Maxwellian gravity and the cosmological constant

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Question

Can Maxwellian gravity be understood as a systematic correction to AdS gravity in an expansion in powers of the cosmological constant?

Motivation

- Maxwell algebra

$$[\mathcal{J}_{AB}, \mathcal{J}_{CD}] = \eta_{[A[C}\mathcal{J}_{D]B]} \,, \qquad [\mathcal{J}_{AB}, \mathcal{Z}_{CD}] = \eta_{[A[C}\mathcal{Z}_{D]B]} \,,$$
$$[\mathcal{J}_{AB}, \mathcal{P}_{C}] = \eta_{C[B}\mathcal{P}_{A]} \,, \qquad [\mathcal{P}_{A}, \mathcal{P}_{B}] = \mathcal{Z}_{AB}$$

- Describes particle systems in constant electromagnetic fields
  [Bacry,Combe,Richard(1970)][Schrader(1972)][Beckers,Hussin(1983)][Negro,Olmo(1990)][Gibbons,Gomis,Pope(2009)][Gomis,Kleinschmidt(2017)]
- Gauging: Extension of GR [de Azcarraga,Kamimura,Lukierski(2010)]
- Cosmological models [Durka,Kowalski-Glikman(2011)][de Azcarraga,Kamimura,Lukierski(2012)][Kibaroglu(2023)]
- Supersymmetric and higher-spin extensions of the Maxwell algebra have been formulated
  [Bonanos,Gomis,Kamimura,Lukierski(2010)][Fedoruk,Lukierski(2013)][Caroca,Concha,Rodríguez,Salgado-Rebolledo(2017)][Peñafiel,Ravera(2017)]

Motivation

- Three-dimensional case

$$[\mathcal{J}_A,\mathcal{J}_B]=\epsilon_{ABC}\mathcal{J}_C,\quad [\mathcal{J}_A,\mathcal{P}_B]=\epsilon_{ABC}\mathcal{P}_C,\quad [\mathcal{J}_A,\mathcal{Z}_B]=\epsilon_{ABC}\mathcal{Z}_C=[\mathcal{P}_A,\mathcal{P}_B]$$

$$\langle\mathcal{J}_A,\mathcal{P}_B\rangle=\alpha\,\eta_{AB},\quad \langle\mathcal{J}_A,\mathcal{J}_B\rangle=\beta\,\eta_{AB},\quad \langle\mathcal{J}_A,\mathcal{Z}_B\rangle=\gamma\,\eta_{AB}=\langle\mathcal{P}_A,\mathcal{P}_B\rangle,$$

- It is possible to define an extended Chern-Simons gravity theory [Szabo, Salgado,Valdivia(2014)]
- Connection and curvature

$$\mathcal{A}=e^A\mathcal{P}_A+\omega^A\mathcal{J}_A+\sigma^A\mathcal{Z}_A,\qquad \mathcal{F}=T^A\mathcal{P}_A+R^A\mathcal{J}_A+F^A\mathcal{Z}_A$$

$$T^A=de^A+\epsilon^{ABC}\omega_B\wedge e_C,\quad R^A=d\omega^A+\frac{1}{2}\epsilon^{ABC}\omega_B\wedge\omega_C,$$

$$F^A=d\sigma^A+\epsilon^{ABC}\omega_b\wedge\sigma_c+\frac{1}{2}\epsilon^{ABC}e_B\wedge e_C$$

- Extended three-dimensional gravity action

$$S=\int CS_{Maxwell}=\int\left\langle\mathcal{A}\wedge d\mathcal{A}+\frac{2}{3}\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}\right\rangle,$$

Motivation

- The explicit form of the action is

$$\int CS_{Maxwell} = \int \left[2\alpha e^A R_A + \beta CS(\omega) + \gamma (2\sigma^A R_A + e^A T_A) \right]$$

