Charged black holes in quadratic gravity

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Tartu 2026

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older works in vacuum with: A. Pravdová, J. Podolský, R. Švarc

Based on

- G. Turner, V. Pravda, A. Pravdová, *On vacuum and charged asymptotically (A)dS black holes in quadratic gravity*, Phys. Rev. D 113 (2026) 2, 024040
- V. Pravda, A. Pravdova, G. Turner, *Charged black holes in quadratic gravity*, Phys. Rev. D 110 (2024) 4, 044069

older works:

- J. Podolský, R. Švarc, V.P., A. Pravdová, *Explicit black hole solutions in higher-derivative gravity*, Phys. Rev. D **98**, 021502 (2018)
- R. Švarc, J. Podolský, V.P., A. Pravdová, *Exact black holes in quadratic gravity with any cosmological constant*, Phys. Rev. Lett. 121, 231104 (2018)
- J. Podolský, R. Švarc, V.P., A. Pravdová, *Black holes and other exact spherical solutions in Quadratic Gravity*, Phys. Rev. D 101, 024027 (2020)
- V.P., A. Pravdová, R. Švarc, J. Podolský, *Black holes and other spherical solutions in quadratic gravity with a cosmological constant*, Phys. Rev. D 103, 064049 (2021)

Prelude

In 2015 [Lu, Perkins, Pope, Stelle, Phys. Rev. Lett. 114, 171601 (2015)] have shown using numerical methods, that in (four-dimensional) quadratic gravity, there are two static, spherically symmetric vacuum black hole solutions - Schwarzschild and non-Schwarzschild.....

QG field equations in spherically symmetric coordinates

[Lu, Perkins, Pope, Stelle, Phys. Rev. D 92, 2015]

$B = h$, $A = 1/f$, rr-component of the original field equations reads in the $\Lambda = 0$ case:

$$
\begin{aligned}
&8r^3A^2B^2B^{(3)}\left(r(\alpha-3\beta)B' - 2(\alpha+6\beta)B\right)\\
&-4r^2AB^2A''\left(r^2(\alpha-3\beta)B'^2 - 4r(\alpha+6\beta)BB' + 4(\alpha-12\beta)B^2\right)\\
&-4r^4(\alpha-3\beta)A^2B^2B''^2\\
&-4r^2ABB''\Big(2rBA'\left(r(\alpha-3\beta)B' - 2(\alpha+6\beta)B\right)\\
&\quad+A\left(3r^2(\alpha-3\beta)B'^2 - 12r(\alpha+3\beta)BB' + 8(\alpha+6\beta)B^2\right)\Big)\\
&+7r^2B^2A'^2\left(r^2(\alpha-3\beta)B'^2 - 4r(\alpha+6\beta)BB' + 4(\alpha-12\beta)B^2\right)\\
&+2r^2ABA'B'\left(3r^2(\alpha-3\beta)B'^2 - 4r(2\alpha+3\beta)BB' + 4(\alpha+24\beta)B^2\right)\\
&+24A^3B^3\left(\gamma r^3B' + B\left(\gamma r^2 - 12\beta\right)\right)\\
&+A^2\Big(7r^4(\alpha-3\beta)B'^4 - 4r^3(5\alpha+12\beta)BB'^3\\
&\quad-4r^2(\alpha-48\beta)B^2B'^2 + 32r(\alpha+6\beta)B^3B' - 16(\alpha-21\beta)B^4\Big)\\
&+8A^4B^4\left(2\alpha-6\beta-3\gamma r^2\right) = 0,
\end{aligned}
$$

We suspected that employing some of our previous results a substantial simplification could be achieved and indeed we arrived to

QG field equations in conformal to Kundt coordinates

$$\Omega\Omega'' - 2\Omega'^2 = \frac{1}{3}k\,\mathcal{B}_1\mathcal{H}^{-1}$$

$$\Omega\Omega'\mathcal{H}' + 3\Omega'^2\mathcal{H} + \Omega^2 - \Lambda\Omega^4 = \frac{1}{3}k\,\mathcal{B}_2$$

where

$$\mathcal{B}_1 \equiv \mathcal{H}\mathcal{H}''''$$

$$\mathcal{B}_2 \equiv \mathcal{H}'\mathcal{H}''' - \frac{1}{2}\mathcal{H}''^2 + 2$$

