Universität Bremen

QuantumFrontiers
Cluster of Excellence

CENTER OF
APPLIED SPACE TECHNOLOGY
AND MICROGRAVITY
ZARM

Center for applied space science and microgravity (ZARM)

Fachbereich 04
Faculty of Production Engineering

Unified and higher spin field theories from Hamiltonian cotangent bundle geometry

Deriving the Einstein-Maxwell System from a Single Phase-Space Scalar Field

Joint work with J. Relancio

IJGMMP 2026
https://arxiv.org/abs/2510.15812
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Dr. Christian Pfeifer
GeomGrav 2026, Tartu, July 2026

1. Unified field theories

2. A scalar field equation for point particle Hamiltonians

3. From the scalar equation on phase space to the Einstein-Maxwell equations

4. Conclusion

1. **Unified field theories**

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Unified field theories - some approaches throughout the years

“a theory joining the gravitational and the electromagnetic field into one single hyper-field whose equations represent the conditions imposed on the geometrical structure of the universe” **[Tonnalet 1995, H. Goenner 2004]**

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- Grand unified theory (GUT)
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Mostly applied as modified gravity not unification

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No consensus how to consistently reproduce the standard model yet

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This Talk

Gravity and Electromagnetism
Unification using the geometry of phase space

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Another perspective - A look on point particle motion

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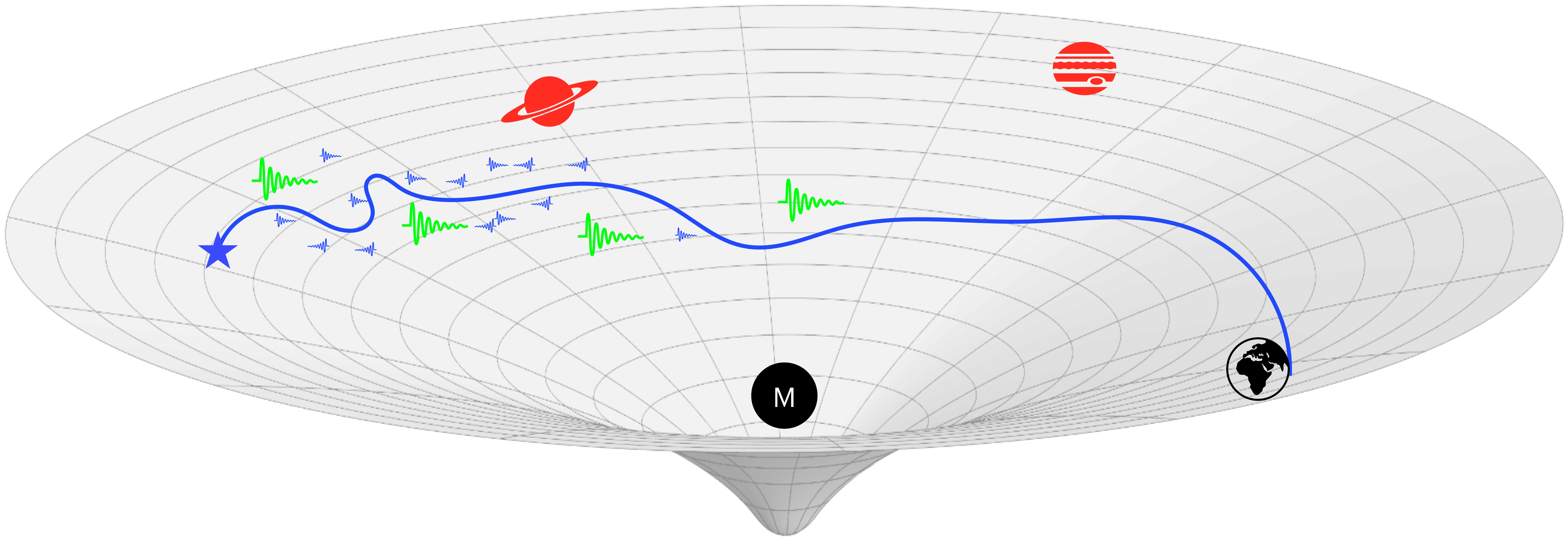
We observe particles and fields to probe the physical world and draw conclusions



