中国科学院近代物理研究所
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International MUON Collider Collaboration

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7th ICFA Beam Dynamics Mini-Workshop

# Simulation Studies of Space Charge Effects in the 6D Rectilinear Muon Cooling Channel


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Special thanks to colleagues from the *International Muon Collider Collaboration*

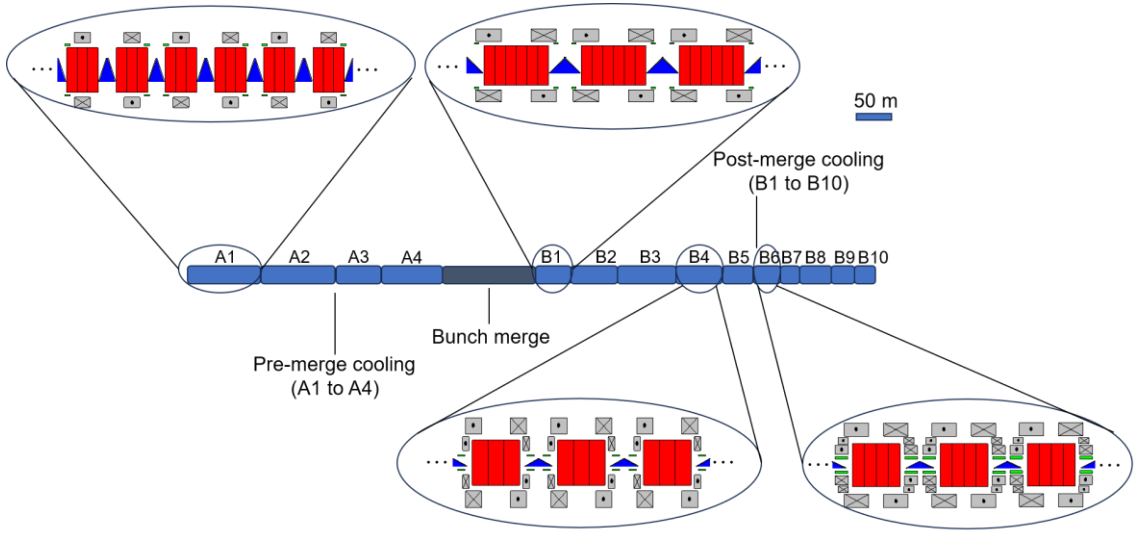

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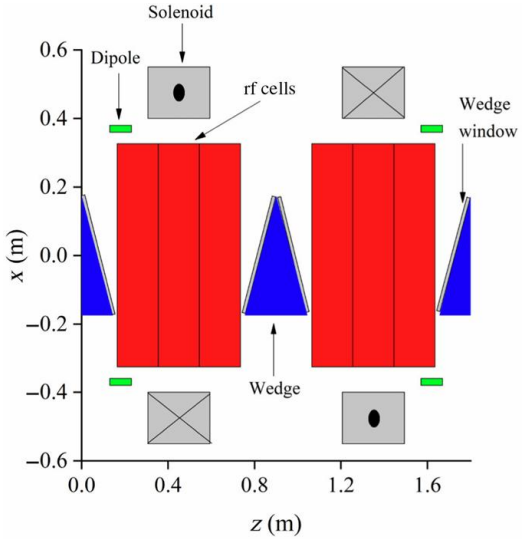

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# Overview of rectilinear cooling channel

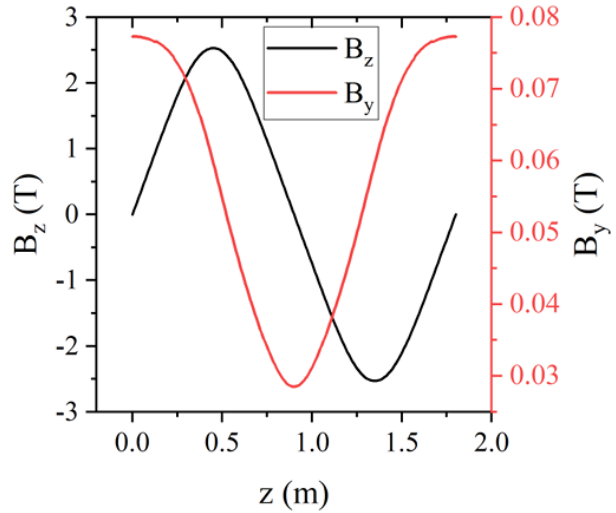
### Rectilinear cooling channel




### Rectilinear cooling cell




### Magnetic field profile




- 6D Rectilinear cooling channel is essential for the high luminosity of a muon collider.
- Wedge absorbers in dispersion-regions provide emittance exchange for 6D cooling.
- Repeated cells include solenoids with opposite polarity, RF cavities and wedge absorbers.
- Successive stages use stronger focusing and smaller equilibrium emittance; the **SC study focuses on B6–B10 cooling stages**[1] in post-merge section.