Quadratic gravity in n dimensions

Action

$$S = \int \mathrm{d}^n x \sqrt{-g}\left(\frac{1}{\kappa}\left(R - 2\Lambda_0\right) + \alpha R^2 + \beta R_{ab}^2 + \gamma\left(R_{abcd}^2 - 4R_{ab}^2 + R^2\right)\right)$$

$\Rightarrow$ field equations [Gullu, Tekin, Phys. Rev. D, 2009]

$$\frac{1}{\kappa}\left(R_{ab} - \tfrac{1}{2}Rg_{ab} + \Lambda g_{ab}\right) + 2\alpha R\left(R_{ab} - \tfrac{1}{4}Rg_{ab}\right) + (2\alpha+\beta)\left(g_{ab}\nabla^c\nabla_c - \nabla_a\nabla_b\right)R$$
$$+2\gamma\left(RR_{ab} - 2R_{acbd}R^{cd} + R_{acde}R_b{}^{cde} - 2R_{ac}R_b{}^c - \tfrac{1}{4}g_{ab}\left(R_{cdef}^2 - 4R_{cd}^2 + R^2\right)\right)$$
$$+\beta\nabla^c\nabla_c\left(R_{ab} - \tfrac{1}{2}Rg_{ab}\right) + 2\beta\left(R_{acbd} - \tfrac{1}{4}g_{ab}R_{cd}\right)R^{cd} = 0.$$

For **Einstein spacetimes** $R_{ab} = \frac{2\Lambda}{n-2} g_{ab}$ the quadratic gravity field equations reduce to [Málek, V.P., Phys. Rev. D, 2011]

$$\mathcal{B} g_{ab} - \gamma \left(C_a{}^{cde} C_{bcde} - \frac{1}{4} g_{ab} C^{cdef} C_{cdef} \right) = 0, \tag{1}$$

where

$$\mathcal{B} = \frac{\Lambda - \Lambda_0}{2\kappa} + \Lambda^2 \left(\frac{(n-4)}{(n-2)^2} (n\alpha + \beta) + \frac{(n-3)(n-4)}{(n-2)(n-1)} \gamma \right). \tag{2}$$

- **For $\mathbf{n = 4}$**, the field equations reduce to $\mathcal{B} = 0 \leftrightarrow \Lambda = \Lambda_0$ and thus **all Einstein spacetimes are solutions to QG**. Obviously, this includes Schwarzschild.
- **For $\mathbf{n > 4}$**, Eq. (1) is nontrivial and (2) is a quadratic equation for Λ. Thus **most Einstein spacetimes are not vacuum solutions to QG**. **Exceptions:** all Weyl type N Einstein spacetimes, universal spacetimes of Weyl types III and II.
- **Non-Einstein** solutions to QG: e.g. the new non-Schwarzschild black hole

Quadratic gravity in four dimensions

Action

$$S = \int \mathrm{d}^4x \sqrt{-g} \left(\gamma \left(R - 2\Lambda \right) - \alpha C_{abcd} C^{abcd} + \beta R^2 \right), \quad \gamma = 1/\kappa$$

Field equations

$$\gamma \left(R_{ab} - \tfrac{1}{2} R g_{ab} + \Lambda g_{ab} \right) - 4\alpha B_{ab} + 2\beta \left(R_{ab} - \tfrac{1}{4} R g_{ab} + g_{ab} \Box - \nabla_b \nabla_a \right) R = 0$$

Bach tensor:

$$B_{ab} \equiv \left(\nabla^c \nabla^d + \tfrac{1}{2} R^{cd} \right) C_{acbd}$$

$$B_{ab} = \tfrac{1}{2} \Box R_{ab} - \tfrac{1}{6} \left(\nabla_a \nabla_b + \tfrac{1}{2} g_{ab} \Box \right) R - \tfrac{1}{3} R R_{ab} + R_{acbd} R^{cd} + \tfrac{1}{4} \left(\tfrac{1}{3} R^2 - R_{cd} R^{cd} \right) g_{ab}$$

$$B_{ba} = B_{ab}, \; B^a{}_a = 0, \; B^{ab}{}_{;b} = 0,$$

only in 4D: $\tilde{B}_{ab} = \Omega^{-2} B_{ab}$ under conformal tr. $\tilde{g}_{ab} = \Omega^2 g_{ab}$

