

Avoiding Instabilities and Resonances with Circular Modes

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From Flat Beams to Circular Modes

- **Collider motivation.** Flat beams offer opportunities to improve luminosity within collider constraints. (Burov, 2013)

$$\mathcal{L} = \frac{f_{\text{coll}} N_p^2}{4\pi \sigma_x^* \sigma_y^*} \quad \text{identical, head-on Gaussian beams}$$

- **Low-energy challenge.** Strong space charge can limit the transport of spatially flat beams.
- **Circular-mode approach.** Retain intrinsic flatness, $\varepsilon_1 \gg \varepsilon_2$, while maintaining a round transverse beam profile. (Burov et al., 2002; Burov, 2013)
- **Recent progress.** Simulations demonstrate transport and acceleration of circular-mode beams in suitable coupled lattices, including strong space-charge regimes. (Gilanliogullari et al., 2026)

This talk: Consequences for tune footprints and resonance excitation.

Motivation

- Space charge produces amplitude-dependent tune shifts in nonuniform beams, with particularly strong constraints in the narrow plane of spatially flat beams.
- Large tune footprints can overlap resonances; the resulting beam degradation also depends on the strength of the resonant excitation.
- Circular-mode beams combine a nearly round spatial distribution with intrinsic flatness: $\varepsilon_2 \ll \varepsilon_1$.

Can transverse-mode control improve resonance tolerance?

Can we control the space-charge tune footprint and weaken the excitation of selected resonances?

What are Circular Modes?

- Coupled transverse motion is a superposition of two normal modes. (Lebedev and Bogacz, 2010)

$$\vec{z} = \text{Re} \left(\sqrt{2J_1} \vec{v}_1 e^{-i\psi_1} + \sqrt{2J_2} \vec{v}_2 e^{-i\psi_2} \right),$$

$$\vec{v}_1 = \begin{pmatrix} \frac{\sqrt{\beta_{1x}}}{-i(1-u)+\alpha_{1x}} \\ \sqrt{\beta_{1x}} \\ \sqrt{\beta_{1y}} e^{i\nu_1} \\ -\frac{i u + \alpha_{1y}}{\sqrt{\beta_{1y}}} e^{i\nu_1} \end{pmatrix}, \quad \vec{v}_2 = \begin{pmatrix} \frac{\sqrt{\beta_{2x}} e^{i\nu_2}}{-i u + \alpha_{2x}} \\ \sqrt{\beta_{2x}} \\ \sqrt{\beta_{2y}} \\ -\frac{i(1-u)+\alpha_{2y}}{\sqrt{\beta_{2y}}} e^{i\nu_2} \end{pmatrix}.$$

- At a field-free waist, a circular-mode basis is obtained with

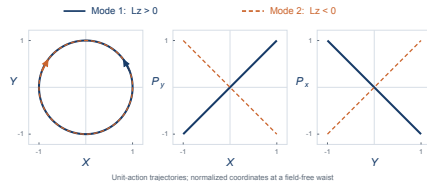
$$u = \frac{1}{2}, \quad \nu_1 = \nu_2 = \frac{\pi}{2}, \quad \alpha_{il} = 0, \quad \beta_{il} = \beta_c.$$

The two modes have equal horizontal and vertical amplitudes, with opposite senses of rotation.

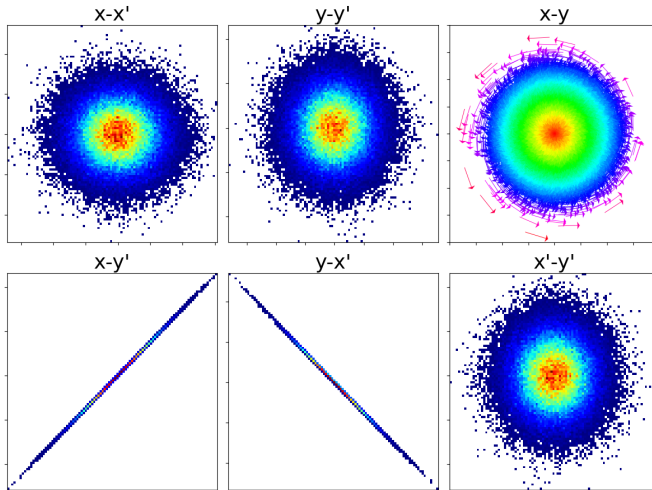
Angular momentum: Substitution cancels the mixed-mode terms, leaving

$$\ell_z \equiv \frac{L_z}{p_0} = xy' - yx' = J_1 - J_2$$

Two pure circular modes



What are Circular Modes?