[1] R. Zhu, C. Rogers, J. Yang, H. Zhao, G. Cheng and J. Li, Design method, performance evaluation, and tolerance analysis of the rectilinear cooling channel for a muon collider, Phys. Rev. Accel. Beams 28, 041003 (2025).

# Why space charge matters?

## Late-stage cooling beam

$p_z \approx 200$ MeV/c

$N \approx (5–6) \times 10^{12}$

$\sigma_z \approx 18–19$ mm

$\varepsilon_{N,\perp} \approx 0.14–0.19$ mm

## Space-charge estimate

- Generalized perveance:

$$K = \frac{2I}{I_{A,\mu}\beta^3\gamma^3} \approx (3.7 - 4.7) \times 10^{-4}$$

- SC tune shift:

$$\frac{\Delta\nu_{\mathrm{SC}}}{\nu_{\mathrm{cell}}} = \frac{N_\mu}{\varepsilon_\perp} \frac{\beta_{\perp,\mathrm{ave}} r_\mu}{2\sqrt{2}\pi\sigma_z\beta\gamma^2} \approx 0.06 - 0.16$$

## Previous study

- Stratakis et al. (2015)[1]
- $\frac{\Delta\nu_{sc}}{\nu_{cell}} \approx 0.11$
- Late-stage Space Charge is non-negligible.

**Space charge should be noticeable in the late cooling stages (B6-B10)**

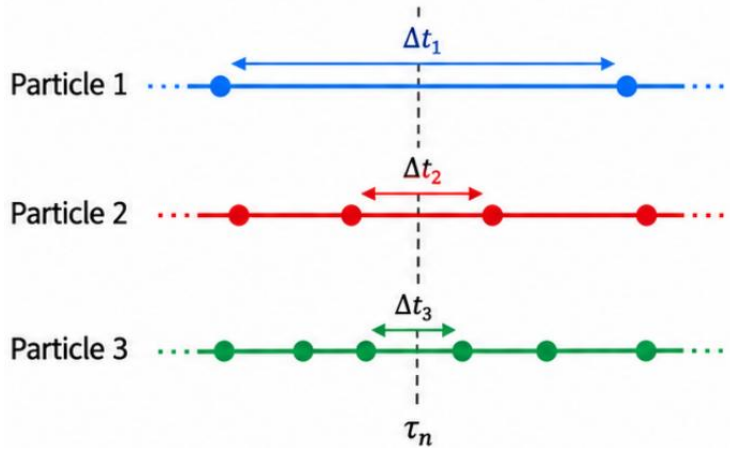
[1] D. Stratakis, R. B. Palmer and D. P. Grote, Influence of space-charge fields on the cooling process of muon beams, Phys. Rev. ST Accel. Beams 18, 044201 (2015).

# Space-charge modeling in G4beamline

## Why G4beamline / Geant4?

- Realistic particle-matter interactions are essential.
- Energy loss and multiple scattering must be preserved.
- Stochastic interaction must remain correct.

## Collective field update requires synchronized beam information




Geant4 transport proceeds with unequal natural steps.

## Why naive synchronization fails (old SC module[1] in G4bl)?

- Suspending particles for synchronization changes the Geant4 step sequence.
- Particle-material interaction can be distorted.
- This leads to **pseudo emittance growth, even without space charge field**.

