# A GUP-inspired Quantum Black Hole: The Static And Rotating Cases, And Their Phenomenology

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Based on: *PRD* 111 (2025) 2, 024048 • *PRD* 111 (2025) 10, 104017 • *JCAP* 06 (2026) 081]

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# Outline

# GUP

Generalized uncertainty principle (GUP): based on modification of algebra

$$\Delta q \Delta p \geq \frac{1}{2} \left| \langle [q, p] \rangle \right| \Rightarrow \text{modification of RHS} \Leftrightarrow \text{modification of LHS}$$

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Our remedy: Analytic extension of interior to exterior

# Interior of BH in AB variables

Classical interior metric in AB variables:

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Classical interior Hamiltonian (constraint) in AB variables

$$H = -\frac{N\left(p_b, p_c\right)}{2G\gamma^2}\left[\left(b^2 + \gamma^2\right)\frac{p_b}{\sqrt{p_c}} + 2bc\sqrt{p_c}\right]$$

with algebra

$$\{b, p_b\} = 2G\gamma, \qquad \{c, p_c\} = G\gamma$$

# GUP-Modified Solution

1. GUP-modify the algebra

$$\{b, p_b\} = 2G\gamma\left(1 + \beta_b b^2\right), \qquad \{c, p_c\} = G\gamma\left(1 + \beta_c c^2\right)$$

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5. Correct classical and asymptotic limits?

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Incorrect Kretschmann fall off

$$K_{\mathrm{GUP}}(r \to \infty) \propto \frac{1}{r^4} \text{ and not } \frac{1}{r^6}$$

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$$\beta \to \bar{\beta}\,(p)$$

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Apply improved scheme to GUP

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In our model

$$\beta_b \to \bar{\beta}_b = \frac{\beta_b L_0^4}{p_b^2}, \qquad \beta_c \to \bar{\beta}_c = \frac{\beta_c L_0^4}{p_c^2}$$

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which means

$$\{b, p_b\} = 2G\gamma \left(1 + \frac{\beta_b L_0^4}{p_b^2} b^2\right), \qquad \{c, p_c\} = G\gamma \left(1 + \frac{\beta_c L_0^4}{p_c^2} c^2\right)$$

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4. Has correct limits:

$$\lim_{\beta_b,\beta_c\to 0} g_{\mu\nu}^{\text{GUP-Imp}} = g_{\mu\nu}^{\text{Schw}}$$

$$\lim_{r\to\infty} g_{\mu\nu}^{\text{GUP-Imp}} = \eta_{\mu\nu}$$

# GUP-Modified Improved Metric

Result: first-ever GUP BH metric derived [Fragomeno, Gingrich, Hergott, Rastgoo, Vienneau, **PRD 111 (2025) 2, 024048**; Gingrich, Rastgoo, **PRD 111 (2025) 10, 104017**; Fragomeno, Hergott, Rastgoo, Vienneau, **JCAP 06 (2026) 081**]

$$g_{00}^{\text{GUP-Imp}} = -\left(1 + \frac{Q_b}{r^2}\right)\left(1 + \frac{Q_c R_s^2}{4r^8}\right)^{-1/4}\left(1 - \frac{R_s}{\sqrt{r^2 + Q_b}}\right)$$

$$g_{11}^{\text{GUP-Imp}} = \left(1 + \frac{Q_c R_s^2}{4r^8}\right)^{1/4}\left(1 - \frac{R_s}{\sqrt{r^2 + Q_b}}\right)^{-1}$$

$$g_{22}^{\text{GUP-Imp}} = r^2\left(1 + \frac{Q_c R_s^2}{4r^8}\right)^{1/4}$$

with dimensionful quantum parameters

$$Q_b = -\,\text{sgn}(\beta_b)|\beta_b|\gamma^2 L_0^2, \qquad Q_c = -\,\text{sgn}(\beta_c)|\beta_c|\gamma^2 L_0^6$$

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Reality of the metric on $r \in [0, \infty)$ dictates

$$Q_b > 0 \Rightarrow \text{sgn}(\beta_b) = -1, \qquad Q_c > 0 \Rightarrow \text{sgn}(\beta_c) = -1$$

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$$g_{22}^{\text{GUP-Imp}} = r^2\left(1 + \frac{Q_c R_s^2}{4r^8}\right)^{1/4}$$

