Space Charge Simulation for the Third Order Resonance Corrections at J-PARC MR

Space Charge 2026, 7th ICFA Mini-workshop

2026/9/30

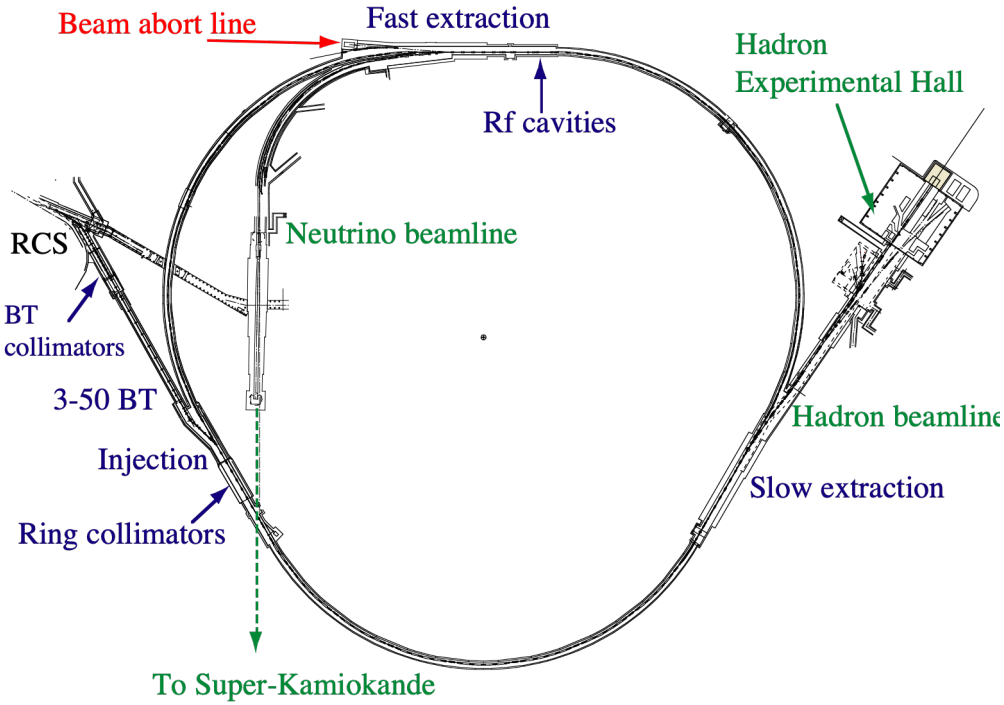
Susumu Igarashi KEK/J-PARC Center

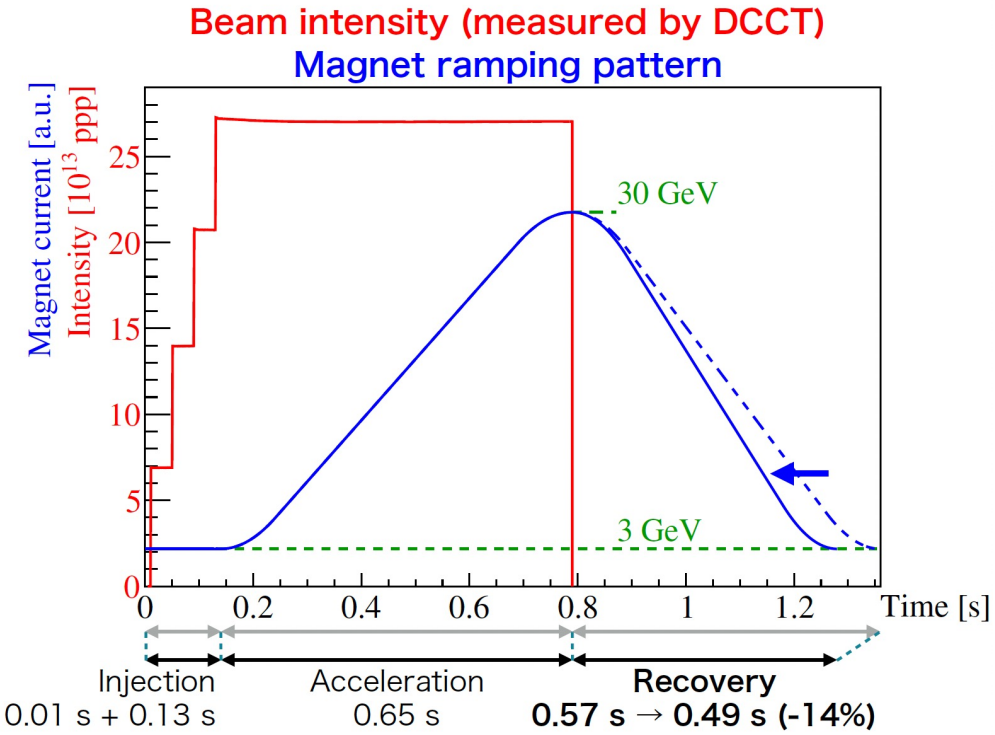
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MR Design and Operation Modes