- Infinite-dimensional asymptotic symmetry [Concha,Merino,Miskovic,Rodríguez,Salgado-Rebolledo,Valdivia(2018)]
- Dimensional reduction leads to Jackiw-Teiteilboim gravity [Cangemi(1992)][Duval,Horvath,Horvathy(2008)]
- Non-relativistic and Carrollian limits [Avilés,Gomis,Frodden,Hidalgo,Zanelli(2018)][Peñafiel,Salgado-Rebolledo(2019)][Concha,Peñafiel,Ravera, Rodríguez(2021)]
- Supersymmetric, hypersymmetric, and higher-spin extensions [Concha,Fierro,Rodríguez,Salgado(2015)][Caroca,Concha,Rodríguez,Salgado-Rebolledo(2017)] [Concha,Ravera,Rodríguez(2019)][Caroca,Concha,Matulich, Rodríguez,Tempo(2021)][Caroca,Peñafiel,Salgado-Rebolledo(2023)]
- Effective model of a topological insulator [Palumbo (2017)]

Outline

1. Infinite semigroup expansion
2. Example: Non-relativistic expansion of Poincaré
3. Expansion of AdS in the cosmological constant
4. Recovering Maxwellian gravity
5. Discussion and perspectives

Lie algebra expansions

- Lie algebra expansions allow to relate Lie algebras of different dimension
  [Hatsuda,Sakaguchi(2003)] [deAzcárraga, Picón, Varela(2003)] [Izaurieta,Rodríguez,Salgado(2006)]
- Consider a Lie algebra $\mathfrak{g}$ and an Abelian semigroup S

$$[T_\alpha, T_\beta] = f^{\gamma}_{\ \alpha\beta} T_\gamma \,, \quad \lambda_i \cdot \lambda_j = K^{k}_{ij} \lambda_k$$

- The direct product $\mathfrak{g} \times S$ is a Lie algebra

$$[T_{(\alpha,i)}, T_{(\beta,j)}] = f^{(\gamma,k)}_{\ (\alpha,i)(\beta,j)} T_{(\gamma,k)}$$

  where

$$T_{(\alpha,i)} = \lambda_i \otimes T_\alpha \,, \quad f^{(\gamma,k)}_{\ (\alpha,i)(\beta,j)} = f^{\gamma}_{\ \alpha\beta} K^{k}_{ij}$$

- We call $\mathfrak{g} \times S$ an expansion of $\mathfrak{g}$ by S
- **Reduced expansion**: if the semigroup has a zero element λ_{zero}

$$\lambda_{\mathrm{zero}} \cdot \lambda_i = \lambda_{\mathrm{zero}} \ \forall i \quad \Rightarrow \quad \{\mathfrak{g} \times S\} - T_{(\alpha,\mathrm{zero})} \text{ is a Lie algebra}$$

Lie algebra expansions

- **Resonant expasion**: consider the case where $\mathfrak{g}$ admits subspace decomposition

$$\mathfrak{g} = \bigoplus_{p \in I} V_p = V_0 \oplus V_1 \oplus \cdots \oplus V_n$$

- where, for each pair of subspaces V_p and V_q, there is a set of indices $i(p,q) \in I$ such that

$$[V_p, V_q] \subset \bigoplus_{r \in i(p,q)} V_r$$

- if the semigroup S admits a "resonant" decomposition $S = S_0 \oplus S_1 \oplus \cdots \oplus S_n$

$$S_p \cdot S_q \subset \bigoplus_{r \in i(p,q)} S_r$$

- The resonant expansion

$$\bigoplus_p V_p \times S_p = \{V_0 \times S_0\} \oplus \{V_1 \times S_1\} \oplus \cdots \oplus \{V_n \times S_n\}$$

- is a Lie algebra

Non-relativistic expansions of Poincaré

- **Warm-up example:** Non-relativistic expansions of $\mathfrak{iso}(D-1,1)$
- Poincaré algebra

$$[\mathcal{J}_{AB},\mathcal{J}_{CD}] = \eta_{[A[C}\mathcal{J}_{D]B]}\,, \quad [\mathcal{J}_{AB},\mathcal{P}_{C}] = \eta_{C[B}\mathcal{P}_{A]}$$

- Splitting indices in the form $A = (0, a = 1, \ldots, D)$

$$\mathcal{P}_A = (\mathcal{P}_0, \mathcal{P}_a), \qquad \mathcal{J}_{AB} = (\mathcal{J}_{0a}, \mathcal{J}_{ab})$$

