Locally Rotationally Symmetric Spacetimes in Einstein Cartan Theory and Their Classification

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Torsion: Spin, Neutron Stars and Cosmology

Torsion: A strong candidate for extending General Relativity (GR).

Why? A Fundamental Improvement!

Inclusion of quantum effects, namely spin, into gravitational physics, achieved by ECSK theory of gravity, by coupling the spin of the matter fluid to torsion.

Role in Astrophysics and Cosmology

- Neutron Stars: Spin stabilises the core $\rightarrow$ Theoretical Models with Torsion
- Explains late-time acceleration of the Universe
- Introduces inflationary periods at very early times
- Avoids Big Bang singularity
- Averts gravitational collapse to a singularity?

Question

Can we develop a theoretical framework which is:

- coordinate-independent
- model-independent

to allow us to study effects of torsion collectively in various settings, using physically descriptive variables?

Covariant Formalism

METHODOLOGY: $(1+1+2)$ Covariant Formalism!

$$g_{ab} = -u_a u_b + h_{ab} = -u_a u_b + e_a e_b + N_{ab} \ , \qquad \eta_{ab} = \eta_{[ab]} = \eta_{dabc} e^c u^d \ ,$$
$$u_a u^a = -1 \ , \qquad e_a e^a = 1 \ , \qquad u_a e^a = 0 \ , \qquad u^a N_{ab} = e^a N_{ab} = 0 \ .$$

- Decompose the quantities describing the spacetime along congruences and the (locally) orthogonal hypersurface.
- Allows one to describe the spacetime via physically meaningful variables while avoiding confinement to a specific coordinate system.

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CLASSIFICATION of such spacetimes for non-classical fluid.
NEW EXAMPLES, some with no corrective in GR!

Local Rotational Symmetry

Exploit symmetries:

Most of the Cosmological & Astrophysical Spacetimes possess local rotational symmetry.

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Implication in the Covariant Decomposition:
Vectors & Tensors on 2-surface N_{ab} vanish: $(u^a \nabla_a u_b) N^b_c = 0$. (but $N^{ab} \nabla_a u_b \neq 0$)

Therefore, spacetime is described by a set of scalars.

Manifold

MANIFOLD $\mathcal{M}$: 4−dimensional (pseudo-)Riemannian manifold equipped with a metric tensor g_{ab} and a metric compatible covariant derivative (∇, $\nabla g = 0$).

TORSION: $\nabla_a \nabla_b \psi - \nabla_b \nabla_a \psi = -T^k{}_{ab} \nabla_k \psi$

RIEMANN CURVATURE: $\nabla_a \nabla_b X_c - \nabla_b \nabla_a X_c + T^k{}_{ab} \nabla_k X_c = -R_{ab}{}^d{}_c X_d$
and its decomposition:

$$\text{Reimann Tensor } R_{abcd} \rightarrow \underbrace{\text{Ricci Tensor } R_{ab}}_{\text{Trace part}} + \underbrace{\text{Weyl Tensor } C_{abcd}}_{\text{Traceless part}}$$

ECSK Theory and Weyssenhoff Fluid

Einstein-Cartan-Sciama-Kibble Theory of Gravity

$$R_{ab} - \frac{1}{2}g_{ab}R = S_{ab}\ , \qquad {T^c}_{ab} + 2\delta^c_{[a}{T^d}_{b]d} = 2{\Delta^c}_{ab}$$

where, S_{ab} is canonical energy momentum tensor, ${\Delta^c}_{ab}$ is hypermomentum tensor.

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$$L_{ab} = -L_{ba} \qquad\qquad u^a L_{ab} = 0$$

where u^a is fluid 4−velocity.

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L_{ab} is postulated to be related to fluid quantities:

$$\Delta^{c}{}_{ab} = u^{c} L_{ab} \ , \qquad S_{ab} = \mu u_a u_b + p(g_{ab} + u_a u_b) - 2u_a L_{bn} u^{m} \nabla_{m} u^{n}$$

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Note:
S_{ab} is not symmetric.
$T^{k}{}_{ak} = 0 \implies T^{c}{}_{ab} = 2\Delta^{c}{}_{ab} = 2u^{c}L_{ab}$

$1+1+2$ Covariant Decomposition and Covariant Variables

REMINDER: $g_{ab} = -u_a u_b + h_{ab} = -u_a u_b + e_a e_b + N_{ab}$, & $\eta_{ab} = \eta_{[ab]} = \eta_{dabc} e^c u^d$.