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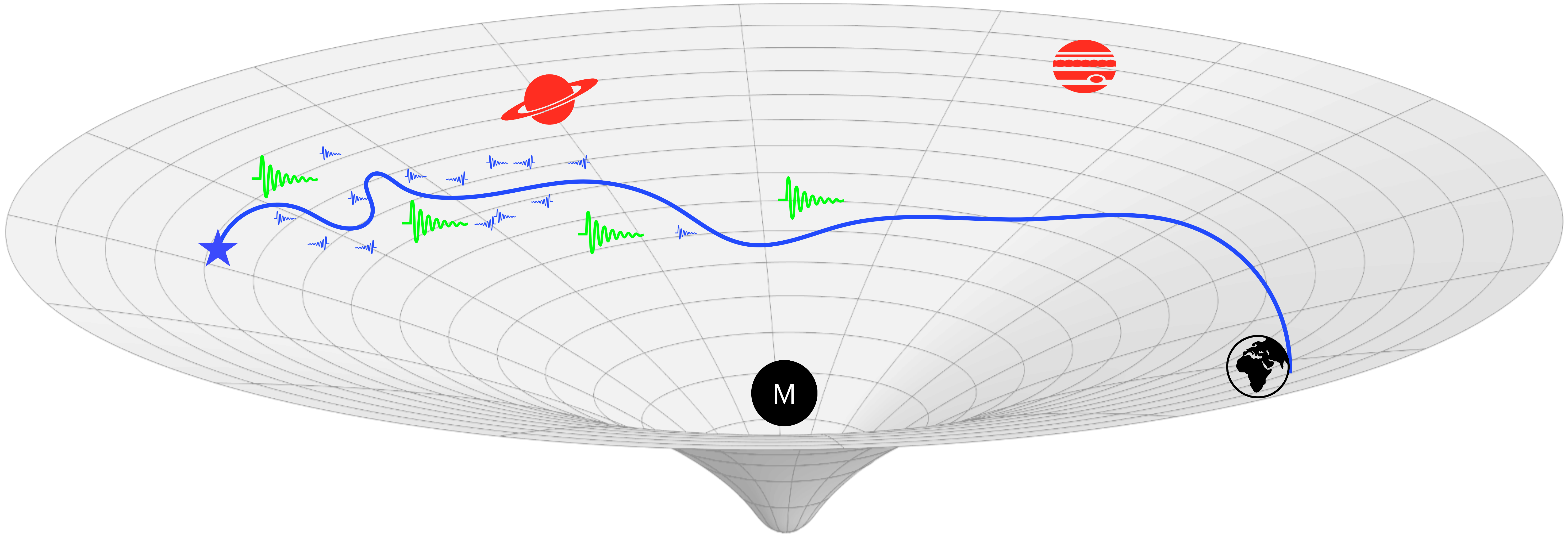
Classical particles



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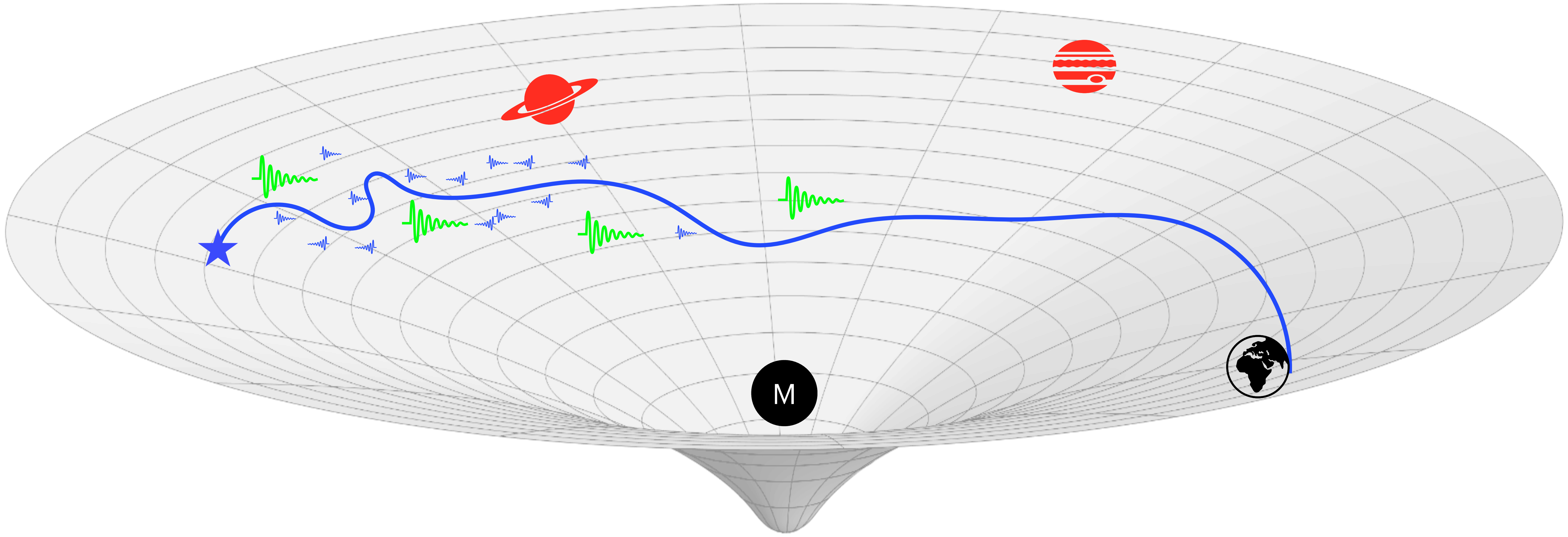
- Particle motion from Hamilton fct.

