

C-2: Instabilities and Space Charge

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DEPARTMENT OF
PHYSICS



Coherent dipole-moment stability:

1. When space charge helps
2. When space charge hurts
3. Living with space charge

Coherent higher-order-moment stability:

4. When space charge becomes the driver

1. When space charge helps

Transverse Mode Coupling Instability:

- ▶ in short-bunch regime (bunch length < wake wavelength): head-tail modes 0 and -1 couple at finite intensity threshold
- ▶ ABS spectrum of space-charge head-tail modes at zero wake: Blaskiewicz, PRSTAB 1, 044201 (1998)

$$\frac{\Delta Q}{Q_s} = -\frac{\Delta Q_{sc}}{2} \pm \sqrt{\left(\frac{\Delta Q_{sc}}{2}\right)^2 + k^2 Q_s^2}$$

- ▶ direct SC detunes coherent modes except for rigid mode $k=0$

⇒ intensity threshold of TMCI increases with SC

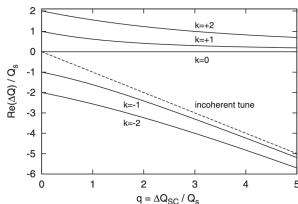


FIG. 1. Tune shifts of five head-tail modes versus space-charge parameter q as given by the airbag theory [Eq. (2)]. The dashed line is the incoherent betatron tune $Q_0 - \Delta Q_{sc}$.

First paper: V. Balbekov, Zh. Tekh. Fiz. 46, 1470 (1976)

“[...] space charge tune spread results in Landau damping which suppresses many of the head-tail modes, much like other sources of the incoherent tune spread.” (Balbekov 2026)

- KV-Gaussian Landau damping:
Kornilov and Boine-Frankenheim,
PRSTAB **13**, 114201 (2010)
- 3D-Gaussian Landau damping:
Macridin et al., PRSTAB **18**
074401 (2015)

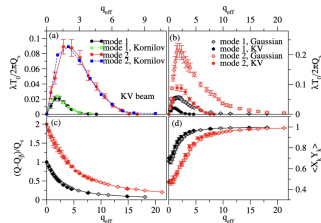


FIG. 9. (a) Landau damping for KV-G bunches. Comparison between our simulations and those of Kornilov and Boine-Frankenheim [9]. The agreement is good. (b)–(d) Comparison between the first two modes of KV-G beams and 3D-G beams. (b) Landau damping. The damping of the 3D-G beams’ modes is much larger. (c) Relative tune shift. (d) Spatial overlap of the mode shape with the space charge harmonic [see Eq. (32)]. Unlike the Landau damping the tune shift and the modes shapes depend very little on the transverse beam distribution, as expected.

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For a KV: $q_{sc} \approx \frac{\Delta Q_{kv}}{Q_s}$

For a 3D-Gaussian: $q_{sc} \approx \frac{\Delta Q_{sc}}{2Q_s}$

- ▶ more Landau damping for 3D-Gaussian vs. KV-Gaussian
- ▶ coherent mode tune shifts are equal
- ▶ space-charge eigenmode overlaps are equal

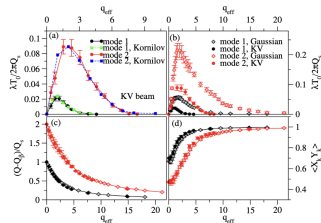
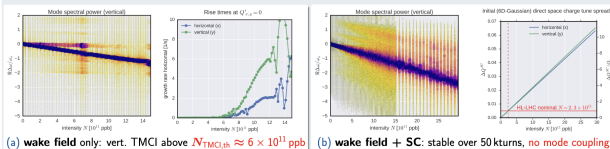