- Introduce a parameter ϵ and the rescaling

$$H = \epsilon^{-1}\mathcal{P}_0, \quad P_a = \mathcal{P}_a, \quad G_a = \epsilon J_{0a}, \quad J_{ab} = \mathcal{J}_{ab}$$

- In the limit $\epsilon \to 0$ the generators $\{H, P_a, G_a, J_{ab}\}$ close into the Galilean algebra

$$[G_a, H] = P_a\,, \qquad [J_{ab}, G_c] = \delta_{c[b}G_{a]}\,,$$
$$[J_{ab}, P_c] = \delta_{c[b}P_{a]}\,, \qquad [J_{ab}, J_{cd}] = \delta_{[a[c}J_{d]b]}$$

- The parameter ϵ can be interpreted as the inverse of the speed of light $\epsilon = 1/c$
- This procedure, known as Inönü-Wigner contraction, can be generalized by considering expansions of Lie algebras

Non-relativistic expansions of Poincaré

- Now we consider non-relativistic expansions of the Poincaré algebra [Bergshoeff,Izquierdo,Ortín,Romano(2019)]

$$[\mathcal{J}_{AB}, \mathcal{J}_{CD}] = \eta_{[A[C}\mathcal{J}_{D]B]}\,, \quad [\mathcal{J}_{AB}, \mathcal{P}_C] = \eta_{C[B}\mathcal{P}_{A]}$$

- Subspace decomposition

$$V_0 = \{\mathcal{P}_0, \mathcal{J}_{ab}\}\,, \qquad V_1 = \{\mathcal{P}_a, \mathcal{J}_{0a}\}\,.$$

- They satisfy $\mathbb{Z}_2$-graded structure

$$[V_0, V_0] \subset V_0\,, \quad [V_0, V_1] \subset V_1\,, \quad [V_1, V_1] \subset V_0\,.$$

- We will consider the semigroup $S_E^{(N)} = \{\lambda_0, \ldots, \lambda_{N+1}\}$

$$\lambda_i \cdot \lambda_j = \begin{cases} \lambda_{i+j} & \text{if} \quad i+j \leqslant N \\ \lambda_{N+1} & \text{if} \quad i+j > N \end{cases}\,, \qquad \lambda_{N+1} \text{ is the zero of } S_E^{(N)}$$

Non-relativistic expansions of Poincaré

- $S_E^{(N)}$ Admits the following subset decomposition

$$
\begin{aligned}
S_0^{(N)} &= s_0^{(N)} \cup \{\lambda_{N+1}\} \\
S_1^{(N)} &= s_1^{(N)} \cup \{\lambda_{N+1}\}
\end{aligned}
$$

$$
\begin{aligned}
s_0^{(N)} &= \{\lambda_{2m}\}\,,\ m = 0, \ldots, \left[\tfrac{N}{2}\right] \\
s_1^{(N)} &= \{\lambda_{2m+1}\}\,,\ m = 0, \ldots, \left[\tfrac{N-1}{2}\right]
\end{aligned}
$$

- Resonant with V_0 and V_1 in the sense that

$$
S_0^{(N)} \cdot S_0^{(N)} \subset S_0^{(N)}, \quad S_0^{(N)} \cdot S_1^{(N)} \subset S_1^{(N)}\,, \quad S_1^{(N)} \cdot S_1^{(N)} \subset S_0^{(N)}
$$

- This leads to a resonant and reduced algebra

$$
\{V_0 \times S_0^{(N)}\} \oplus \{V_1 \times S_1^{(N)}\} - \{\mathcal{P}_A \otimes \lambda_{N+1}\} \oplus \{\mathcal{J}_{AB} \otimes \lambda_{N+1}\}
$$

Non-relativistic expansions of Poincaré

- Case $S_E^{(1)}$

λ_2	λ_2	λ_2	λ_2
λ_1	λ_1	λ_2	λ_2
λ_0	λ_0	λ_1	λ_2
	λ_0	λ_1	λ_2

$$S_0^{(1)} = \{\lambda_0, \lambda_2\}$$
$$S_1^{(1)} = \{\lambda_1, \lambda_2\}$$

