Cosmology and Inflation with torsion and nonmetricity

Metric Affine Gravity

GeomGravX 29/06/2026

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1) Metric Affine Gravity (MAG)

2) Cosmology in MAG

3) Inflation in MAG

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General relativity

- Metric: $g_{\mu\nu}$

Riemannian geometry

General relativity

- Metric: $g_{\mu\nu}$

Curvature R

General relativity

- Metric: $g_{\mu\nu}$

Curvature R

What lagrangian?

$$\mathcal{L}_g = \tilde{R}[\tilde{\Gamma}[g]]$$

General relativity

$T_{\mu\nu}$

Stress - energy tensor

$$T^{\alpha\beta} \equiv +\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_M)}{\delta g_{\alpha\beta}}$$

Metric affine gravity

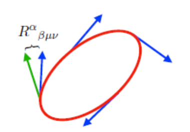
- $g_{\mu\nu}$
- Γ

Non Riemannian geometry

Metric affine gravity

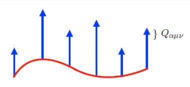
- $g_{\mu\nu}$
- Γ

Curvature R



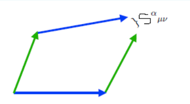
$+$

Nonmetricity Q



$+$

Torsion S



Metric affine gravity

- $g_{\mu\nu}$
- Γ

What lagrangian?

$$\mathcal{L}_g = R[g, \Gamma]$$

$$R[g, \Gamma] = \tilde{R}[g] + (Q..) + (S..)$$

Metric affine gravity

$T_{\mu\nu}$

Stress - energy tensor

$$T^{\alpha\beta} \equiv +\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_M)}{\delta g_{\alpha\beta}}$$

$\Delta_{\lambda}^{\mu\nu}$

HYPERMOMENTUM

$$\Delta_{\lambda}{}^{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_M)}{\delta \Gamma^{\lambda}{}_{\mu\nu}}$$

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Standard cosmological model

ΛCDM model

Standard cosmological model

ΛCDM model

- Matter + CDM + Λ
- GR as theory of gravity
- CP = homogeneity and isotropy
- 6 free parameters
- Flat universe
- Inflation

Going beyond ΛCDM implies breaking one or more of the above points!

Standard cosmological model

ΛCDM model

- Matter + CDM + Λ
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Going beyond ΛCDM implies breaking one or more of the above points!

General relativity

$$\mathcal{L} = \tilde{R}[\tilde{\Gamma}[g]] + \mathcal{L}_m[g]$$

$$\downarrow$$

$$\delta_{g}\mathcal{L} = 0$$

Friedmann equations

$$3\,H^2 = \kappa\,\rho$$

$$2\dot{H} + 3H^2 = -\kappa\,\mathrm{w}\,\rho$$

$$\dot{\rho} + 3\,H(1+\mathrm{w})\rho = 0$$

$$p = \mathrm{w}\,\rho$$

Perfect fluid

$$\dot{\rho} \pm \sqrt{3\,\kappa}\,(1 + \mathrm{w})\,\rho^{3/2} = 0$$

$$\rho(t) = \frac{\rho_0}{\left(1 \pm \frac{\sqrt{3\,\kappa\,\rho_0}}{2}\,(1 + \mathrm{w})\,(t - t_0)\right)^2}$$

How the energy density dilutes

$$2\dot{H} + 3\,(1 + \mathrm{w})\,H^2 = 0$$

$$H(t) = \frac{H_0}{1 + \frac{3H_0}{2}\,(1 + \mathrm{w})\,(t - t_0)}$$

How the universe expands

Metric affine gravity

$$\mathcal{L} = R[g, \Gamma] + \mathcal{L}_m[g, \Gamma]$$

$$\delta_{g}\mathcal{L} = 0 \qquad \delta_{\Gamma}\mathcal{L} = 0$$

Modified Friedmann equations

$$3H^2 = \kappa\rho + \kappa\rho_h$$

$$2\dot{H} + 3H^2 = -\kappa p - \kappa p_h$$

$$\dot{\rho} + 3H(\rho + p) = -\dot{\rho}_h - 3H\left(\rho_h + p_h\right)$$

$$p = \mathrm{w}\,\rho$$

$$p_h = \mathrm{w}_h\,\rho_h$$

perfect fluid

+

hypermomentum

Modified Friedmann equations

$$3H^2 = \kappa\rho + \kappa\rho_h$$

$$\rho_h[\sigma, \Sigma_1, \Sigma_2]$$

$$2\dot{H} + 3H^2 = -\kappa p - \kappa p_h$$

$$p_h[\sigma, \Sigma_1, \Sigma_2]$$

$$\dot{\rho} + 3H(\rho + p) = -\dot{\rho}_h - 3H\left(\rho_h + p_h\right)$$

$$p = \mathbf{w}\,\rho$$

$$p_h = \mathbf{w}_h\,\rho_h$$

$$w_{eff} \equiv \frac{p + p_h}{\rho + \rho_h}$$

$$\dot{\rho} \pm \sqrt{3\kappa}\,(1 + \mathbf{w})\,\rho^{3/2} = 0 \qquad 2\dot{H} + 3\,(1 + \mathbf{w})\,H^2 = 0$$

