Evolution of superhorizon perturbations in early Universe with anisotropic solid remnant

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Structure of the talk

1. Motivation
2. Solid inflation and symmetry reduction
3. Background evolution with an anisotropic remnant
4. Superhorizon perturbations
5. Observational consequences

Motivation: why look beyond ΛCDM?

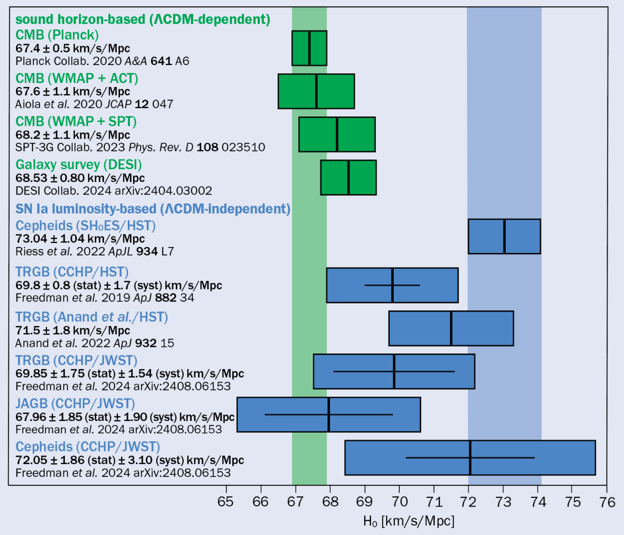
ΛCDM succeeds

$$\Lambda\text{CDM} + \text{inflation} + \text{reheating}$$

- expansion history
- CMB anisotropies
- large-scale structure

But tensions remain

$$H_0^{\text{CMB}} \neq H_0^{\text{local}}$$



Source: CERN Courier, "The Hubble tension", figure credit: V. Poulin.

Motivation: a controlled anisotropic phase

Inflation

solid sector

$\longrightarrow$

Post-inflationary phase

radiation dominated

$$\mathcal{L}_{\rm m} = \mathcal{L}_{\rm rad.} + \lambda f(\Theta, \Omega)$$

$$\lambda \ll 1$$

$\longrightarrow$

Standard era

radiation

ΛCDM

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Standard era

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Guiding idea

A small anisotropic remnant temporarily perturbs the radiation-dominated evolution.

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$\lambda \ll 1$

$\longrightarrow$

Standard era
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ΛCDM

Guiding idea

A small anisotropic remnant temporarily perturbs the radiation-dominated evolution.

Question

How does this stage affect superhorizon perturbations?

Solid inflation

The Lagrangian

$$\mathcal{L}_{\mathrm{m}} = F(X, Y, Z), \quad X = \operatorname{Tr} B, \quad Y = \frac{\operatorname{Tr} B^2}{X^2}, \quad Z = \frac{\operatorname{Tr} B^3}{X^3}, \quad B^{ij} = g^{\mu\nu} \Phi^i_{,\mu} \Phi^j_{,\nu},$$

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Homogeneity and isotropy of the solid

$$\Phi^i \to R^i{}_j \Phi^j + T^i, \quad R \in SO(3), \quad T \in \mathbb{R}^3.$$

Quantities X, Y,and Z are invariant under $E(3) = SO(3) \ltimes T(3)$

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Main idea: Introduce anisotropy by reducing the symmetry group

$$SO(3) \ltimes T(3) \longrightarrow SO(2) \ltimes T(3)$$

Model under consideration

Spacetime metric

$$ds^2 = -dt^2 + a(t)^2 (dx)^2 + b(t)^2 \left(dy^2 + dz^2\right)$$

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Anisotropy in the matter Lagrangian

$$\Theta = g^{\mu\nu}\Phi^1_{,\mu}\Phi^1_{,\nu} \quad \Omega = \frac{1}{2}g^{\mu\nu}\delta_{AB}\Phi^A_{,\mu}\Phi^B_{,\nu} \quad X = \Theta + 2\Omega$$

Model under consideration

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$$\mathcal{L}_\mathrm{m} = F(X, Y, Z) \propto \begin{cases} X^\epsilon = (\Theta + 2\Omega)^\epsilon & \text{during} \quad \text{inflation} \\ \mathcal{L}_\text{rad.} + \lambda f(\Theta, \Omega) & \text{during} \quad \text{reheating} \\ \mathcal{L}_\text{rad.} & \text{during} \quad \text{the standard radiation-dominated era} \end{cases}$$

where $\epsilon = -\frac{\dot{H}}{H^2}$ and $\lambda \ll 1$ controls the amount of anisotropic solid remnant.

Background: controlled anisotropy

Instead of using two scale factors directly, write

$$ds^2 = e^{2\alpha}\left[-d\tau^2 + e^{-4\sigma}dx^2 + e^{2\sigma}(dy^2 + dz^2)\right].$$

$$\alpha = \alpha_0 + \lambda\alpha_1, \qquad \sigma = \lambda\sigma_1, \qquad \lambda \ll 1.$$

Interpretation

$$\alpha : \text{average expansion}, \qquad \sigma : \text{anisotropic deformation}.$$

Strategy

Solve the standard radiation era at zeroth order, then compute the leading anisotropic correction.