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$$\nabla_a u_b = -\mathcal{A} u_a e_b + \left(\frac{\Theta}{3} + \Sigma\right) e_a e_b + \left(\frac{\Theta}{3} - \frac{\Sigma}{2}\right) N_{ab} + \Omega \eta_{ab}$$

Spacetime Quantities & decomposition to Covariant Variables

- $\nabla_a u_b$, $\nabla_a e_b \rightarrow \{\mathcal{A}, \Theta, \Sigma, \Omega, \phi, \xi\}$ — Kinematic Variables
- $C_{abcd} \rightarrow \{\mathcal{E}, \tilde{\mathcal{E}}, \mathcal{H}_r, \mathcal{H}_t, \overline{\mathcal{H}}_r, \overline{\mathcal{H}}_t\}$ — Weyl Variables
- R_{ab} and $T^a{}_{bc}$: Replaced with S_{ab} and $\Delta^a{}_{bc}$ using field equations.

Matter

$$S_{ab} = \mu u_a u_b + p(e_a e_b + N_{ab}) + \Pi(e_a e_b - N_{ab}/2) + 2Q e_{(a} u_{b)} + 2\tilde{Q} e_{[a} u_{b]} + M\eta_{ab}$$

For u^a is parallel to the fluid 4-velocity of the Weyssenhoff fluid.

$$L_{ab} = \tau\eta_{ab} \implies S_{ab} = \mu u_a u_b + p(e_a e_b + N_{ab}) \ , \quad \Delta^a{}_{bc} = \tau u^a \eta_{bc} = \frac{1}{2} T^a{}_{bc}$$

$S_{ab}, \Delta^a{}_{bc} \to \{\mu, p, \tau\}$. Matter variables

Why Covariant Variables? And not 10 components of a generic metric?

- While not frame independent, we are still coordinate independent.
- Effective use of properties of manifold and fluid to eliminate variables, by choosing (e^a, u^a). In fact, imposing required behaviour is easier.
  - Exploiting LRS and choosing e^a: eliminated vectors and tensors on N_{ab}.
  - Exploiting properties of Weyssenhoff fluid and choosing u^a: eliminated 11 out of 14 covariant variables from decomposition of S_{ab} and $\Delta^a{}_{bc}$.

While for metric tensor, typically, one can only remove dependence of components on certain coordinates of a suitable coordinate system.

Governing Equations: Covariant Equations

$$\text{Following Equations} \xrightarrow[\text{combinations of } \{u_a, e_a, N_{ab}, \eta_{ab}\}]{\text{projections along different}} \text{Covariant Equations}$$

Ricci Identity: $R_{abcd} n^d = \nabla_a \nabla_b n_c - \nabla_b \nabla_a n_c + T^k{}_{ab} \nabla_k n_c \;, \qquad n^a = \{u^a, e^a\}$

Alg. Bianchi Identity: $R_{[ab}{}^n{}_{c]} = \nabla_{[a} T^n{}_{bc]} - T^k{}_{[ab} T^n{}_{c]k}$

Der. Bianchi Identity: $\nabla_{[a} R_{bc]}{}^k{}_l = T^n{}_{[ab} R_{c]n}{}^k{}_l$

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DER. BIANCHI IDENTITY: $\nabla_{[a} R_{bc]}{}^k{}_l = T^n{}_{[ab} R_{c]n}{}^k{}_l$

Result: Set of 20 equations among scalars, at most first-order derivative.

CONSTRAINTS
Algebraic (4)

EVOLUTION EQN
Time derivative
$u^a \nabla_a \psi = \dot{\psi}$

PROPAGATION EQN
Radial derivative
$e^a \nabla_a \psi = \hat{\psi}$

And the 6 CONSISTENCY CONDITIONS: $(\Omega - \tau)\dot{\psi} = \xi \hat{\psi}$

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Constraints	Evolution Eqn	Propagation Eqn
Algebraic (4)	Time derivative	Radial derivative
	$u^a \nabla_a \psi = \dot{\psi}$	$e^a \nabla_a \psi = \hat{\psi}$

And the 6 Consistency Conditions: $(\Omega - \tau)\dot{\psi} = \xi \hat{\psi}$

Allows study of geometry of the spacetime, like Raychaudhuri equation (actually completes the set of equations), and does not impose fluid equation (yet).