$Q_b$: controls the exterior/near horizon quantum effects

$Q_c$: controls deep interior/high curvature quantum effects

# Outline

# Limits and Singularity Resolution

Singularity is resolved

- Radius of 2-spheres

$$\sqrt{g_{22}^{\text{GUP-Imp}}} = \left(r^8 + \frac{Q_c R_s^2}{4}\right)^{\frac{1}{8}}, \qquad \sqrt{g_{22}^{\text{GUP-Imp}}}\Big|_{r=0} = \left(\frac{Q_c R_s^2}{4}\right)^{\frac{1}{8}}$$

- Kretschmann

$$K = \frac{\cdots}{\sqrt{r^8 + \frac{1}{4} Q_c R_s^2}} + \frac{\cdots}{\left(r^2 + Q_b\right)^5 \left(r^8 + \frac{1}{4} Q_c R_s^2\right)^{9/2}}, \qquad \lim_{r\to 0^+} K = \frac{8}{R_s \sqrt{Q_c}}$$

- Null expansion scalar

$$\text{Exterior: } \theta_+ \left(r > r_H\right) > 0, \quad \theta_- \left(r > r_H\right) < 0,$$
$$\text{Interior: } \theta_+ \left(r \leq r_H\right) < 0, \quad \theta_- \left(r \leq r_H\right) < 0,$$
$$\text{At core: } \theta_\pm \left(r = 0\right) = 0$$

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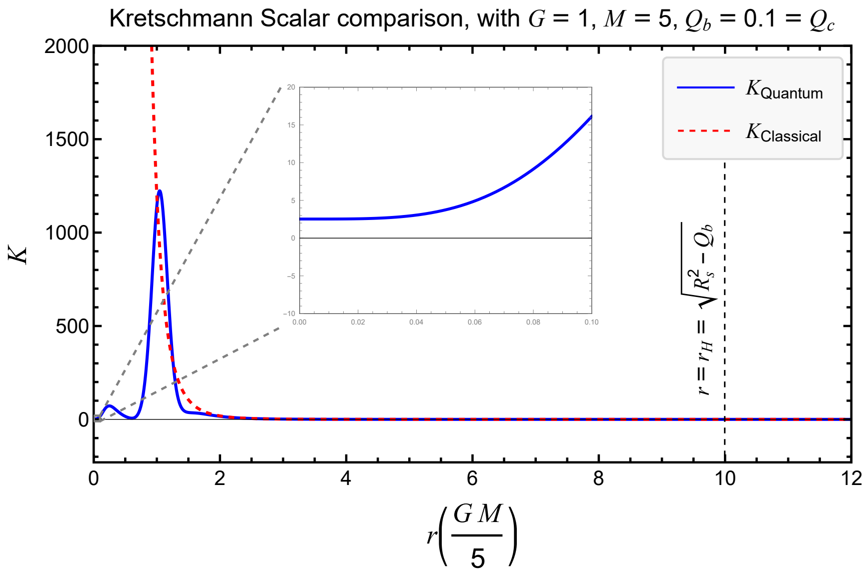

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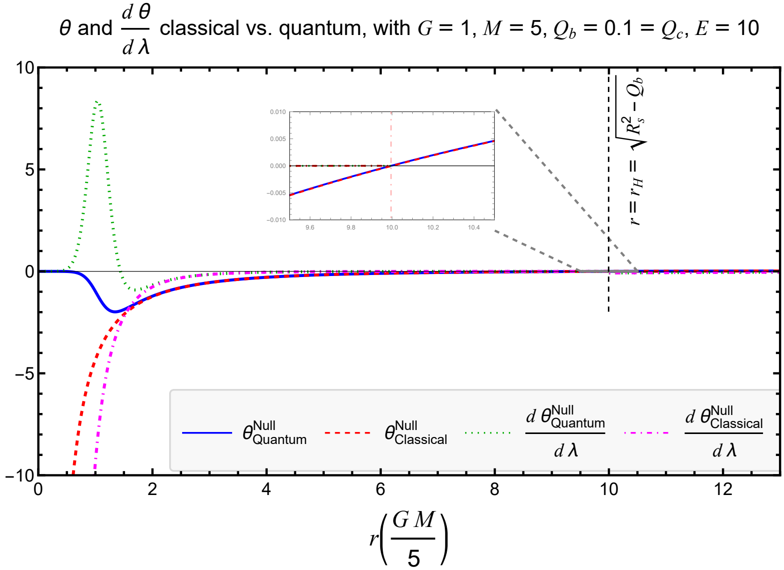

