Space-charge effects in multi-turn extraction

M. Giovannozzi

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Outline

- Introduction
- PS Multi-turn extraction
- Space-charge effects: measurements
- Space charge effects: numerical simulations
- Digression: crossing a 2D resonance
- Conclusions and outlook

Introduction

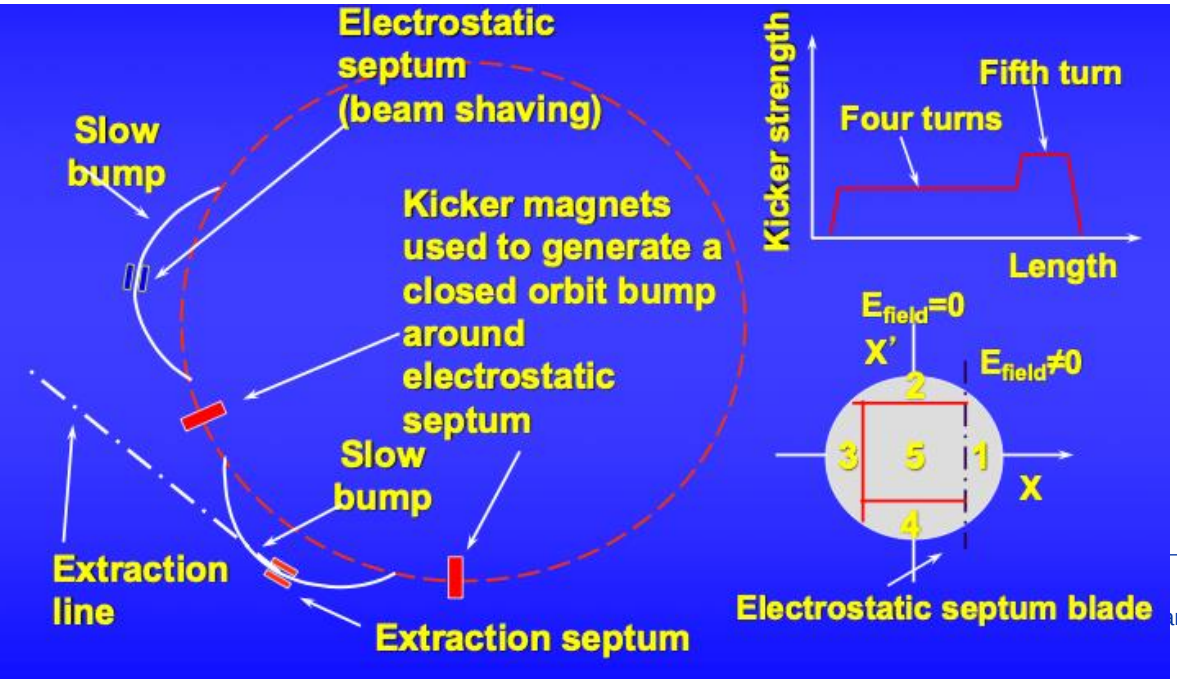
CERN PS MULTI TURN EXTRACTION

Why multi-turn extraction?

- To transfer **efficiently** a beam between circular accelerators of different circumference?
- **Efficiently** ➡ maximising the flux of particles for fixed-target physics at the receiving accelerator.

The CERN context

- High-intensity proton beams (about 3e13 p/extraction)
- C_{SPS}=11 C_{PS}
- Solution proposed in the 70s ➡ Continuous Transfer (CT).

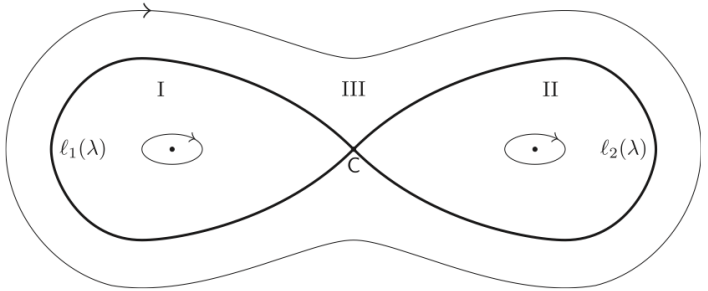


How to overcome extraction losses

- The CT is based on shaving the beam onto the foil of an electrostatic septum.
- An alternative consists of using the (**non-linear**) dynamics to change the transverse beam structure.