Highlights in Equations

Relation between Conformal Structure $\leftrightarrow$ Torsion & kinematic variables revealed via algebraic Bianchi identity

$$\mathcal{H}_r = 3\xi\Sigma - (2\mathcal{A} - \phi)\,\Omega + 2\mathcal{A}\tau \qquad \overline{\mathcal{H}}_t = 2\mathcal{A}\tau - \mathcal{H}_r \qquad 2\overline{\mathcal{H}}_r = -\overline{\mathcal{H}}_t - \mathcal{H}_t$$

$$\hat{\tau} = -\tau\phi + \frac{\mathcal{H}_r}{2} + \frac{\mathcal{H}_t}{2}$$

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Equations for quantities which determine admittance of foliation remain same.

General Relativity	Torsion Spacetimes
$\dot{\Omega} = \xi\,(\mathcal{A} - \phi)$	$\dot{\Omega} - \dot{\tau} = \xi\,(\mathcal{A} - \phi)$
$\hat{\Omega} = \Omega\,(\mathcal{A} - \phi)$	$\hat{\Omega} - \hat{\tau} = (\Omega - \tau)\,(\mathcal{A} - \phi)$
$\dot{\xi} = \xi\left(2\Sigma - \frac{\Theta}{3}\right)$	$\dot{\xi} = \xi\left(2\Sigma - \frac{\Theta}{3}\right)$
$\hat{\xi} = \Omega\left(2\Sigma - \frac{\Theta}{3}\right)$	$\hat{\xi} = (\Omega - \tau)\left(2\Sigma - \frac{\Theta}{3}\right)$

Classification of TLRS spacetimes - Scheme

GR: Three Possible Solutions of $\Omega\xi = 0$, derived for perfect fluid and $\mu + p > 0$ (NEC).

$\Omega \neq 0, \xi = 0$ $\quad\quad$ $\Omega = 0, \xi = 0$ $\quad\quad$ $\Omega = 0, \xi \neq 0$

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Torsion Spacetimes: In general, $(\Omega - \tau)\xi \neq 0$

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Key Variables: $(\Omega - \tau)$ and ξ (vanishing or not)
Key Features (maintained from GR):

- Separation of Classes: Local values of these quantities lead to global implications.
- Foliations: Spacetime belonging to each class has a characteristic foliation.

Benefits: Simplification of the governing equations, Developing a taxonomy of exact solutions

Classification for TLRS Spacetimes - Classes

1. TLRS Class I: $\xi = 0$, $\Omega - \tau \neq 0 \implies \dot{\psi} = 0 = \Sigma = \Theta$
   Two subclasses:
   1. TLRS Class IA: $\mu + p(\mu) \neq 0$
   2. TLRS Class IB: $\tau \neq 0$
      Cannot be reduced to General Relativity. Notice absence on condition on matter fluid.

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2. TLRS Class II: $(\Omega - \tau), \xi = 0$
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3. TLRS Class III: $\Omega - \tau = 0,\ \xi \neq 0 \implies \hat{\psi} = 0 = \phi = \mathcal{A}$.
   Must satisfy $\mu + p(\mu) \neq 0$, $\tau = 0$.
   Reduces to General Relativity (LRS class III)

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4. TLRS Class IV: $(\Omega - \tau), \xi \neq 0$ or $\mu + p = 0,\ \tau = 0,\ (\Omega - \tau)^2 + \xi^2 \neq 0$
   is true (at a point, over a patch or globally).
   Or simply: spacetimes which do not belong to TLRS classes I, II, or III.

Examples of TLRS class I

- Properties: $\xi = \Sigma = \Theta = 0$ & $\dot{\psi} = 0$, $\Omega - \tau \neq 0$.
  No expansion or distortion. Admits foliation orthogonal to e^a. Stationary spacetimes, with the fluid-4 velocity u^a being killing field. No notion of global time.
- Very few differential equations remain!!!

$$\hat{\Omega} - \hat{\tau} = (\Omega - \tau)(\mathcal{A} - \phi) \qquad \hat{\tau} = -\tau\phi + \frac{\mathcal{H}_r}{2} + \frac{\mathcal{H}_t}{2}$$

$$\hat{\phi} = -\frac{1}{2}\phi^2 + \mathcal{A}\phi + 2\Omega^2 - \mu - p \qquad \hat{\mathcal{A}} = -\mathcal{A}\left(\mathcal{A} + \phi\right) - 2\Omega^2 + \frac{\mu}{2} + \frac{3}{2}p$$

$$\hat{p} = -\mathcal{A}\left(\mu + p\right) - 2\tau\overline{\mathcal{H}}_r$$

- Conformal Structure is determined algebraically, including Electric part, except $\mathcal{H}_t$.
- To close the system: $p = p(\mu)$ & $\tau = \tau(\mu)$... or impose a behaviour on the magnetic Weyl scalars.
  Remember: either $\mu + p \neq 0$ or $\tau \neq 0$ at every point $x \in \mathcal{M}$.