$$\mathbf{w} \longrightarrow \mathbf{w}_\rho \qquad \mathbf{w} \longrightarrow \mathbf{w}_{\mathrm{eff}}$$

$$\dot{\rho} \pm \sqrt{3\kappa}\,(1 + \mathbf{w}_\rho)\,\rho^{3/2} = 0 \qquad 2\dot{H} + 3\,(1 + \mathbf{w}_{\mathrm{eff}})\,H^2 = 0$$

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How the energy density dilutes

How the universe expands

Only SPIN σ

$$\sigma \propto b\sqrt{\rho}$$

$$\mathbf{w}_\rho = \mathbf{w} \pm b \qquad \mathbf{w}_{\text{eff}} = \frac{2\,\mathbf{w} \mp b}{2 \pm 3b}$$

$$\rho(t) = \frac{\rho_0}{\left(1 \pm \frac{\sqrt{3\,\kappa\,\rho_0}}{2}\,(1 + \mathbf{w}_\rho)\,(t - t_0)\right)^2}$$

$$H(t) = \frac{H_0}{1 + \frac{3H_0}{2}(1 + \mathbf{w}_{\mathrm{eff}})(t - t_0)}$$

New possibilities to model cosmological dynamics!

1) Metric Affine Gravity (MAG)

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Metric theory

$$\mathcal{L} = \mathcal{A}(\phi)\,\tilde{R} + K(\phi)\,\partial_\mu\phi\partial^\mu\phi + V(\phi)$$

Metric theory

$$\mathcal{L} = \mathcal{A}(\phi)\,\tilde{R} + K(\phi)\,\partial_\mu\phi\partial^\mu\phi + V(\phi)$$

↓

conformal transformations

$$\mathcal{L} = \tilde{R} + \hat{K}(\phi)\,\partial_\mu\phi\partial^\mu\phi + V(\phi)$$

$$\hat{K}(\phi) = \frac{3}{2}\frac{(\partial_\phi\mathcal{A}(\phi))^2}{\kappa\mathcal{A}(\phi)^2} + \frac{K(\phi)}{\mathcal{A}(\phi)} = \text{standard}$$

Metric theory

$$\mathcal{L} = \tilde{R} + \hat{K}(\phi)\,\partial_\mu\phi\partial^\mu\phi + V(\phi)$$

$\hat{V}(\varphi)$

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$\downarrow$

field ridefinition

$$\mathcal{L} = \tilde{R} + \partial_\mu\varphi\partial^\mu\varphi + \hat{V}(\varphi)$$

Metric affine gravity

$$\mathcal{L} = \mathcal{A}(\phi)\, R + K(\phi)\, \partial_\mu \phi \partial^\mu \phi + V(\phi)$$
$$+ C_i\, (Q^\mu, S^\mu)\, \partial_\mu \phi + c_i(QS) + a_i(QQ) + b_i(SS)$$

Metric affine gravity

$$\mathcal{L} = \mathcal{A}(\phi)\, R + K(\phi)\, \partial_\mu\phi\partial^\mu\phi + V(\phi) + C_i\left(Q^\mu, S^\mu\right)\partial_\mu\phi + c_i(QS) + a_i(QQ) + b_i(SS)$$

$\downarrow$

$\delta_\Gamma(\mathcal{L}) = 0$

+ conformal transformations

$$\mathcal{L} = \tilde{R} + \hat{K}(\phi)\, \partial_\mu\phi\partial^\mu\phi + V(\phi)$$

Metric affine gravity

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$$\downarrow$$

$$\mathcal{L} = \tilde{R} + \hat{K}(\phi)\, \partial_\mu\phi\partial^\mu\phi + V(\phi)$$

$$\hat{K}(\phi) \longrightarrow \hat{V}(\varphi)$$

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$\hat{K}(\phi)$

Metric affine gravity

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$\hat{K}(\phi)$ $=$ standard

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$\hat{K}(\phi)$ $=$ NOT standard

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$$\hat{K}(\phi) \quad = \quad \text{standard}$$

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$$+ C_i\, (Q^\mu, S^\mu)\, \partial_\mu \phi + c_i(QS) + a_i(QQ) + b_i(SS)$$

$$\hat{K}(\phi) \longrightarrow \hat{V}(\varphi)$$

NOT ALWAYS !

Why do we need $\hat{V}(\varphi)$?