- Te expanded generators

$$J_{ab} = \lambda_0 \otimes \mathcal{J}_{ab}\,, \quad G_a = \lambda_1 \otimes \mathcal{J}_{0a}\,,$$
$$H = \lambda_0 \otimes \mathcal{P}_0\,, \quad P_a = \lambda_1 \otimes \mathcal{P}_a\,,$$

- satisfy the Galilei algebra in D dimensions

$$[G_a, H] = P_a\,,$$
$$[J_{ab}, P_c] = \delta_{c[b}P_{a]}\,,$$
$$[J_{ab}, G_c] = \delta_{c[b}G_{a]}\,,$$
$$[J_{ab}, J_{cd}] = \delta_{[a[c}J_{d]b]}$$

- This is known as the infinite speed of light limit of the Poincaré symmetry. This will be useful later to obtain a physical interpretation of λ_i.

Non-relativistic expansions of Poincaré

- Case $S_E^{(2)}$

λ_3	λ_3	λ_3	λ_3	λ_3
λ_2	λ_2	λ_3	λ_3	λ_3
λ_1	λ_1	λ_2	λ_3	λ_3
λ_0	λ_0	λ_1	λ_2	λ_3
	λ_0	λ_1	λ_2	λ_3

$$S_0^{(2)} = \{\lambda_0, \lambda_2, \lambda_3\}$$
$$S_1^{(2)} = \{\lambda_1, \lambda_3\}$$

- There are two new expanded generators

$$S_{ab} = \lambda_2 \otimes \mathcal{J}_{ab} \,, \qquad M = \lambda_2 \otimes \mathcal{P}_0 \,.$$

- The expanded algebra is given by the previous commutators plus

$$[G_a, P_b] = \delta_{ab} M \,, \quad [G_a, G_b] = S_{ab} \,, \quad [J_{ab}, S_{cd}] = \delta_{[a[c} S_{d]b]}$$

- In 2+1 dimensions this reduces to the Extended Bargmann algebra.
- Further extensions introduce more generators
- The $S_E^{(3)}$ case has been used to define an action for Newton-Cartan gravity

[Hansen,Hartong,Obers(2018)]

Non-relativistic expansions of Poincaré

- These algebras can be summarized in one infinite-dimensional algebra
- Consider the infinite semigroup [Peñafiel,Ravera(2017)]

$$S^{(\infty)} = \{\lambda_0, \lambda_1, \lambda_2, \dots\} , \qquad \lambda_i \cdot \lambda_j = \lambda_{i+j}$$

- In this we can consider the non-reduced expansion

$$\Big(\{\lambda_{2m}\} \times \{\mathcal{J}_{ab}, \mathcal{P}_0\} \Big) \oplus \Big(\{\lambda_{2m+1}\} \times \{\mathcal{J}_{0a}, \mathcal{P}_a\} \Big)$$

- Expanded generators

$$\begin{aligned} J_{ab}^{(m)} &= \lambda_{2m} \otimes \mathcal{J}_{ab} , \quad & B_a^{(m)} &= \lambda_{2m+1} \otimes \mathcal{J}_{0a} , \\ H^{(m)} &= \lambda_{2m} \otimes \mathcal{P}_0 , \quad & P_a^{(m)} &= \lambda_{2m+1} \otimes \mathcal{P}_a , \end{aligned}$$

- They satisfy the $\mathfrak{G}_\infty$ algebra [Hansen,Hartong,Obers(2019)] [Gomis,Kleinschmidt,Palmkvist(2019)]

$$\begin{aligned} [J_{ab}^{(m)}, P_c^{(n)}] &= \delta_{c[b} P_{a]}^{(m+n)} , \\ [J_{ab}^{(m)}, J_{cd}^{(n)}] &= \delta_{[a[c} J_{d]b]}^{(m+n)} , \\ [B_a^{(m)}, P_b^{(n)}] &= \delta_{ab} H^{(m+n+1)} , \end{aligned}$$

$$\begin{aligned} [B_a^{(m)}, H^{(n)}] &= P_a^{(m+n)} , \\ [J_{ab}^{(m)}, B_c^{(n)}] &= \delta_{c[b} B_{a]}^{(m+n)} , \\ [B_a^{(m)}, B_b^{(n)}] &= J_{ab}^{(m+n+1)} \end{aligned}$$