How to Preserve Circular Modes?

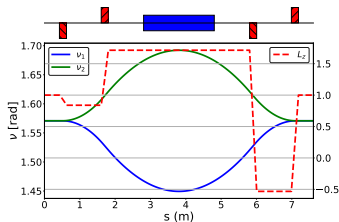
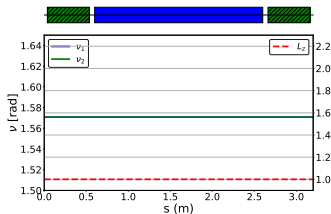
- For linear map $\mathcal{M}$, and S the symplectic unit matrix:

$$L = \begin{pmatrix} 0 & S \\ -S & 0 \end{pmatrix}, \quad \Rightarrow \mathcal{M}^T L \mathcal{M} = L$$

- The linear transfer map:

$$\mathcal{M} = R(\theta) \begin{pmatrix} T & 0 \\ 0 & T \end{pmatrix}.$$

- For quadrupole based periodic cells, the symmetry is broken.
- Mirror symmetric designs: **Angular momentum is conserved discretely.**
- Solenoids, indexed dipoles. **Angular momentum is conserved continuously.**



Single Particle

Sextupole Resonances and Mode Actions

The cubic potential generates products of the two mode amplitudes:

$$V_s = \frac{S}{3}(x^3 - 3xy^2), \quad \mathbf{r}_\perp = \sqrt{J_1} \mathbf{f}_1 + \sqrt{J_2} \mathbf{f}_2.$$

$\mathbf{r}_\perp = (x, y)^T$; $\mathbf{f}_i$ contain the mode optics and phases.

Leading third-order harmonics

$$J_i \sim \varepsilon_i, \quad \rho = \varepsilon_2/\varepsilon_1 = 0.01$$

Resonance family	Action factor	Relative action factor
$3Q_1 = p$	$J_1^{3/2}$	1
$2Q_1 \pm Q_2 = p$	$J_1 \sqrt{J_2}$	$\rho^{1/2} = 0.1$
$Q_1 \pm 2Q_2 = p$	$\sqrt{J_1} J_2$	$\rho = 0.01$
$3Q_2 = p$	$J_2^{3/2}$	$\rho^{3/2} = 0.001$

Action factors normalized by $\varepsilon_1^{3/2}$; lattice coefficients factored out.

$$\mathcal{H}_{\text{res}} = 2|h_{03}|J_2^{3/2} \cos \Theta, \quad \dot{J}_2 = 6|h_{03}|J_2^{3/2} \sin \Theta.$$

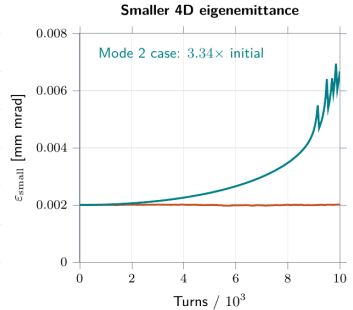
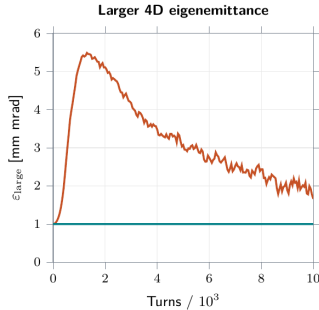
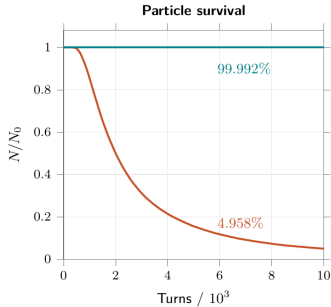
For $3Q_2 = p$: h_{03} is the resonance coefficient, Θ its phase, and $\dot{J}_2 = dJ_2/d\theta$ with ring azimuth θ .