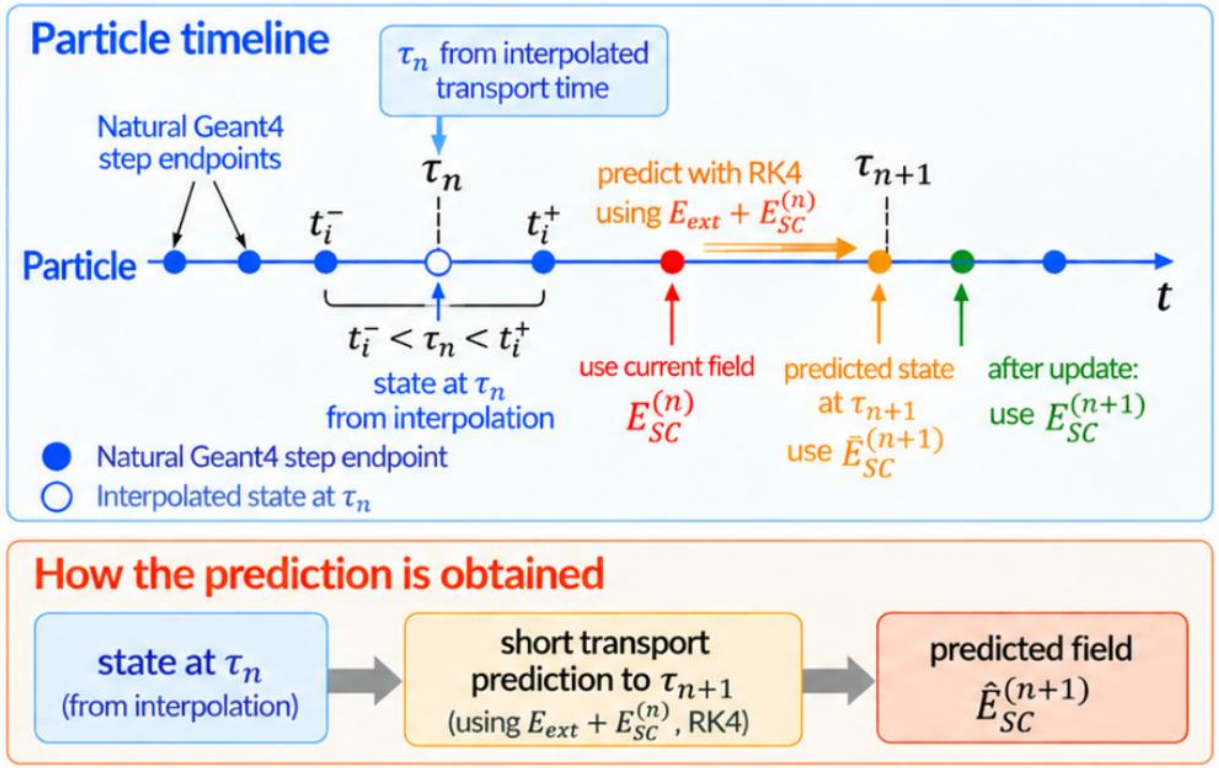
**The goal is to synchronize the space charge field update without artificially interrupting Geant4 transport**

[1] Thomas J. Roberts, G4BEAMLINE CODE DEVELOPMENT, Proceedings of IPAC2012.