The solution

- **Sextupoles** and **octupoles** are used to create **stable islands** in the horizontal phase space.
- A **tune variation** is introduced to cross the fourth-order resonance.
- All this relies on **adiabatic trapping and transport**.
- A well-established theory exists for 2D Hamiltonian systems.



$$\mathcal{P}_{\mathrm{III}\to\mathrm{I}} = \frac{\Theta_{\mathrm{I}}}{\Theta_{\mathrm{I}} + \Theta_{\mathrm{II}}} \qquad \mathcal{P}_{\mathrm{III}\to\mathrm{II}} = 1 - \mathcal{P}_{\mathrm{III}\to\mathrm{I}},$$

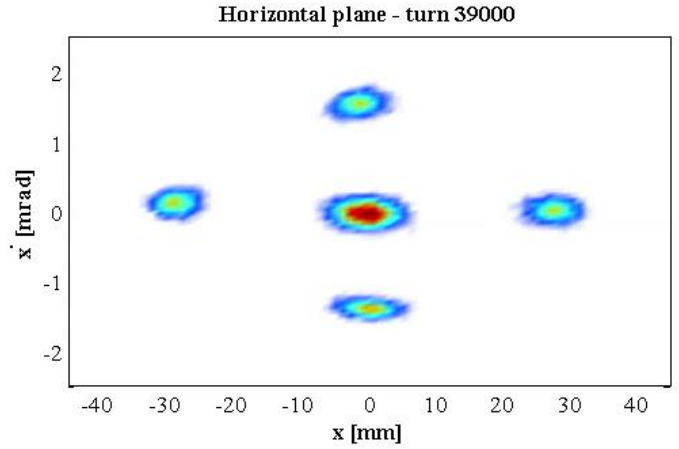
$$\Theta_i = \left.\frac{\mathrm{d}A_i}{\mathrm{d}\lambda}\right|_{\tilde{\lambda}} = \oint_{\partial A_i} \mathrm{d}t \left.\frac{\partial \mathcal{H}}{\partial \lambda}\right|_{\tilde{\lambda}} \qquad i = \mathrm{I},\ \mathrm{II},$$

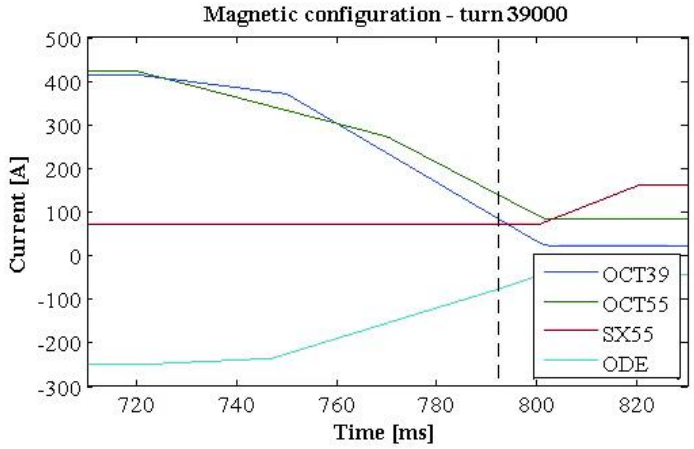
A. Neishtadt. “Passage through a separatrix in a resonance problem with a slowly-varying parameter”. J. Appl. Math. Mech. 39(4) (1975), pp. 594–605.
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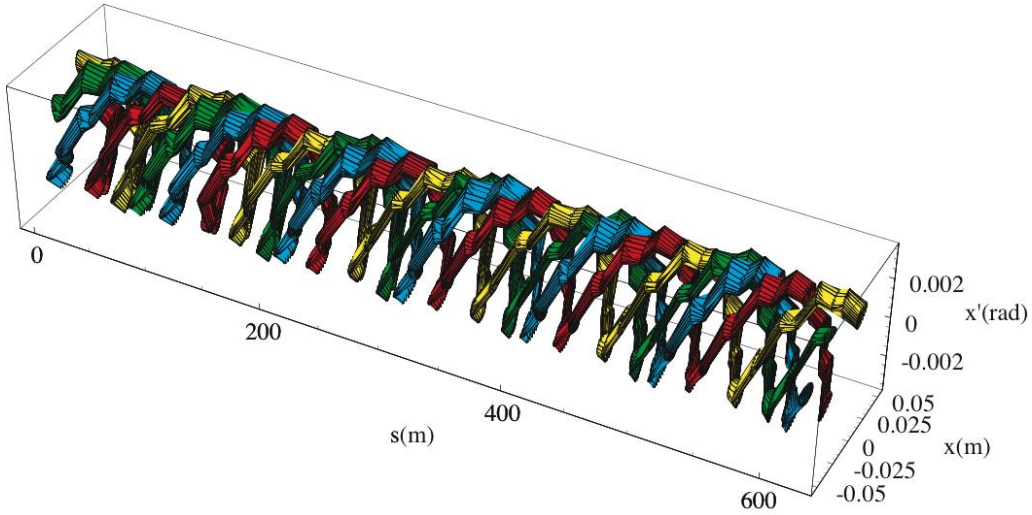
PS multi-turn extraction (transverse)

