Event horizon termination and the emergence of Lorentz signature

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Phys. Rev. D 113, 084025 (2026) arXiv:2602.02646, *and ongoing work*

GeomGravX: 10 Years of Geometric Foundations of Gravity and eXtensions, University of Tartu, Tartu, Estonia **2nd July 2026**

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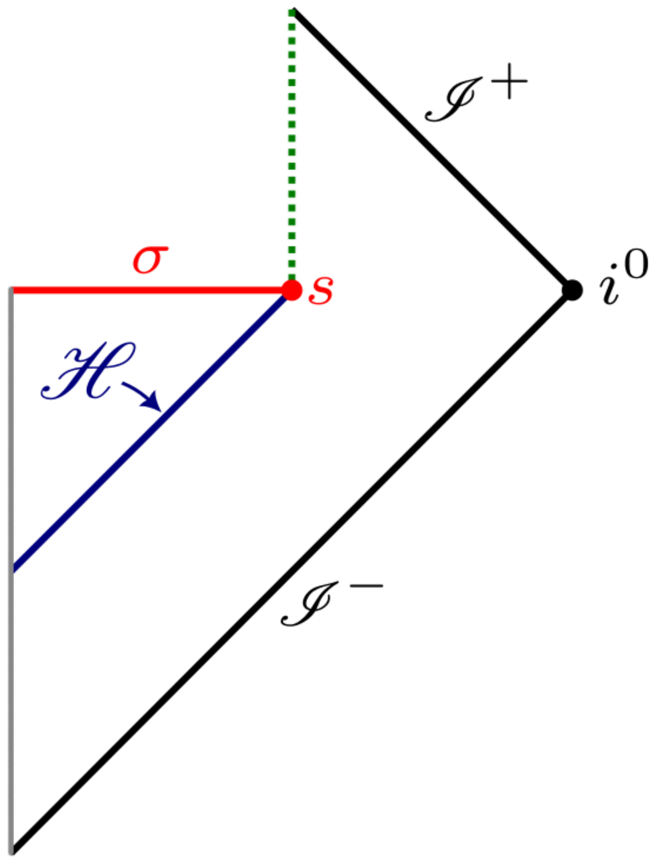
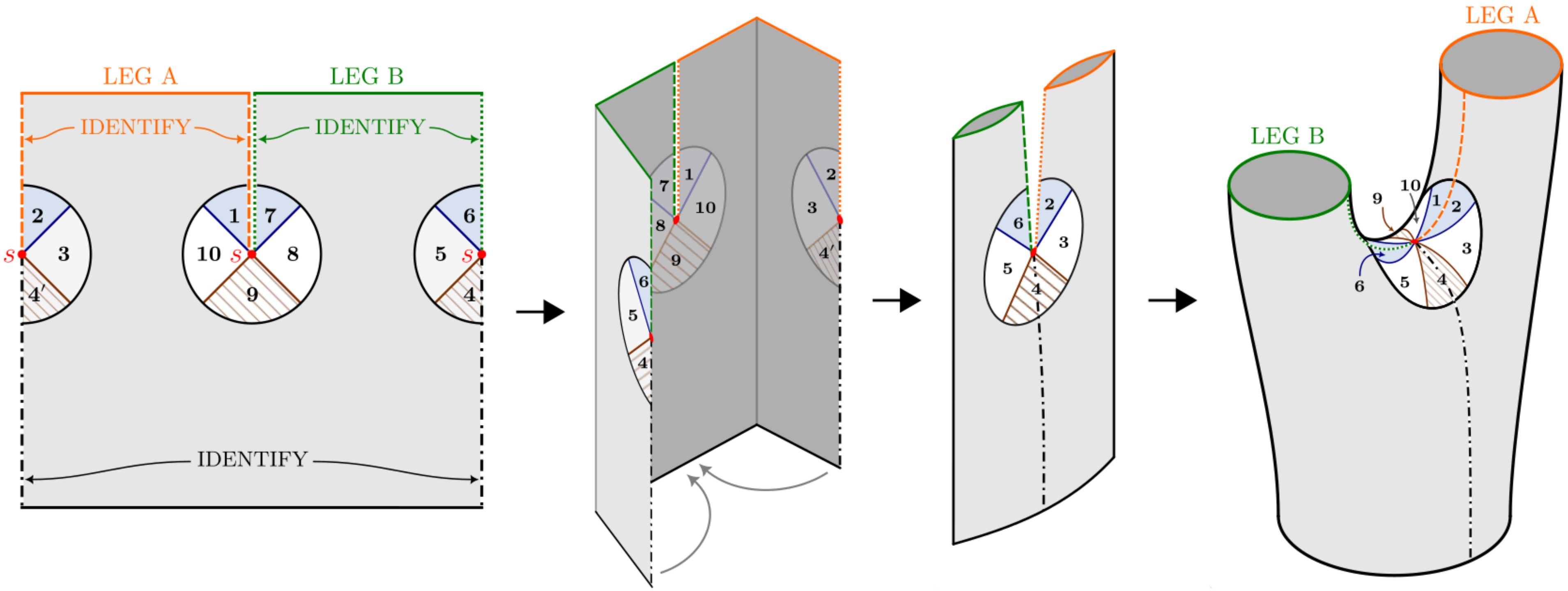
End state of a Black Hole



