to *define* physics?

Cartan-khronon flowchart



luxi.zheng@ut.ee Lucy Zheng

Perturbations in GR limit $\beta = 0$: FLRW + Λ + ideal dust

- Standard cosmology

  Gravitational wave

  $$\ddot{h}_{ij} + 3\frac{\dot{a}}{a}\dot{h}_{ij} + \frac{1}{a^2}\nabla^2 h_{ij} = 0 \xrightarrow{\text{Wick}} -h''_{ij} - 3Hh'_{ij} + \frac{1}{a^2}\nabla^2 h_{ij} = 0 \qquad (25)$$

  No propagation for vector perturbation

  $$\begin{aligned} &\nabla^2 V_i = 0 \\ &V'_i + 3HV_i = 0 \end{aligned} \qquad (26)$$

  Continuity equation under scalar perturbation

  $$\delta\rho' + 3H\left(\delta\rho + \delta p\right) = \left(\rho + p\right)\left[\nabla^2\left(\frac{v}{a} + c' - \frac{\Phi}{a^2}\right) - 3\Psi'\right] \qquad (27a)$$

  Euler equation

  $$a\left(\rho + p\right)\left(v' + 4aHv\right) + \left(\rho + p\right)' v = \delta p - \frac{2}{3}\nabla^2\Pi \qquad (27b)$$

- + effective ideal dust

  Continuity equation

  $$\delta\hat{\rho}' + 3H\delta\hat{\rho} = \Phi'\hat{\rho}' + 3\left(H\Phi' - \Psi' + \frac{1}{3}\nabla^2 c'\right)\hat{\rho} \qquad (28a)$$

  Density evolution of CDM
  ($\hat{\delta} \equiv \delta\hat{\rho}/\hat{\rho}$)

  $$\hat{\delta}'' + 2H\hat{\delta}' = \frac{\hat{\rho}}{2m_P^2}\hat{\delta} \qquad (28b)$$

luxi.zheng@ut.ee Lucy Zheng

$g_+{}^{+}\boldsymbol{F} - g_-{}^{-}\boldsymbol{F}$: nature of $\boldsymbol{M}^a$

- effective pressure
$$\hat{p} = \beta \left(\frac{H}{t} - \frac{1}{t^2} \right)$$
- effective energy density sourced by its own pressure
$$\hat{\rho}' + 3H\hat{\rho} = -3\hat{p}/t$$
- acceleration driven by time lapse fluctuation
$$\hat{v}' \sim \frac{\partial \varphi}{\partial \phi}$$
- clustering
$$\delta\hat{\rho}' + 3H\delta\hat{\rho} = \frac{\nabla^2}{a^2} \left[2\beta m_P^2 \left(\Psi' - H\Phi' \right) + \left(\hat{\rho} - 2\beta m_P^2 H' + \frac{2\beta m_P^2}{t^2} \right) \Phi \right]$$
- vanishing sound speed
$$\hat{c}_s \equiv \frac{\hat{p}'}{\hat{\rho}'} = 0$$