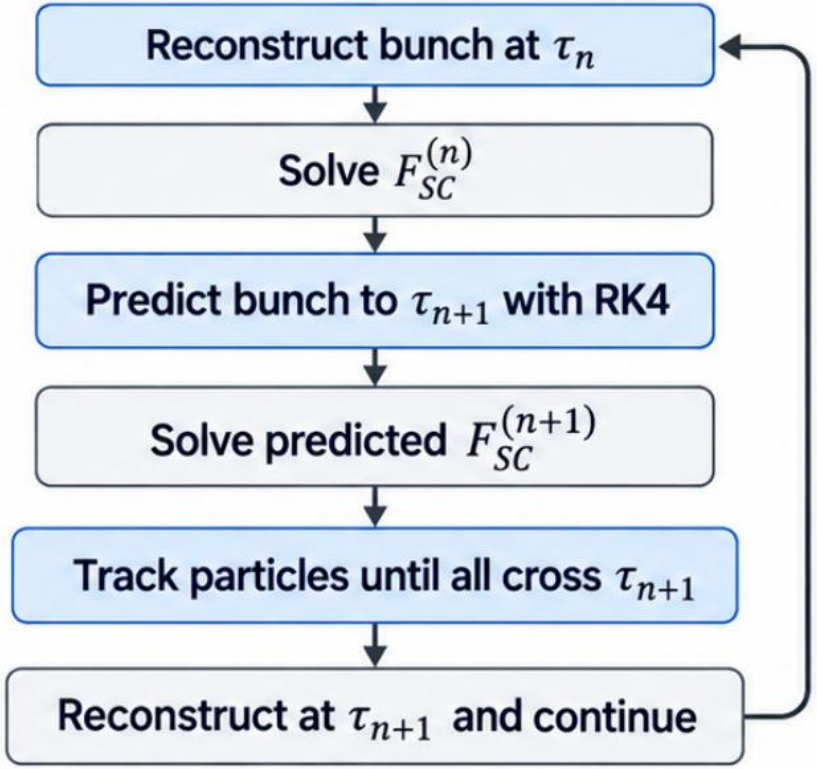
# Newly-implemented space charge module

## ➢ Transport algorithm

### Particle transport timeline




### Algorithm flow

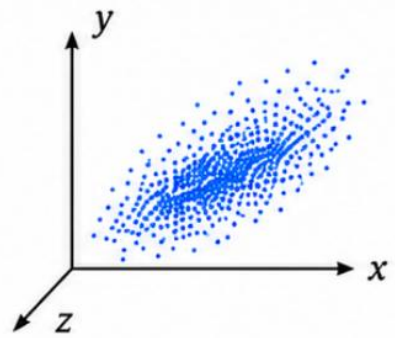



**The bunch is synchronized only for field update, and the natural Geant4 steps are preserved.**

# Newly-implemented space charge module

## ➢ Field solver algorithm

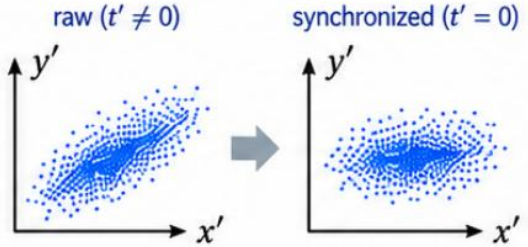
**1 Common-time virtual snapshot**




Particle distribution from PIC at a common time step.

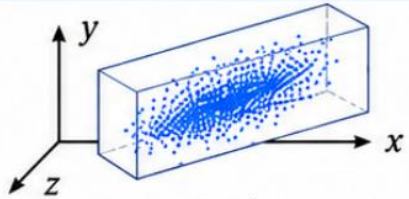
**2 Beam-frame synchronization**

$$x'_{sync} = x'_{raw} - v't'$$




Transform to the beam frame and synchronize all particles to $t' = 0$.

**3 Adaptive physical bounds**

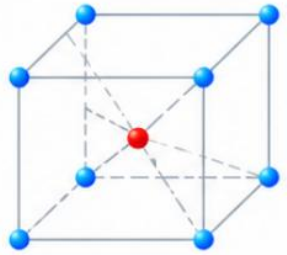



$$x_{min} = \min x'_i - \text{margin}$$
$$x_{max} = \max x'_i + \text{margin}$$
$$\Delta x = \frac{x_{max} - x_{min}}{N_x}$$

(similarly for y and z: $\Delta y = \frac{y_{max} - y_{min}}{N_y}$, $\Delta z = \frac{z_{max} - z_{min}}{N_z}$)

**margin = mesh padding,** not outlier removal

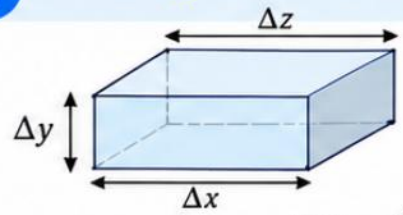
**4 3D CIC deposition**



$$Q_{ijk} \mathrel{+}= q w_x w_y w_z$$

Deposit each particle's charge to 8 neighboring nodes (CIC).

**5 Cell-integrated Green function**




$$\tilde{G}(r) = \frac{1}{\Delta x \Delta y \Delta z} \iiint_{\text{cell}} \frac{d^3 r'}{|r - r'|}$$

**Preserves near-field accuracy for highly unequal mesh scales.**

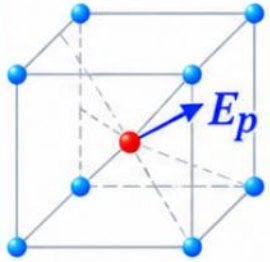
**6 FFT convolution**

$$\tilde{E}_\alpha(\mathbf{k}) = \tilde{\rho}(\mathbf{k}) \tilde{G}_\alpha(\mathbf{k})$$
$$\alpha = x, y, z$$

$$E_\alpha(\mathbf{r}) = \mathcal{F}^{-1}\left[\tilde{\rho}(\mathbf{k}) \tilde{G}_\alpha(\mathbf{k})\right]$$

Compute electric field on the mesh via FFT convolution.

**7 Field interpolation + transform back**




Interpolate $E$ from the mesh to particle positions (CIC), and transform back to the lab frame.

- Full 3D SC solver
- Handle anisotropic mesh (e.g., Δz >> Δx, Δy)

# Free-space drift benchmark

## Beam parameters:

- Species: μ+
- Real particles: $1\times10^{12}$
- Momentum: $P_z$=30 MeV/c
- Sphere beam: a=b=c=100 mm
- Ellipsoid beam: a=b=50 mm, c=200 mm