CERN PS MULTI TURN EXTRACTION

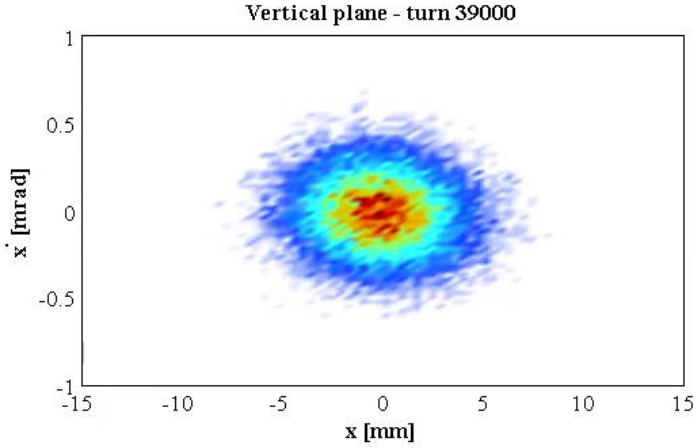
Numerical simulations performed without space-charge effects included

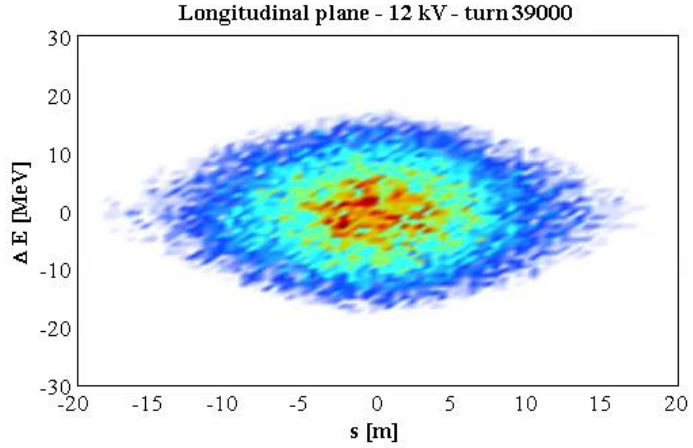






Courtesy A. Huschauer



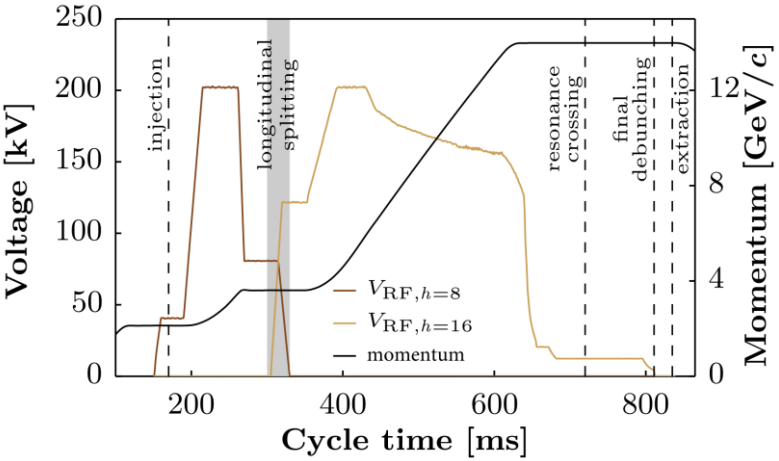


Since 2015, the CERN PS delivers beam to the SPS using Multi-turn Extraction (MTE)

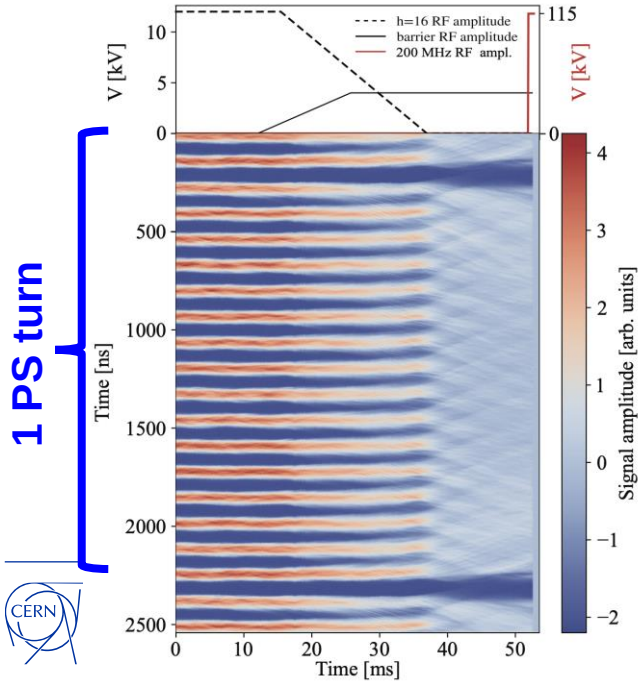
PS multi-turn extraction (longitudinal)

CERN PS MULTI TURN EXTRACTION

MTE is lossless (transverse), but the beam is debunched prior to extraction: beam losses are unavoidable during the risetime of the extraction kickers.