- Circumference 1567.5 m
- Three-fold symmetry
- Target Beam Power: 1.3 MW (for fast extraction)
- Fast extraction mode (FX) for the neutrino Facility: 1 turn extraction.
  - Cycle : 1.28 s → 1.16 s
  - Beam Intensity 2.7×10^{14} ppp → 3.3×10^{14} ppp
- Slow extraction mode (SX) for the hadron hall: 2 s – spill extraction.
  - Cycle : 4.24 s



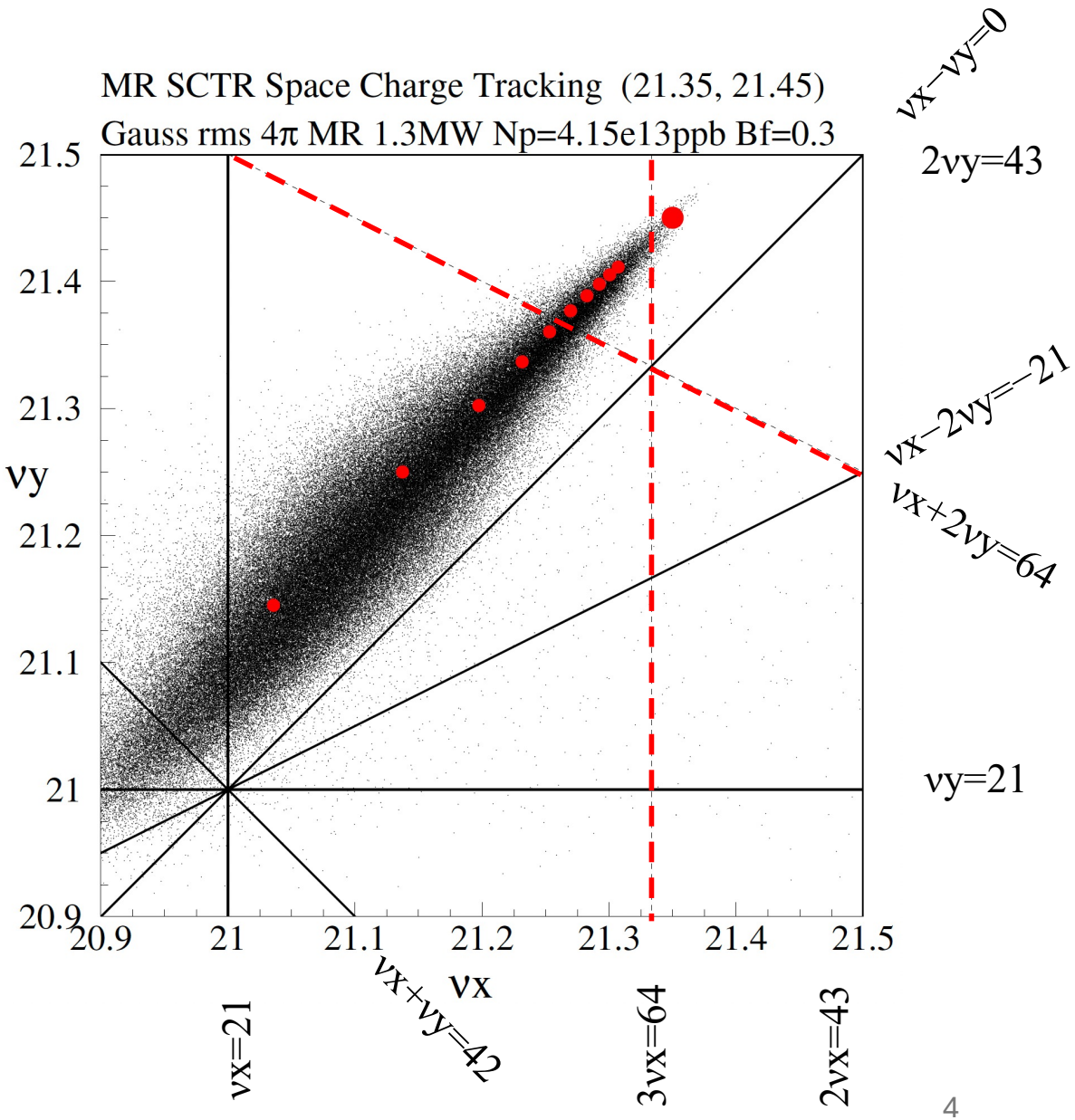


Space Charge Tune Spread and Resonances

- MR Power 1.3 MW
- Number of protons: 4.15e13 protons per bunch
- Typical operation tune : (21.35, 21.45)

- Beam loss reduction is necessary for the high intensity operation.

- Resonances of
  3νx = 64 and
  νx+2νy = 64
  are corrected for the FX operation.



3rd Order Resonances Corrections with Trim Coils of Sextupole Magnets

- $\nu_x+2\nu_y=64$, (1+2)
- Beam survival recovers with SFA048: +0.5 A, SFA055: +0.2 A.
