One Lagrangian to rule them all

Work (2601.19734) with P. Álvarez and C. Quinzacara

Fernando Izaurieta

Universidad San Sebastián, Concepción, Chile

GeomGravX, University of Tartu
June 30th, 2026


“One Ring to rule them all, One Ring to find them,
One Ring to bring them all and in the darkness bind them.”

One Lagrangian to rule them all

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How not to build a Theory of Everything

Work (2601.19734) with P. Álvarez and C. Quinzacara

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Feynman’s Game: *Look at it as a Martian!*
Let us look at all known physics as if we don’t know anything!

$$\mathcal{L}^{(4)}_{\text{almost everything}} = \frac{1}{\kappa_4}\left(\frac{1}{2}R - \Lambda\right) - \frac{1}{4}F^{A}{}_{\mu\nu}F_{A}{}^{\mu\nu} + i\bar{\psi}\gamma^{\mu}\mathrm{D}_{\mu}\psi - \Phi\bar{\psi}y\psi - \mathrm{D}_{\mu}\Phi^{\dagger}\mathrm{D}^{\mu}\Phi - V(\Phi)$$

Does it look as a coherent whole?

Neither Unification nor QG $\rightarrow$

Neither Unification nor QG → Should you print that into a T-shirt?

$$\mathcal{L}^{(4)}_{\text{almost everything}} = \frac{1}{\kappa_4}\left(\frac{1}{2}R - \Lambda\right) - \frac{1}{4}F^{A}{}_{\mu\nu}F_{A}{}^{\mu\nu} + i\bar{\psi}\gamma^{\mu}\mathrm{D}_{\mu}\psi - \Phi\bar{\psi}y\psi - \mathrm{D}_{\mu}\Phi^{\dagger}\mathrm{D}^{\mu}\Phi - V(\Phi)$$

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Quadratic
in gauge curvature

Linear contraction
in gauge curvature

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Fields in multiplet reps.

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Quadratic in gauge curvature

Scalar 🤔

Linear contraction in gauge curvature

A constant?

Fields in multiplet reps.

$$\mathcal{L}^{(4)}_{\text{almost everything}} = \frac{1}{\kappa_4}\left(\frac{1}{2}R - \Lambda\right) - \frac{1}{4}F^{A}{}_{\mu\nu}F_{A}{}^{\mu\nu} + i\bar{\psi}\gamma^{\mu}\mathrm{D}_{\mu}\psi - \Phi\bar{\psi}y\psi - \mathrm{D}_{\mu}\Phi^{\dagger}\mathrm{D}^{\mu}\Phi - V(\Phi)$$

Quadratic in gauge curvature

Scalar

Challenge (a dare):

Is it possible to write all of that as a coherent whole in $d = 4$?

$$\mathcal{L}_{\text{YM}}^{(4)} = -\frac{1}{2}\langle \mathbb{F} \wedge *\mathbb{F} \rangle$$

Unify everything in a single gauge principle? What about Yang-Mills gravity?

$$\mathbb{A} = \frac{1}{2}\omega^{ab}\mathbb{J}_{ab} + \frac{1}{l}e^{a}\mathbb{P}_{a}.$$

$$\mathbb{F} = \mathrm{d}\mathbb{A} + \frac{1}{2}\left[\mathbb{A}, \mathbb{A}\right],$$
$$= \frac{1}{2}\left(R^{ab} + \frac{1}{l^2}e^a \wedge e^b\right)\mathbb{J}_{ab} + \frac{1}{l}T^a\mathbb{P}_a,$$

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{2}\langle \mathbb{F} \wedge *\mathbb{F} \rangle$$

$$* : \Omega^{p}\left(M^{(d)}\right) \rightarrow \Omega^{d-p}\left(M^{(d)}\right)$$

$$* = *(e)$$

First crack: The gauge invariance is spoiled!

$$\delta e = -\mathrm{D}\lambda \implies \delta(*) \neq 0.$$

It almost works!

$$\mathcal{L}_{\text{YM}}(\omega, e) = \frac{1}{4l^2}\epsilon_{abcd}R^{ab}\wedge e^c\wedge e^d + \frac{1}{8l^4}\epsilon_{abcd}e^a\wedge e^b\wedge e^c\wedge e^d + \frac{1}{4}R^{ab}\wedge *R_{ab} - \frac{1}{2l^2}T^a\wedge *T_a.$$

