Renewed hope in new general relativity with boost violation

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Metric teleparallelism assumes both the curvature of the affine connection and non-metricity to vanish. Within this framework it is possible to construct an action which is quadratic in torsion and parity-preserving (known as new general relativity). For decades it was thought that only two subcases of this theories would be ghost-free. One being equivalent with general relativity, and the other has strongly coupled degrees of freedom. Recently, the perturbation analysis was revisited and it has been confirmed in three independent works that there is another theory (named type 3 NGR) which is ghost-free. It also has strongly coupled degrees of freedom in Minkowski, while all degrees of freedom has been found to be ghost-free around cosmological backgrounds. The theory propagate tensor, vector, and scalar degrees of freedom, and I will demonstrate the scales for which they appear.

Metric teleparallel equivalent to GR

General relativity described by the Einstein-Hilbert action assumes a connection that is torsion-free (symmetric in its last two indices) and metric compatible:

$$\overset{L.C}{\nabla}_{\rho} g_{\mu\nu} = 0, \quad \overset{L.C}{\Gamma}{}^{\rho}{}_{[\mu\nu]} = 0. \tag{1}$$

The Einstein-Hilbert action is then given by

$$S_{\rm EH} = \frac{1}{2\kappa}\int \mathrm{d}^4x\sqrt{-g}R(\overset{L.C}{\Gamma}). \tag{2}$$

Definition: The *teleparallel condition* assumes that the Riemannian curvature tensor of an affine connection vanishies:

$$R^{\alpha}{}_{\beta\mu\nu} = 2\partial_{[\mu}\Gamma^{\alpha}{}_{|\beta|\nu]} + 2\Gamma^{\alpha}{}_{\lambda[\mu}\Gamma^{\lambda}{}_{|\beta|\nu} = 0. \tag{3}$$

Definition: *Metric teleparallelism* assumes the teleparalell condition together with *vanishing non-metricity*, i.e.

$$\nabla_{\rho} g_{\mu\nu} = 0. \tag{4}$$

With vanishing non-metricity we can define the *contortion* tensor $K^{\rho}{}_{\mu\nu}$ via the relation

$$\Gamma^{\rho}{}_{\mu\nu} = \overset{L.C}{\Gamma}{}^{\rho}{}_{\mu\nu} + K^{\rho}{}_{\mu\nu}, \tag{5}$$

it can after some calculations be shown that **under the teleparallel condition**

$$S_{\rm EH} = \frac{1}{2\kappa}\int \mathrm{d}^4x\sqrt{-g}\left(2K^{\mu}{}_{\tau[\rho}K^{\tau}{}_{|\nu|\sigma]} - 2\overset{L.C}{\nabla}_{[\mu}K^{\mu\nu}{}_{\nu]}\right). \tag{6}$$

The last term is a boundary term and can the *teleparallel equivalent to general relativity* differs from the Einstein-Hilbert action by exactly this term:

$$S_{\rm TEGR} = \frac{1}{2\kappa}\int \mathrm{d}^4x\sqrt{-g}2K^{\mu}{}_{\tau[\rho}K^{\tau}{}_{|\nu|\sigma]}. \tag{7}$$

New general relativity and perturbations around Minkowski

Torsion can be written as

$$T^{A}{}_{\mu\nu} = 2\partial_{[\mu}e^{A}{}_{\nu]}, \qquad K^{\rho}{}_{\mu\nu} = \frac{1}{2}g^{\rho\lambda}\left(T_{\mu\lambda\nu} + T_{\nu\lambda\mu} + T_{\lambda\mu\nu}\right). \tag{8}$$

New general relativity was recently shown to have a larger ghost-free class than previously stated[2] and is given by the Lagrangian

$$S_{\rm NGR} = \frac{1}{2\kappa}\int \mathrm{d}^4x\sqrt{-g}\left(c_1T_{\mu\nu\rho}T^{\mu\nu\rho} + c_2T_{\mu\nu\rho}T^{\rho\mu\nu} - T_{\mu}T^{\mu}\right). \tag{9}$$

TEGR is obtained in the special case $c_1 = -\frac{1}{4}$, $c_2 = \frac{1}{2}$, $c_3 = 1$. For the weaker condition $2c_1 + c_2 = 0$ (known as NGR of Type 3) we know from the Hamiltonian analysis[3] that the theory propagates 5 degrees of freedom non-linearly. Around Minkowski we get the Lagrangian on the form

$$\mathcal{L}_{\text{Type 3}} = \mathcal{L}_{hh} + \mathcal{L}_{hb} + \cancelto{0}{\mathcal{L}_{bb}}, \tag{10}$$

where we have split the perturbation into symmetric h and antisymmetric b perturbations. A detailed analysis[1,2] reveals that Type 3 has one scalar mode propagating in addition to the tensor mode of GR. Still two more degrees of freedom are needed to match the 5 degrees of freedom of the full nonlinear theory.

