Analysis of adiabatic trapping and transport phenomena for an accelerator model with space-charge forces

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Introduction

Multi-Turn Extraction (MTE) [1] is a technique used to split beams in the transverse plane that has undergone experimental validation at the CERN Proton-Synchrotron (PS). Experimental measurements [2] showed peculiar behaviour that could be explained by space-charge effects. MTE is based on the theory of *adiabatic invariance* [3], which was developed for two-dimensional (one degree-of-freedom) Hamiltonian systems. Its applicability to four-dimensional symplectic maps has been confirmed via single-particle tracking [4]. This poster presents simulations of a beam undergoing adiabatic manipulations, with space-charge effects computed in the Vlasov approximation.

Model

- Fully coherent beam, nonrelativistic tranverse motion, non-collisional approx. → 2D Vlasov equation ($\mathbf{x} := (x, y)$, $\mathbf{x}' := d\mathbf{x}/ds$, $\mathbf{F}$ = external force)

$$\left[\frac{\partial}{\partial t} + v\,\mathbf{x}'\cdot\frac{\partial}{\partial \mathbf{x}} + \frac{1}{mv}\left(\mathbf{F} - \frac{e}{\gamma^2}\nabla\phi\right)\cdot\frac{\partial}{\partial \mathbf{x}'}\right]\rho = 0$$

- Numerical integration: leapfrog algorithm [5]

$$\mathcal{H} = \mathcal{H}_M + \mathcal{H}_{SC} \qquad D_{\mathcal{H}}(f) := \{f, \mathcal{H}\}_{P.B.}$$
$$\implies e^{\Delta t D_{\mathcal{H}}} = e^{\frac{\Delta t}{2} D_{\mathcal{H}_M}}\, e^{\Delta t D_{\mathcal{H}_{SC}}}\, e^{\frac{\Delta t}{2} D_{\mathcal{H}_M}} + \mathcal{O}\left(\Delta t^3\right)$$

  with $\mathcal{H}_M$ = magnetic element and $\mathcal{H}_{SC}$ = "space-charge element"

- Discretisation of ρ on a grid: PIC with Triangle-Shaped Cloud [5]

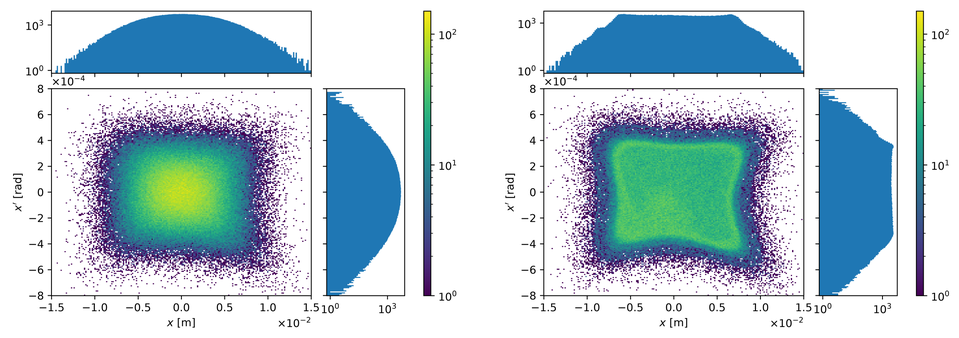
$$S(x) := \left(\left[1/2 - {}^{x}/_{\Delta x}\right]^2/2,\ {}^3/_4 - {}^{x^2}/_{\Delta x^2},\ \left[1/2 + {}^{x}/_{\Delta x}\right]^2/2\right) \qquad x \in\ ]-{}^{\Delta x}/_2,\ {}^{\Delta x}/_2[$$

- Solution of the discretised $\nabla^2\phi = -\rho/\epsilon_0$: spectral method [5] (here shown in 1D), guarantees $\phi = 0$ on the rectangular boundary

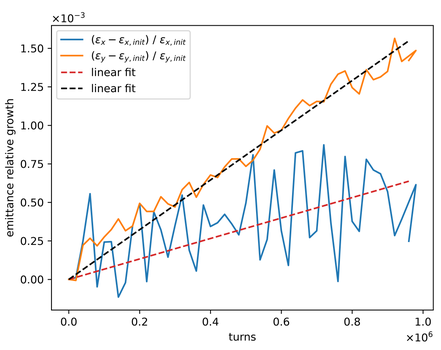
$$\widetilde{\rho}_k := \frac{2}{N}\sum_{j=1}^{N-1}\rho(jL/N)\sin\frac{\pi jk}{N} \qquad \rho(0) = \rho(L) = 0$$