It almost works!

$$\mathcal{L}_{\mathrm{YM}}\left(\omega,e\right)=\frac{1}{4l^{2}}\epsilon_{abcd}R^{ab}\wedge e^{c}\wedge e^{d}+\frac{1}{8l^{4}}\epsilon_{abcd}e^{a}\wedge e^{b}\wedge e^{c}\wedge e^{d}+\frac{1}{4}R^{ab}\wedge *R_{ab}-\frac{1}{2l^{2}}T^{a}\wedge *T_{a}.$$

$$\mathrm{d}*\mathrm{d}\neq 0.$$

What about fermions?

"Plurality is not to be posited without necessity."

—William of Ockham (1287 – 1347)

Ockham's razor-*mania*:

$$\mathbb{A} = \underbrace{\frac{1}{2}\omega^{ab}\mathbb{J}_{ab} + \frac{1}{l}e^{a}\mathbb{P}_{a}}_{\text{AdS}} + \cdots + \underbrace{A^{A}\mathbb{T}_{A}}_{\text{Internal}} + \underbrace{\frac{1}{2\sqrt{l}}\left(\bar{\mathbb{Q}}\Psi_{(3/2)} - \bar{\Psi}_{(3/2)}\mathbb{Q}\right)}_{\text{Super}}.$$

What if the only ingredient is a gauge connection in $d = 4$?

Clifford rep., $\mathrm{osp}\,(n|4)$ or $\mathrm{su}\,(2,2|n)$

$$\mathcal{L}_{\mathrm{YM}}\big|_{\psi} = -\frac{1}{2}\left\langle \mathbb{F} \wedge *\mathbb{F} \right\rangle ?$$

Add the constraint

$$\Psi_{(3/2)} = \not{e}\psi, \qquad \not{e} = e^a\Gamma_a.$$

It almost works!

$$\mathcal{L}_{\mathrm{YM}}\big|_{\psi} = \sqrt{|g|}\mathrm{d}x^4\left(\frac{3!}{l^{11/2}}\left[\frac{1}{2}i\left(\bar{\psi}\Gamma^m\nabla_m\psi - \nabla_m\bar{\psi}\Gamma^m\psi\right) - \frac{3!}{l^{5/2}}\bar{\psi}\psi\right] + \right.$$
$$\left. + \frac{1}{l^3}\nabla_m\bar{\psi}\Gamma^{mn}\nabla_n\psi + \frac{3}{l^3}\nabla_m\bar{\psi}\nabla^m\psi\right) + \text{torsion terms.}$$

Solutions?

MacDowell-Mansouri gravity

$$* \longrightarrow \star$$

$$\star\left(e_a \wedge e_b\right) = *\left(e_a \wedge e_b\right) = \frac{1}{2}\epsilon_{abcd}e^c \wedge e^d,$$

$$\star R_{ab} = \frac{1}{2}\epsilon_{abcd}R^{cd},$$

$$R^{ab} \wedge *R_{ab} \to R^{ab} \wedge \star R_{ab} = \frac{1}{2}\epsilon_{abcd}R^{ab} \wedge R^{cd}.$$

Does it work too well?

Unconventional SUSY (Super-MacDowell-Mansouri 2005.04178)

$$\mathcal{L}_{\text{USUSY}} = \langle \mathbb{F} \wedge \circledast \mathbb{F} \rangle \,.$$

Does it work too well?

Contents

What if the Yang-Mills term is only the first one from a series?