# Horizon

There is a single horizon $g^{11}(r_H) = 0$ or $g_{00}(r_H) = 0$ located at

$$r_H = R_s \sqrt{1 - \frac{Q_b}{R_s^2}} = R_s - \frac{1}{2}\frac{Q_b}{R_s} + \mathcal{O}\left(\frac{Q_b^2}{R_s^3}\right)$$

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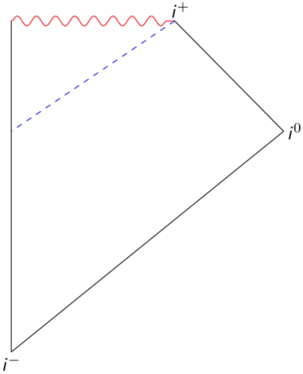
Based on the value of $M$ and $Q_b$

$$\begin{cases} R_s^2 > Q_b \Rightarrow M > \frac{\sqrt{Q_b}}{2G}, & \text{Black hole} \\ R_s^2 = Q_b \Rightarrow M = \frac{\sqrt{Q_b}}{2G}, & \text{Min mass, remnant} \\ R_s^2 < Q_b \Rightarrow M < \frac{\sqrt{Q_b}}{2G}, & \text{No horizon, wormhole} \end{cases}$$

# Penrose Diagram



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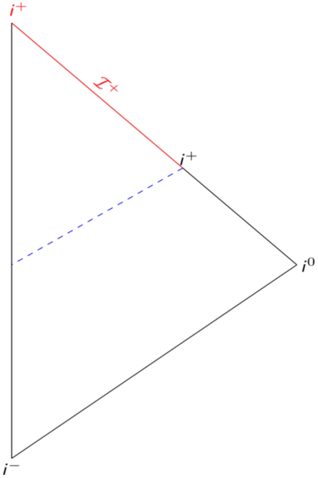

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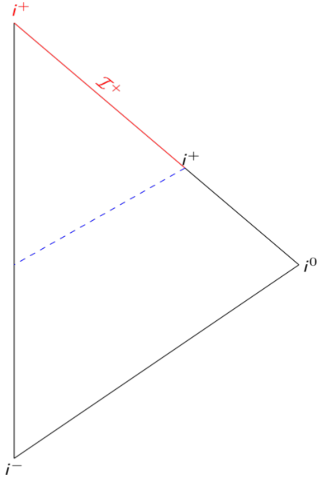


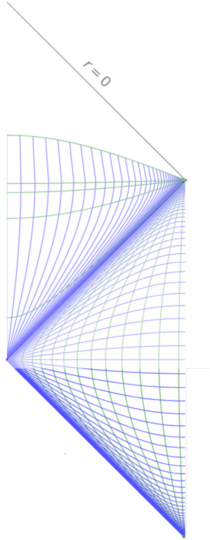

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Based on [Eur. Phys. J. C (2014) 74:2865]:

1) Starting from the static metric

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5) Make a transformation (introduce rotation deformation)

$$r' = r + ia\cos(\theta)\,, \qquad \theta' = \theta$$
$$u' = u - ia\cos(\theta)\,, \qquad \phi' = \phi$$

under which

$$l^{\tilde{\mu}} \to l^{\bar{\mu}},\ n^{\tilde{\mu}} \to n^{\bar{\mu}},\ m^{\tilde{\mu}} \to m^{\bar{\mu}}$$

and

$$g_{\mu\nu} \to \breve{g}_{\mu\nu}$$

and later we demand $\lim_{a\to 0} \breve{g}_{\mu\nu} = g_{\mu\nu}$

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6) Find $g^{(\mathrm{EF})\mu'\nu'} = -l^{\mu'} n^{\nu'} - l^{\nu'} n^{\mu'} + m^{\mu'} \bar{m}^{\nu'} + m^{\nu'} \bar{m}^{\mu'}$ and then find $g^{(\mathrm{EF})}_{\mu'\nu'}$

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7) Make a Boyer-Lindquist transformation

$$\begin{aligned} du' &= d\bar{t} + \lambda\left(\bar{r}\right) d\bar{r} \\ d\phi' &= d\bar{\phi} + \chi\left(\bar{r}\right) d\bar{r} \end{aligned}$$