- What is end state of evaporating black hole?
- Penrose diagram for a scenario in a spherically symmetric setting
    i. Black hole forms
    ii. Undergoes Hawking evaporation
    iii. Horizon terminates when area = 0
- Not too difficult to regularize singularity† σ
- But what about s?

†Ex: A Simpson, M Visser, JCAP 02 (2019) 042 [arxiv:1812.07114]

Trousers spacetime: a digression



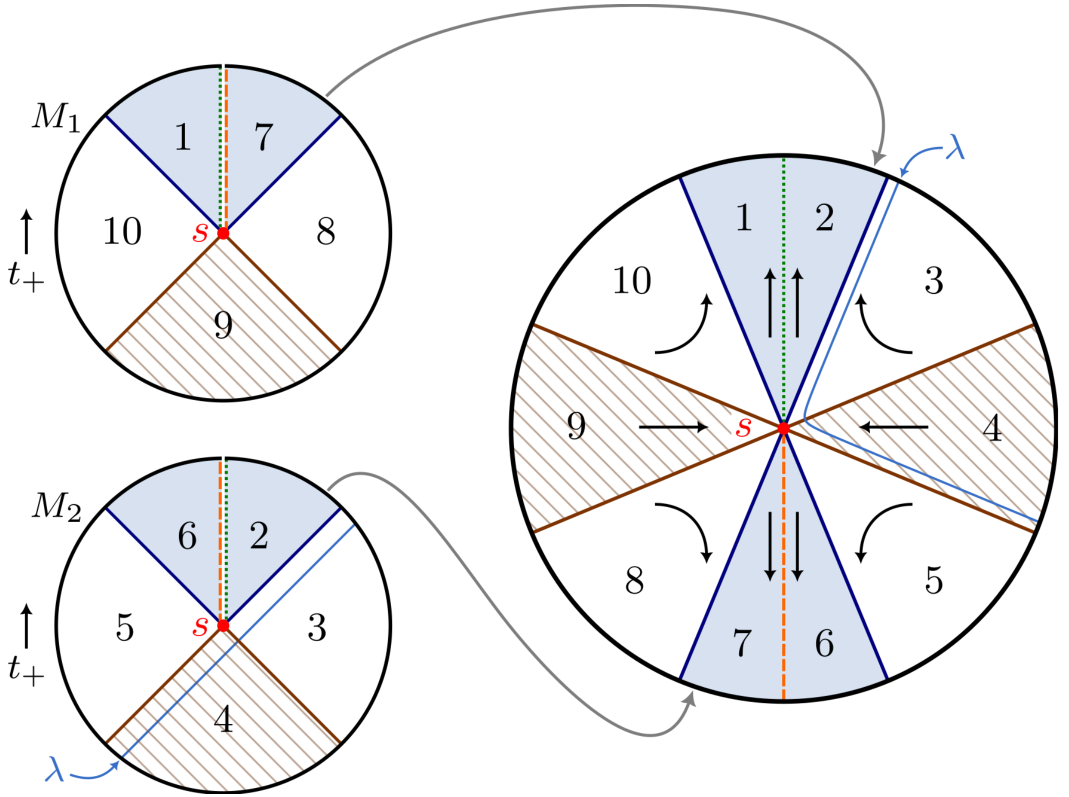
Normally, spacetime points have one future and one past light cone

s is a quasiregular singularity, characterized by 2 future and 2 past light cones*

*G F R Ellis and B G Schmidt, Gen. Rel. Grav. 8, 915 (1977).

Quasiregular singularities

They generalize conical singularities, construct via cut and paste methods.