Spacetimes with vanishing Bach tensor

- For Einstein spaces Bach tensor vanishes identically:

$$R_{ab} = \frac{R}{4} g_{ab} \implies B_{ab} = 0$$

- Due to $\tilde{B}_{ab} = \Omega^{-2} B_{ab}$ Bach tensor also vanishes for conformally Einstein spaces.
- Note that not conformally–Einstein spacetimes with $B_{ab} = 0$ also exist [Liu, Lu, Pope, Vazquez-Poritz, CQG, 2013]

In four dimensions all Einstein spaces are vacuum solutions to quadratic gravity.

Field equations for constant Ricci scalar

Static and spherically symmetric asymp. flat. solutions to QG with a horizon have $R = 0$ [Lu et al 2015].

Field equations for constant Ricci scalar

$R = \text{const}$:

trace(FE): $\Rightarrow R = 4\Lambda$

Field equations: $(\gamma + 8\beta\Lambda)(R_{ab} - \Lambda g_{ab}) = 4\alpha B_{ab}$

- $R_{ab} = \Lambda g_{ab}$ Einstein spacetimes ($B_{ab} = 0$) $\Longrightarrow$ trivial solutions
- $\Lambda = -\gamma/(8\beta)$ and non-Einstein spacetimes with $B_{ab} = 0$
- general solution $B_{ab} \neq 0$

From now on, we assume $R = \text{const}$.

Field equations: $R_{ab} - \Lambda g_{ab} = 4kB_{ab}$, $k = \frac{\alpha}{\gamma + 8\beta\Lambda}$

Robinson-Trautman spacetimes are conformal to Kundt

Proposition [V.P., Pravdová, Podolský, Švarc, 2017],
see also [Robinson, Trautman, 1983]

All Robinson-Trautman spacetimes are conformal to Kundt.

$$\mathrm{d}\tilde{s}^2_{\mathrm{RT}} = \Omega^2(u,r,x)\,\mathrm{d}s^2_{\mathrm{Kundt}}$$

- Weyl type is preserved
- same PND ($\tilde{\ell}_a = \ell_a$)
  $\tilde{\theta} = \frac{1}{n-2}\,(\tilde{g}^{ab}\ell_b)_{;a} = \frac{\theta}{\Omega^2} + \frac{\boldsymbol{\ell}(\Omega)}{\Omega^3} = \frac{\Omega_{,r}}{\Omega^3},\ \tilde{\sigma}^2 = \frac{\sigma^2}{\Omega^4} = 0,$
  $\tilde{\omega}^2 = \frac{\omega^2}{\Omega^4} = 0$
- Ricci type may differ - highest b.w. comp. is generated by Ω
  $\tilde{R}_{ab}\,\tilde{\ell}^a\tilde{\ell}^b = \tilde{g}^{ac}\tilde{g}^{bd}\,\tilde{R}_{ab}\,\ell_c\ell_d = \Omega^{-4}\,\tilde{R}_{rr} =$
  $-(n-2)\,\Omega^{-6}(\Omega\Omega_{,rr} - 2\Omega^2_{,r})$

Static spherically symmetric spacetimes

Static spherically symmetric metric in spherical coordinates

$$\mathrm{d}s^2 = -h(\bar{r})\,\mathrm{d}t^2 + \frac{\mathrm{d}\bar{r}^2}{f(\bar{r})} + \bar{r}^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)$$

conformal to type D direct-product Kundt form

$$\mathrm{d}s^2 = \Omega^2\,\mathrm{d}s^2_{\text{Kundt}} = \Omega^2(r)\left[\,\mathcal{H}(r)\,\mathrm{d}u^2 - 2\,\mathrm{d}u\,\mathrm{d}r + \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\,\right]$$

coordinate transformation

$\bar{r} = \Omega(r)\,, \qquad t = u - \int \mathcal{H}(r)^{-1}\mathrm{d}r$

$h(\bar{r}) = -\Omega^2\,\mathcal{H}, \quad f(\bar{r}) = -\left(\frac{\Omega'}{\Omega}\right)^2\mathcal{H}$

The key idea: since in 4d quadratic gravity with $R =$const **all corrections to Einstein** equations are **proportional to Bach** tensor, which transforms nicely under conformal transformations, the resulting **field equations** should be **simpler in the conformal to Kundt form**.