Physical interpretation

- This algebraic construction can be reproduced by expanding the gravitational gauge fields in powers of the speed of light c

$$\lambda_m \qquad \sim \; 1/c^m$$

- By considering a connection one-form $\mathcal{A}$ takes values on $\mathfrak{G}_\infty$

$$\begin{aligned}\mathcal{A} &= \sum_{m=0}^{\infty}\left(e^a_{(m)}P^{(m)}_a + \omega^a_{(m)}G^{(m)}_a + \tau_{(m)}H^{(m)} + \frac{1}{2}\omega^{ab}_{(m)}J^{(m)}_{ab}\right)\\ &= E^0\mathcal{P}_0 + E^a\mathcal{P}_a + \Omega^{0a}\mathcal{J}_{0a} + \frac{1}{2}\Omega^{ab}\mathcal{J}_{ab}\end{aligned}$$

- The Poincaré gauge fields are expanded in powers of the speed of light

$$\begin{aligned} E^0 &= \textstyle\sum_{m=0}^{\infty} c^{-2m+1}\tau_{(m)}, \qquad & E^a &= \textstyle\sum_{m=0}^{\infty} c^{-2m}e^a_{(m)}\\ \Omega^{0a} &= \textstyle\sum_{m=0}^{\infty} c^{-2m-1}\omega^a_{(m)}, \qquad & \Omega^{ab} &= \textstyle\sum_{m=0}^{\infty} c^{-2m}\omega^{ab}_{(m)}\end{aligned}$$

- This NR expansion of the gravitational fields allow to construct the post-Newtonian expansion of General Relativity and CS gravity

[Hansen,Hartong,Obers(2019)][Gomis,Kleinschmidt,Palmkvist,Salgado-Rebolledo(2019)]]

Adding Λ-corrections to Poincaré

- Inspired in the previous construction we look at the contraction of AdS that leads to the Poincaré algebra
- Consider the AdS algebra $\mathfrak{so}(D-1,2)$

$$[\mathbb{J}_{AB},\mathbb{J}_{CD}]=\eta_{[A[C}\mathbb{J}_{D]B]}\,,\quad [\mathbb{J}_{AB},\mathbb{P}_{C}]=\eta_{C[B}\mathbb{P}_{A]}\,,\quad [\mathbb{P}_{A},\mathbb{P}_{B}]=\mathbb{J}_{AB}$$

- As before, we introduce a parameter σ and the following rescaling of the generators

$$\mathcal{P}_A=\sigma\mathbb{P}_A,\qquad \mathcal{J}_{AB}=\mathbb{J}_{AB}$$

- In the limit $\sigma\to 0$ we find the Poincaré algebra

$$[\mathcal{J}_{AB},\mathcal{J}_{CD}]=\eta_{[A[C}\mathcal{J}_{D]B]}\,,\quad [\mathcal{J}_{AB},\mathcal{P}_{C}]=\eta_{C[B}\mathcal{P}_{A]}$$

- The parameter σ is related to the cosmological constant Λ or the AdS radius ℓ

$$\sigma=1/\ell\sim\sqrt{|\Lambda|}$$

Adding Λ-corrections to Poincaré

- We can now generalize this contraction using the expansion mechanism
- The subspace separation for AdS in this case reads

$$V_0 = \{\mathbb{J}_{AB}\}, \qquad V_1 = \{\mathbb{P}_A\}.$$

- satisfies a $\mathbb{Z}_2$-graded structure
- We will consider again the semigroup $S_E^{(N)} = \{\lambda_0, \ldots, \lambda_{N+1}\}$

$$\lambda_i \cdot \lambda_j = \begin{cases} \lambda_{i+j} & \text{if} \quad i+j \leqslant N \\ \lambda_{N+1} & \text{if} \quad i+j > N \end{cases}$$

- and the resonant and reduced algebra

$$\{V_0 \times S_0^{(N)}\} \oplus \{V_1 \times S_1^{(N)}\} - \{\mathbb{P}_A \otimes \lambda_{N+1}\} \oplus \{\mathbb{J}_{AB} \otimes \lambda_{N+1}\}$$