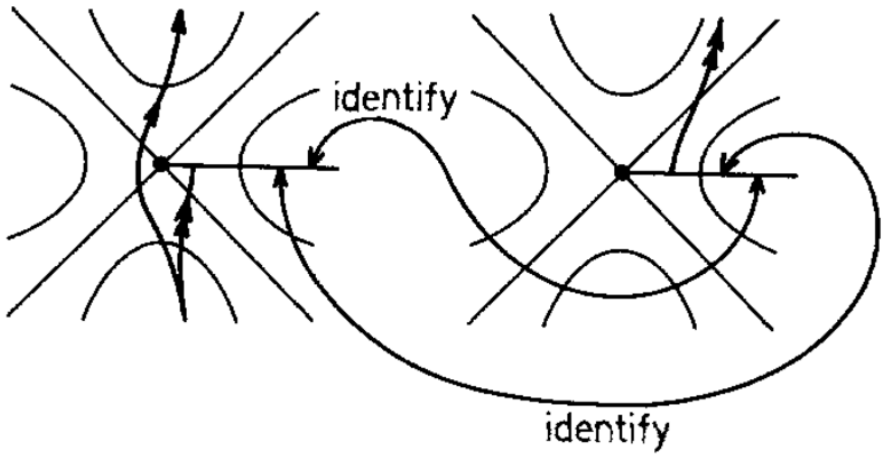
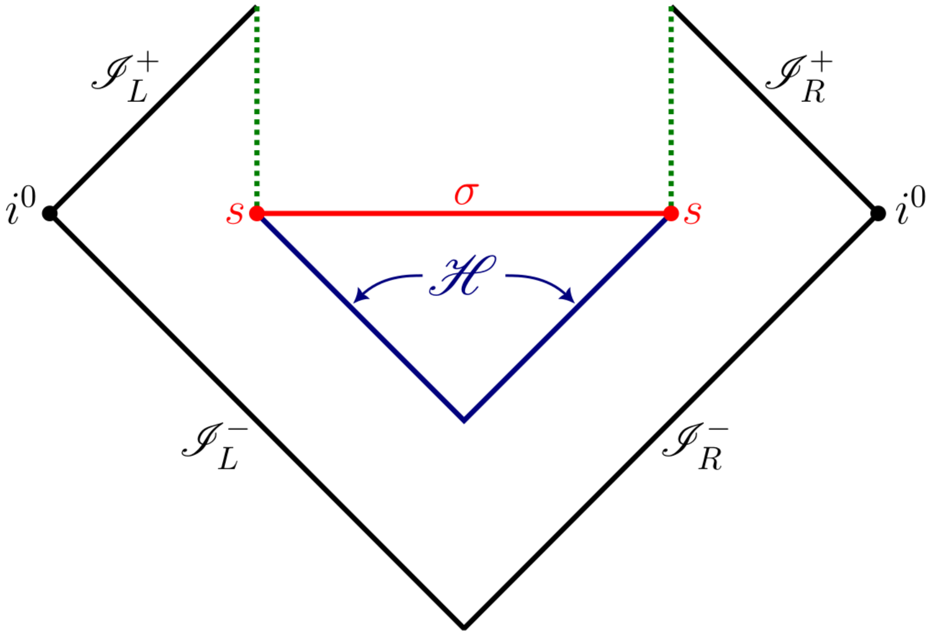