Field equations in Kundt coordinates

Autonomous system

$\Omega\Omega'' - 2\Omega'^2 = \frac{1}{3} k\, \mathcal{B}_1 \mathcal{H}^{-1}$

$\Omega\Omega'\mathcal{H}' + 3\Omega'^2\mathcal{H} + \Omega^2 - \Lambda\Omega^4 = \frac{1}{3} k\, \mathcal{B}_2$

where

$\mathcal{B}_1 \equiv \mathcal{H}\mathcal{H}''''$

$\mathcal{B}_2 \equiv \mathcal{H}'\mathcal{H}''' - \frac{1}{2}\mathcal{H}''^2 + 2$

Bach curvature invariant

$$B_{ab}\, B^{ab} = \frac{1}{72}\, \Omega^{-8} \big[(\mathcal{B}_1)^2 + 2(\mathcal{B}_1 + \mathcal{B}_2)^2\big]$$

Solution with vanishing Bach $\to$ Schwarzschild

Vanishing B_{ab}

$\mathcal{B}_1 = 0 = \mathcal{B}_2 \to$ Einstein's theory

Solution: Schwarzschild-(A)dS

$$\Omega(r) = \bar{r} = -\frac{1}{r}, \quad \mathcal{H}(r) = \frac{\Lambda}{3} - r^2 - 2m\, r^3$$

$$f = h = 1 - 2m\, \bar{r}^{(-1)} - \frac{1}{3}\Lambda\, \bar{r}^2$$

Non-vanishing Bach - power series

Autonomous system - solution in power series in r expanded around *any* point r_0

$[n, p]$ solution around r_0

$$\Omega(r) = \Delta^n \sum_{i=0}^{\infty} a_i \, \Delta^i, \qquad \mathcal{H}(r) = \Delta^p \sum_{i=0}^{\infty} c_i \, \Delta^i$$

$\Delta \equiv r - r_0, \; n, \; p \in \mathbb{R}$

$[N, P]^\infty$ solutions for $r \to \infty$

$$\Omega(r) = r^N \sum_{i=0} A_i \, r^{-i}, \qquad \mathcal{H}(r) = r^P \sum_{i=0} C_i \, r^{-i}$$

$N, \; P \in \mathbb{R}$

Lowest orders constrain n and p

By substituting the series into the first field equation

$$\Omega\Omega'' - 2\Omega'^2 = \frac{1}{3} k\, \mathcal{H}''''$$

we obtain

$$\sum_{l=2n-2}^{\infty} \Delta^l \sum_{i=0}^{l-2n+2} a_i\, a_{l-i-2n+2}\, (l-i-n+2)(l-3i-3n+1)$$

$$= \frac{1}{3} k \sum_{l=p-4}^{\infty} \Delta^l\, c_{l-p+4}\, (l+4)(l+3)(l+2)(l+1)$$

For $p-4 < 2n-2$ this gives

$$c_0 p(p-1)(p-2)(p-3) = 0$$

and thus $p \in \{0, 1, 2, 3\}$. Further constraints follow from the second equation and from higher orders.

Table: The only admitted parameters[n, p]
(in the last column, $n \neq -1, -1/2$.)

n	0	0	1	-1	-1	0	0	< 0
p	1	0	0	2	0	2	≥ 2	$2n+2$
Λ	any	any	any	0	$\neq 0$	$\neq 0$	$\frac{3}{8k}$	$\frac{11n^2+6n+1}{1-4n^2}\frac{3}{8k}$

Possible values of $[n, p]/[N, P]$ for $\Lambda = 0$.

From now on we set $\Lambda = 0$.