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and demand

- $g_{\bar{t}\bar{r}} = 0 = g_{\bar{r}\bar{\phi}}$,
- $\lambda$ and $\chi$ are only functions or $\bar{r}$,
- $\lim_{a\to 0} \breve{g}_{\mu\nu} = g_{\mu\nu}$

# Rotating GUP BH Metric

Newman-Janis algorithm $\Rightarrow$ rotating metric

$$g_{\bar{0}\bar{0}} = -\frac{\rho^2}{\Sigma^2}\left(\Sigma - \Lambda\right), \quad g_{\bar{0}\bar{3}} = -\frac{a\sin^2(\theta)\,\rho^2}{\Sigma^2}\Lambda, \quad g_{\bar{1}\bar{1}} = \frac{\rho^2}{\Delta},$$

$$g_{\bar{2}\bar{2}} = \rho^2, \quad g_{\bar{3}\bar{3}} = \frac{\rho^2\sin^2(\theta)}{\Sigma^2}\left[\Sigma^2 + a^2\sin^2(\theta)\left(\Sigma + \Lambda\right)\right],$$

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where

$$\rho^2 = g_{22} + a^2\cos^2(\theta)$$

$$\Delta = \frac{g_{22}}{g_{11}} + a^2$$

$$\Sigma = \frac{g_{22}}{\sqrt{-g_{00}g_{11}}} + a^2\cos^2(\theta)$$

$$\Lambda = \Sigma - \Delta + a^2\sin^2(\theta).$$

# Rotating GUP BH Metric

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where

$$\rho^2 = g_{22} + a^2\cos^2(\theta) = r^2\left(1 + \frac{Q_c R_s^2}{4r^8}\right)^{\frac{1}{4}} + a^2\cos^2(\theta)$$

$$\Delta = \frac{g_{22}}{g_{11}} + a^2 = r^2\left(1 - \frac{R_s}{\sqrt{r^2 + Q_b}}\right) + a^2$$

$$\Sigma = \frac{g_{22}}{\sqrt{-g_{00}g_{11}}} + a^2\cos^2(\theta) = \left(\rho^2 - a^2\cos^2(\theta)\right)\left(1 + \frac{Q_b}{r^2}\right)^{-\frac{1}{2}} + a^2\cos^2(\theta)$$

$$\Lambda = \Sigma - \Delta + a^2\sin^2(\theta).$$

# Limits

Correct limits:

$$\text{Static limit: } \lim_{a\to 0} g_{\bar{\mu}\bar{\nu}} = g_{\mu\nu}^{\text{GUP-Imp}}$$

$$\text{Classical limit: } \lim_{Q_b\to 0, Q_c\to 0} g_{\bar{\mu}\bar{\nu}} = g_{\mu\nu}^{\text{Kerr}}$$

$$\text{Asymptotic expansion: } g_{\bar{\mu}\bar{\nu}}\big|_{Q_b\to 0, Q_c\to 0, r\to\infty} = g_{\mu\nu}^{\text{Kerr}}\big|_{r\to\infty}$$

# Horizons

Horizons found by $g^{11} = 0$ (up to 1st order in $Q_b, Q_c$)

$$r_{\text{Inner}} = \frac{1}{2}\left(R_s - \sqrt{R_s^2 - 4a^2}\right) + \frac{Q_b R_s}{R_s\left(\sqrt{R_s^2 - 4a^2} - R_s\right) + 4a^2}$$

$$r_{\text{outer}} = \frac{1}{2}\left(R_s + \sqrt{R_s^2 - 4a^2}\right) - \frac{Q_b R_s}{R_s\left(\sqrt{R_s^2 - 4a^2} + R_s\right) - 4a^2}$$

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Horizons found by $g^{11} = 0$ (up to 1st order in $Q_b,\ Q_c$)

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$r_{\text{Inner}}$ expands and $r_{\text{outer}}$ shrinks due to quantum effects ($Q_b$)