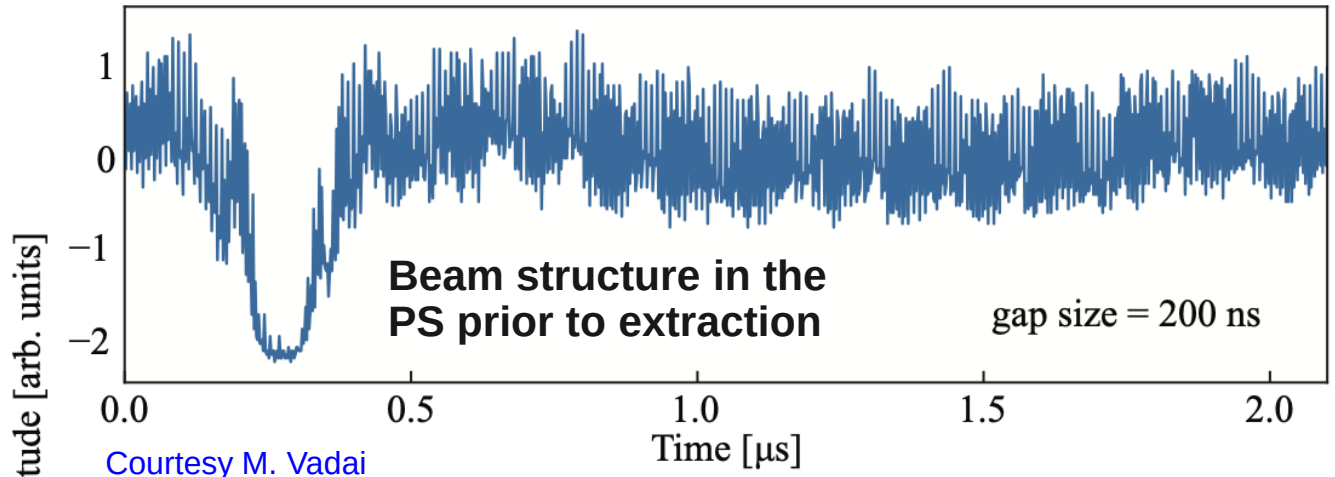
Standard longitudinal manipulations performed on the MTE beam



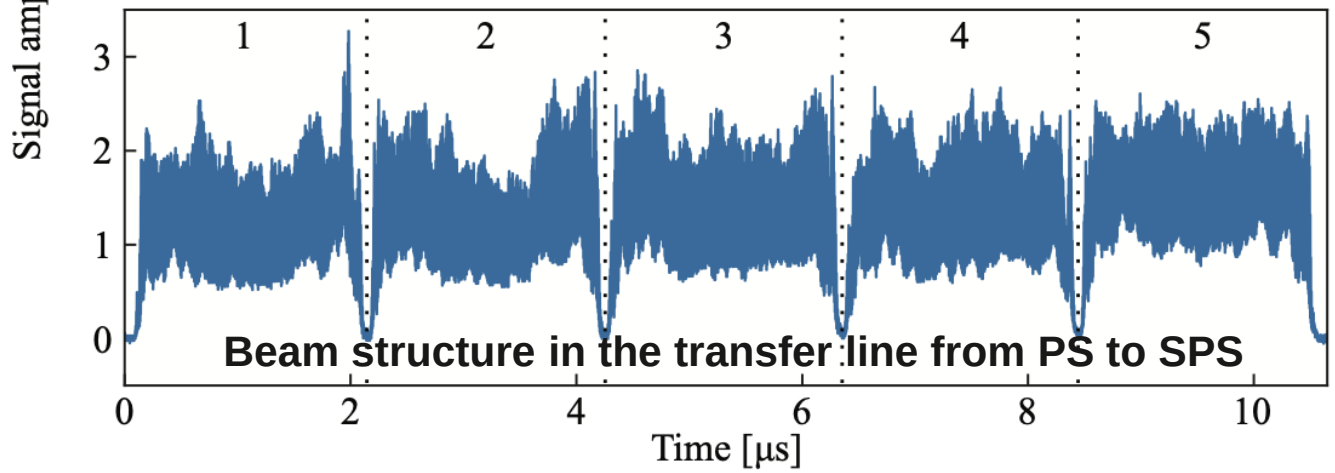
RF manipulations after splitting

- **Reduction of voltage on main RF system.**
- **Increase of voltage in wideband cavity.**
- **Partial bunching at 200 MHz (for SPS transfer).**

Barrier bucket solves this!