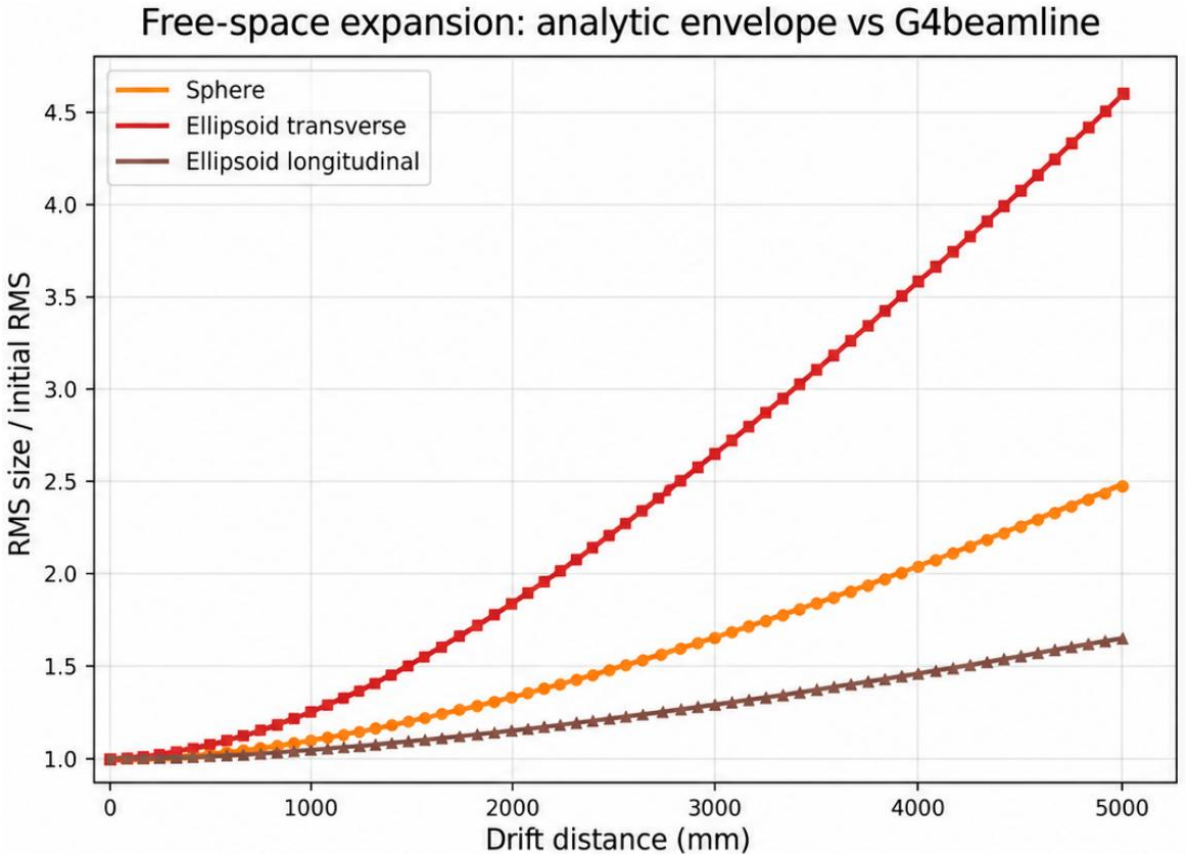
## Solver parameters:

- Grid: 128×128×128
- Macroparticles: $1\times10^{5}$
- deltaZ: 25 mm

## RMS beam envelope theory:

$$\frac{d^2a_i}{dz^2} = \frac{3N_\mu r_\mu}{\beta^2\gamma^2}\frac{A_i a_i}{abc} \quad A_i = \frac{abc}{2}\int_0^\infty \frac{ds}{(a_i^2+s)\sqrt{\prod_j (a_j^2+s)}}, \quad \sum_i A_i = 1.$$

$$\sigma_{x,y} = \frac{a,b}{\sqrt{5}}, \qquad \sigma_z = \frac{c}{\gamma\sqrt{5}}.$$




**Maximum discrepancy is around 0.5%!**

# FODO lattice benchmark

## Beam and lattice parameters:

- Species: μ+
- Real particles: $1\times10^{11}$
- Momentum: $P_z$=200 MeV/c
- Ellipsoid beam: a=30 mm, b=20 mm, c=10 mm
- FODO: $L_{cell}$=4 m, $L_{quad}$=0.4 m, $|G| = 0.3\ \mathrm{T/m}$