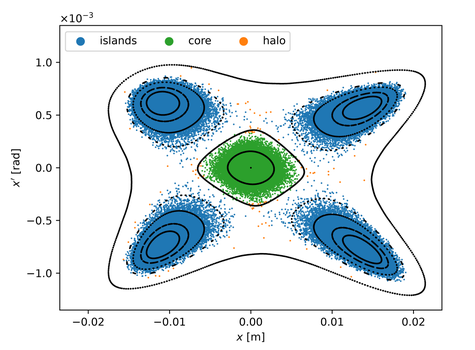
$$\implies \phi(jL/N) \simeq \frac{L^2}{\epsilon_0\pi^2}\sum_{k=1}^{N-1}\frac{\widetilde{\rho}_k}{k^2}\sin\frac{\pi jk}{N}$$

- Fully parallel code, using `C++20`. Performance portability provided by the `alpaka` library [6] (also used in the CMS experiment): the code can be compiled for CPU with any parallelisation library (e.g. `OpenMP`, `TBB`) or for any GPU (Nvidia, AMD)



Initial conditions of the simulation: obtained after relaxation starting from a gaussian adapted to the linear accelerator.
Left: no space-charge forces. **Right**: 3 times the nominal intensity (see below for def.).

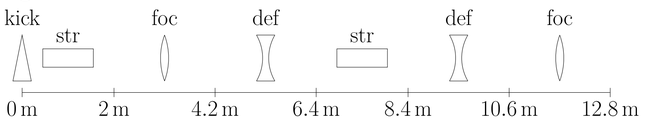




Left: Numerical validation: emittance growth over time for a beam with 5 times the nominal intensity (see below for def.), with constant magnetic fields.
Right: Theoretical validation: beam final state (color-coded) for MTE splitting done over 40 000 turns for a flat beam ($y_j(s) = y_j'(s) = 0\ \forall j,\ \forall s$) without space-charge forces. See below for the simulation parameters.

Results

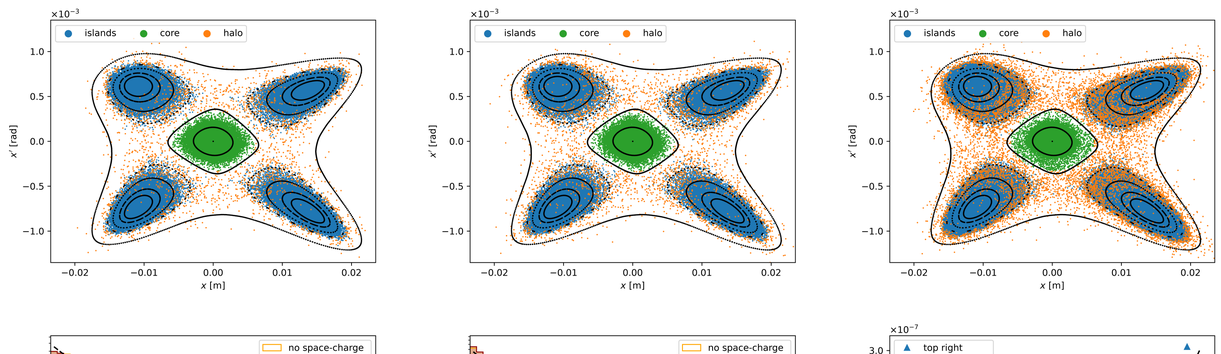
- Single-cell PS model:



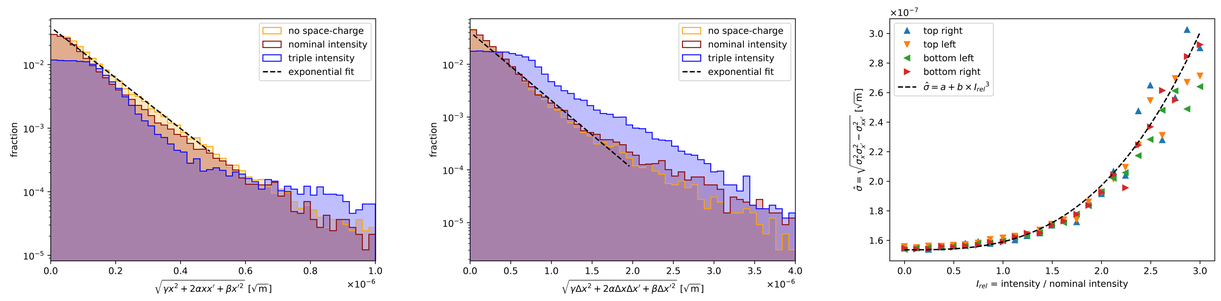
- Nominal (i.e. standard operating) beam intensity: 3×10^{13} total protons. Longitudinal density approximated as constant: 4.78×10^9 protons per meter
- Simulation parameters ($K_{1,\ldots}$ are slowly changed):