- |G| = 0.076

$$G_{1,2,64} = \frac{\sqrt{2}}{8\pi}\beta_x^{1/2}\beta_y k_2 \exp[i(\phi_x + 2\phi_y)]$$

- $3\nu_x=64$, (3+0)
- Beam survival recovers with SFA048 +0.5 A, SFA055 0.0 A.
- |G| = 0.055

$$G_{3,0,64} = \frac{\sqrt{2}}{24\pi}\beta_x^{3/2}k_2 \exp[i(3\phi_x)]$$

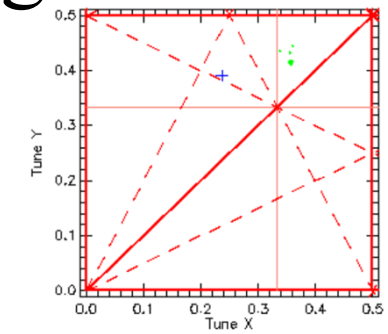
- Simultaneous equations for 4 sextupole trim coils to correct both resonances.

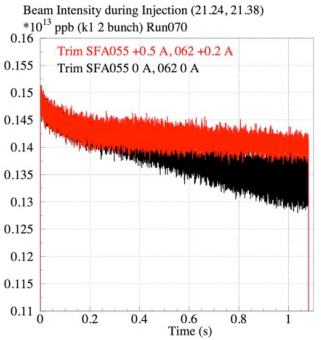
$$\sum_{j=1}^{4}\frac{\sqrt{2}}{24\pi}\beta_x^{\frac{3}{2}}(j)\ k_2(j)\cos[3\phi_x(j)] = \sum_{i=1}^{2}\frac{\sqrt{2}}{24\pi}\beta_x^{\frac{3}{2}}(i)\ k_2(i)\ \cos[3\phi_x(i)]$$

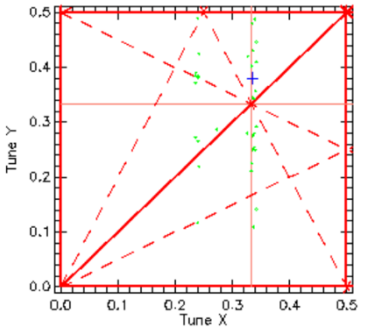
$$\sum_{j=1}^{4}\frac{\sqrt{2}}{24\pi}\beta_x^{\frac{3}{2}}(j)\ k_2(j)\sin[3\phi_x(j)] = \sum_{i=1}^{2}\frac{\sqrt{2}}{24\pi}\beta_x^{\frac{3}{2}}(i)\ k_2(i)\ \sin[3\phi_x(i)]$$