Two Possible Cases: $\phi(\mu + p - 2\tau\Omega) = 0$

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Case with $\phi = 0$ solution:

$$\mathcal{A} = \mathcal{H}_r = \mathcal{H}_t = \overline{\mathcal{H}}_r = \overline{\mathcal{H}}_t = 0 \qquad\qquad \Omega^2 = p = -\frac{3}{2}\mathcal{E} = \mu \ .$$

- μ determines all variables with same relations as in GR, except...
- τ is completely independent and non-zero.
- Effects on the geodesics? Closed timelike curves? Novel observables?

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Case with $\phi \neq 0$ solution:

$$\tau^2 = 3\frac{(\mu + p)^2}{7\mu + 5p} \qquad p > -\frac{7}{5}\mu$$

belongs to both TLRS class IA and IB.

Gravitationally Silent Torsional Spacetime

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Case 1 $\Omega \neq 0$: Leads to Case 1 of Gödel's Universe with torsion.

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CASE 1 $\Omega \neq 0$: Leads to Case 1 of Gödel's Universe with torsion.

CASE 2 $\Omega = 0$:

- Weyl tensor vanishes, and fluid follows geodesics: $\mathcal{E} = \mathcal{A} = 0$.
- Dark Radiation Equation of state: $p = -\frac{\mu}{3}$, with $\hat{p} = 0 \implies \mu = \mu_0, p = p_0$
- For area-radius coordinate r ($e_a = \frac{2}{r\phi} dr$):

$$\tau = \frac{\tau_0}{r^2} \qquad\qquad \phi^2 = 4p_0 + \frac{2c}{r^2}$$

Note: The fluid properties are obtained as an outcome!!!

Canonical Vacuum

Metric EM Tensor:

$s^{ab} = s^{ba} = \frac{\delta \mathcal{L}_m}{\delta g_{ab}} = S^{ab} + (\nabla_c + T^m{}_{cm})(\Delta^{abc} - \Delta^{cab} - \Delta^{bca})$

$s_{ab} \to \{\overline{\mu}, \overline{p}, \overline{\Pi}, \overline{Q}\}$

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Canonical Vacuum: $S_{ab} = 0 \implies s^{ab} = -4\Omega\tau u^a u^b - 2\Omega\tau N^{ab}$

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Canonical Vacuum: $S_{ab} = 0 \implies s^{ab} = -4\Omega\tau u^a u^b - 2\Omega\tau N^{ab}$

- Matter fluid is NOT absent!!!
- Ricci Tensor & "geometrical" energy density and pressure vanish.
- Weyl tensor &, thus, Riemann curvature tensor remain non-vanishing.

For TLRS class IB: $\tau \neq 0$, we obtain closed set of equations for $\hat{\Omega}, \hat{\tau}, \hat{\phi}, \hat{\mathcal{A}}$ which with fluid properties ($\overline{p}_r = \overline{p} + \overline{\Pi},\ \overline{p}_t = \overline{p} - \frac{1}{2}\overline{\Pi}$):

$$\overline{p} = \frac{\overline{\mu}}{3}\ , \qquad \overline{\Pi} = -\frac{\overline{\mu}}{3} \qquad \textbf{or} \qquad \overline{p}_r = 0\ , \qquad \overline{p}_t = \frac{\overline{\mu}}{2}\ .$$

Similar to Florides solution?

Conclusions

- A covariant framework to study spacetimes of astrophysical and cosmological relevance, in a coordinate and model independent way.
- Benefits: better understanding conformal structure, differential structure, ease of discovering/imposing physical properties via covariant variables.
- Classification of TLRS spacetimes based on admittance of foliation.
- Shown effectiveness of the framework in studying physically interesting spacetimes via examples.

THANK YOU FOR YOUR ATTENTION

Refer to Phys. Rev. D 112, 064075 for details.