- $[-1, 2]$ - Schwarzchild BH
- $[0, 1]$ - Schwa-Bach BH (around horizon, S)
- $[0, 0]$ - generic regular point, contains Schwa-Bach BH (S)
- $[1, 0]$ - Bachian singularity (around origin, NS)

- $[-1, 2]^\infty$ - regular origin - Bachian vacuum (NS)
- $[-1, 3]^\infty$ - Schwa-Bach BH II (around origin, singularity, S)

$[0,1]$ Schwarzschild-Bach BH around horizon

$$\Omega(r) = -\frac{1}{r} - \frac{b}{r_h}\sum_{i=1}^{\infty}\alpha_i\left(1-\frac{r}{r_h}\right)^i$$
$$\mathcal{H}(r) = (r-r_h)\left[\frac{r^2}{r_h} + 3b\,r_h\sum_{i=1}^{\infty}\gamma_i\left(\frac{r}{r_h}-1\right)^i\right]$$

- $b = 0$: Schwarzschild BH
- $b \neq 0$: Schwa-Bach BH

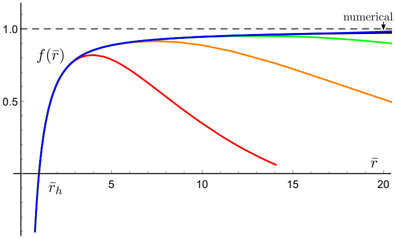
Recursive formula

$$\alpha_0 \equiv 0,\ \ \alpha_1 \equiv 1\,, \quad \gamma_1 = 1,\ \gamma_2 = \frac{1}{3}\left(4 - \frac{1}{2kr_h^2} + 3b\right)$$
$$\alpha_l = \frac{1}{l^2}\Big[\alpha_{l-1}(2l^2-2l+1) - \alpha_{l-2}(l-1)^2$$
$$-3\sum_{i=1}^{l}(-1)^i\gamma_i\,(1+b\,\alpha_{l-i})\big(l(l-i)+\tfrac{1}{6}i(i+1)\big)\Big]$$
$$\gamma_{l+1} = \frac{(-1)^l}{kr_h^2\,(l+2)(l+1)l(l-1)}\sum_{i=0}^{l-1}\left(\alpha_i + \alpha_{l-i}(1+b\,\alpha_i)\right)(l-i)(l-1-3i)$$
$l \geq 2$

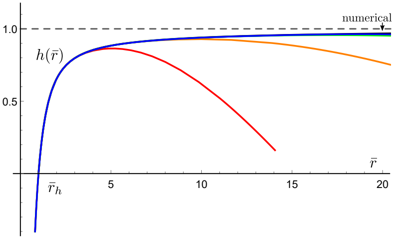
$[0, 1]$ Schwa-Bach BH

static spherically symmetric Schwa-Bach BHs

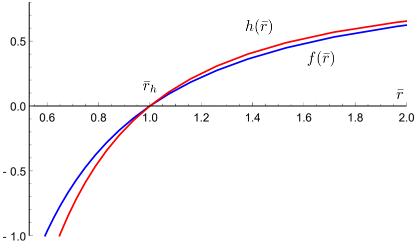
- $r = r_h$ - Killing horizon, $\mathcal{H}(r_h) = 0$
- b - Bach parameter ($b = 0 \Rightarrow$ Schwarzschild BH)
- For a given r_h the black hole is asymtotically flat only for $b = 0$ and for one additional particular value of $b = b_{AF} = b_{AF}(r_h)$ [Lu et al 2015] - numerics/series.