$\hat{V}(\varphi)$

Inflation

$\downarrow$

- Slow roll parameters $\epsilon[\hat{V}(\varphi)]$, $\eta[\hat{V}(\varphi)]$

$\downarrow$

- Tensor to scalar ratio r
- Scalar tilt n_s

Thank you

Cosmology:

- Phys.Rev.D 111 (2025) 6, 064063
- J.Phys.Conf.Ser. 3177 (2026) 1, 012033

Inflation:

- soon in arXiv
(theory and comparison with data)

I also have been victim

of academic harassment

$$
\begin{aligned}
S[g,\Gamma,\Phi] =& \frac{1}{2\kappa}\int \mathrm{d}^4x\sqrt{-g}\Big[\mathcal{A}(\phi)R - \kappa\mathcal{B}(\phi)\partial_\mu\phi\partial^\mu\phi - 2\kappa\mathcal{V}(\phi) \\
&+ \kappa\left(\mathcal{C}_1(\phi)Q^\mu + \mathcal{C}_2(\phi)q^\mu + \mathcal{C}_3(\phi)S^\mu + \mathcal{C}_4(\phi)t^\mu\right)\partial_\mu\phi \\
&+ b_1(\phi)S_{\alpha\mu\nu}S^{\alpha\mu\nu} + b_2(\phi)S_{\alpha\mu\nu}S^{\mu\nu\alpha} + b_3(\phi)S_\mu S^\mu \\
&+ b_5(\phi)S_\mu t^\mu + b_6(\phi)\varepsilon^{\alpha\beta\gamma\delta}S_{\alpha\beta\mu}S_{\gamma\delta}{}^\mu \\
&+ a_1(\phi)Q_{\alpha\mu\nu}Q^{\alpha\mu\nu} + a_2(\phi)Q_{\alpha\mu\nu}Q^{\mu\nu\alpha} + a_3(\phi)Q_\mu Q^\mu \\
&+ a_4(\phi)q_\mu q^\mu + a_5(\phi)Q_\mu q^\mu + a_6(\phi)\varepsilon^{\alpha\beta\gamma\delta}Q_{\alpha\beta\mu}Q_{\gamma\delta}{}^\mu \\
&+ c_1(\phi)Q_{\alpha\mu\nu}S^{\alpha\mu\nu} + c_2(\phi)Q_\mu S^\mu + c_3(\phi)q_\mu S^\mu \\
&+ c_4(\phi)Q_\mu t^\mu + c_5(\phi)q^\mu t_\mu + c_6(\phi)\varepsilon^{\alpha\beta\gamma\delta}Q_{\alpha\beta\mu}S_{\gamma\delta}{}^\mu\Big]
\end{aligned}
$$

$$t^{\alpha} = \epsilon^{\alpha bca} S_{bca} \qquad S_{\alpha} = S_{\alpha\beta}{}^{\beta}$$

$$Q_{\mu} = g^{\alpha\lambda} Q_{\mu\alpha\lambda} \qquad q_{\mu} = g^{\alpha\lambda} Q_{\alpha\lambda\mu}$$

Metric affine gravity

$$\mathcal{L}_g[g, \Gamma] + \mathcal{L}_M[g, \Gamma, \phi_M]$$

3

$$\delta_g(\mathcal{L}_g + \mathcal{L}_M) = 0 \qquad \delta_\Gamma(\mathcal{L}_g + \mathcal{L}_M) = 0$$

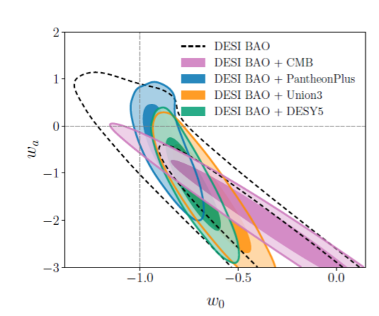
1

2

Constrains!!!

Why we need to go beyond ΛCDM?

- H_0 tension
- S_8 tension
- Dark energy
- Dark matter



$$w(z) = w_0 + w_a \frac{z}{1+z}$$

$$\text{DESI} \rightarrow w_0 \neq -1, w_a \neq 0$$

Modified Friedmann equations assumptions:

- Cosmological principle
- Projective invariance
- Parity even

Cosmological principle

- $Q \to 3$ dofs
- $S \to 2$ dofs
- $\Delta \to 5$ dofs

- ~~*Weyssenhoff fluid*~~
- FLRW metric
- Perfect fluid

Projective invariance

$$\Gamma^{\lambda}_{\mu\nu} \mapsto \Gamma^{\lambda}_{\mu\nu} + \delta^{\lambda}_{\mu}\xi_{\nu}$$

$$\mathcal{L}_g = R[\tilde{\Gamma}] \qquad \Longrightarrow \qquad \Delta_\alpha = 0$$

- $\Delta \rightarrow 4$ dofs
- $Q \rightarrow 2$ dofs

Parity even

$$\mathcal{L}_g = R[\tilde{\Gamma}] \qquad \Longrightarrow \qquad \mathcal{L}_m \text{ even}$$

- $\Delta \to 3$ dofs
- $S \to 1$ dof

Assumptions for the lagrangian for inflation studies?

None of the above!