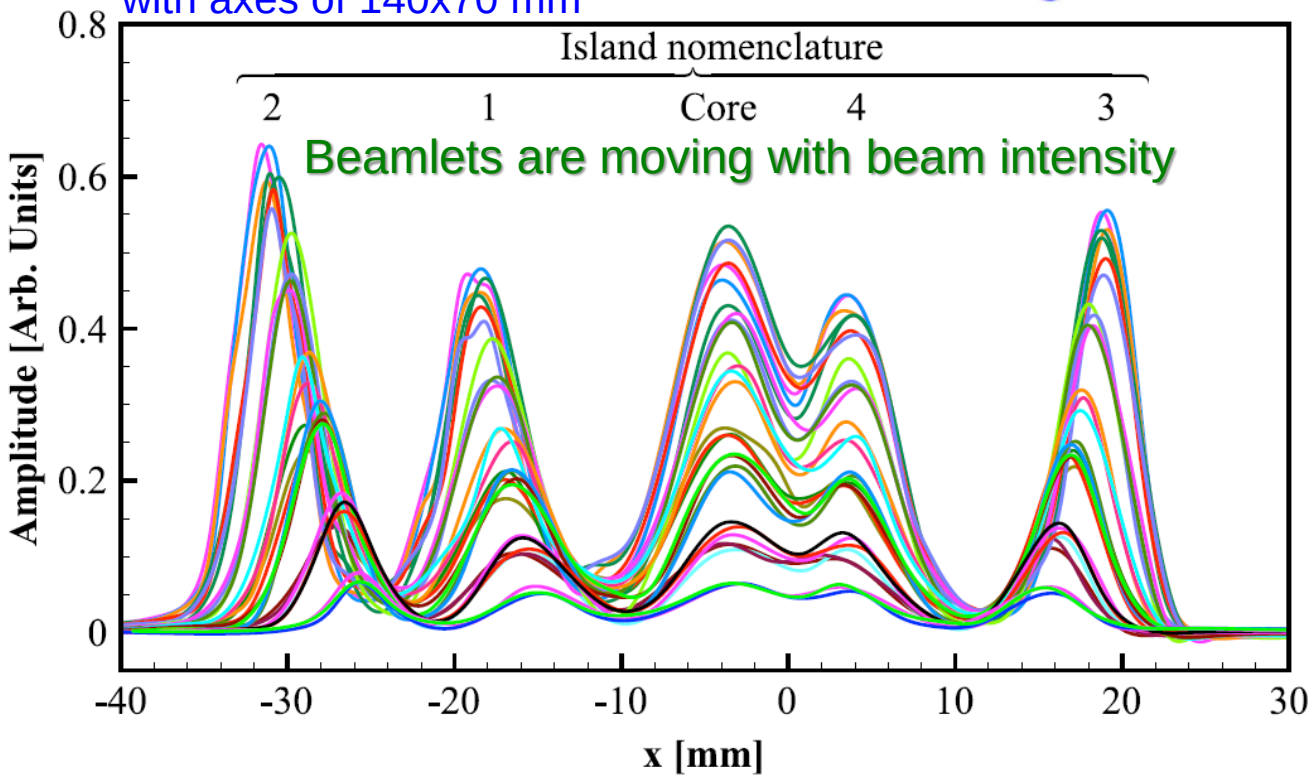
Courtesy M. Vadai



Space-charge effects: measurements

- **Experimental configuration**
  - **Magnetic status of PS frozen**.
  - Total beam intensity (N_{total}) varied between 2e12 p to 2e13 p.
  - Wire scanner used to determine beamlets properties vs beam intensity.
  - Bunched and de-bunched versions of the beam used (NB: bunching fact approx. 1.6).
- **Experimental observations**
  - Globally: all beamlets properties are **independent** on bunching state.
  - Properties of the core beam: **independent** on N_{total}.
  - Trapping fraction: **independent** on N_{total}.
  - Properties of beamlets: position **linearly dependent** on N_{total}; sigma **independent** on N_{total}.
- **Interpretation**
  - The linear ring tune is changed by the beam intensity, which moves beamlets towards high amplitudes.
  - The numerical model of the PS ring allowed “calibrating” this effect.