Tracking Results

Third-integer resonance: the populated mode matters

No space charge | $\varepsilon_1/\varepsilon_2 = 500$ | $N_0 = 50\,000$ | 10 000 turns

— Mode 1 resonant — Mode 2 resonant



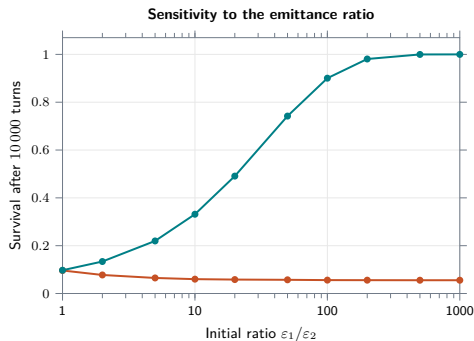
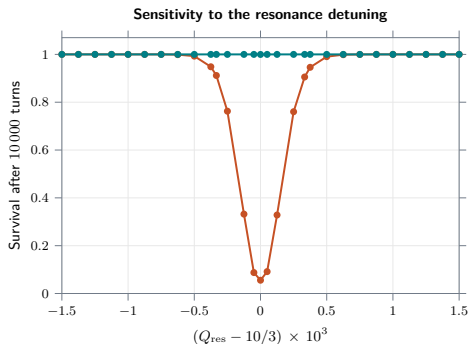
$Q_{\text{res}} = 10/3$, $Q_{\text{other}} = 3.945$; $\beta = 10$ m; $K_2 L = 0.1 \text{ m}^{-2}$; aperture radius $a = 30$ mm.
Emittances describe the particles still inside the aperture at each turn.

Tracking Results

The benefit depends on tune, emittance ratio, and tracking time

No space charge | 4000 particles per point | Lines connect sampled working points

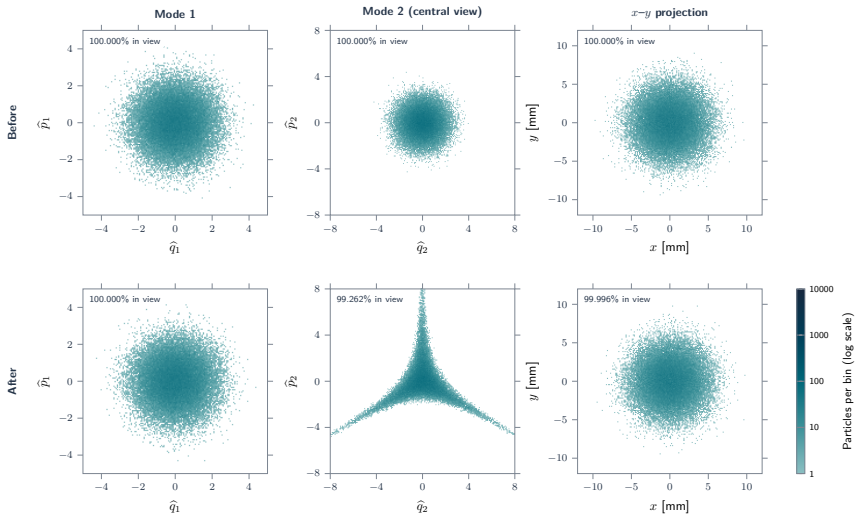
— Mode 1 resonant — Mode 2 resonant



Left: ratio 500. Right: exact $Q_{\text{res}} = 10/3$, fixed $\varepsilon_1 + \varepsilon_2 = 1.002$ mm mrad.

Both scans keep $\beta = 10$ m, $K_2 L = 0.1$ m⁻², $a = 30$ mm and $Q_{\text{other}} = 3.945$.

Tracking Results



Resilience

- Coupling the lattice for circular modes changes the RDT structure.
- The intrinsic flatness suppresses the resonance strength for the smaller mode.
- The resonance effects are present but suppressed by the dominant mode.

Space-Charge Effects

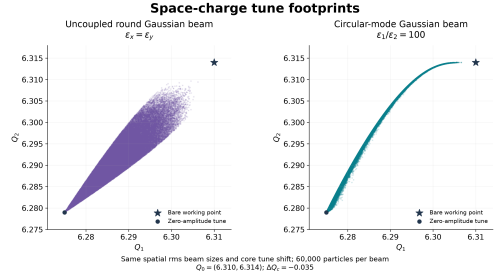
Space Charge Effects

- The Hamiltonian

$$\mathcal{H} = Q_1 J_1 + Q_2 J_2 + V_{sc}(J, \psi; s), \quad \Delta Q_i = \frac{\partial \mathcal{H}}{\partial J_i}.$$

- Here, V_{sc} is the Gaussian potential and ΔQ_i the incoherent tune shifts.
- The space-charge induced tune spreads for circular mode approximation:

$$\Delta Q_i = \Delta Q_c \int_0^1 e^{-t(u_1+u_2)} \left[I_0(\xi) - \sqrt{\frac{u_j}{u_i}} I_1(\xi) \right] dt$$



From Tune Spread to Resonance Excitation

How does circular-mode symmetry constrain space-charge-driven coupling between the modes?