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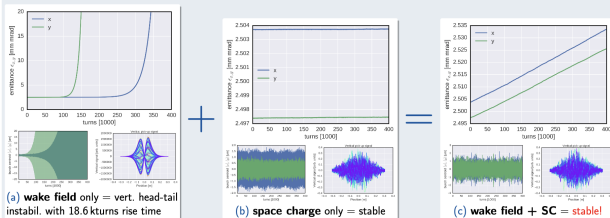
HL-LHC: Transverse Mode Coupling Instability (TMCI) + Space Charge

HL-LHC injection plateau at $Q'_{x,y} = 0$, $\sigma_z = 10.4$ cm and $\epsilon_{x,y} = 2.5$ mmrad:



HL-LHC: Head-Tail Instability + Space Charge

HL-LHC injection plateau at $Q'_{x,y} = 5$, $\sigma_z = 10.4$ cm and $N = 4 \times 10^{11}$ ppb $< N_{\text{TMCI,th}}$:



Oeftiger and Li, Benevento Workshop 2017

2. When space charge hurts

SC compresses coherent spectrum:

- ▶ Normal instability: an eigenmode grows, $\Im m \Delta Q > 0$
- ▶ Non-normal instability: stable eigenmodes ($\Im m \Delta Q = 0$) collectively produce large transient growth

⇒ beam response to perturbations can become enormous through head-to-tail amplification

- ▶ Burov, PRAB **22**, 034202 (2019)
- ▶ Alexahin, Conf. Proc. MCBI 2019
- ▶ Buffat et al., PRAB **24**, 060101 (2021)

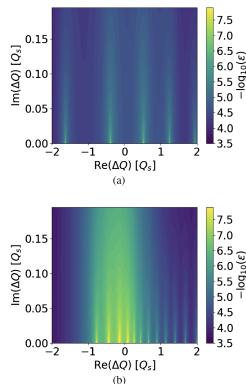


FIG. 5. Pseudospectrum of the one turn matrix with weak (above) and strong (below) space charge forces, both being above the decoupling of the original TMCI by space charge. The normalized intensity is 20. (a) $\Delta Q_{SC} = 6Q_s$, (b) $\Delta Q_{SC} = 50Q_s$.

Figure: Sensitivity to perturbations, Buffat 2021

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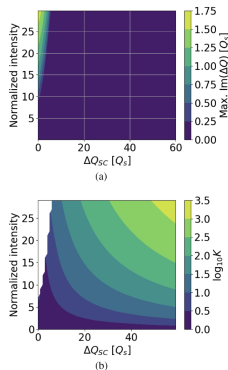


FIG. 4. Maximum tune shift (above) and the Kreiss constant (below) obtained for an airbag distribution for various strength of space charge forces and wakefields. The Kreiss constant becomes infinite for configurations featuring unstable eigenvalues which is marked by a white area. (a) Maximum tune shift (b) Kreiss constant.

Figure: Stability (airbag d.) in Buffat et al. 2021

SC compresses coherent spectrum:

- ▶ M
 - ▶ E
 - ▶ M
 - ▶ E
 - ▶ F
 - ⇒ E
 - ⇒ E
 - ⇒ F
- ## Comparison to “usual” absolute instability

 - ▶ growth rate changes with perturbation amplitude (!)
 - ▶ spatial amplification along bunch increases overall amplitude in time but then decays again
- ⇒ strong transient amplification (growth rates of several Q_s) can lead to beam loss
 - ⇒ fast beam loss might not be easy to distinguish from conventional (no-SC) head-tail instability

▶ Alexanth, Conf. Proc. MCBT 2019

- ▶ Buffat et al., PRAB **24**, 060101 (2021)

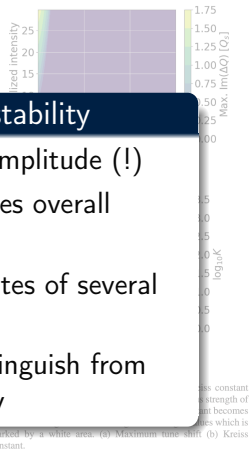


Figure: Stability (airbag d.) in Buffat et al. 2021

Intermediate SC increases TMCI threshold.