$K_{1,\mathrm{foc,init}}$	$0.057\,21\ \mathrm{m}^{-2}$	$\epsilon_{\mathrm{rms},x,\mathrm{init}}$	6.7×10^{-7}m
$K_{1,\mathrm{foc,fin}}$	$0.056\,66\ \mathrm{m}^{-2}$	$\epsilon_{\mathrm{rms},y,\mathrm{init}}$	3.35×10^{-7}m
$K_{1,\mathrm{def,init}}$	$-0.057\,27\ \mathrm{m}^{-2}$	perveance	1.39×10^{-8}
$K_{1,\mathrm{def,fin}}$	$-0.056\,90\ \mathrm{m}^{-2}$	grid cells	32 768
$\widetilde{K}_{2,\mathrm{kick}}$	$2.307\ \mathrm{m}^{-2}$	sim. particles	327 680
$\widetilde{K}_{3,\mathrm{kick}}$	$-217.3\ \mathrm{m}^{-3}$	chamber size	136×70 mm

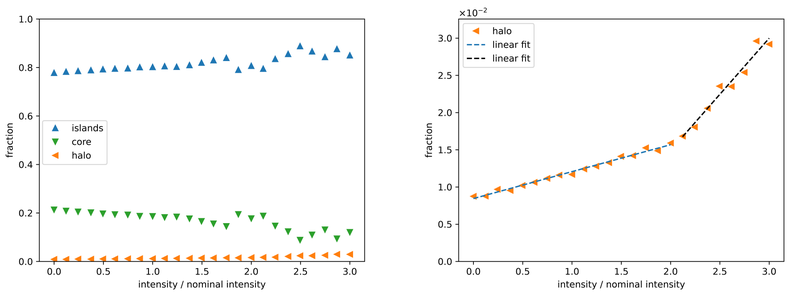
- Core/islands/halo classification based on the tune [7]: given ν_x calculated over 4096 turns, a particle is
  - ⋆ island if $|\nu_x - 0.25| < 10^{-6}$
  - ⋆ core if not island and $\sqrt{\gamma_x x^2 + 2\alpha_x x x' + \beta_x x'^2} < 10^{-6}\sqrt{\mathrm{m}}$ ($\alpha_x, \beta_x, \gamma_x$ being the Twiss parameters)
  - ⋆ halo otherwise
- Islands' area calculated as $\sqrt{\sigma_x^2\sigma_{x'}^2 - (\sigma_{xx'})^2}$ ($\sigma_{\ldots}$ being the covariances)



Beam final state (color-coded) for MTE splitting done over 40 000 turns.
Left: no space-charge forces. **Centre**: nominal intensity (see text for def.). **Right**: 3 times the nominal intensity.



Beam final state for MTE splitting done over 40 000 turns.
Left, **centre**: emittance distribution of the core, top-right island respectively, at different intensities. $(\Delta x, \Delta x')$ are relative to the island centre. **Right**: island area (see text for def., color-coded) vs. intensity.



Beam final state for MTE splitting done over 40 000 turns.
Left: islands, core, halo population (color-coded) vs. intensity. **Right**: zoom of the left plot.

Conclusions

We show that the particle distribution in the core and the islands changes as the intensity increases, becoming flatter and wider. The fraction of trapped particles increases with intensity, but the same happens for the halo. In particular, we show that there are two regimes of linear growth of the halo, the threshold being at approximately twice the operational beam intensity in the PS. These results show that space charge is not seriously affecting MTE around nominal beam intensities. The situation changes for much higher beam intensities. The next steps of this study are simulations using the real PS lattice and the usage of elliptical boundary conditions in the Poisson equation. For more information about MTE, see prof. Giovannozzi's talk.

References

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