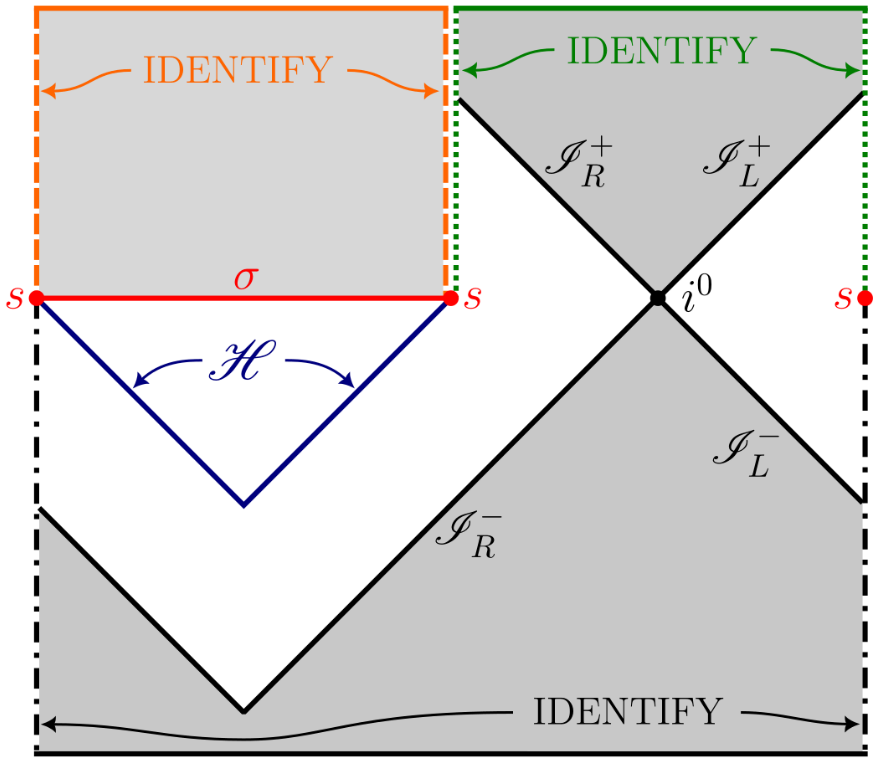
Fig 4(e) from Ellis & Schmidt†

†G F R Ellis and B G Schmidt, Gen. Rel. Grav. 8, 915 (1977).

$1+1$ Black hole and trousers† spacetime

Evaporating $1+1$ BH is conformal to trousers spacetime: s is a singularity





†A Anderson, B S DeWitt, Found.Phys. 16 (1986) 91-105 F. Dowker, S. Surya, PRD 58, 124019 (1998) [arXiv:gr-qc/9711070]; Buck et al., Class.Quant.Grav. 34 (2017) 5, 055002 [arXiv:1609.03573]

Things that look like quasiregular singularities

- Consider saddle-like scalar field, Euclidean metric:

$$\varphi = (u^2 - v^2)/(2L_0)$$
$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = du^2 + dv^2 + dy^2 + dz^2$$

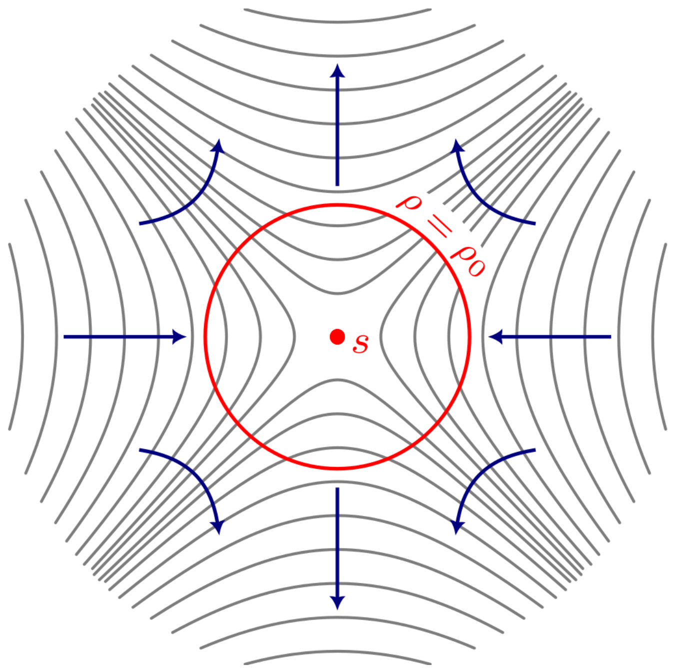
- Can construct an effective metric:

$$\mathbf{g}_{ab} = g_{ab} - \nabla_a \varphi \nabla_b \varphi / M^4$$

if gradients $\nabla_a \varphi$ sufficiently large, $\mathbf{g}_{ab}$ is Lorentzian.

- Zooming out, this looks like a point with two future and two past light cones

- There is at least one scalar-tensor theory that admits this as a solution[1]



Arrows indicate $\nabla_a \varphi$ direction, i.e. direction of time

[1] JCF, S. Mukohyama, S. Carloni, Phys. Rev. D 109, 024040 (2024)

Higher derivative shift sym. Euclidean scalar-tensor theory

- Postulate[1] Euclidean-signature $g_{\mu\nu}$ with scalar-tensor action bounded below:

$$S = \int_M d^4x\sqrt{|g|}L, \quad \varphi_a := \nabla_a\varphi, \quad \varphi_{ab} := \nabla_a\nabla_b\varphi, \quad X := \varphi^a\varphi_a$$

$$L = c_1R^2 + c_2R_{ab}R^{ab} + c_3R_{abcd}R^{abcd} + c_4XR + c_5R^{ab}\varphi_a\varphi_b$$
$$+ c_6\mathrm{X}^2 + c_7(\Box\varphi)^2 + c_8\varphi_{ab}\varphi^{ab} + c_9R + c_{10}X + c_{11}$$

- Postulate matter couplings that reduce to effective metric at long distances[2]

$$\mathbf{g}_{ab} = g_{ab} - \varphi_a\varphi_b/M^4$$

- Renormalizable[3] (expected, since this is the case for quadratic gravity[4])
- Can argue it reduces to Lorentzian scalar-tensor theory in long-distance limit.[1]

