LONGITUDINAL PHASE-SPACE RECONSTRUCTION USING DIFFERENTIABLE SPACE-CHARGE SOLVERS

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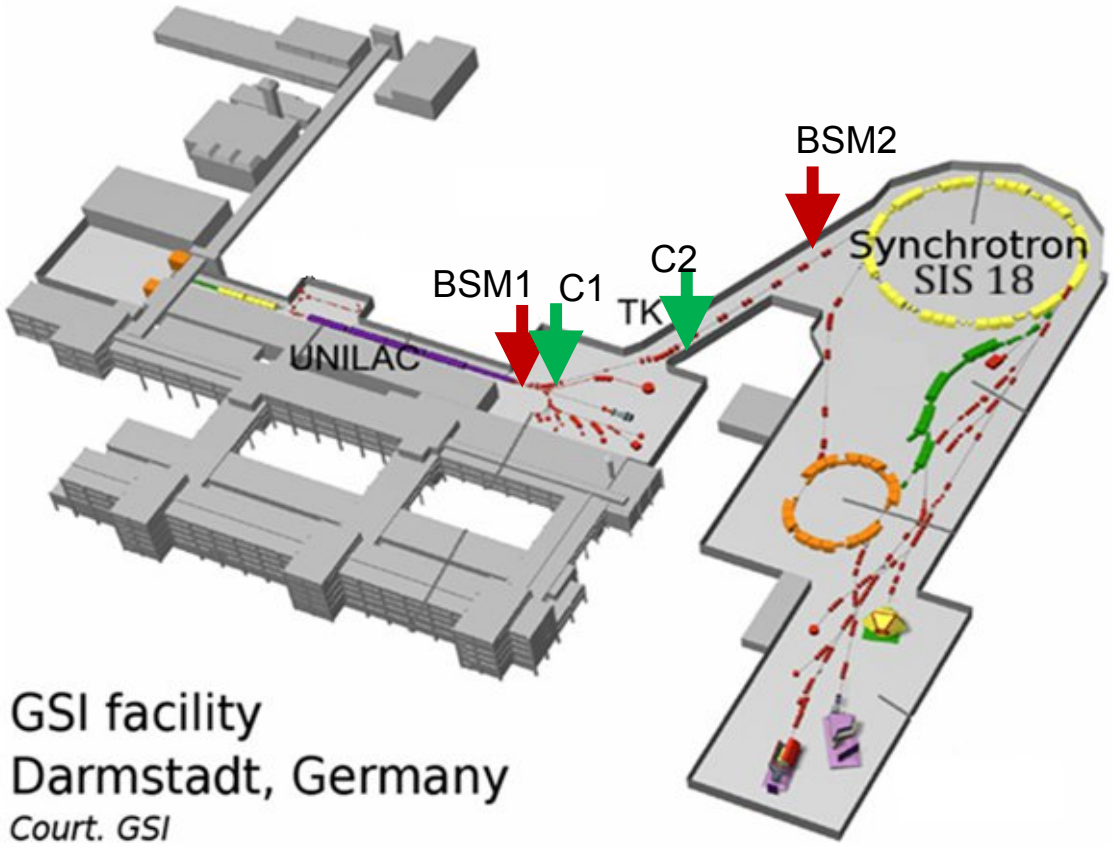
2 Differentiable space charge solvers

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MOTIVATION

- Injection into SIS18 from UNILAC/TK requires $\frac{\Delta p}{p} \leq 10^{-3}$ to avoid losses during RF capture and acceleration.
- First step: Reconstruct bunch distribution before SIS18 injection in the TK.
- Second step: Optimize the bunch phase-space distribution using the available RF systems in the TK.
- Fast enough not to disrupt routine operation.