NB: the vacuum chamber if quasi-elliptical with axes of 140x70 mm^2

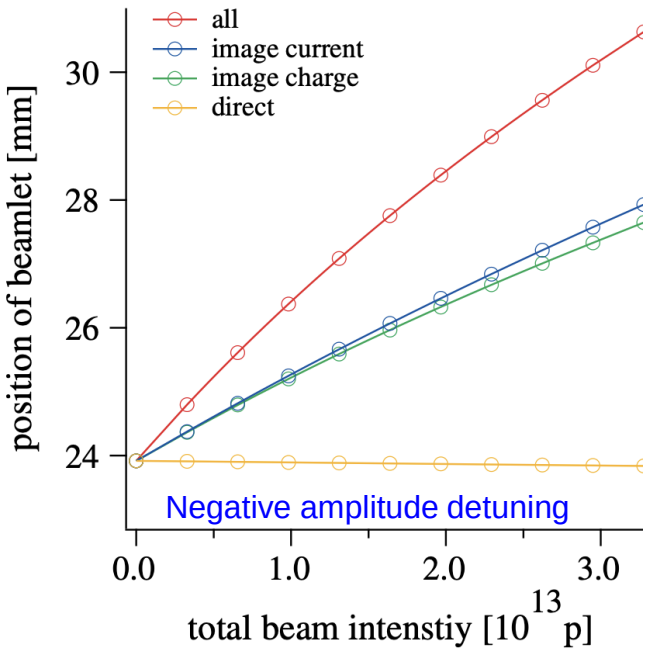


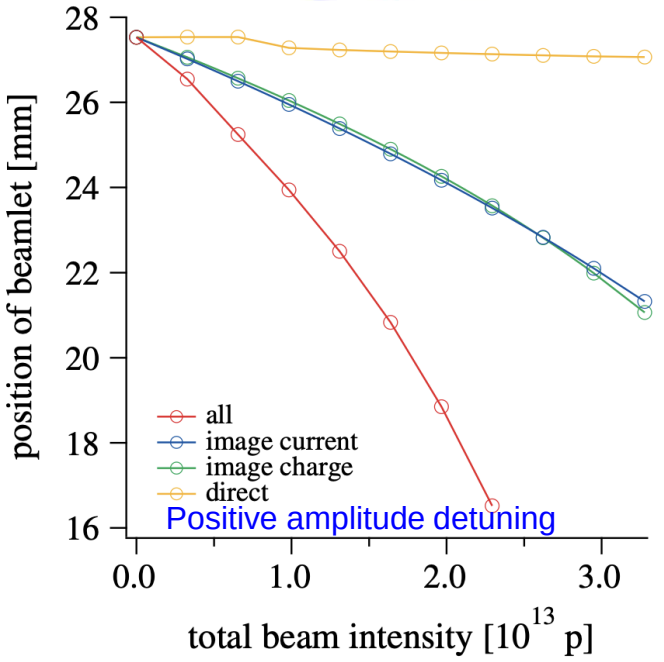
$$Q_x(N_{\text{total}}) = m_{Q_x} N_{\text{total}} + Q_{x,0}$$

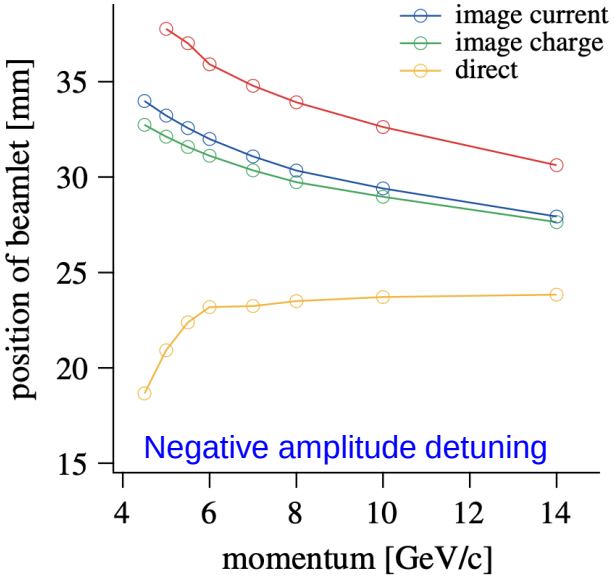
$$m_{Q_x} = (15 \pm 4) \times 10^{-7}, \quad Q_{x,0} = 0.257 \pm 0.033$$

Space-charge effects: numerical simulations

- **Simulation configuration**
  - SIMPSON code with special features added.
  - The goal is to study the behaviour of the position of the fixed points in the presence of space-charge effects.
  - Several aspects considered:
    - Interplay between space charge and non-linearities
    - Boundary conditions (parallel plates or rectangular)
    - Impact of beam energy
- **Numerical observations**
  - At 14 GeV/c **direct** effects are **independent** on N_{total}.
  - At 14 GeV/c **indirect** effects are **dependent** on N_{total}.
  - Beyond 6 GeV/c **direct** effect are **constant** vs beam energy.
  - **Indirect** effect are **decreasing** vs beam energy.
  - Small difference between parallel plate or rectangular boundaries as beamlets are not
- **Conclusions**
  - Qualitative agreement between
    experimental observations.
  - Indirect effects are the origin of the displacement observed.







More detail on space-charge simulations for MTE and plans for future studies have been given by F. O. Pancaldi in his poster (and flash talk).

Digression: crossing a 2D resonance

- **Going beyond MTE**
  - MTE and its extensions are all based on resonance crossing in a single plane, i.e. $m\ \omega_x \approx p$.
  - The transverse dynamics has two degrees of freedom, each characterised by a tune value, and a generic 2D resonance reads $m\ \omega_x + n\ \omega_y \approx p$.
  - **Adiabatic crossing** of a 2D **difference** resonance induces **redistribution** of transverse emittances.
  - The Hamiltonian of the generic system is

Amplitude-detuning terms

$$\mathcal{H}(\phi_x, J_x, \phi_y, J_y) = \omega_x J_x + \omega_y J_y + \alpha_{xx} J_x^2 + 2\alpha_{xy} J_x J_y + \alpha_{yy} J_y^2 + \\ + G J_x^{m/2} J_y^{n/2} \cos(m\phi_x - n\phi_y)$$