Adding Λ-corrections to Poincaré

- Case $S_E^{(1)}$

λ_2	λ_2	λ_2	λ_2
λ_1	λ_1	λ_2	λ_2
λ_0	λ_0	λ_1	λ_2
	λ_0	λ_1	λ_2

$$S_0^{(1)} = \{\lambda_0, \lambda_2\}$$
$$S_1^{(1)} = \{\lambda_1, \lambda_2\}$$

- Expanded generators

$$\mathcal{J}_{AB} = \lambda_0 \otimes \mathbb{J}_{AB}\,, \quad \mathcal{P}_A = \lambda_1 \otimes \mathbb{P}_A$$

- They satisfy the Poincaré algebra in D dimensions $\mathfrak{iso}(D-1,1)$

$$[\mathcal{J}_{AB}, \mathcal{J}_{CD}] = \eta_{[A[C}\mathcal{J}_{D]B]}\,, \quad [\mathcal{J}_{AB}, \mathcal{P}_C] = \eta_{C[B}\mathcal{P}_{A]}$$

- As expected, the simplest example of the expansion reproduces the contraction previously discussed

Adding Λ-corrections to Poincaré

- Case $S_E^{(2)}$

λ_3	λ_3	λ_3	λ_3	λ_3
λ_2	λ_2	λ_3	λ_3	λ_3
λ_1	λ_1	λ_2	λ_3	λ_3
λ_0	λ_0	λ_1	λ_2	λ_3
	λ_0	λ_1	λ_2	λ_3

$$S_0^{(2)} = \{\lambda_0, \lambda_2, \lambda_3\}$$
$$S_1^{(2)} = \{\lambda_1, \lambda_3\}$$

- We introduce a new expanded generator

$$\mathcal{Z}_{AB} = \lambda_2 \otimes \mathbb{J}_{AB}$$

- The expanded algebra is the Maxwell algebra

$$[\mathcal{J}_{AB}, \mathcal{J}_{CD}] = \eta_{[A[C}\mathcal{J}_{D]B]}\ , \qquad [\mathcal{J}_{AB}, \mathcal{Z}_{CD}] = \eta_{[A[C}\mathcal{Z}_{D]B]}\ ,$$
$$[\mathcal{J}_{AB}, \mathcal{P}_C] = \eta_{C[B}\mathcal{P}_{A]}\ , \qquad [\mathcal{P}_A, \mathcal{P}_B] = \mathcal{Z}_{AB}$$

- As in the non-relativistic case, we can generate infinite-dimensional algebras

Adding Λ-corrections to Poincaré

- Consider the infinite semigroup [Peñafiel,Ravera(2017)]

$$S^{(\infty)} = \{\lambda_0, \lambda_1, \lambda_2, \dots\} \,, \qquad \lambda_i \cdot \lambda_j = \lambda_{i+j}$$

- In this we can consider the non-reduced expansion

$$\Big(\{\lambda_{2m}\} \times \{\mathbb{J}_{AB}\}\Big) \oplus \Big(\{\lambda_{2m+1}\} \times \{\mathbb{P}_A\}\Big)$$

- and define

$$\mathcal{J}^{(m)}_{AB} = \lambda_{2m} \otimes \mathbb{J}_{AB} \,, \quad \mathcal{P}^{(m)}_{A} = \lambda_{2m+1} \otimes \mathbb{P}_A$$

- These generators satisfy the Poincaré$_\infty$ algebra [Gomis,Kleinschmidt,Roest,Salgado-Rebolledo(2020)]

$$[\mathcal{J}^{(m)}_{AB}, \mathcal{J}^{(n)}_{CD}] = \delta_{[A[C}\mathcal{J}^{(m+n)}_{D]B]} \,, \qquad [\mathcal{P}^{(m)}_{A}, \mathcal{P}^{(n)}_{B}] = \mathcal{J}^{(m+n+1)}_{AB} \,,$$

$$[\mathcal{J}^{(m)}_{AB}, \mathcal{P}^{(n)}_{C}] = \delta_{C[B}\mathcal{P}^{(m+n)}_{A]}$$