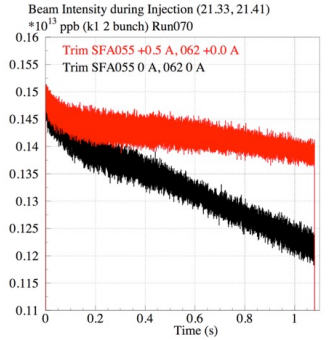
$$\sum_{j=1}^{4}\frac{\sqrt{2}}{8\pi}\beta_x^{\frac{1}{2}}(j)\beta_y(j)\ k_2(j)\cos[\phi_x(j)+2\phi_y(j)] = \sum_{i=1}^{2}\frac{\sqrt{2}}{8\pi}\beta_x^{\frac{1}{2}}(i)\beta_y(i)\ k_2(i)\ \cos[\phi_x(i)+2\phi_y(i)]$$

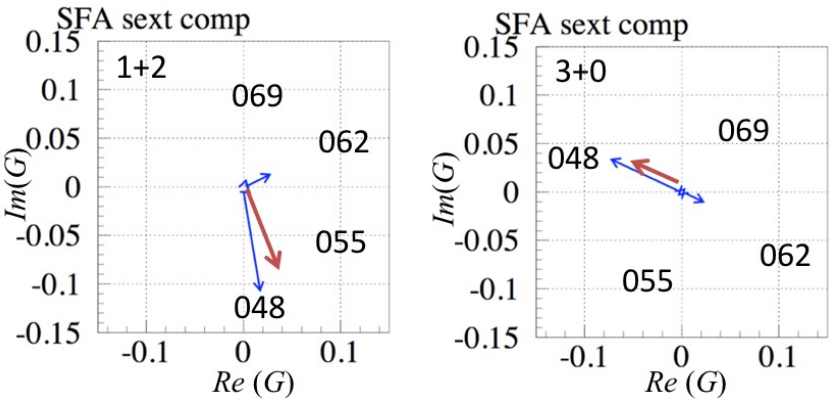
$$\sum_{j=1}^{4}\frac{\sqrt{2}}{8\pi}\beta_x^{\frac{1}{2}}(j)\beta_y(j)\ k_2(j)\sin[\phi_x(j)+2\phi_y(j)] = \sum_{i=1}^{2}\frac{\sqrt{2}}{8\pi}\beta_x^{\frac{1}{2}}(i)\beta_y(i)\ k_2(i)\ \sin[\phi_x(i)+2\phi_y(i)]$$