$$H(x, p)$$



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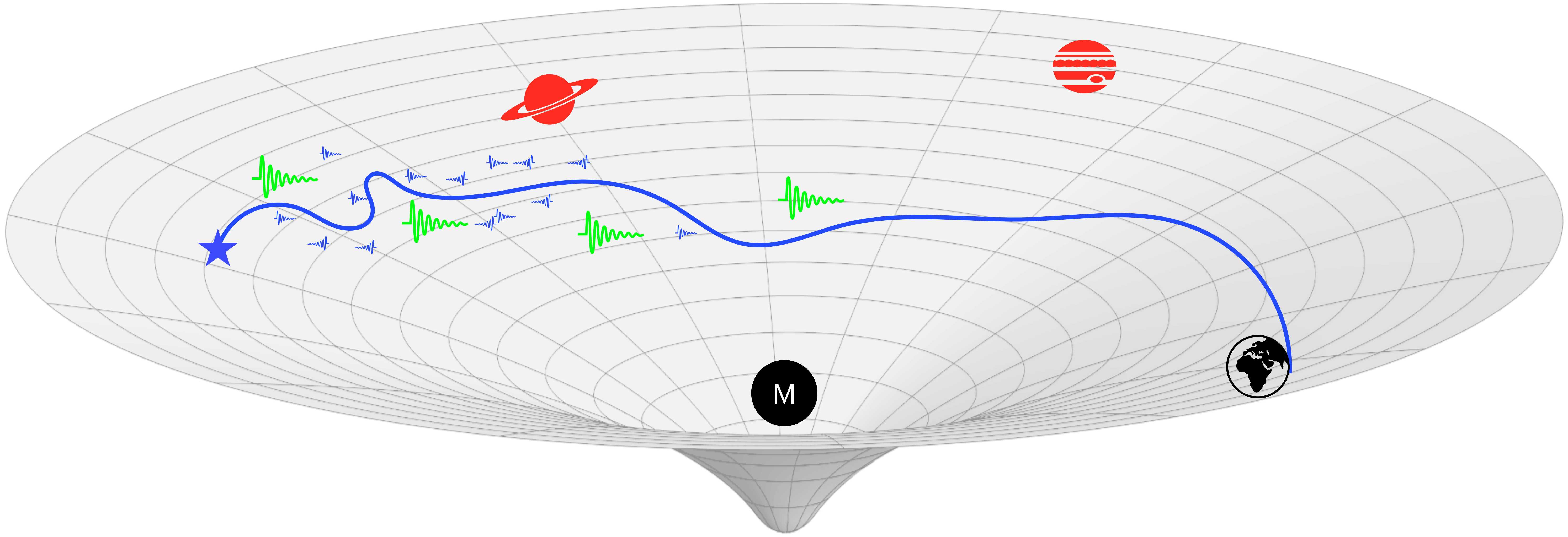