Background

$$f(\Theta, \Omega) = K\Theta^P\Omega^Q, \qquad \Theta_0 \sim \Omega_0 \sim a^{-2}.$$

$$\mathcal{L}_{\text{rad.}} \sim a^{-4}, \qquad \lambda f(\Theta_0, \Omega_0) \sim a^{-2(P+Q)}.$$

Condition	Relative behaviour
$P + Q < 2$	remnant grows
$P + Q = 2$	limiting case
$P + Q > 2$	remnant decays

Choose

$$P + Q = 2$$

Strongest effect of the remnant while avoiding its growth.

Why superhorizon perturbations?

Observationally relevant modes crossed outside the horizon during inflation:

$$k \ll aH.$$

During the early post-inflationary phase we therefore work in the limit

$$k\tau \ll 1.$$

Consequence

The perturbation equations simplify, and the dominant time-dependence can be extracted analytically.

Physical point

We are not following acoustic oscillations or subhorizon dynamics; we are asking how primordial large-scale modes are reshaped before standard radiation evolution takes over.

Perturbations: what changes?

Standard FLRW

Full spatial symmetry:

$$SO(3) \ltimes T(3)$$

so perturbations split into

scalar   vector   tensor

Anisotropic background

$$SO(2) \ltimes T(3)$$

Residual symmetry in $y - z$ plane allows us a decomposition into

2D scalars   2D vectors

Perturbations: qualitative result

Mode	standard radiation	metric anisotropy	solid remnant
scalar	$0, -1, -3$	$-2, -4$	$\mathcal{Y}_1 - 2 \simeq 1.85$
vector	-2	-3	$\mathcal{Y}_1 - 1 \simeq 2.85$
tensor	$0, -1$	-3	no effect

$\mathcal{Y}_1 \simeq 3.85$ is the real root of $\mathcal{Y}(\mathcal{Y}+1)(\mathcal{Y}-3) = 16$

Main physical message

The solid remnant produces growing scalar and vector modes.

Why this matters

The tensor sector is unnafected, while scalar and vector perturbations can be amplified.

Observational consequences: what changes?

Track two ratios

$$r \equiv \frac{\text{tensor amplitude}}{\text{scalar amplitude}}, \qquad s \equiv \frac{\text{vector amplitude}}{\text{scalar amplitude}}.$$

Standard radiation era

$$r_{\text{inf.}} \longrightarrow r_{\text{reh.}} \quad \text{approximately unchanged}$$

$$s_{\text{inf.}} \longrightarrow s_{\text{reh.}} \sim a^{-4} s_{\text{inf.}}$$

Vector modes are strongly suppressed.

With solid remnant

scalars grows, vectors grows

tensors are unaffected

Therefore:

$$r_{\text{reh.}} < r_{\text{inf.}}, \qquad s_{\text{reh.}} \text{ is enhanced.}$$

Observational consequences: qualitative estimates

Let

$$\tilde{a} = \frac{a_{\mathrm{reh.}}}{a_{\mathrm{inf.}}}, \qquad \mathcal{Y}_1 \simeq 3.85.$$

Ignoring decaying modes and keeping only order-of-magnitude factors:

$$\frac{\text{scalar}_{\mathrm{reh.}}}{\text{scalar}_{\mathrm{inf.}}} \sim 1 + \lambda\left(\tilde{a}^{\mathcal{Y}_1 - 2} - 1\right),$$

$$\frac{\text{vector}_{\mathrm{reh.}}}{\text{vector}_{\mathrm{inf.}}} \sim \tilde{a}^{-2} + \lambda\left(\tilde{a}^{\mathcal{Y}_1 - 1} - 1\right), \qquad \frac{\text{tensor}_{\mathrm{reh.}}}{\text{tensor}_{\mathrm{inf.}}} \sim 1.$$

Resulting trend

$$\frac{r_{\mathrm{reh.}}}{r_{\mathrm{inf.}}} < 1, \qquad \frac{s_{\mathrm{reh.}}}{s_{\mathrm{inf.}}} \text{ is enhanced relative to the standard case.}$$

Observational consequences: interpretation

Tensor sector

- no growing tensor mode
- tensor-to-scalar ratio is suppressed
- for $\lambda \sim 0.1$ or 0.01, suppression remains moderate

Vector sector

- vector-to-scalar ratio increases
- vector perturbations remain more relevant than in standard radiation
- in a limiting case, vector modes decay as a^{-1}, not a^{-2}

Physical message: the remnant improves the scalar/tensor phenomenology while keeping vector effects potentially interesting.

Outlook

Open direction

The growth of superhorizon modes suggests that nonlinear effects may become important.

- nonlinear dynamics could stabilize the remnant;
- the decay of the remnant may be tied to the instability itself;
- possible production of gravitational waves;
- possible links to dark-matter production mechanisms.

Summary

- Studied a short post-inflationary phase with

$$\mathcal{L}_{\rm m} = \mathcal{L}_{\rm rad.} + \lambda f(\Theta, \Omega), \qquad \lambda \ll 1.$$

- The solid remnant introduces a controlled anisotropy:

$$SO(3) \ltimes T(3) \longrightarrow SO(2) \ltimes T(3).$$

- On superhorizon scales:

$$\text{scalar/vector modes grow}, \qquad \text{tensor modes do not grow}.$$

- Observationally:

$$r_{\rm reh.} < r_{\rm inf.}, \qquad s_{\rm reh.} \text{ is enhanced}.$$

Main message

A short anisotropic phase can leave scalar/vector imprints without producing growing tensor modes.

Thank you!