Ingredients:

1. $\mathbb{A} = \frac{1}{2}\omega^{ab}\mathbb{J}_{ab} + \frac{1}{l}e^{a}\mathbb{P}_{a} + A^{A}\mathbb{T}_{A} + \frac{3}{\sqrt{2l}}\bar{\mathbb{Q}}\not{e}\psi,$
2. $* : \Omega^{p} \to \Omega^{d-p},$
3. $\not{e} = e^{a}\Gamma_{a} = e^{a} = 2e^{a}\boldsymbol{P}_{a}$ (The e^{a} is already inside the Hodge, so it comes for free!

$$\mathcal{L}^{(4)} = \langle W_0 \wedge *W_0 \rangle + \langle W_1 \wedge *W_1 \rangle + \langle W_2 \wedge *W_2 \rangle + \cdots$$

$$W_n = W_n \left(\mathbb{F}, \not{\phi}\right)$$

Such series is not infinite in $d = 4$!

$$\mathcal{L}^{(4)} = \lambda_0 \langle \mathbb{F} \wedge *\mathbb{F} \rangle + \lambda_1 \langle \not{\phi} \wedge \mathbb{F} \wedge * (\mathbb{F} \wedge \not{\phi}) \rangle + \lambda_2 \left\langle \not{\phi}^2 \wedge \mathbb{F} \wedge * \left(\mathbb{F} \wedge \not{\phi}^2 \right) \right\rangle + \\ + \lambda_{[1]} \langle [\not{\phi}, \mathbb{F}] \wedge * [\mathbb{F}, \not{\phi}] \rangle + \lambda_{[2]} \left\langle \left[\not{\phi}^2, \mathbb{F} \right] \wedge * \left[\mathbb{F}, \not{\phi}^2 \right] \right\rangle .$$

Such series is not infinite in $d = 4$!

$$\mathcal{L} = \lambda_0 \langle \mathbb{F} \wedge *\mathbb{F} \rangle + \lambda_1 \langle \not{\phi} \wedge \mathbb{F} \wedge * (\mathbb{F} \wedge \not{\phi}) \rangle + \lambda_2 \left\langle \not{\phi}^2 \wedge \mathbb{F} \wedge * \left(\mathbb{F} \wedge \not{\phi}^2 \right) \right\rangle +$$
$$+ \lambda_{[1]} \langle [\not{\phi}, \mathbb{F}] \wedge * [\mathbb{F}, \not{\phi}] \rangle + \lambda_{[2]} \left\langle \left[\not{\phi}^2, \mathbb{F} \right] \wedge * \left[\mathbb{F}, \not{\phi}^2 \right] \right\rangle .$$

Unless you choose the coeficients in a particular way!

$$\mathcal{L}_{\mathrm{LL}}^{(4)} = \frac{1}{2}\Big[-\langle \mathbb{F} \wedge *\mathbb{F}\rangle + \langle \not\phi \wedge \mathbb{F} \wedge *\left(\mathbb{F} \wedge \not\phi\right)\rangle + \frac{1}{4}\left\langle \not\phi^2 \wedge \mathbb{F} \wedge *\left(\mathbb{F} \wedge \not\phi^2\right)\right\rangle + \\ + \frac{2}{3}\left(\langle [\not\phi, \mathbb{F}] \wedge *[\mathbb{F}, \not\phi]\rangle + \frac{1}{4}\left\langle \left[\not\phi^2, \mathbb{F}\right] \wedge *\left[\mathbb{F}, \not\phi^2\right]\right\rangle \right)\Big]$$

$$\mathcal{L}^{(4)}_{\mathrm{LL}} = \frac{1}{8}\epsilon_{abcd}\left(R^{ab} + \frac{1}{l^2}e^a \wedge e^b\right) \wedge \left(R^{cd} + \frac{1}{l^2}e^c \wedge e^d\right) +$$

$$-\frac{i}{l^3}\left(\nabla\bar{\psi}^I \wedge \not{e} + \frac{1}{2l^{5/2}}\bar{\psi}^I \not{e}^2\right) \wedge \left(\not{e}\Gamma_5 \wedge \nabla\psi_I + \frac{1}{2l^{5/2}}\not{e}^2\Gamma_5\psi_I\right) +$$

$$-\frac{1}{2}\left\langle \mathbb{F}_{\mathrm{int}} \wedge *\mathbb{F}_{\mathrm{int}} \right\rangle + \text{curvature} \wedge \text{matter} + \text{torsion} \wedge \text{matter}$$

$$+ \text{torsion} \wedge *\text{torsion-terms.} + \text{torsion} \wedge *\text{matter}$$

$$\mathrm{d}^2 = 0.$$

Contents

A path to the One Ring or to Mount Doom?

$$\mathbb{A} = \underbrace{\frac{1}{2}\omega^{ab}\mathbb{J}_{ab} + \frac{1}{l}e^{a}\mathbb{P}_{a}}_{\text{AdS}} + \cdots + \underbrace{A^{A}\mathbb{T}_{A}}_{\text{Internal}} + \underbrace{\frac{1}{2\sqrt{l}}\left(\bar{\mathbb{Q}}\Psi_{(3/2)} - \bar{\Psi}_{(3/2)}\mathbb{Q}\right)}_{\text{Super}}.$$

We got surprisingly far with surprinsingly little!
Why this naive “unification” works so well?