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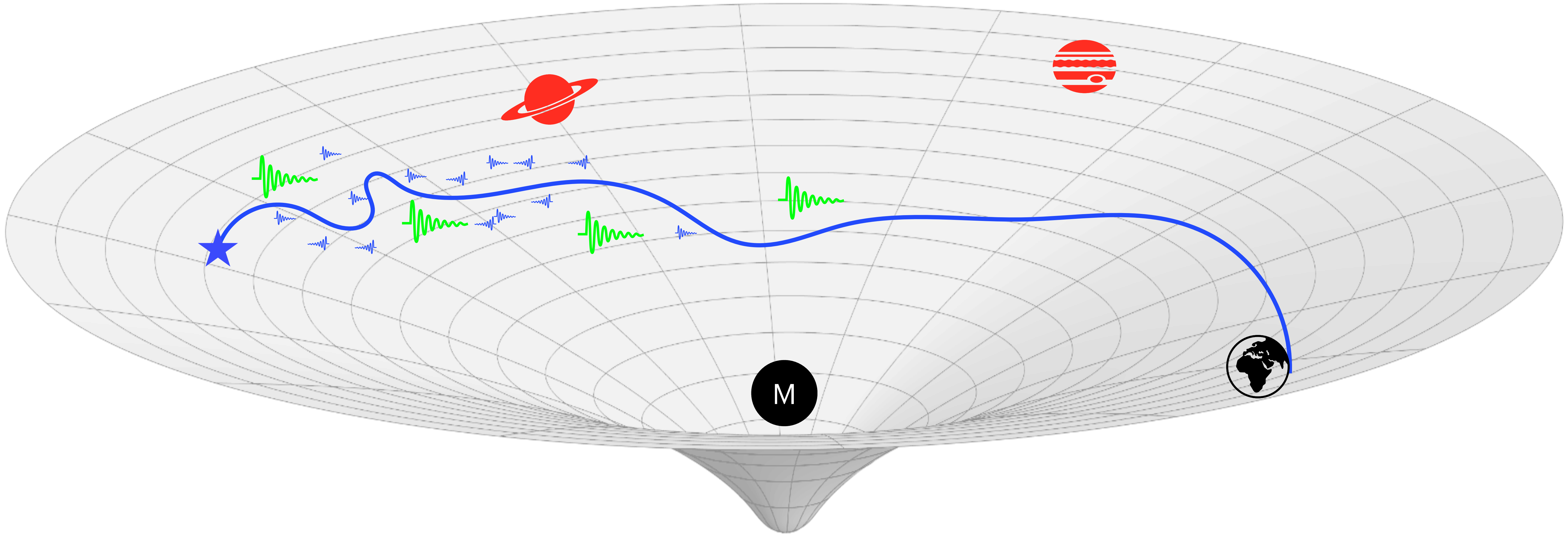
Quantum particles

- Hamiltonian main ingredient in Schroedinger equation
  $$\frac{d}{dt}\Psi = \hat{H}\Psi$$



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[Addazzi 2022, Läänemets 2024]

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[Rubilar 2007; Hehl, Obukhov 2003]

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$$H = g^{\mu\nu}(x)(p_\mu - eA_\mu(x))(p_\nu - eA_\nu(x)) + V(x) + \dots$$

- Further terms - ED in moving plasma

$$H = g^{\mu\nu}(x)(p_\mu - eA_\mu(x))(p_\nu - eA_\nu(x)) + V(x) + (n^2(x,p) - 1)(p_\rho v^\rho(x))^2 + \dots$$

[Synge 1960; Tsupko 2025]

Another perspective - A look on point particle motion

We observe particles and fields to probe the physical world and draw conclusions

Classical particles

- Particle motion from Hamilton fct.

$$H(x,p)$$

- determining dispersion relation

$$H(x,p) = m^2$$

- and Hamilton equations of motion

$$\dot{x}^\mu = \partial_{p_\mu} H, \quad \dot{p}_\mu = -\partial_{x^\mu} H$$

Quantum particles

- Hamiltonian main ingredient in Schroedinger equation

$$\frac{d}{dt}\Psi = \hat{H}\Psi$$

Which Hamiltonian?

- Hamiltonian chosen depending on particle - background field interaction
- Freely falling in a gravitational field

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"…joining the gravitational and the electromagnetic field into one single hyperfield…"
[Tonnalet 1995, H. Goenner 2004]

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- Identify the **Hamiltonian** as the **hyperfield** that joins (all) bosons
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Can we determine the dynamical equations for background fields from a dynamical equation for H?