Montague Resonance and Circular Modes

A round space-charge potential contains a fourth-order term:

$$V_4 = c_4(s)(x^2 + y^2)^2.$$

Planar modes: difference coupling

The x^2y^2 term produces

$$V_M \propto J_x J_y \cos[2(\psi_x - \psi_y)].$$

Near the Montague resonance,

$$\boxed{2Q_x - 2Q_y \simeq 0},$$

this term can exchange action between planes:

$$\dot{J}_x = -\dot{J}_y.$$

Hofmann and Franchetti, 2006.

Circular modes: sum-phase dependence

For the circular-mode coordinates,

$$r^2 = 2\beta_c \left[J_1 + J_2 + 2\sqrt{J_1 J_2} \sin(\psi_1 + \psi_2) \right].$$

Hence a radial V_{sc} depends on $\psi_1 + \psi_2$, giving

$$\dot{J}_1 = \dot{J}_2,$$

and therefore

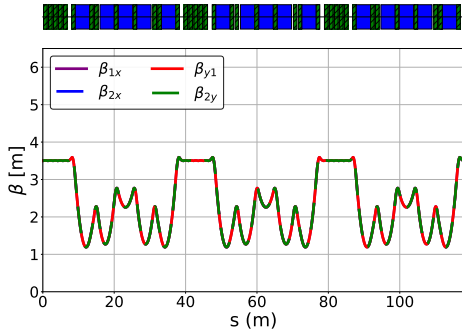
$$\boxed{J_1 - J_2 = \ell_z = \text{const.}}$$

Consequence of rotational symmetry

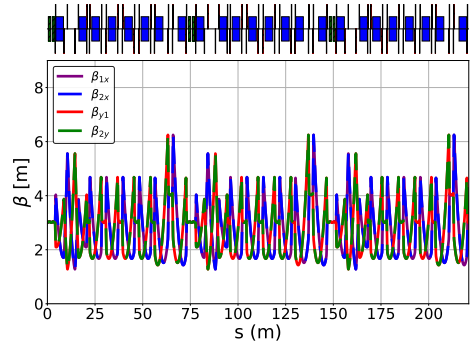
The circular-mode difference harmonic $\cos[2(\psi_1 - \psi_2)]$ is absent, even for $\varepsilon_1 \gg \varepsilon_2$. Sum-phase resonances remain allowed.

Tracking

Solenoid Based Ring

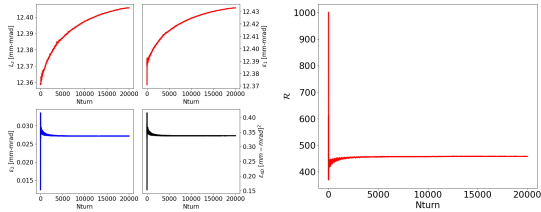


Quadrupole Based Ring



Tracking

Solenoid Based Ring



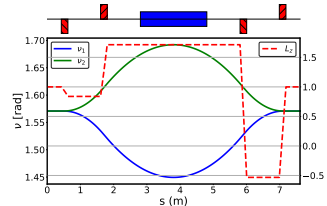
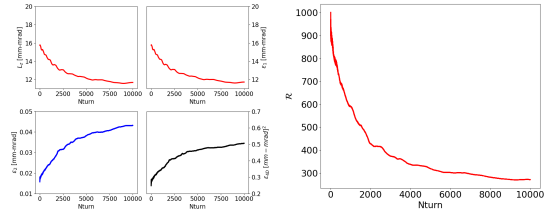
- The eigenmode emittances:

$$\epsilon_1 = L_z \left(1 + \frac{\epsilon_4 D}{L_z^2}\right), \quad \epsilon_2 = \frac{\epsilon_4 D}{L_z}$$

- In quadrupole based: $V_{sc}(r) \mapsto V_{sc}(r, \phi)$:

- The Montague resonance is back on the table: $2Q_1 - 2Q_2 = 0$
- Demagnetization.

Quad Based Ring



Conclusion

- Circular modes are great candidates for enabling flat beams at high energies.
- Show resilience to resonance due to mode suppression.
- Smaller incoherent tune footprint.

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Discussion Q

- **Fun Question:** Everywhere we look we see 3D, what is an object that you see everyday life that is perfectly 2D.
- Are there benefits for collective effects?

Mitigation of Space-Charge-Driven Resonance and Instability in High-Intensity Linear Accelerators via Beam Spinning

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Thank you

BackUP

Tune Spread from Tracking