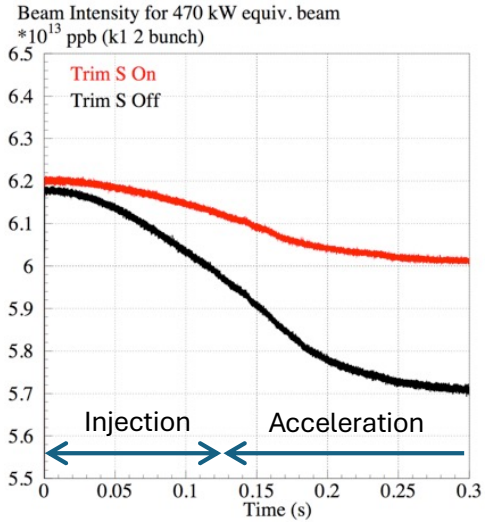








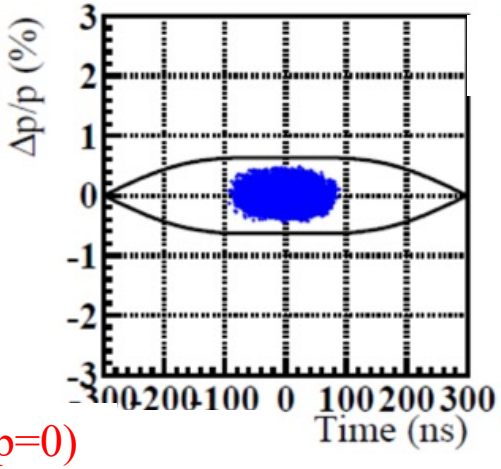


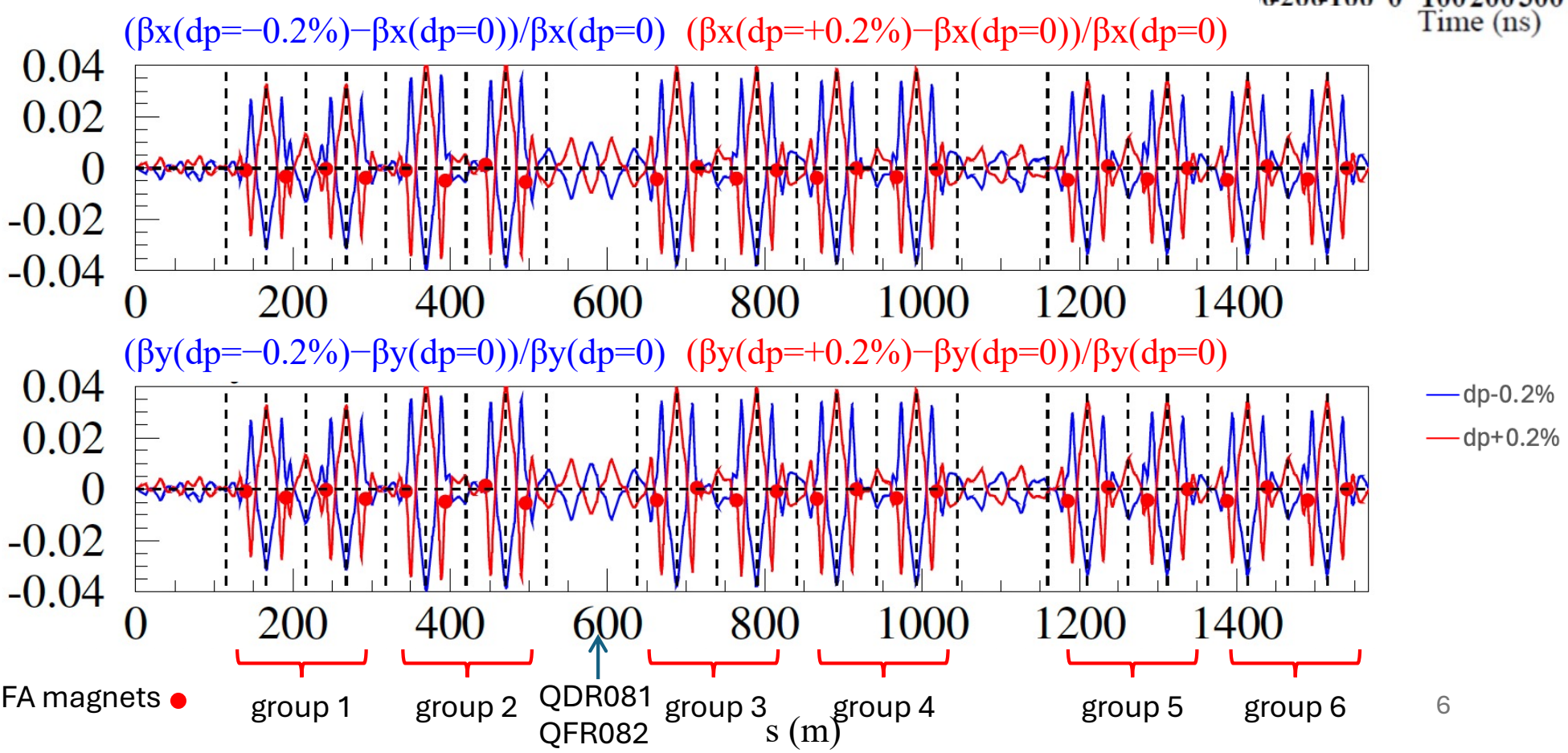


Beam optics for off-momentum particles

- Beam optics for the off-momentum particles (dp/p = ±0.2%, ±0.4%,) were calculated with SAD with respect to the optics of on-momentum (dp/p = 0) particle .