At stronger SC, TMCI returns for Gaussian-distributed beams: higher- k mode coupling!

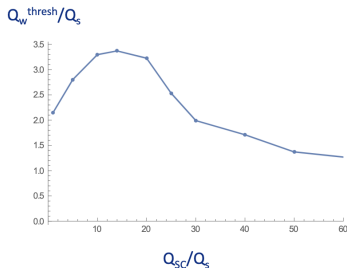


Figure: TMCI threshold vs. SC,
Alexahin 2019 (note $q_{sc} = x\text{-axis}/2$)

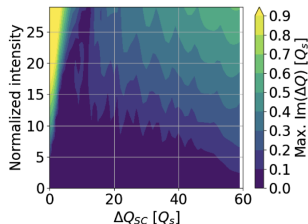


FIG. 8. Maximum imaginary tune shift for a Gaussian distribution obtained with 100 slices and 50 rings for different strengths of the wakefields and the space charge forces.

Figure: Stability vs. SC,
Buffat et al. 2021

Examples of detrimental SC impact for real machine parameters:

1. CERN SPS Q26, $q_{sc} = 0.27/0.004/2 \approx 34$, measurements by Oeftiger 2018

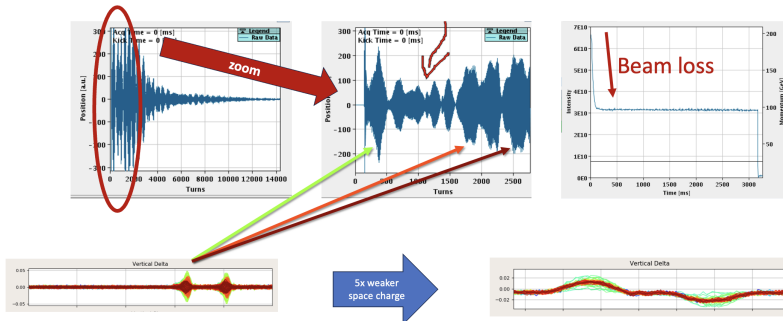


Figure: SPS Q26 measurements with vs. without SC

Examples of detrimental SC impact for real machine parameters:

1. CERN SPS Q26, $q_{sc} = 0.27/0.004/2 \approx 34$, measurements by Oeftiger 2018
2. EIC HSR, $q_{sc} = 0.05/0.00018/2 \approx 140$, PIC simulations by Oeftiger 2026 in EIC Preliminary Design Report

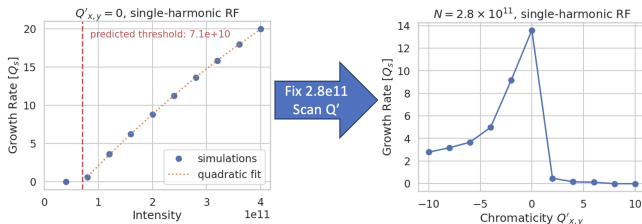


Figure: EIC HSR **without** SC, TMCI intensity scan & chromaticity scan

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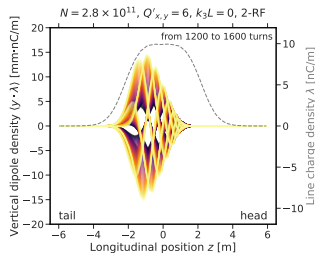
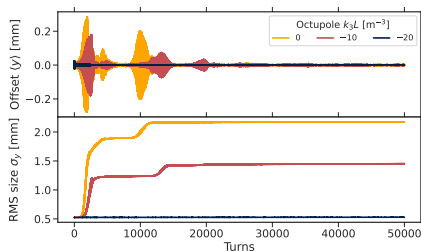


Figure: EIC HSR with flat beams ($\epsilon_y/\epsilon_x = 1/10$)