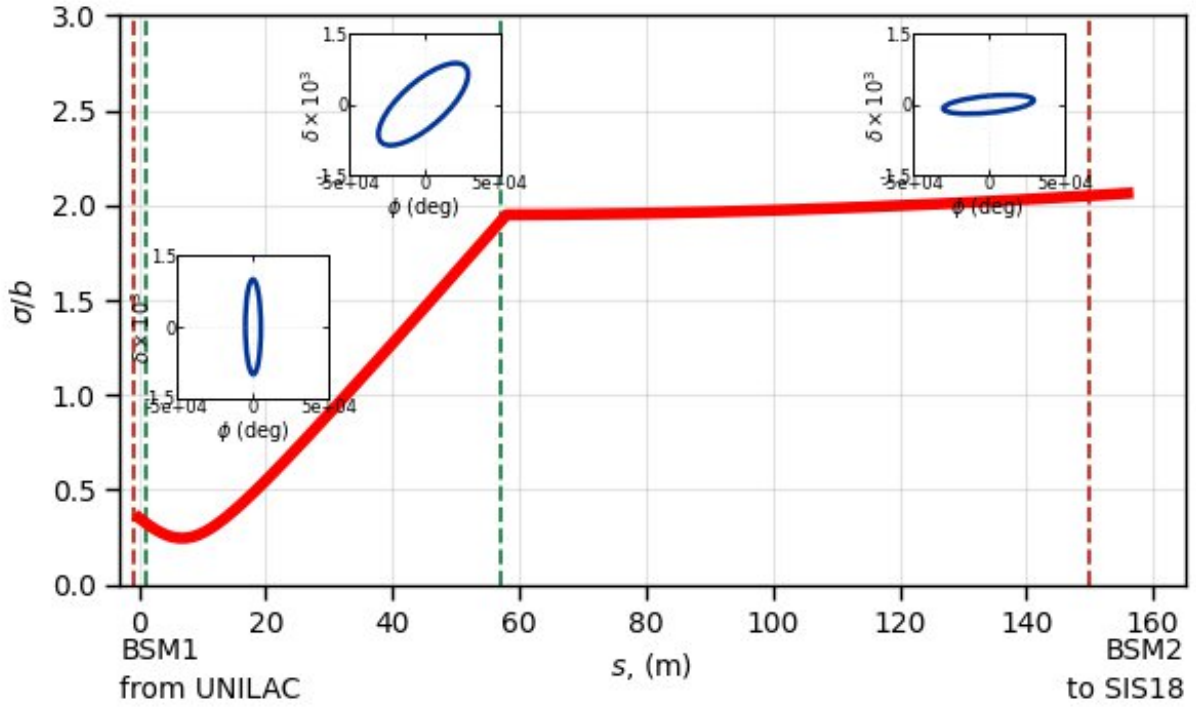
TK: TransferKanal (transfer channel), a 140 m beamline, connecting UNILAC and SIS18

S. Appel and O. Boine-Frankenheim, Phys. Rev. ST Accel. Beams 15, 054201 (2012).

BUNCH LENGTH EVOLUTION ALONG THE TK

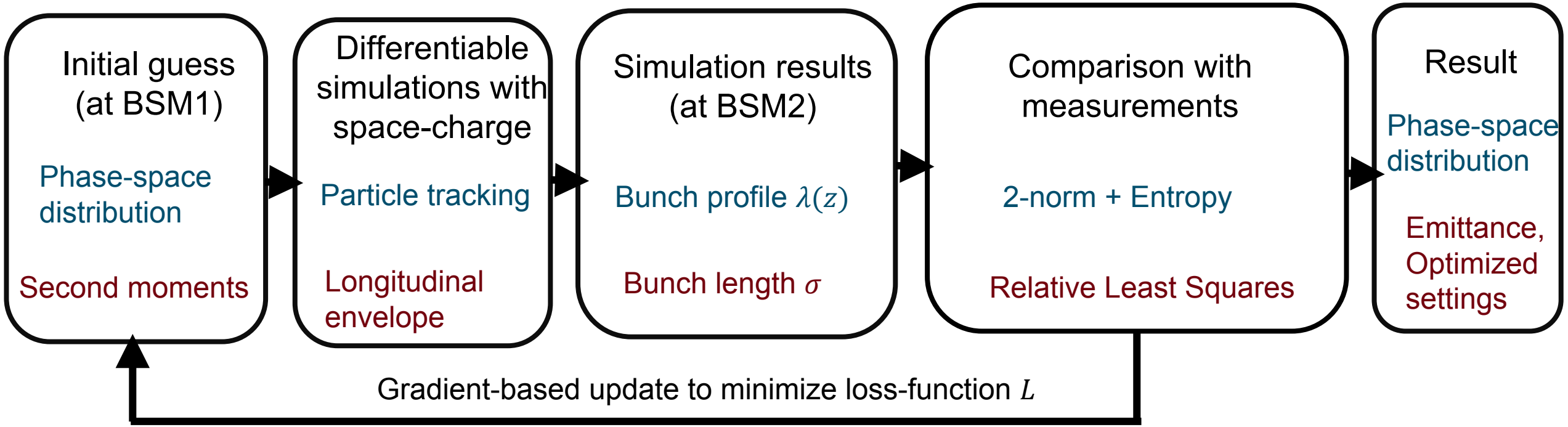
- The bunch lengthens along the TK (from 1.4 mm to 7-15 cm).