# Horizons

Extremal case (single horizon with radius $r_{ext}$; up to 1st order in $Q_b$)

$$r_{ext} = \frac{1}{4}\left(R_s + \sqrt{R_s^2 + 8Q_b}\right)$$

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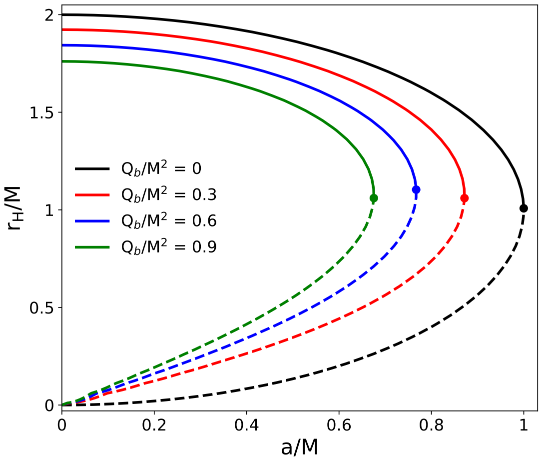
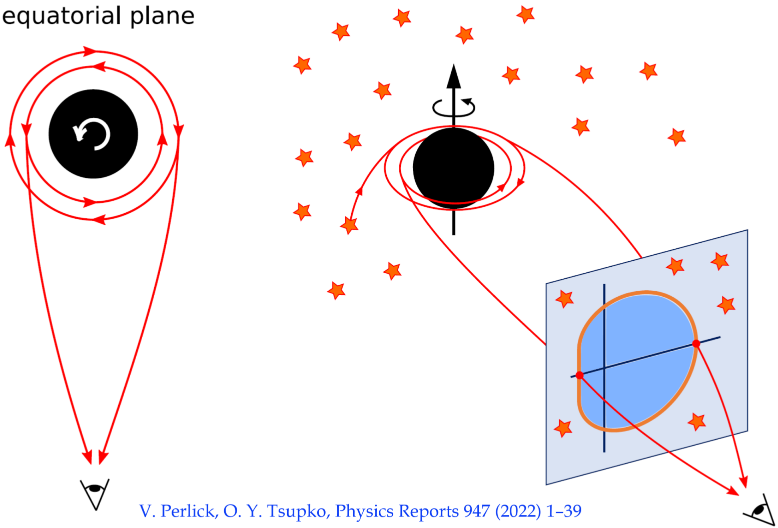


# Outline

# Shadow - Axisymmetric - Planar

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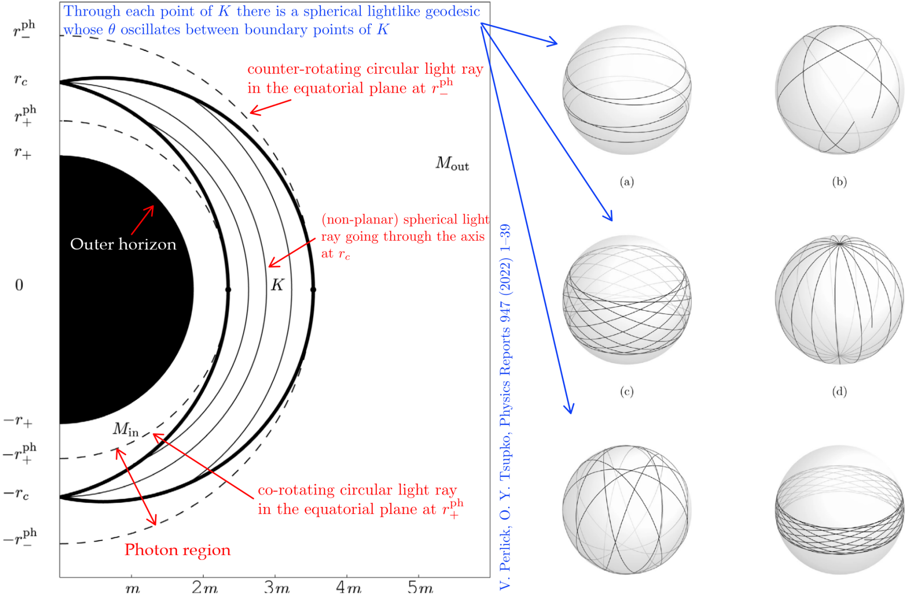
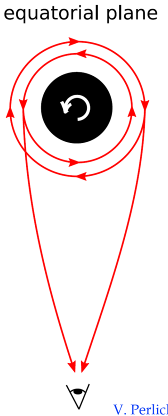
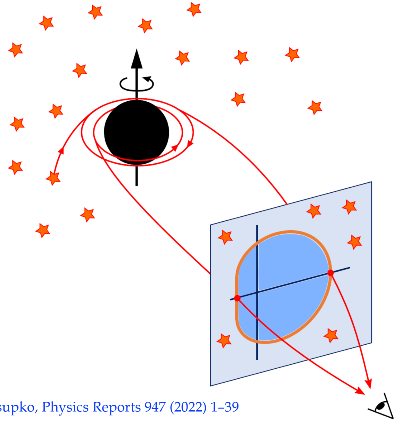
# Shadow - Axisymmetric