Resonant term

  - The following canonical transformation

$$J_x = mJ_1\,, \qquad \phi_1 = m\phi_x - n\phi_y\,,$$
$$J_y = J_2 - nJ_1\,, \quad \phi_2 = \phi_y\,,$$

  - Allows recasting the Hamiltonian in the following from:

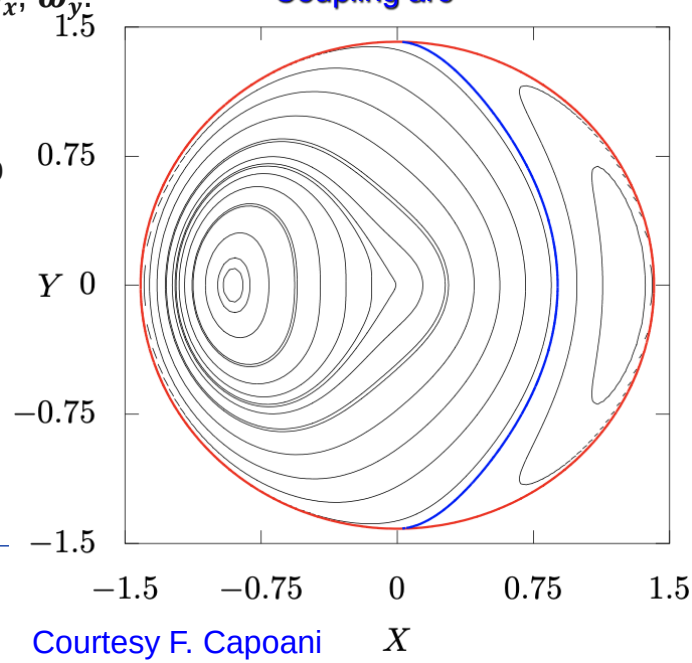
$$\mathcal{H}(\phi_1, J_1) = (\delta + \alpha_{12} J_2)\, J_1 + \alpha_{11} J_1^2 + G(mJ_1)^{\frac{m}{2}} (J_2 - nJ_1)^{\frac{n}{2}} \cos\phi_1 + \left[\omega_y J_2 + \alpha_{22} J_2^2\right]$$

Observations

- $\delta = m\ \omega_x - n\ \omega_y$ distance from resonance.
- α_{11}, α_{12} are function of amplitude-detuning.
- J_2 is a constant of motion.
- The dynamics is confined to the circle $J_1 < J_2/n$

Consequences

- Let us consider the case $m=1$, $n=2$.
- Hyperbolic fixed points exist, as well as separatrices (the coupling arc).
- These structures depends on ω_x, ω_y.
- Varying δ, the coupling arc sweeps the phase space.
- When δ changes from $-\delta_{max}$ to δ_{max} then
  - $J_{y,f} = n/m\ J_{x,i}$
  - $J_{x,f} = m/n\ J_{y,i}$
- **Redistribution of actions**