- The model must account for space charge with pipe boundary throughout this evolution.



RECONSTRUCTION AS OPTIMIZATION

- Two operation modes: phase-space reconstruction and determination of second moments.



R. Roussel et al., Phys. Rev. Lett. 130, 145001 (2023).

LONGITUDINAL SPACE-CHARGE MODEL

- Averaging over the transverse bunch density reduces the electric field calculation to 1D.
- Axisymmetric Green's function in a round conducting pipe averaged over respective density:

$$G(r, z, r', z') = \frac{1}{\epsilon_0 b} \sum_{n=1}^{\infty} \frac{J_0(\frac{\alpha_n}{b} r) J_0(\frac{\alpha_n}{b} r')}{\alpha_n J_1^2(\alpha_n)} e^{-\frac{\alpha_n}{b}|z-z'|} \qquad J_0(\alpha_n) = 0$$

$$E_{sc,z}(z_i) = \frac{Q}{\epsilon_0 b^2} \sum_{n=1}^{\infty} \frac{\left\langle J_0\left(\frac{\alpha_n}{b} r\right)\right\rangle_r^2}{J_1^2(\alpha_n)} \left\langle \operatorname{sgn}(z-z')\, e^{-\frac{\alpha_n}{b}|z-z'|} \right\rangle_{z'}$$

- The field reduces to a linear combination of exponentials — differentiable, supporting automatic differentiation.
- With drift and cavity elements, this solver is used in the differentiable particle-tracking model.

LONGITUDINAL ENVELOPE MODEL

- Obtained from kinetic model for Gaussian bunches using averaging (along z).
- rms bunch length evolution (drifts):

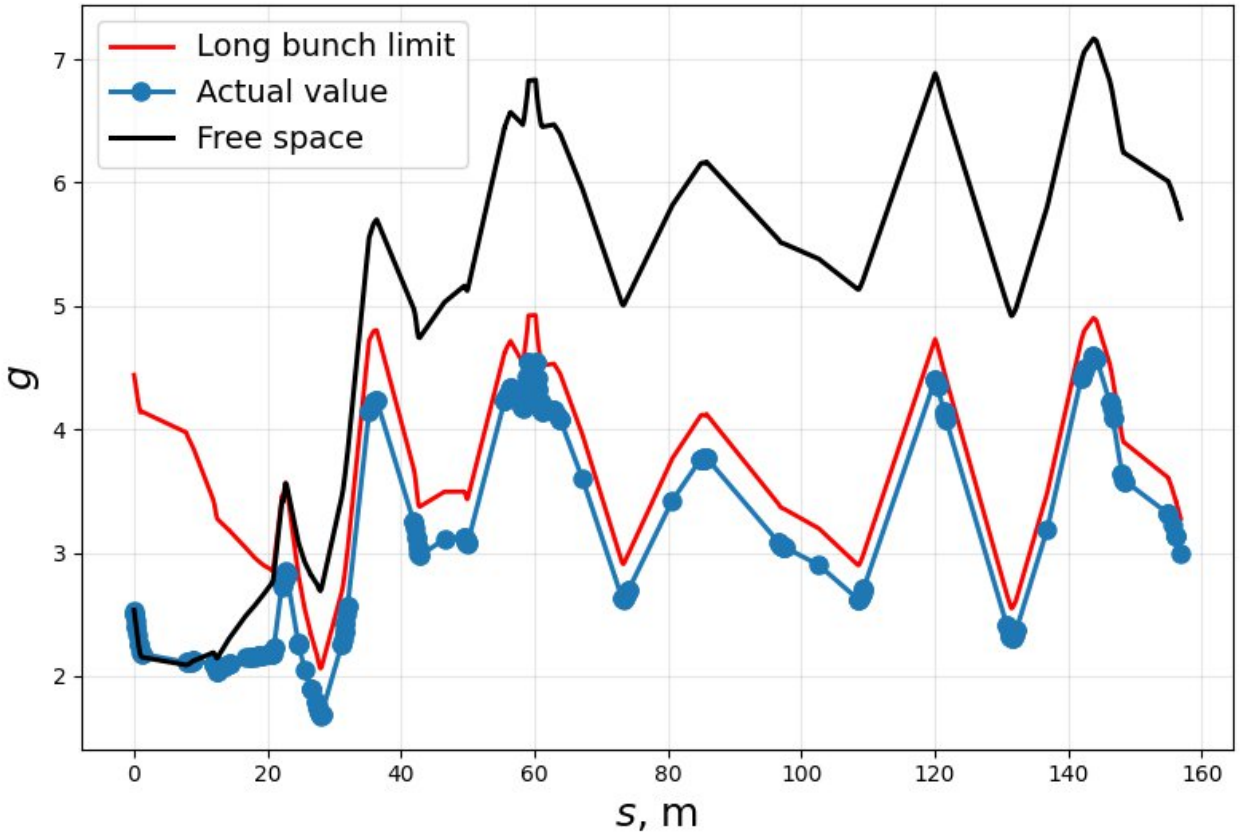
$$\sigma'' - \frac{\varepsilon^2}{\sigma^3} - \frac{q\eta}{m\beta^2\gamma^3 c^2}\frac{\langle zE_{sc,z}\rangle_z}{\sigma} = 0$$