- This symmetry has been used to add curvature corrections to particle models on Minkowski space
- Finite truncations of these algebras have been studied in the context of (super)gravity and have been dubbed B_n-algebras or Generalized Poincaré algebras

[Concha,Peñafiel,Rodríguez,Salgado(2015)]

Physical interpretation

- We proceed as in the non-relativistic example
- Since the leading order of this expansion reproduces the vanishing cosmological constant limit of AdS, we identify

$$\lambda_m \sim 1/\ell^m$$

- In order to expand a gravity theory, we define a connection taking values on Poincaré$_\infty$

$$\begin{aligned}\mathcal{A} &= \sum_{m=0}^{\infty}\left(e^A_{(m)}\mathcal{P}^{(m)}_A + \frac{1}{2}\omega^{AB}_{(m)}\mathcal{J}^{(m)}_{AB}\right)\\ &= E^A\mathbb{P}_A + \frac{1}{2}\Omega^{AB}\mathbb{J}_{AB}\end{aligned}$$

- The AdS gauge fields are expanded in powers of the cosmological constant or the AdS radius

$$E^A = \sum_{m=0}^{\infty}\ell^{-2m-1}e^A_{(m)}, \qquad \Omega^{AB} = \sum_{m=0}^{\infty}\ell^{-2m}\omega^{AB}_{(m)}$$

- We will use this prescription to expand AdS gravity in three dimensions

ℓ-Expansion of AdS gravity

- We start with the Chern-Simons action invariant under $\mathfrak{so}(2,2)$

$$S_{CS} = \int CS_{AdS} = \int \left\langle \mathcal{A} \wedge d\mathcal{A} + \frac{2}{3}\mathcal{A} \wedge \mathcal{A} \wedge \mathcal{A} \right\rangle, \quad \mathcal{A} = E^A \mathbb{P}_A + \Omega^A \mathbb{J}_A$$

$$\langle \mathbb{J}_A, \mathbb{P}_B \rangle = \mu\, \eta_{AB}, \quad \langle \mathbb{J}_A, \mathbb{J}_B \rangle = \nu\, \eta_{AB}, \quad \langle \mathbb{P}_A, \mathbb{P}_B \rangle = \nu\, \eta_{AB}$$

- where we have dualized the spin connection and the Lorentz generators

$$\Omega^{AB} = -\epsilon^{ABC}\Omega_C, \qquad \mathbb{J}_{AB} = \epsilon_{ABC}\mathbb{J}_C$$

- We consider the simplest case of ℓ-expansion

$$E^A = \ell^{-1} e^A, \qquad \Omega^A = \omega^A + \ell^{-2}\sigma^A$$

- A similar expansion is considered at the level of the invariant tensor constants

$$\mu = \ell^{-1}\alpha, \qquad \nu = \beta + \ell^{-2}\gamma$$

Recovering Maxwellian gravity

- The resulting action takes the form

$$S_{CS} = \beta \int CS(\omega) + \ell^{-2} \int CS_{Maxwell} + O(\ell^{-4})$$

- where $CS_{Maxwell}$ is the Chern-Simons form associated to the Maxwell algebra in 2+1 dimensions [Szabo,Salgado,Valdivia(2014)]

$$\int CS_{Maxwell} = \int \left[2\alpha e^A R_A + \gamma CS(\omega) + \gamma(2\sigma^A R_A + e^A T_A) \right]$$

- The procedure can be generalized to higher orders in $\ell^{-1} = \sqrt{\Lambda}$
- The resulting Chern-Simons actions will be invariant under Generalized Poincaré algebras
- This provides a systematic way to add ℓ-corrections to gravitational theories obtained from gauging the Poincaré symmetry

Recovering Maxwellian gravity

- The result can be generalized to $D = 4$ by considering the MacDowell-Mansouri action

$$S_{MM} = \frac{\kappa}{2}\int \epsilon^{ABCD}\mathcal{F}^{AB}\wedge\mathcal{F}^{CD}$$