1. Unified field theories

2. **A scalar field equation for point particle Hamiltonians**

3. From the scalar equation on phase space to the Einstein-Maxwell equations

4. Conclusion

A scalar field equation for point particle Hamiltonians on phase space

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The strategy

- Point particle Hamiltonians as scalar functions on phase space - technically the cotangent bundle

$$H : T^*M \to \mathbb{R}$$
$$(x, p) \mapsto H(x, p)$$

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$$\Sigma = d^4x \wedge d^4p$$

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- To construct an action principle

$$S[H] = \int_{T^*M} \Sigma \mathscr{L}(H, \partial_x H, \partial_p H, \partial_x\partial_x H, \partial_p\partial_x H, \partial_p\partial_p H, \dots)$$

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- Variation yields desired scalar field equation for H

How to construct well behaved
(spacetime coordinate invariant, …)
scalar field Lagrange function $\mathscr{L}$?

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The answer: Hamilton geometry - Derive the geometry of phase space from H

[Miron; Barcaroli 2015; CP, J. Relancio 2026]

A scalar field equation for point particle Hamiltonians on phase space

[CP, J. Relancio 2026]

The action

$$S[H] = \int_{T^*M} d^4x d^4p \left(\alpha_1 \frac{\mathcal{R}}{H} + \kappa \left(\alpha_2 \frac{1}{H^2} g^{\mu\nu} \delta_\mu H \delta_\nu H + \alpha_3 \frac{1}{H} (\bar{\partial}^\rho \delta_\mu H)(\bar{\partial}^\mu \delta_\rho H) \right) \right)$$

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$$\delta_H S = 0$$

The scalar field equation

$$\alpha_1 \left(\frac{1}{2} \bar\partial^\mu \bar\partial^\nu \left(\frac{\mathcal{R}_{\mu\nu}}{H} + Q^\rho{}_\rho g_{\mu\nu} - Q_{\mu\nu} \right) - \frac{\mathcal{R}}{H^2} \right) + \kappa \left\{ \alpha_2 \left(\frac{1}{2} \bar\partial^\mu \bar\partial^\nu \left(\frac{\delta_\mu H \delta_\nu H}{H^2} \right) - 2 \frac{g^{\mu\nu} \delta_\mu H \delta_\nu H}{H^3} - 2 \tilde\nabla_\mu \left(\frac{g^{\mu\nu} \delta_\nu H}{H^2} \right) \right) + \alpha_3 \left(- \frac{1}{H^2} (\bar\partial^\rho \delta_\mu H)(\bar\partial^\mu \delta_\rho H) + 2 \tilde\nabla_\mu \bar\partial^\rho \left(\frac{\bar\partial^\mu \delta_\rho H}{H} \right) \right) \right\} + \frac{1}{4} X = 0$$

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Universität Bremen

A scalar field equation for point particle Hamiltonians on phase space

[CP, J. Relancio 2026]

The action

$$S[H] = \int_{T^*M} d^4x d^4p \left(\alpha_1 \frac{\mathscr{R}}{H} + \kappa \left(\alpha_2 \frac{1}{H^2} g^{\mu\nu} \delta_\mu H \delta_\nu H + \alpha_3 \frac{1}{H} (\bar{\partial}^\rho \delta_\mu H)(\bar{\partial}^\mu \delta_\rho H) \right) \right)$$

$$\delta_H S = 0$$

The scalar field equation

$$\alpha_1 \left(\frac{1}{2} \bar{\partial}^\mu \bar{\partial}^\nu \left(\frac{\mathscr{R}_{\mu\nu}}{H} + Q^\rho{}_\rho g_{\mu\nu} - Q_{\mu\nu} \right) - \frac{\mathscr{R}}{H^2} \right) + \kappa \left\{ \alpha_2 \left(\frac{1}{2} \bar{\partial}^\mu \bar{\partial}^\nu \left(\frac{\delta_\mu H \delta_\nu H}{H^2} \right) - 2 \frac{g^{\mu\nu} \delta_\mu H \delta_\nu H}{H^3} - 2 \tilde{\nabla}_\mu \left(\frac{g^{\mu\nu} \delta_\nu H}{H^2} \right) \right) + \alpha_3 \left(-\frac{1}{H^2} (\bar{\partial}^\rho \delta_\mu H)(\bar{\partial}^\mu \delta_\rho H) + 2 \tilde{\nabla}_\mu \bar{\partial}^\rho \left(\frac{\bar{\partial}^\mu \delta_\rho H}{H} \right) \right) \right\} + \frac{1}{4} X = 0$$

