Static Spherically Symmetric Black Holes in regularised 4D Einstein-scalar-Gauss-Bonnet theory

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Does the known $h = 1$ solution exhaust the static spherical sector?

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$$D = 4 : \qquad \mathcal{G} \quad \text{topological.}$$

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$$\boxed{D = 4 \text{ dynamical GB} \Rightarrow \text{scalar-tensor theory.}}$$

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- General $h \neq 1$ solutions studied mainly numerically, and for $\Lambda = 0$ (Lu-Pang '20).

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checked by matching $\mathcal{E}_f$,$\mathcal{E}_h$,$\mathcal{E}_\phi$ to covariant combinations.

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Takeaway: For $\Lambda \neq 0$ branches, the logarithmic scalar seed is selected asymptotically.

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Takeaway: ϕ is finite at the horizon, but not necessarily analytic in $\Delta = r - r_h$.

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$$r \longrightarrow r_h: \quad \boxed{(1,1,0)_{1/2},} \quad F \sim \Delta, \quad H \sim \Delta, \quad \phi \sim c_0.$$

Finite order local candidates in $h \neq 1$

Region	Seed	Candidate	Interpretation
$r \to \infty$	$\lambda \ln r$	$(2,2,0)_{\lambda=1}$	non-AF, $\Lambda \neq 0$
$r \to \infty$	$\lambda = 0, 2$	$(0,0,0)_{\lambda=0,2}$	only $\Lambda = 0$
$r \to r_h$	half-integer	$(1,1,0)_{1/2}$	non-extremal horizon
$r \to r_h$	analytic	$(1,1,0)_1$	$c_{2k+1} = 0$ sub-branch

Table: Candidate local families classified by $(s,t,u) \sim (F,H,\phi)$ at a finite order truncation in the generic gauge.

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- derive full recursion relations and convergence radius,
- distinguish gauge data from physical data,
- investigate physical properties once a global branch is under control.