- AdS curvature along $\mathbb{J}_{AB}$

$$\mathcal{F}^{AB} = R^{AB}(\Omega) + E^A\wedge E^B, \qquad R^{AB}(\Omega) = d\Omega^{AB} + \Omega^A{}_C\wedge\Omega^{CB}$$

- As before, up to order ℓ^{-2}, the cosmological constant expansion reads

$$E^A = \ell^{-1}e^A, \qquad \Omega^{AB} = \omega^{AB} + \ell^{-2}\sigma^{AB}$$

- By also introducing $\kappa = \ell^2\kappa_0 + \kappa_1$ we find the Maxwellian gravity action proposed by de Azcarraga, Kamimura and Lukierski as the subleading term in the expansion

$$S_{MM} = \kappa_0\int L_{EH} + \ell^{-2}\int L_{Maxwell} + O(\ell^{-4})$$

$$L_{EH} = \epsilon_{ABCD}R^{AB}(\omega)\wedge e^C\wedge e^D, \qquad L_C = \epsilon_{ABCD}e^A\wedge e^B\wedge e^C\wedge e^D$$

$$L_{Maxwell} = \kappa_1 L_{EH} + \frac{\kappa_0}{2}L_C + \kappa_0\epsilon_{ABCD}\left(D\sigma^{AB}\wedge e^C\wedge e^D + \frac{1}{2}D\sigma^{AB}\wedge D\sigma^{CD}\right)$$

A lower-dimensional test: Maxwell BF gravity in $D = 2$

- In two dimensions, antisymmetric Lorentz tensors can be dualized to scalars:

$$J = -\frac{1}{2}\epsilon^{AB} J_{AB}, \qquad Z = -\frac{1}{2}\epsilon^{AB} Z_{AB}.$$

- The Maxwell algebra becomes

$$[J, P_A] = \epsilon_A{}^B P_B, \qquad [P_A, P_B] = \epsilon_{AB} Z.$$

- The Maxwell-valued connection and curvature are

$$A = e^A P_A + \omega J + \sigma Z, \qquad F = T^A P_A + RJ + F_Z Z,$$

with

$$T^A = de^A - \epsilon^A{}_B \omega \wedge e^B, \qquad R = d\omega, \qquad F_Z = d\sigma + \epsilon_{AB} e^A \wedge e^B.$$

ℓ-expansion of the AdS_2 BF theory

- Start from the AdS_2 BF action

$$S_{\mathrm{BF}} = \int \langle B, F \rangle, \qquad B = B^A P_A + BJ.$$

- Use the minimal truncation of the cosmological expansion:

$$E^A = \ell^{-1} e^A, \qquad \Omega = \omega + \ell^{-2}\sigma,$$

together with

$$B^A = \ell^{-1} b^A, \qquad B = \varphi + \ell^{-2}\chi, \qquad \mu = \alpha + \ell^{-2}\beta.$$

- Then the Lagrangian organizes as

$$L = \alpha L_0 + \frac{1}{\ell^2}\left(\alpha L_1 + \beta L_0\right) + O(\ell^{-4}),$$

where

$$L_0 = \varphi R, \qquad L_1 = \chi R + \varepsilon\, b_A T^A + \varphi\left(d\sigma + \frac{\varepsilon}{2}\epsilon_{AB} e^A \wedge e^B\right).$$

The subleading term reproduces the Maxwell BF sector; in second-order form it is related to the CGHS (Callan-Giddings-Harvey-Strominger) model.

Summary and future directions

- Infinite Lie algebra expansions with the semigroup S_∞ can applied to physical systems can reproduce expansions on a small physical parameter.
- The procedure of adding $1/c$ post-Galilean or post-Minkowskian corrections to non-relativistic gravity is captured by the S_∞-expansions.
- The results be generalized to the case of an expansion in powers of the cosmological constant.
- Maxwellian gravity in $D = 3$ and $D = 4$ dimensions can be obtained as the subleading term in the cosmological constant expansion of AdS gravity.
- *Work in progress*: Computation of the asymptotic symmetries of CS invariant under generalized Poincaré algebras using the ℓ-expansion and the results known for the Maxwell case. [Concha,Merino,Miskovic,Rodríguez,Salgado-Rebolledo,Valdivia(2018)]

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