[1] S Mukohyama, PRD 87, 085030 (2013) [2] S Mukohyama, J Uzan, PRD 87:065020 (2013); J Kehayias, S Mukohyama, J Uzan, PRD 89, 105017 (2014)
[3] K Muneyuki, N Ohta, Phys. Lett. B 725 (2013) 495-499 [4] K S Stelle, PRD 16, 953 (1977)

A Lorentzian dispersion relation emerges[1]

- **Theory is fundamentally Euclidean---can one be fooled into thinking that it is Lorentzian?**
- Rewrite L (C_{abcd} is Weyl, E is Gauss-Bonnet):

$$\tilde{L} := P_0 + (X - X_0)^2 - Z_0 R + \eta_0(\chi^2 - \chi R) + \frac{1}{2\lambda_0} C_{abcd}C^{abcd} + \gamma_0 G^{ab}\varphi_a\varphi_b + \alpha_0 {\varphi_a}^a {\varphi_b}^b + \beta_0 \varphi_{ab}\varphi^{ab} + \sigma_0 E$$

- Set $P_0 = 0$, have flat background soln:

$$\bar{g}_{ab} = \mathrm{diag}(1,1,1,1), \qquad \bar{\varphi} = t\sqrt{X_0}, \qquad \bar{\chi} = 0$$

- Perturb, restrict to modes in x-direction:

$$\delta g_{ab} = s\begin{pmatrix} U & 0 & B_y & B_z \\ 0 & \psi & 0 & 0 \\ B_y & 0 & \psi + h_+ & h_\times \\ B_z & 0 & h_\times & \psi - h_+ \end{pmatrix}, \qquad \delta\varphi = s\phi, \qquad \delta\chi = s\xi.$$

tensor: $(h_+, h_\times)$, vec.: (B_y, B_z), scalar: (ϕ, ξ, U, ψ)

- Pick mode: $F = F_1(t)\sin(kx) + F_2(t)\cos(kx)$
- In long distance limit $\dot{h}, k \ll M_{\mathrm{Pl}}$, $\ddot{h} \ll M_{\mathrm{Pl}}^2$, tensor sector action has form:

$$\underline{S}_h \approx \frac{s^2 v_0}{8}\int dt\left[\mu_3\,\dot{h}^2 - \mu_2\,k^2\,h^2\right], \qquad \mu_3 := X_0(\beta_0 + \gamma_0) + Z_0, \quad \mu_2 := \gamma_0 X_0 - Z_0$$

If $\mu_2\mu_3 > 0$, tensor modes satisfy massless Lorentzian dispersion relation, $\mu_3 > 0$ needed to avoid ghosts.