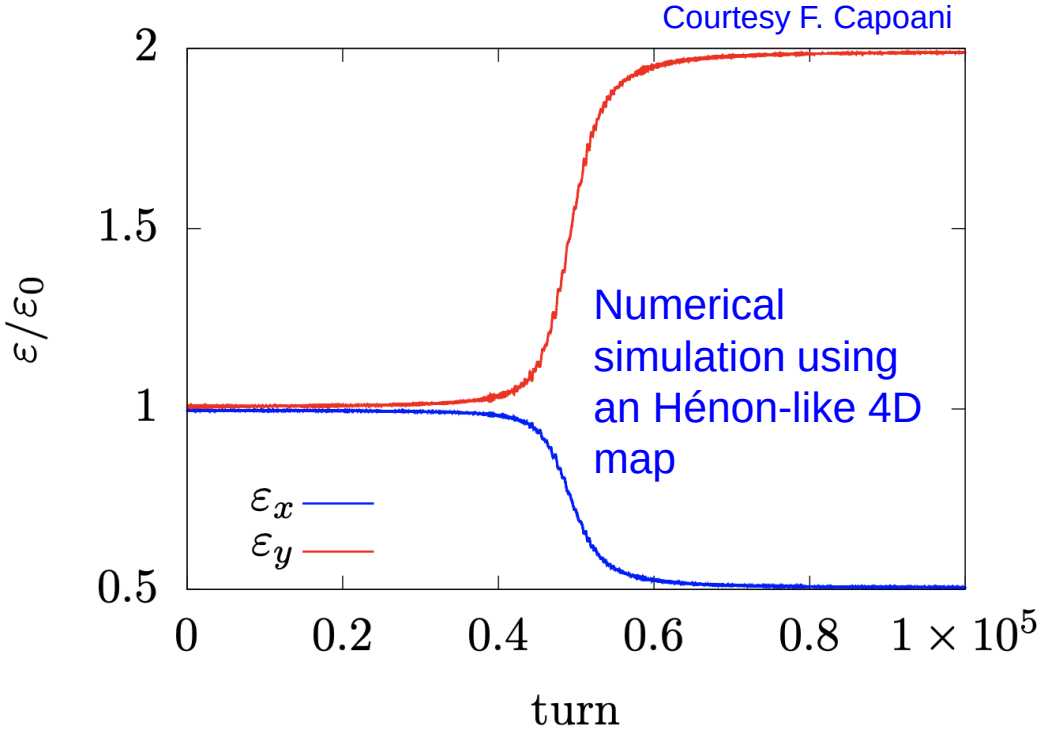


Courtesy F. Capoani

Digression: crossing a 2D resonance

- **Numerical simulations**
  - Numerical simulations using maps confirm the analytical predictions.

Courtesy F. Capoani



  - **Higher-order resonances have b… separatrices generate more phas… might affect redistribution of emi…**

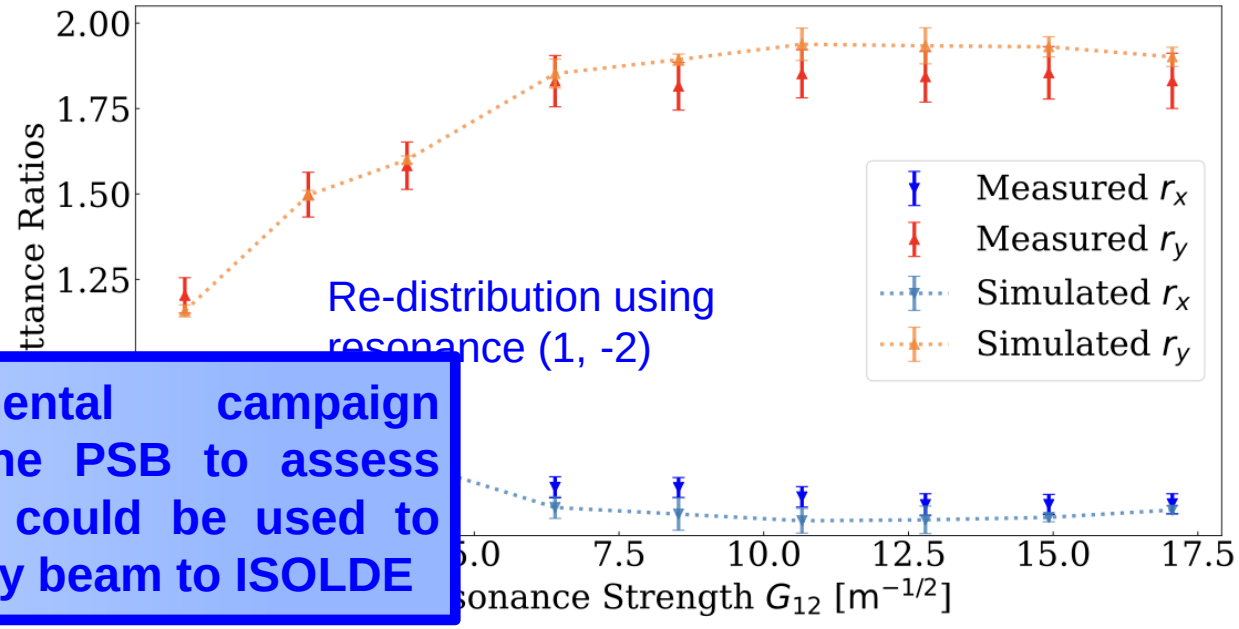
How to apply this to accelerator rings

- When moving from 4D to 6D: tune modulation due to chromaticities that generates trapping and de-trapping phenomena.
- When adding space-charge effects: new non-linearities adding to those that excite resonance. Additional tune spread that might prevent particle from crossing the resonance.

Measurements at CERN PS

- Single bunch at injection energy.
- Sextupoles (normal and skew) and octupoles used to excite a third-order resonance.

Courtesy O. Naumenko



Extended experimental campaign performed also at the PSB to assess whether the method could be used to deliver a high-intensity beam to ISOLDE

Conclusions and outlook

- **MTE has proven to be a challenging and useful beam manipulation, also in terms of space-charge effects.**
- **Studies are on-going to assess adiabatic invariance in the presence of space charge and possibly when it fails.**
- **Crossing a 2D non-linear resonance has been recently proposed to re-distribute transverse emittances. Experimental tests at the CERN PS have shown that it nicely works even in the presence of space charge. Numerical simulations are on-going.**
- **In recent years, intense efforts have been devoted to study novel beam manipulations based on single-particle non-linear dynamics, such as MTE, MTI, transition jumping in stable islands, halo cleaning using stable islands, cooling of annular distributions using stable islands, slow extraction with stable islands and crystal…**

could any of these approaches be profitable also for beams under the influence of space charge?

Thank you for your attention!!!



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