3. Living with space charge

1. impact of Synchrotron Motion

- ▶ SPS Q26 $\rightarrow$ Q20, higher $\eta \Rightarrow$ higher Q_s , less dense spectrum!
- ▶ ISIS RCS, double-harmonic RF with asymmetric profile (leaning towards head)
- ▶ Landau damping from RF bucket nonlinearity

2. transverse *intra-bunch* feedback systems

- ▶ SPS wideband feedback (GHz! Höfle et al., IPAC 2018)
- $\rightarrow$ bunch-by-bunch feedback systems might be ineffective or even drive instability (Burov 2019: amplified tail projects to head)

3. octupole magnets, chromaticity

4. indirect space charge

Effectiveness of octupoles to provide Landau damping for head-tail modes is heavily affected by direct space charge.

- ▶ Kornilov et al., IPAC 2023
- ▶ Oeftiger and Kornilov, IPAC 2025: implemented FAIR SIS100

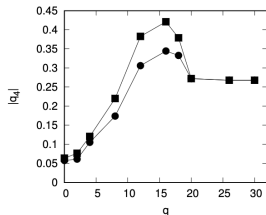


Figure 3: Results of the simulations scans for the $k = 0$ mode: stability thresholds of the octupole power in a dependency from the space-charge parameter q . The circles are for the octupole polarity $q_4 > 0$, the squares are for the octupole polarity $q_4 < 0$.

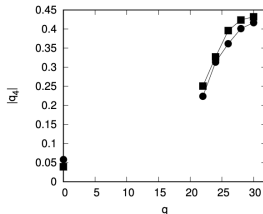


Figure 4: Results of the simulations scans for the $k = 1$ mode: stability thresholds of the octupole power in a dependency from the space-charge parameter q . The circles are for the octupole polarity $q_4 > 0$, the squares are for the octupole polarity $q_4 < 0$.

Figure: Required normalised octupole strength vs. SC strength

J-PARC studies revealed beneficial effect of *indirect space charge*:

- J-PARC RCS: Saha et al., PRAB **21**, 024203 (2018)

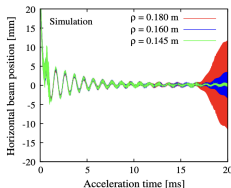


FIG. 16. Simulation results of beam instability growth dependence on the vacuum chamber radius ρ . The indirect space charge effect is reduced by an increase of ρ , and the beam tends to destabilize with increasing ρ .

Figure: Fig. 16 in Saha et al.

J-PARC studies revealed beneficial effect of *indirect space charge*:

- ▶ J-PARC RCS: Saha et al., PRAB **21**, 024203 (2018)
- ▶ J-PARC MR: Yoshimura et al., PRAB **28**, 101003 (2025)

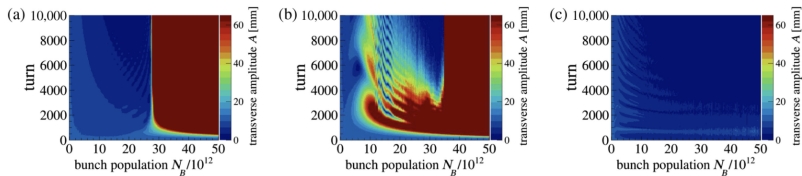


FIG. 10. Transverse amplitude $\tilde{A}_n$ scanned by varying beam intensity N_B for the case of chromaticity $\xi_x = 0$. (a) Only the transverse wakefield, (b) the transverse wakefield and the direct space charge, and (c) the transverse wakefield and the direct and indirect space charge.

Figure: Fig. 10 in Yoshimura et al.