# Shadow Boundary



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**Boundary of the shadow:**

Governed by two impact parameters:

- $\xi = \frac{L}{E}$, photon's perpendicular distance from the rotation axis as seen by a distant observer
- $\eta = \frac{\mathcal{K}}{E}$, measures motion out of the equatorial plane

# Shadow

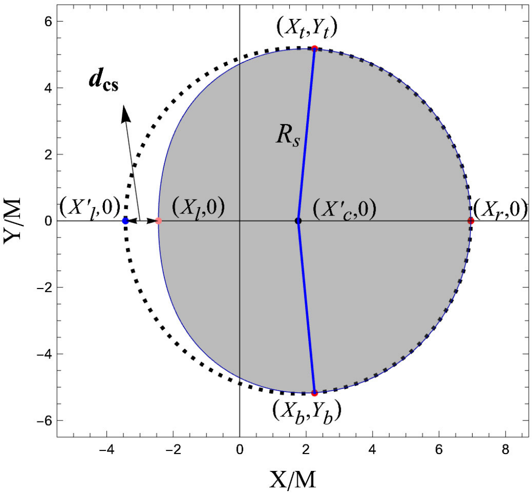
Apparent shape of the shadow

- measured by a distant observer
- located at asymptotically flat infinity
- inclined at an angle $\theta_0$

described by the celestial coordinates

$$\alpha\left(r_{\mathrm{ph}}\right) = -\xi\left(r_{\mathrm{ph}}\right)\csc\left(\theta_0\right),$$
$$\beta\left(r_{\mathrm{ph}}\right) = \pm\sqrt{\eta\left(r_{\mathrm{ph}}\right) + a^2\cos^2\left(\theta_0\right) - \xi^2\cot^2\left(\theta_0\right)}.$$

# Shadow of GUP Rotating Metric



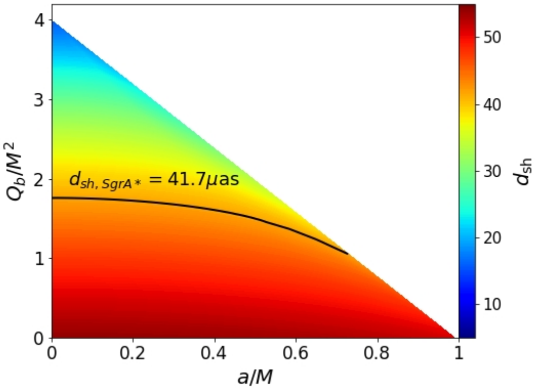

- $D$: distance of observer to BH,
- Shadow area $A$

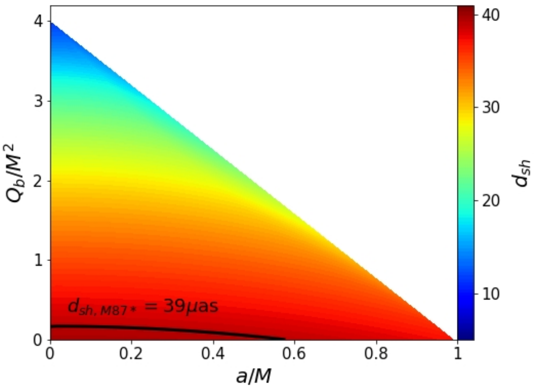
$$A = 2 \int_{r_{ph}^{-}}^{r_{ph}^{+}} \beta\left(r_{ph}\right) \frac{d\alpha\left(r_{ph}\right)}{dr_{ph}} dr_{ph}$$

- Angular shadow diameter

$$d_{sh} = \frac{2}{D}\sqrt{\frac{A}{\pi}},$$

# Comparison to EHT Measurements: Sgr A* vs. M87*



# Summary

- 1st static and NJ-rotated GUP BH metric derived since the beginning of GUP
- Two quantum parameters $Q_b,\ Q_c$
- LQG tricks used (improved scheme, interior-to-exterior)
- Comparing theoretical shadow with EHT data allows putting bounds on $Q_b$
- Limits the spin parameter to $\frac{a}{M} < 1$ for naked singularities
- Considering systematics, etc., limits M87* spin to $\frac{a}{M} \lesssim 0.6$