## Solver parameters:

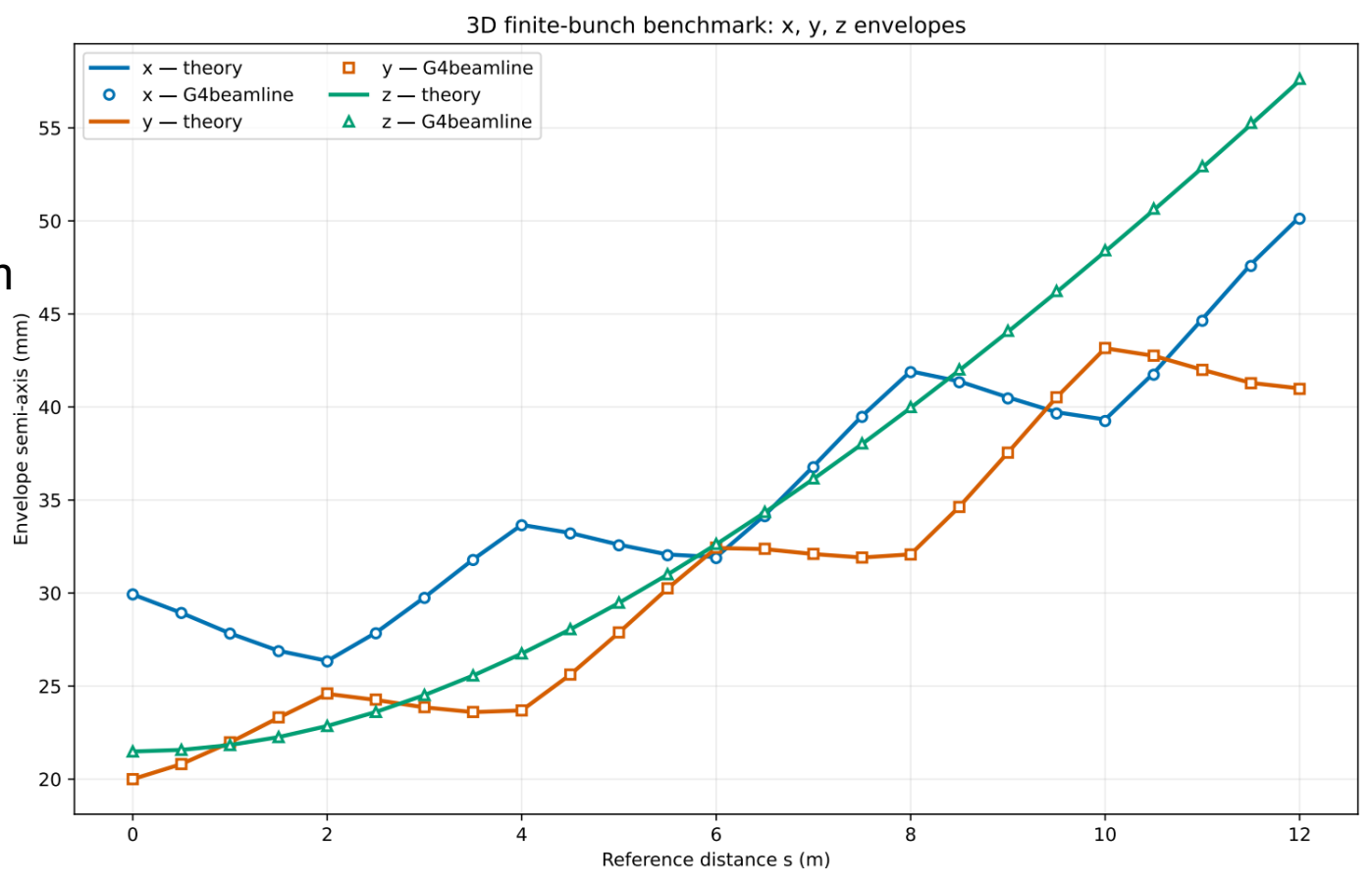
- Grid: 128×128×128
- Macroparticles: $1\times10^{5}$
- deltaZ: 25 mm

## RMS beam envelope theory:

$$a'' + k_x(s)a - \frac{3Nr_\mu}{\beta^2\gamma^2}\frac{A_x a}{abc} = 0,$$

$$b'' + k_y(s)b - \frac{3Nr_\mu}{\beta^2\gamma^2}\frac{A_y b}{abc} = 0,$$

$$c'' - \frac{3Nr_\mu}{\beta^2\gamma^2}\frac{A_z c}{abc} = 0, \quad A_x + A_y + A_z = 1$$