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- ▶ J-PARC RCS: Saha et al., PRAB **21**, 024203 (2018)
 - ▶ J-PARC MR: Yoshimura et al., PRAB **28**, 101003 (2025)
- ⇒ explanation: reactive coherent tune shift separates modes

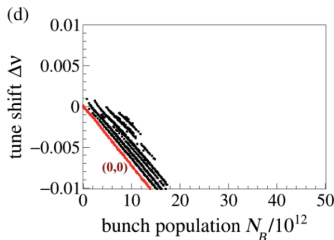
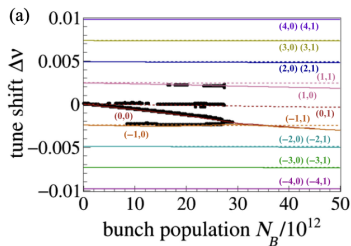


Figure: Figs. 7 (no SC) and 9 (direct + indirect SC) in Yoshimura et al.

4. When space charge becomes
the driver

SC can drive *quadrupole moment* = well-known envelope instability. However, *sextupole*, *octupole*, and higher moments of Gaussian / “realistic” beams are observed to be **stable**.

- Landau Damping of nonlinear moments: Hofmann, Oeftiger and Boine-Frankenheim, PRAB **24**, 024201 (2021)

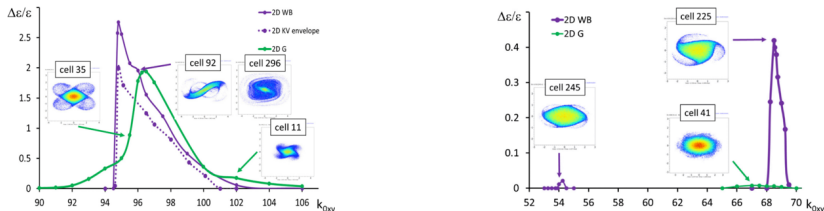


Figure: Emittance growth of coasting Gaussian vs. waterbag distributions

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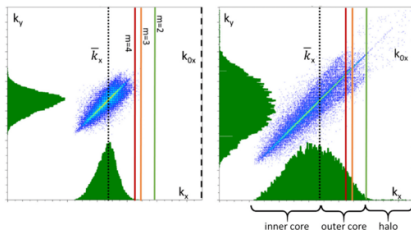
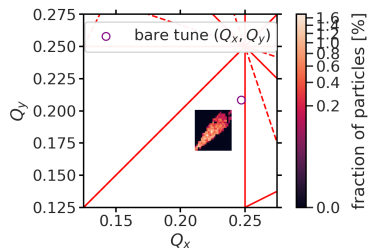
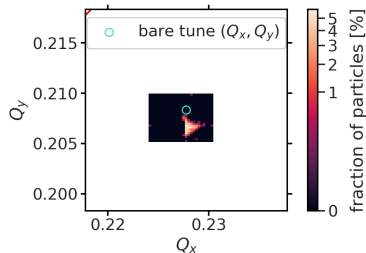


Figure: Coherent mode tunes vs. incoherent spectrum from SC

Ongoing simulation studies on compensating incoherent SC tune spread with Gaussian e-lenses show potential **loss of Landau Damping**, uncovering nonlinear moment instabilities:



(a) only space charge



(b) e-lens compensation

Figure: Comparison of incoherent tune footprints without and with space charge compensation by Gaussian electron lenses.

Ongoing simulation studies on compensating incoherent SC tune spread with Gaussian e-lenses show potential **loss of Landau Damping**, uncovering nonlinear moment instabilities:

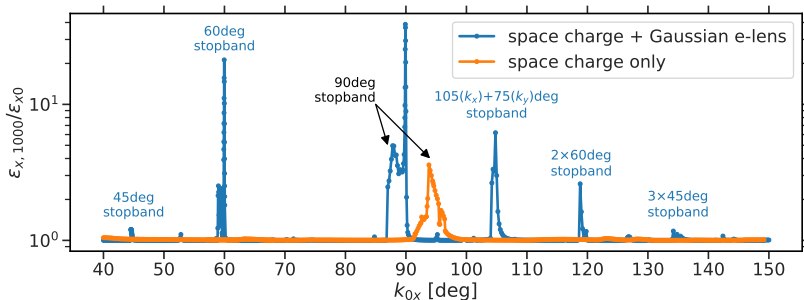


Figure: Rms emittance growth due to instabilities vs. phase advance per FODO cell.