numerical
1.0
0.5
$f(\bar{r})$
$\bar{r}$
$\bar{r}_h$
5
10
15
20

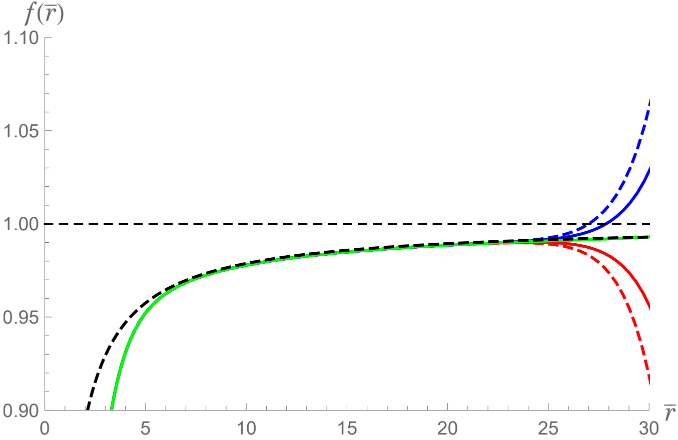


numerical
1.0
0.5
$h(\bar{r})$
$\bar{r}$
$\bar{r}_h$
5
10
15
20

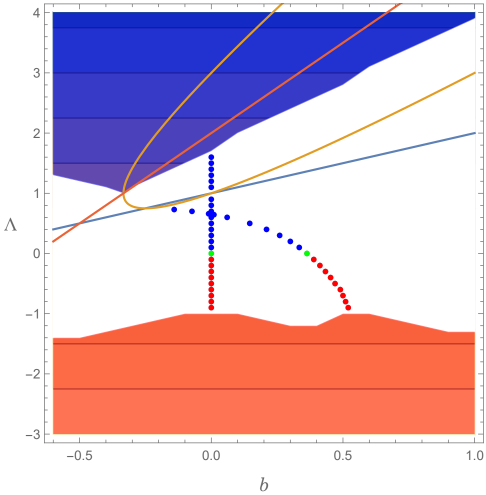


0.5
0.0
- 0.5
- 1.0
$h(\bar{r})$
$f(\bar{r})$
$\bar{r}_h$
$\bar{r}$
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0

Fine tuning the Bach parameter for $\Lambda = 0$



Fine tuning the Bach parameter with Λ



Adding charge

Autonomous system

$$\Omega\Omega'' - 2\Omega'^2 = \frac{1}{3} k\, \mathcal{B}_1 \mathcal{H}^{-1}$$

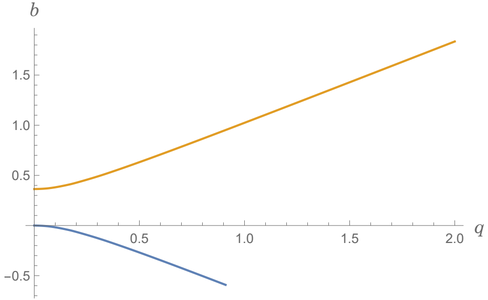
$$\Omega\Omega'\mathcal{H}' + 3\Omega'^2\mathcal{H} + \Omega^2 - \Lambda\Omega^4 = \frac{1}{3} k\, \mathcal{B}_2 + \frac{\kappa'}{2} q^2$$

where

$$\mathcal{B}_1 \equiv \mathcal{H}\mathcal{H}''''$$

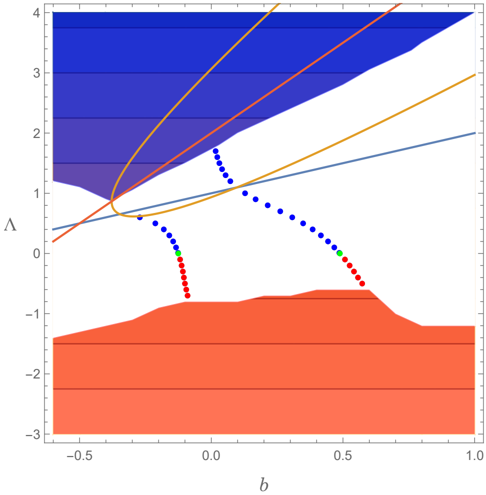
$$\mathcal{B}_2 \equiv \mathcal{H}'\mathcal{H}''' - \frac{1}{2}\mathcal{H}''^2 + 2$$

$$\mathrm{A} = \mathrm{q}\ \mathrm{r}$$



charged Schwarzschild in QG $\neq$ Reissner-Nordström!

charging Schwarzschild and Schwarzschild-Bach with $\Lambda \neq 0$



Conclusions

Static spherically symmetric solutions in QG

• Robinson-Trautman (e.g., static spher. sym. BHs) $\stackrel{\Omega}{\longleftrightarrow}$ Kundt

• Together with $R =$const this leads to a considerable simplification of the quadratic gravity field equations.

• Besides Schwarzschild, QG allows for another SSSAF black hole - Schwarzschild-Bach. Bach parameter has to be fine-tuned for asymptotic flatness.

• For sufficiently large $|\Lambda|$, fine-tuning not necessary and for a contionuous interval of values of b, the black hole is asymptotically (A)dS

• These black holes can be charged - charged Schwarzschild and charged Schwarzschild-Bach