- Generalized g-factor:

$$\bar{g} = \frac{40\sqrt{5}\epsilon_0\pi\sigma}{3Q}\langle zE_{sc,z}\rangle_z$$

- Long bunch limit (red line), holds approx. after 30m:

$$\lim_{\sigma\to\infty}\bar{g} = 0.5 + 2\ln b/a$$



F. J. Sacherer, IEEE Trans. Nucl. Sci. 18, 1105–1107 (1971).

C. K. Allen, M. Reiser, and N. Brown, Part. Accel. 45, 149–165 (1994).

LONGITUDINAL ENVELOPE MODEL

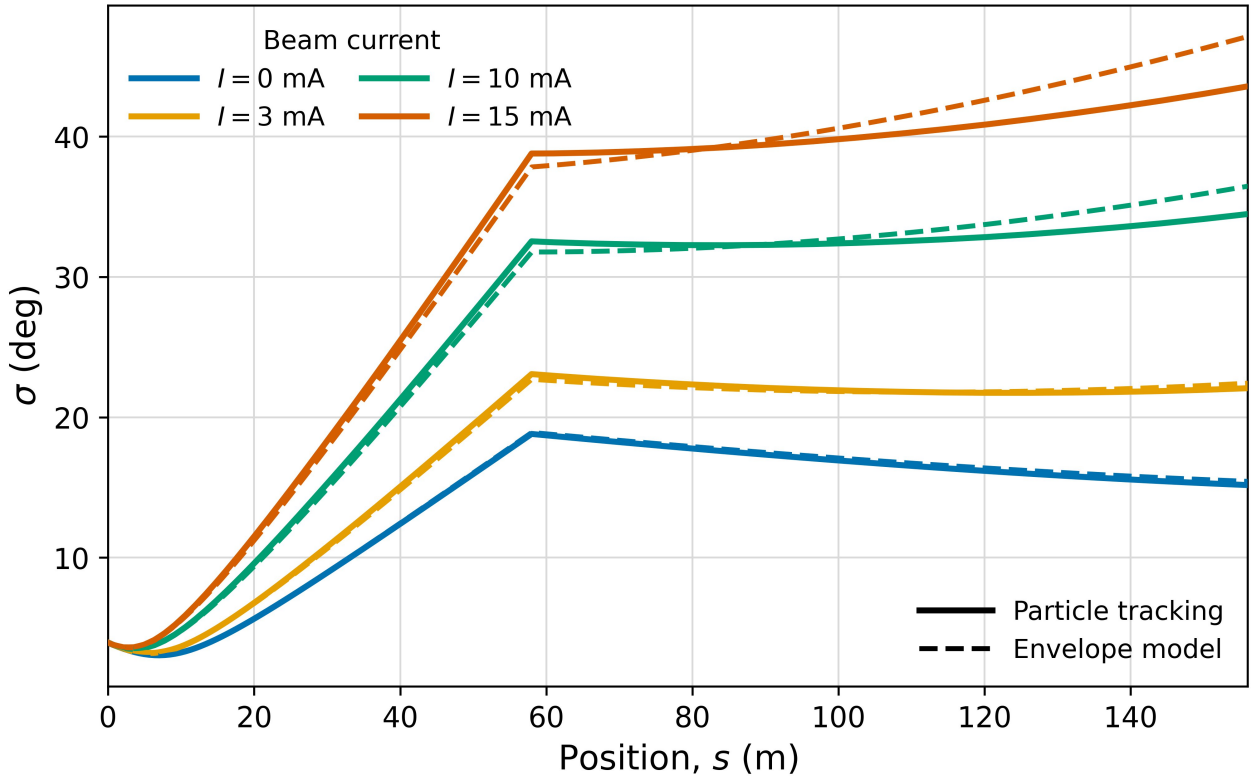
- In (thin) cavities:

$$\sigma'' - \frac{\varepsilon^2}{\sigma^3} - \frac{q\eta}{m\beta^2\gamma c^2}\frac{\langle zE_{RF}\rangle_z}{\sigma} = 0$$