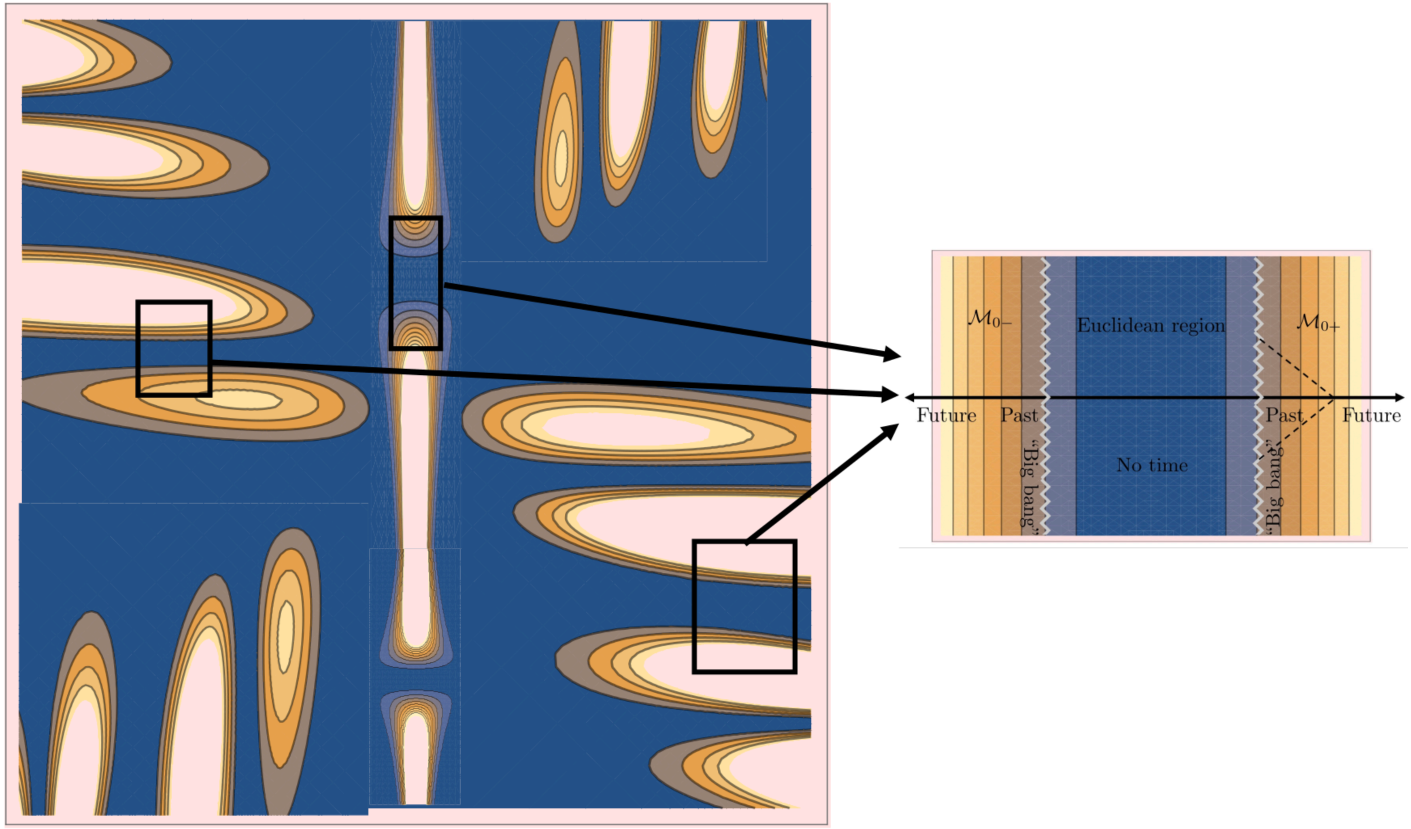
- Large parameter space in which all remaining modes satisfy Euclidean dispersion relations.
  - Scalar sector complicated---all but one have a large tachyonic mass, all can be set to zero with appropriate BCs.

[1] JCF, S. Mukohyama, S. Carloni, Phys. Rev. D 112 024066 (2025) arXiv:2505.00112v1

A Euclidean reality with temporal pockets?

The picture:

- Reality is fundamentally Riemannian (Euclidean sig.)
- Have regions where an effective Lorentzian geometry emerges
- Could our universe be one of these "pockets" of Lorentzian geometry?
  - Big bang as a boundary of "pocket"



JCF, S Mukohyama, and J-P Uzan, Phys. Rev. D 113, 084025 (2026) arXiv:2602.02646

Big Bang solutions

For Simplicity, consider 2nd order model:

$$S_1 = \int d^4x\sqrt{g_{\rm E}}\,\{G_4(X_{\rm E})R_{\rm E} + \mathcal{K}_{\rm E}(X_{\rm E}) - 2\dot{G}_4(X_{\rm E})\left[(\nabla^2_{\rm E}\phi)^2 - (\nabla^{\rm E}_\mu\nabla^{\rm E}_\nu\phi)^2\right]\}$$

Planar symmetry:

$$ds^2 = N_{\rm E}^2(z)dz^2 + a^2(z)\delta_{ij}dx^idx^j, \quad \psi(z) := \partial_z\phi/N_{\rm E}(z)$$

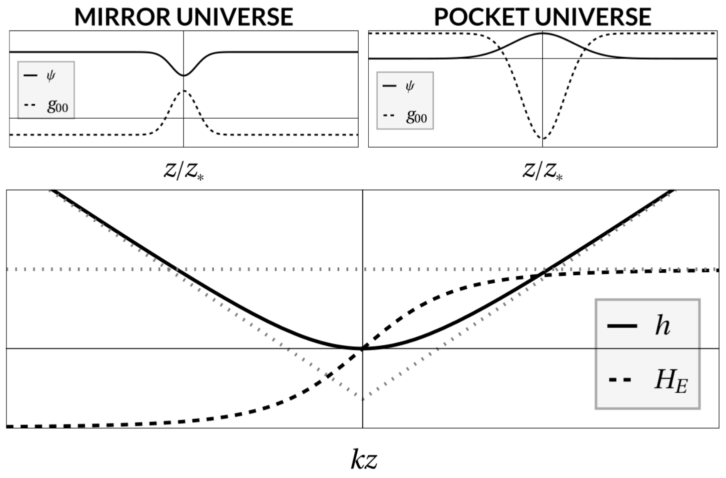
Ansatz: $G_4 = g_0 + g_1 X_E, \quad \mathcal{K} = k_0 + \frac{1}{2}k_2(X_0 - X_E)^2$