Cosmological perturbations

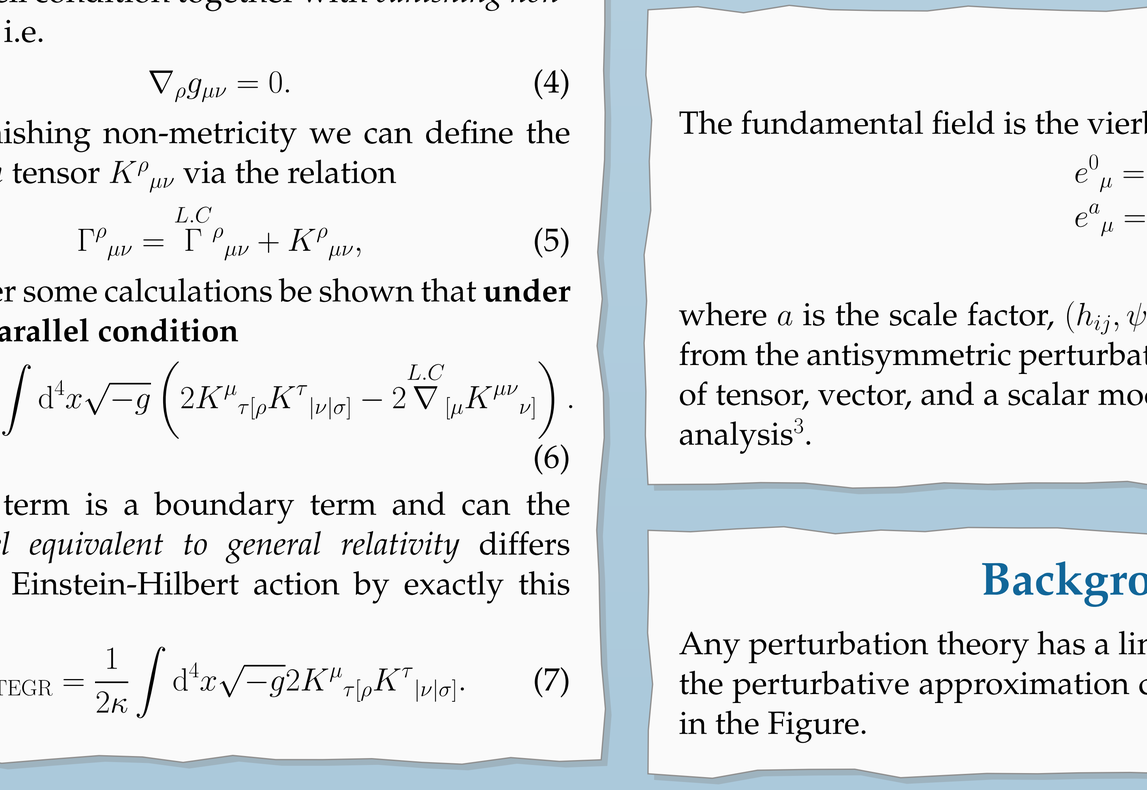
The fundamental field is the vierbein which yields the following cosmological perturbations:

$$\begin{aligned} e^{0}{}_{\mu} &= (1+\psi)\ \delta^{0}{}_{\mu} + a\ (\partial_i\alpha + \alpha_i)\ \delta^{i}{}_{\mu}\,, \\ e^{a}{}_{\mu} &= a\ (1-\varphi)\ \delta^{a}{}_{\mu} + \delta^{ai}\left(\partial_iF + G_i\right)\delta^{0}{}_{\mu} \\ &\quad + a\,\delta^{ai}\ \left[h_{ji} + \partial_j\partial_iB + C_{ij} + \epsilon_{ijk}\,\delta^{kl}\ \left(\partial_l\tilde{\sigma} + \tilde{V}_l\right)\right]\ \delta^{j}{}_{\mu}\,, \end{aligned} \tag{11}$$

where a is the scale factor, $(h_{ij}, \psi, \phi, F, B, G_i, C_i)$ comes from the symmetric perturbations, and $(\alpha, \alpha_i, \tilde{\sigma}, \tilde{V}_i)$ comes from the antisymmetric perturbations. We find for this perturbation 5 degrees of freedom $(h_{ij}, \alpha, \alpha_i)$ which consists of tensor, vector, and a scalar mode. This matches the nonlinear degrees of freedom known from the Hamiltonian analysis[3].

Background-Hierarchy bounds/Strong coupling

Any perturbation theory has a limited scale of validity in energy level. Once the system exceeds this energy scale, the perturbative approximation ceases to be reliable. Scan the QR code and see section V for the details resulting in the Figure.



Background-hierarchy bounds in NGR Type 3 cosmology

The figure on the left demonstrates the conditions of the parameters and scales for when perturbation theory is valid in cosmology. Region III is limited from the right by the ghost-free condition $x < 4$ and from the left by $x = 2$ is TEGR, and $x < 2$ also has ghosts.

References

1. "Gauge-invariant cosmological perturbations in Type 3 New General Relativity and background-hierarchy bounds´´ K. Tomanari, S. Bahamonde and D.B.
2. "Revisiting stability in new general relativity´´ S. Bahamonde, D.B, K. Dialektopoulos and A. Hell
3. "Degrees of freedom of new general relativity: Type 2, type 3, type 5, and type 8´´ K. Tomanari and D.B.

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