- For a Gaussian profile:

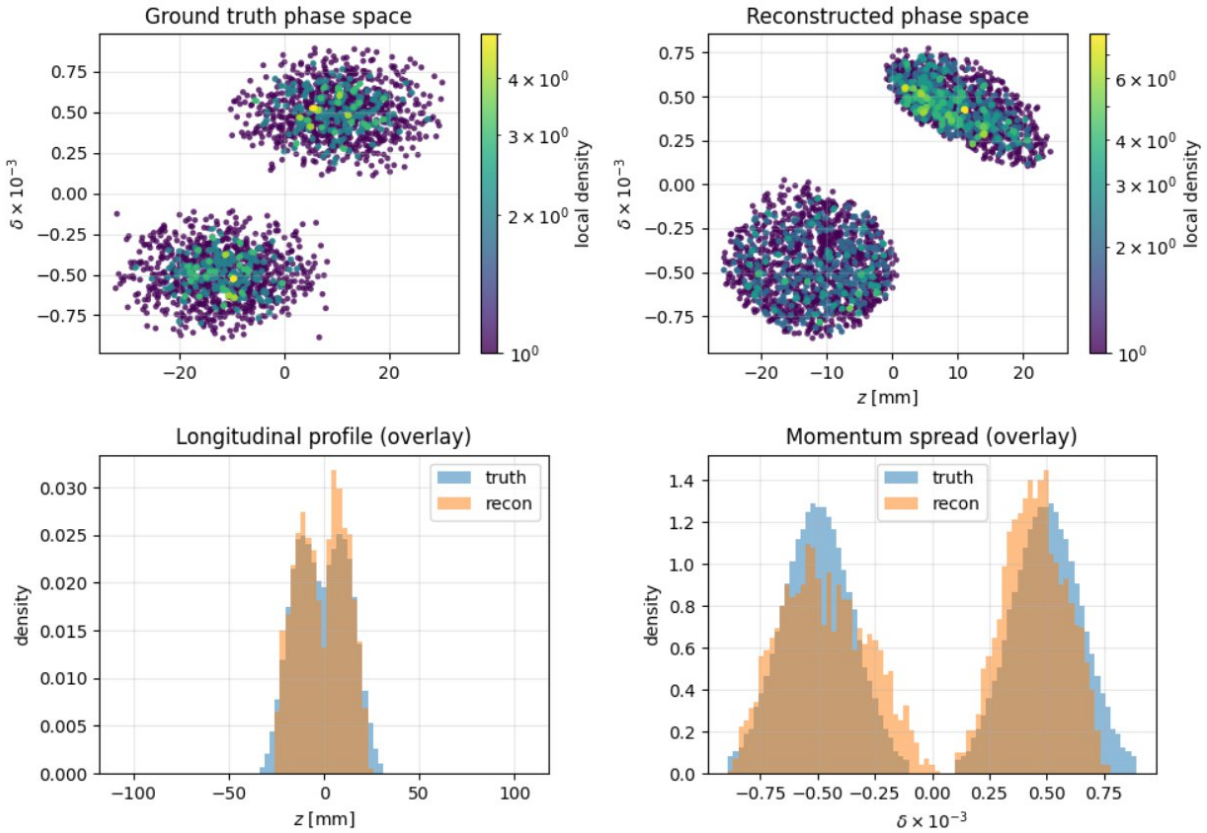
$$\Delta\sigma' = \frac{2\pi qV\eta}{m\beta^3\gamma\lambda c^2}\exp\{-\frac{1}{2}\left(\frac{2\pi\sigma}{\beta\lambda}\right)^2\}$$

- For the long TK bunches we have to account for nonlinear rf kicks in the envelope model.