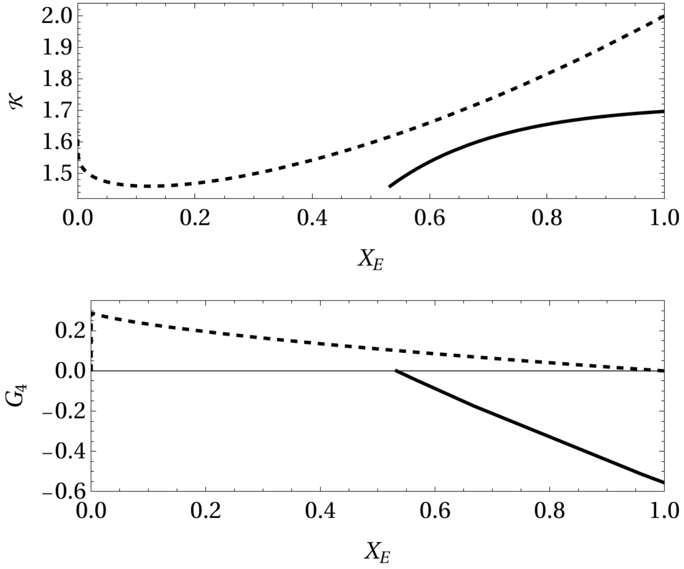
Can have "Euclidean pocket" soln ($\mathbf{g}_{00} = N_{\rm E}^2 - \psi^2/M^4$)

- ψ increases away from $z = 0 \quad \Rightarrow \quad$ sig change!
- $H_E = \frac{\dot{a}_E(z)}{a_E(z)}$ vanishes @ $z = 0 \quad \Rightarrow \quad$ bounce-like

Can obtain "Lorentzian pocket" and more general solutions via reconstruction of $G_4(X_E)$ and $\mathcal{K}(X_E)$

- Can read EoMs as ODEs for $G_4(X_E)$ and $\mathcal{K}(X_E)$





JCF, S Mukohyama, and J-P Uzan, Phys. Rev. D 113, 084025 (2026) arXiv:2602.02646

Compact objects: stealth black hole condition (in progress)

- Consider again 2nd order model:

$$S_1 = \int d^4x \sqrt{g_{\rm E}} \left\{ G_4(X_{\rm E}) R_{\rm E} + \mathcal{K}_{\rm E}(X_{\rm E}) - 2G_4'(X_{\rm E}) \left[(\nabla_{\rm E}^2 \phi)^2 - (\nabla_\mu^{\rm E} \nabla_\nu^{\rm E} \phi)^2 \right] \right\}$$

- Strategy: Demand $\mathbf{g}_{ab} = g_{ab} - \varphi_a \varphi_b / M^4$ is Schwarzschild, then "reconstruct"
- Can find stealth BH solutions if $\exists$ root $\mathcal{K}_{\rm E}(X_{\rm E}) = \mathcal{K}_{\rm E}'(X_{\rm E}) = 0.$
- In spherical sym., EoMs can also be written as ODEs for $G_4(X_E)$ and $\mathcal{K}(X_E)$
  - Done for Lorentzian counterpart[†]
  - Can employ a similar reconstruction strategy for compact objects
- Ongoing work---stay tuned!

[*] Ongoing work with W Barker, S Mukohyama, and J-P Uzan

[†] T Kobayashi, N Tanahashi, PTEP 2014 (2014) 073E02 [arXiv:1403.4364]

Take aways

- Endpoint of BH evaporation resembles quasiregular singularities
  - Causal discontinuity: two future directed light cones, two past directed
- $\exists$ at least one Euclidean scalar-tensor theory that admits such a structure
  - Open question: how to ensure coupling to same effective $\mathbf{g}_{ab}$?*
- Theory can yield a tensor mode with Lorentzian dispersion relation
- Picture: universe as pocket of (effective) Lorentzian geometry in a fundamentally Euclidean manifold
- Admits stealth BH solutions; this is ongoing work---stay tuned!

* S. Chadha and H. B. Nielsen, Nucl. Phys. B 217, 125 (1983).