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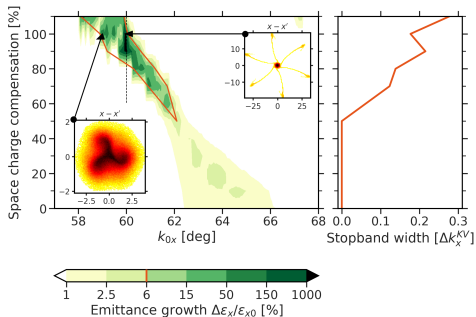


Figure: 60° stopband, unstable sextupole moment above 50% SC comp.

Space Charge provides:

- + negative tune shift of *non-rigid* dipole modes
 $\Rightarrow$ TMCI intensity threshold of 0 and -1 coupling increased
- + Landau Damping for non-rigid head-tail modes
- non-normal eigenmodes $\Rightarrow$ large transient amplitude growth, growth rate depends on perturbation
- transverse higher- k modes coupling instabilities at large q_{sc}
- driving terms for higher-order moments, potentially limits compensation strategies for incoherent SC tune spread

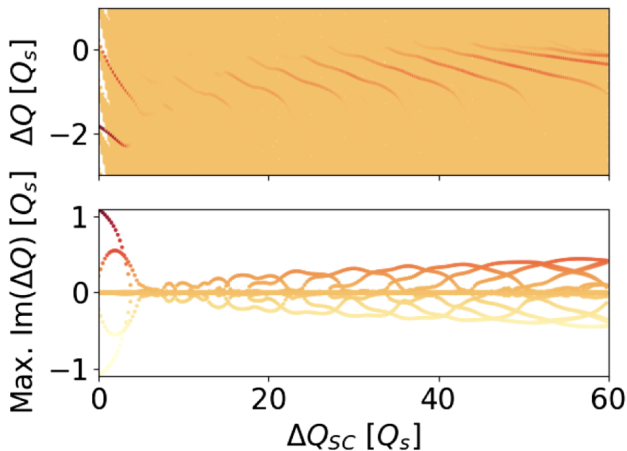
$\Rightarrow$ How can present and future high-intensity proton accelerator facilities **exploit space-charge-induced Landau damping** to intrinsically stabilise bunches when pushing the brightness frontier?

I.e. can we deliberately design with Q_s keeping intermediate $q_{sc} \approx 3$?
 How beneficial would such a scheme be to achieve higher intensity?

An aerial photograph of the Radcliffe Camera in Oxford, England, taken at dusk. The building is a large, circular, neoclassical structure with a prominent dome and many windows that are illuminated from within. It is surrounded by other historic buildings of the University of Oxford, some with blue roofs and others with red. The sky is a clear, deep blue. The text "Thank you for your attention!" is overlaid in the center of the image.

**Thank you
for your attention!**

Appendix



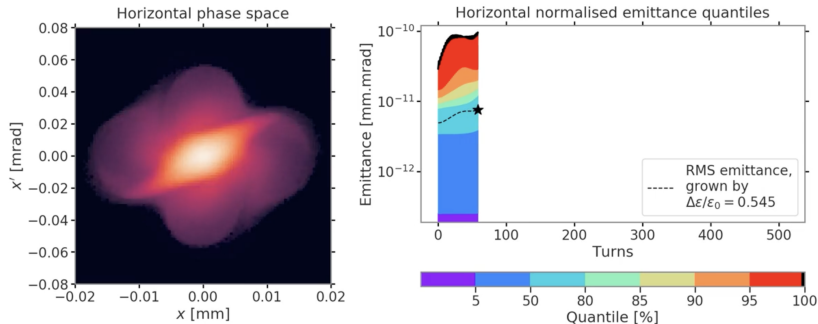


Figure: 90° stopband, envelope instability and 4th-order resonance