Do we need only a single ingredient for a unified theory of everything in $d = 4$?

?

1. Focus on Ostragradsky stability in grav sector?
   (Reproduce 80's Sezgin + van Nieuwenhuizen!)

2. Worth it? Nonminimal couplings?

3. SUSY-Wave operators?

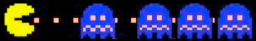
- Is there an underlying math structure (invariant tensor)? Symmetry? Why torsion is always in the way?

- Higgs as paraparticles? Work in progress.

- Torsion + Non minimal couplings = Dark Matter and Dark Energy?

$$\mathcal{L}_{\mathrm{LL}}^{(4)} = \frac{1}{2}\Big[-\langle \mathbb{F} \wedge * \mathbb{F}\rangle + \langle \not{\phi} \wedge \mathbb{F} \wedge *(\mathbb{F} \wedge \not{\phi})\rangle + \frac{1}{4}\Big\langle \not{\phi}^2 \wedge \mathbb{F} \wedge *\Big(\mathbb{F} \wedge \not{\phi}^2\Big)\Big\rangle + \frac{2}{3}\Big(\langle [\not{\phi}, \mathbb{F}] \wedge * [\mathbb{F}, \not{\phi}]\rangle + \frac{1}{4}\Big\langle \Big[\not{\phi}^2, \mathbb{F}\Big] \wedge * \Big[\mathbb{F}, \not{\phi}^2\Big]\Big\rangle\Big)\Big]$$

?



$$\mathcal{L}_{\text{LL}}^{(4)} \ldots ?$$

Lucía Izaurieta

Lizzie Álvarez

Aitäh!

$$
\begin{aligned}
\mathcal{L}_{\mathrm{LL}}^{(4)} = \frac{1}{2}\Bigg(&\frac{1}{3!}\left[\epsilon_{abcd}F^{ab}\wedge F^{cd}+\frac{1}{2}\epsilon_{abcd}F^{ab}\wedge\tilde{F}^{cd}\right]+\\
&+\frac{1}{12}e^{m}\wedge \mathrm{D}T_{m}\wedge *\left(e^{p}\wedge \mathrm{D}T_{p}\right)-\frac{7}{3}\left(\mathrm{D}T^{b}\wedge e^{a}\right)\wedge *\left(\mathrm{D}T_{a}\wedge e_{b}\right)+\\
&+\frac{2}{l^{2}}\left[2T^{b}\wedge *T_{b}-T^{(3)}\wedge *T^{(3)}-T^{b}\wedge e^{a}\wedge *\left(T_{a}\wedge e_{b}\right)\right]\Bigg),\\
F^{ab} = R^{ab}&+\frac{1}{l^{2}}e^{a}\wedge e^{b}.
\end{aligned}
$$

$$\mathcal{L}_{\mathrm{LL}}^{(4)} = \sqrt{|\mathrm{g}|}\mathrm{d}x^4 \left(\frac{1}{3!l^2}\mathcal{R} + \frac{1}{l^4} + \frac{1}{4!} \left(\mathcal{R}^2 - 4\mathcal{R}^a{}_b\mathcal{R}^b{}_a + R^{ab}{}_{cd}R^{cd}{}_{ab} \right) + \right.$$
$$+ \frac{1}{3}\mathcal{R}^{ab} \left(\mathcal{R}_{ab} - \mathcal{R}_{ba} \right) - \frac{1}{3!}R^{abcd}R_{abcd} + \frac{1}{3}R^{abcd}R_{acbd} +$$
$$\left. + \frac{1}{3l^2}T_{abc}T^{bac} + \frac{1}{3l^2}\mathcal{T}_a\mathcal{T}^a \right)$$