- The Hamilton function

$$H(x,p)$$

- x and p derivatives

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1. Unified field theories
2. A scalar field equation for point particle Hamiltonians
3. **From the scalar equation on phase space to the Einstein-Maxwell equations**
4. Conclusion

A scalar field equation for point particle Hamiltonians on phase space

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The Einstein vacuum equations from a scalar field equation on the tangent bundle

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$$\delta_\mu H = -eF_{\mu\nu} \tilde{p}^\nu$$

A scalar field equation for point particle Hamiltonians on phase space

[CP, J. Relancio 2026]

The action

$$S[H] = \int_{T^*M} d^4x d^4p \left(\alpha_1 \frac{\mathcal{R}}{H} + \kappa \left(\alpha_2 \frac{1}{H^2} g^{\mu\nu} \delta_\mu H \delta_\nu H + \alpha_3 \frac{1}{H} (\bar{\partial}^\rho \delta_\mu H)(\bar{\partial}^\mu \delta_\rho H) \right) \right)$$

The scalar field equation

$$\alpha_1 \left(\frac{1}{2} \bar{\partial}^\mu \bar{\partial}^\nu \left(\frac{\mathcal{R}_{\mu\nu}}{H} + Q^\rho{}_\rho g_{\mu\nu} - Q_{\mu\nu} \right) - \frac{\mathcal{R}}{H^2} \right) + \kappa \left\{ \alpha_2 \left(\frac{1}{2} \bar{\partial}^\mu \bar{\partial}^\nu \left(\frac{\delta_\mu H \delta_\nu H}{H^2} \right) - 2 \frac{g^{\mu\nu} \delta_\mu H \delta_\nu H}{H^3} - 2 \tilde{\nabla}_\mu \left(\frac{g^{\mu\nu} \delta_\nu H}{H^2} \right) \right) + \alpha_3 \left(-\frac{1}{H^2} (\bar{\partial}^\rho \delta_\mu H)(\bar{\partial}^\mu \delta_\rho H) + 2 \tilde{\nabla}_\mu \bar{\partial}^\rho \left(\frac{\bar{\partial}^\mu \delta_\rho H}{H} \right) \right) \right\} + \frac{1}{4} X = 0$$

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The Einstein Maxwell equations from a scalar field equation on the tangent bundle

1. Unified field theories

2. A scalar field equation for point particle Hamiltonians

3. From the scalar equation on phase space to the Einstein-Maxwell equations

4. **Conclusion**

A scalar field equation for point particle Hamiltonians on phase space

[CP, J. Relancio 2026]

“a theory joining the gravitational and the electromagnetic field into one single hyperfield whose equations represent the conditions imposed on the geometrical structure of the universe”

[Tonnalet 1995, H. Goenner 2004]

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The idea

- Identify the **Hamiltonian** as the **hyperfield** that joins (all) bosons
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Hamilton Geometry of Phase Space determines the geometric structure of the universe.

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Variation yields non linear, geometric scalar field equation for H

- Decompose field equation into n-th order powers of p

$$\sum_{i=0}^{n} G^{\mu_1 \cdots \mu_n}(x) p_{\mu_1} p_{\mu_n} = 0$$

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Einstein Vacuum Equations

- $H = g^{\mu\nu}(x) p_\mu p_\nu$

$$(n=2) \quad G^{\mu\nu}(x) = 0 \Leftrightarrow \mathring{R}^{\mu\nu} = 0$$

Einstein Maxwell Equations

- $H = g^{\mu\nu}(x)(p_\mu - eA_\mu)(p_\nu - eA_\nu)$

$$(n=3) \quad G^{\mu\nu\rho}(x) = 0 \Leftrightarrow \nabla_\mu F^{\mu\nu} = 0$$

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The next step: General polynomial Hamiltonians

$$H = V(x) + A^\mu(x) p_\mu + g^{\mu\nu}(x) p_\mu p_\nu$$

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Scalar field equation on phase space yield coupled non-linear tensorial equations for higher spin fields

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The next step: General polynomial Hamiltonians

$$H = V(x) + A^\mu(x)p_\mu + g^{\mu\nu}(x)p_\mu p_\nu + \sum_{i=3}^{n} \Psi^{\mu_1\dots\mu_i}(x)p_{\mu_1}\dots p_{\mu_i}$$

Scalar field equation on phase space yield coupled non-linear tensorial equations for higher spin fields

Thank you for your attention!

Aitäh :)

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