**Maximum discrepancy is around 0.2%!**

# Validation in the cooling channel

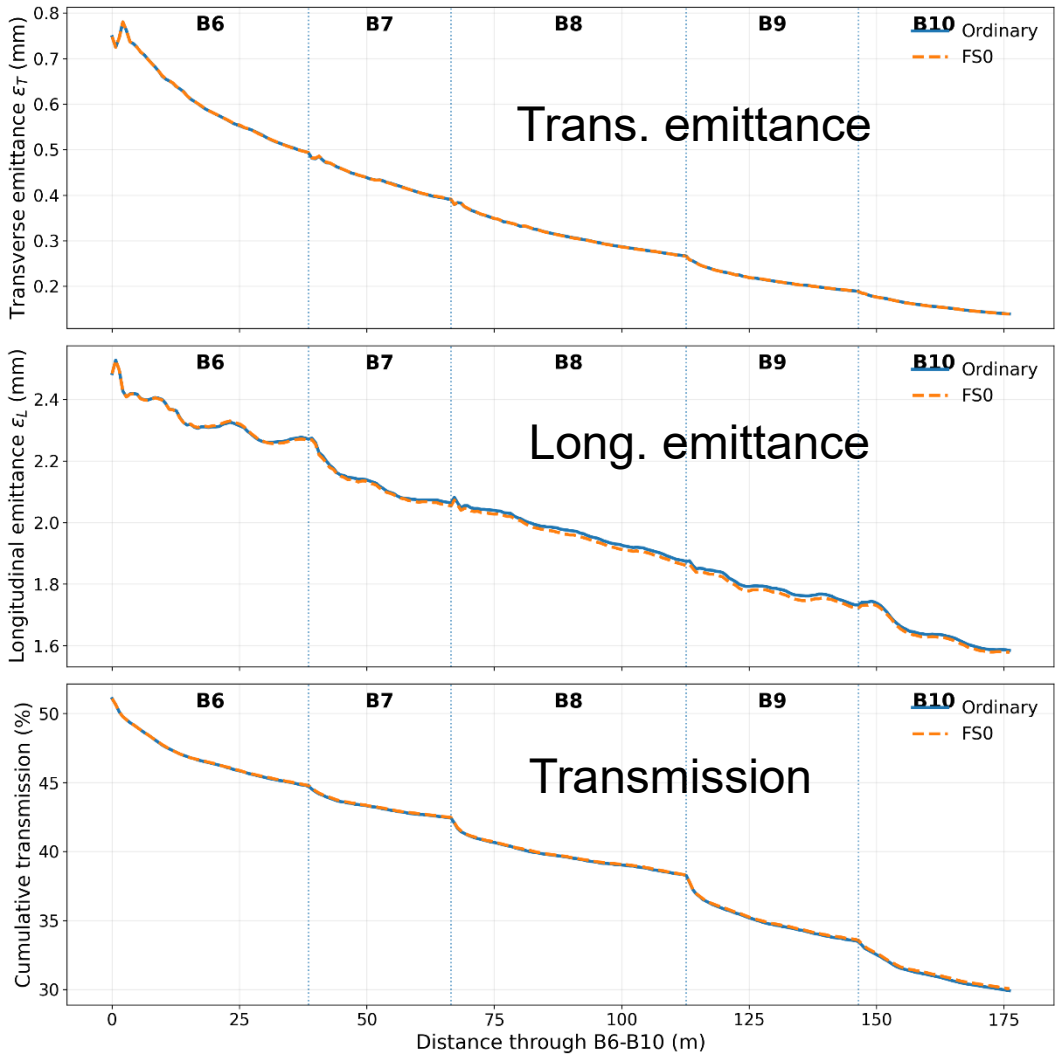
## ➢ Zero-SC transport consistency check

- Collective transport enabled, with the **space charge field switched off**.
- Compare with ordinary G4beamline transport through B6-B10.
- This checks the collective transport algorithm does not significantly perturb the Geant4 transport.
- The new collective transport algorithm introduces very small differences.

■ **At exit of B10:**

$\Delta T$ = +0.263 pp, $\Delta\varepsilon_T$ = -0.072%,

$\Delta\varepsilon_L$ = +1.35%, $\Delta\varepsilon_{6D}$ = +1.20%



# Space charge effect in B6-B10

## Simulation setup (B6-B10):

- Macro-particle number: $1.73 \times 10^5$
- Real particle number: $9.6 \times 10^{12}$ ($1.9 \times 10^{13}$ at B1)
- Average Pz ~ 200 MeV/c
- Grid: 128×128×128, deltaZ=25 mm

Space charge becomes more visible in late cooling stages.

