Differentiable Self-Consistent Space-Charge Mapping Using Differential Algebra and a Truncated Green's Function Solver

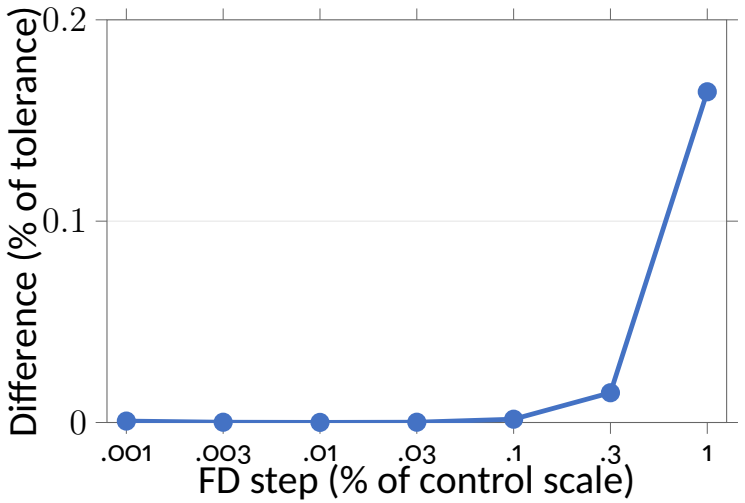
Track the beam and its response together.

TGF: $\widehat{G}_R(k) = [1 - \cos(kR)]/k^2, \quad \widehat{G}_R(0) = R^2/2; \quad \widehat{\phi}_j = \widehat{G}_R \widehat{\rho}_j/\varepsilon_0$

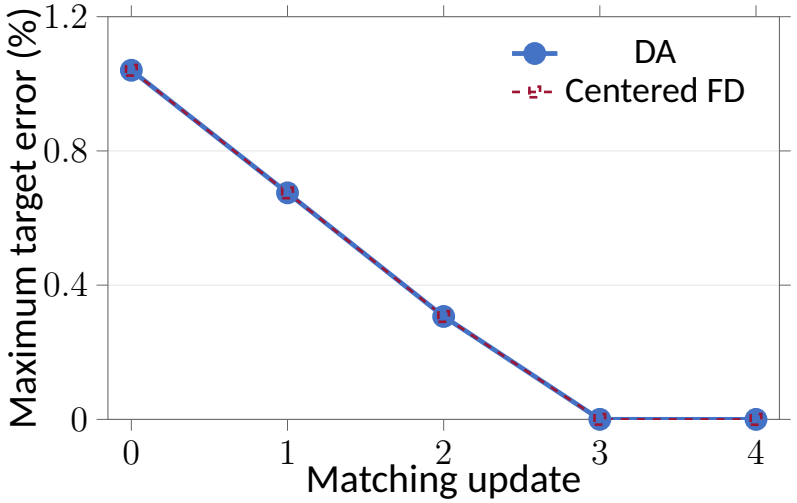
$k = |\boldsymbol{k}|$; R covers required source–target separations; j labels a DA coefficient.

10 MeV protons • 10 pC • 0.20 m FODO • 64 particles • 0.75 mm Gaussian clouds

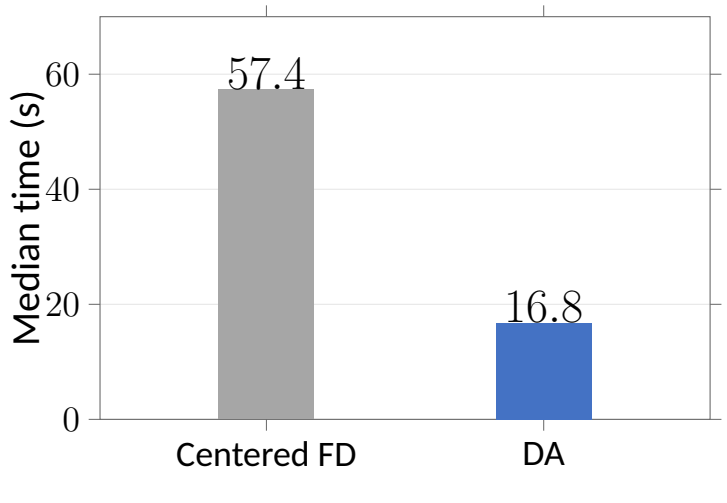
Derivative agreement



Beam-size matching



Response runtime



All seven steps pass

4 updates to the targets

3 controls • 1 CPU thread

Validated at 0.75 mm cloud width. Smaller clouds and new particle ensembles remain open.