BACKUP SLIDES

Scalar sector analysis[1]

- Scalar sector is more complicated. Introduce Lagrange multiplier term $\Theta(\sqrt{2}\Omega - \Box\varphi)$, to get:

$$\begin{aligned} \underline{L}_\Phi = &\dot{Y}^T \cdot \underline{\mathbb{K}} \cdot \dot{Y} + Y^T \cdot \underline{\mathbb{V}} \cdot Y + \dot{Y}^T \cdot \underline{\mathbb{M}}^T \cdot Y \\ &+ Y^T \cdot \underline{\mathbb{M}} \cdot \dot{Y} + Y^T \cdot \underline{\mathbb{A}}^T \cdot Z + Z^T \cdot \underline{\mathbb{A}} \cdot Y \\ &\dot{Y}^T \cdot \underline{\mathbb{B}}^T \cdot Z + Z^T \cdot \underline{\mathbb{B}} \cdot \dot{Y} + Z^T \cdot \underline{\mathbb{C}} \cdot Z \end{aligned}$$

  where $Y := (\psi, \xi, \phi, \Theta)$ and $Z := (\Omega, U)$.

- Can integrate out auxiliary fields to get:

$$L_\Phi = \dot{Y}^T \cdot \mathbb{K} \cdot \dot{Y} + Y^T \cdot \mathbb{V} \cdot Y + \dot{Y}^T \cdot \mathbb{M}^T \cdot Y + Y^T \cdot \mathbb{M} \cdot \dot{Y}$$

- Assuming $Y = Y_0 \exp(i\omega t)$ can get EoMs:

$$\mathbb{E} \cdot Y = 0, \qquad \mathbb{E} := \omega^2 \mathbb{K} + i\omega(\mathbb{M} - \mathbb{M}^T) + \mathbb{V}.$$

  Can identify masses from $\lim_{k^2 \to 0} \det(\mathbb{E}) = 0$

- Find one massless dof. Remaining masses distinct, $\exists$ large parameter space where they are tachyonic
- Can obtain low k^2 dispersion relation by setting $\omega^2 = m^2 + Gk^2 + \mathcal{O}(k^4)$, expanding in k^2 $\exists$ large parameter space they are Euclidean ($G < 0$)
- Have a region of parameter space where massless scalar mode has Lorentzian disp. rel. ($G > 0$), but is necessarily a ghost if $\mu_3 > 0$.
- Can avoid ghost if massless scalar mode satisfies Euclidean disp. rel. ($G < 0$)
  - Function becomes approximately harmonic (with corrections suppressed by $1/M_{\mathrm{Pl}}^2$).

[1] JCF, S. Mukohyama, S. Carloni, arXiv:2505.00112v1

Approximate solution in Riemann normal coordinates

Explicitly, the approximate solution near the origin has the form:

$$\varphi = (u^2 - v^2)/(2L_0) + \varphi_{\mu\nu} x^\mu x^\nu + O(x^{\cdot 3})$$

$$g_{\mu\nu} = \delta_{\mu\nu} - \frac{1}{3}\left[R_{\mu\alpha\nu\beta}\right]_0 x^\alpha x^\beta - \frac{1}{6}\left[\nabla_\gamma R_{\mu\alpha\nu\beta}\right]_0 x^\alpha x^\beta x^\gamma$$
$$- \left[\frac{2}{45} R_{\mu\alpha\lambda\beta} R^\lambda{}_{\gamma\delta\nu} + \frac{1}{20}\nabla_\gamma \nabla_\delta R_{\mu\alpha\nu\beta}\right]_0 x^\alpha x^\beta x^\gamma x^\delta + O(x^{\cdot 5})$$

with coefficients satisfying:

$$\Big[4\{(c_2 + 4c_3)\Box R_{\mu\nu} - (2c_1 + c_2 + 2c_3)\nabla_\mu \nabla_\nu R\} + 4c_{10}\varphi_{\mu\nu}$$
$$+ 8\{c_3 R_\mu{}^{\alpha\beta\gamma} R_{\nu\alpha\beta\gamma} - (c_2 + 2c_3)(R_{\mu\alpha} R_\nu{}^\alpha - R^{\alpha\beta} R_{\mu\alpha\nu\beta})\}$$
$$+ 8\{c_2 R_{\mu\alpha} R_\nu{}^\alpha - c_4 \varphi_{\alpha\mu} \varphi^\alpha{}_\nu\} + 4(c_9 + 2c_1 R) R_{\mu\nu}$$
$$+ 2g_{\mu\nu}\Big\{(4c_1 + c_2)\Box R - c_{11} - c_9 R - c_1 R^2 - c_2 R_{\alpha\beta} R^{\alpha\beta}$$
$$- c_3 R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} - (c_8 - 4c_4 - c_5)\varphi_{\alpha\beta}\varphi^{\alpha\beta}\Big\} + T_{\mu\nu}\Big]_0 = 0$$