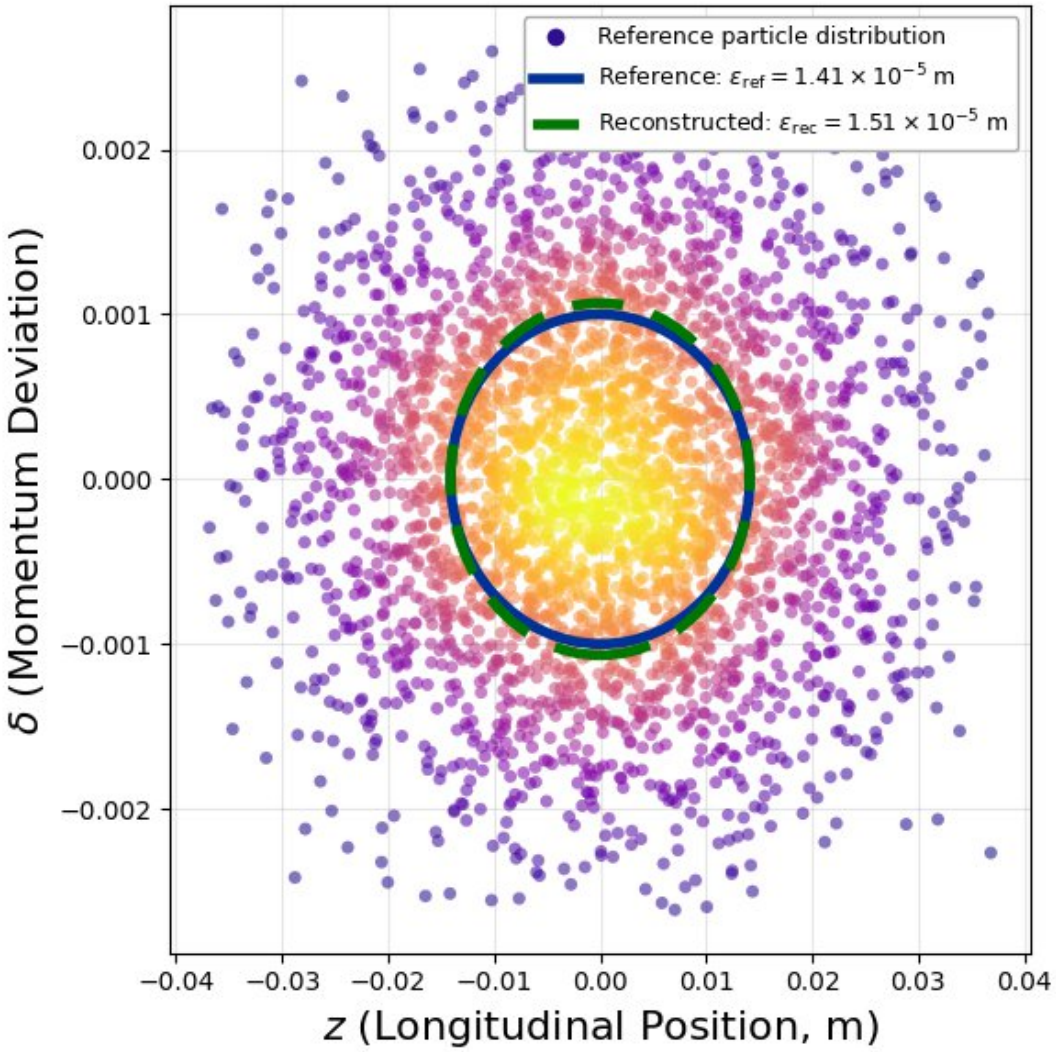
TWO-CORE LONGITUDINAL DISTRIBUTION

- Profiles simulated by the particle tracking model were used as input.
- Reconstructed distribution is described by a neural network.
- Can resolve internal structure of phase space distribution.



RECONSTRUCTED RMS BUNCH PARAMETERS

- Profiles simulated by the particle tracking model from the reference Gaussian bunch.
- The envelope model fitted the generated profile lengths and inferred the rms phase-space ellipse.
- Smooth envelope model avoids macroparticle sampling.



SUMMARY

- Two models were developed, both propagate gradients and can be used for phase-space reconstruction and beamline optimization.

- Particle tracking model can be used to reconstruct arbitrary distributions. It is sensitive to sampling noise.

- Envelope model is a faster alternative for reconstruction of beam’s rms phase space ellipse. It requires assumptions about longitudinal profile.

BACKUP

OPTIMIZATION OF VOLTAGES

- Used the envelope model.
- Cavity voltages that minimize momentum spread at SIS18 injection.
- At high currents achievable spread is limited by the second cavity, at low currents by the first one.