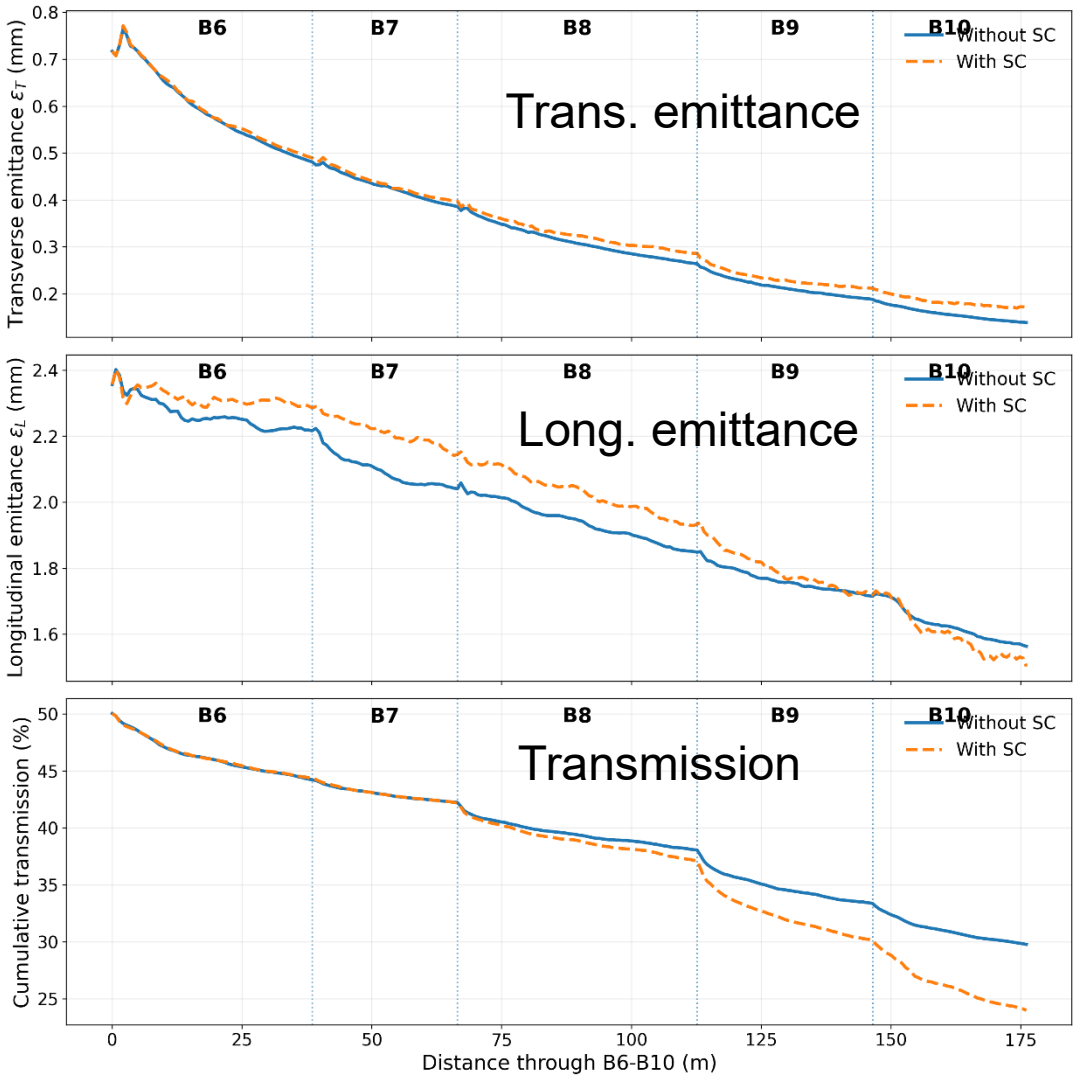
**With space charge, the transverse emittance is systematically larger. The longitudinal emittance is increased over most of B6-B10.**

### At exit of B10:

$\Delta T$ = -5.8 pp (29.8% → 24%),

$\Delta\varepsilon_T$ = +22.9% (0.14 mm → 0.17 mm),

$\Delta\varepsilon_L$ = -12.8% (1.56 mm → 1.36 mm)



# Summary and outlook

## ■Summary:

- A new 3D space-charge model was implemented and validated in G4beamline.
- Space-charge effects become more important in the late cooling stages.
- At B10, $\varepsilon_T$ increases by 22.9% and transmission decreases by 5.8 pp.

## ■Outlook:

- Understand late-stage particle losses caused by space charge.
- Re-optimize the channel including space charge.
- Cross-check with other codes (e.g., RF-Track).

# Reference


- C. Accettura et al., Towards a muon collider, *Eur. Phys. J. C 83, 864 (2023).*
- D. Stratakis and R. B. Palmer, Rectilinear six-dimensional ion ization cooling channel for a muon collider: A theoretical and numerical study, *Phys. Rev. ST Accel. Beams 18, 031003 (2015).*
- R. Zhu, C. Rogers, J. Yang, H. Zhao, G. Cheng and J. Li, Design method, performance evaluation, and tolerance analysis of the rectilinear cooling channel for a muon collider, *Phys. Rev. Accel. Beams 28, 041003 (2025).*
- D. Stratakis, R. B. Palmer and D. P. Grote, Influence of space-charge fields on the cooling process of muon beams, *Phys. Rev. ST Accel. Beams 18, 044201 (2015).*
- Thomas J. Roberts, G4BEAMLINE CODE DEVELOPMENT, Proceedings of IPAC2012.
- R. Zhu, J. Yang, C. Rogers, Design of 6D cooling and final cooling channels for a muon collider, *71st ICFA Adv. Beam Dyn. Workshop High-Intensity High-Bright. Hadron Beams Series.*

# Thank you for your attention.

# Any question or comment?