$$G_{3,0,64} = \frac{\sqrt{2}}{24\pi}\beta_x^{3/2}k_2\exp[i(3\phi_x)]$$





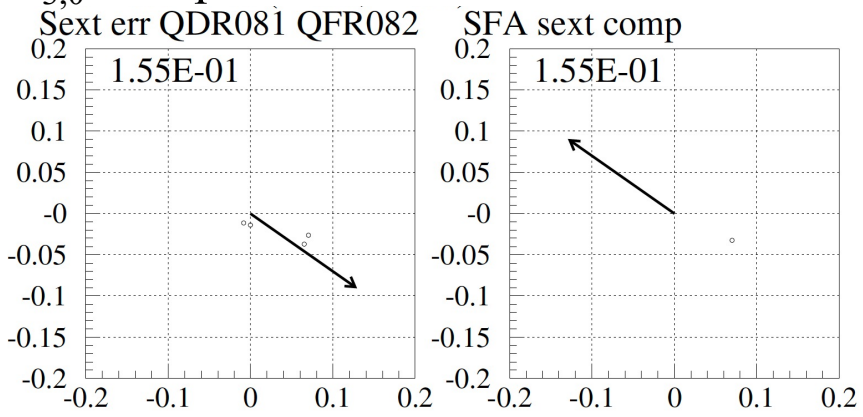
Resonance correction for 3+0 for off-momentum particles

$$G_{3,0,64} = \frac{\sqrt{2}}{24\pi}\beta_x^{3/2}k_2\exp[i(3\phi_x)]$$

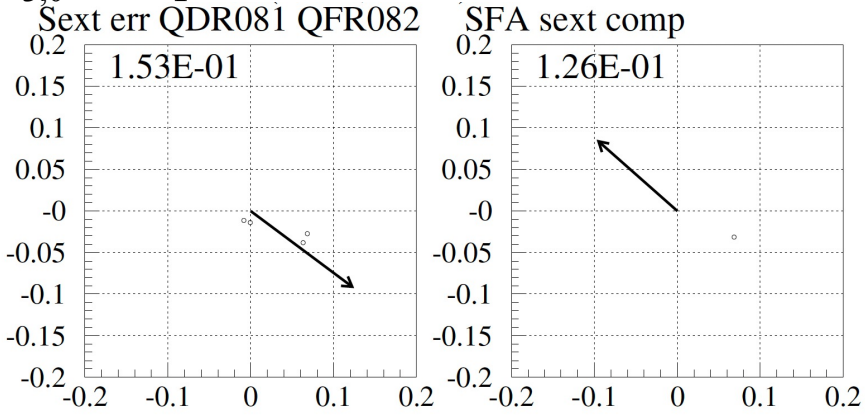
- Error at QDR081 and QFR082
- Resonance correction with trim coils of 4 SFA magnets
- Analysis is repeated for the 1+2 resonance.

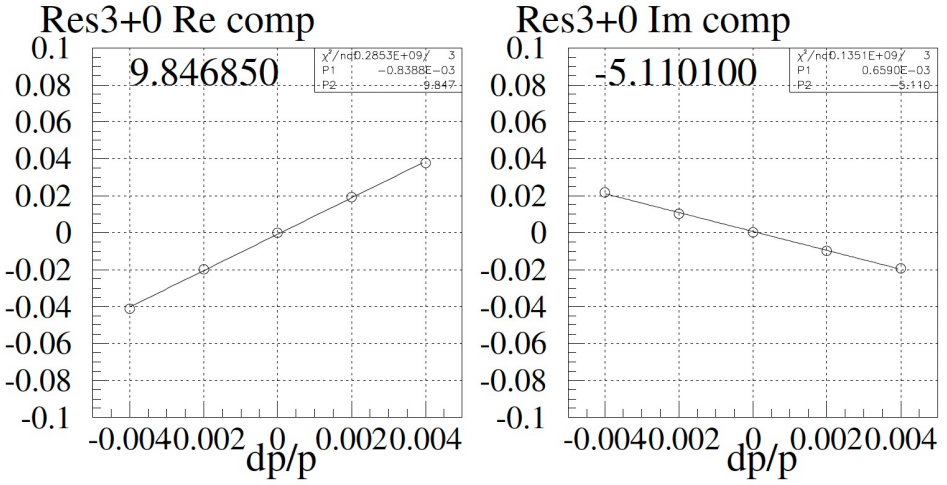
SFA group 1	Trim Coil Current (A)
020	−1.6328
027	+2.8697
034	−4.7788
041	+3.2752

$G_{3,0}$ for dp = 0



$G_{3,0}$ for dp = +0.2%





The real part and imaginary part of $\frac{d\Delta G_{3+0}}{d(\Delta p/p)}$ were derived for group 1.

Resonance correction for off-momentum particles

- Find the solution for the following simultaneous equations.

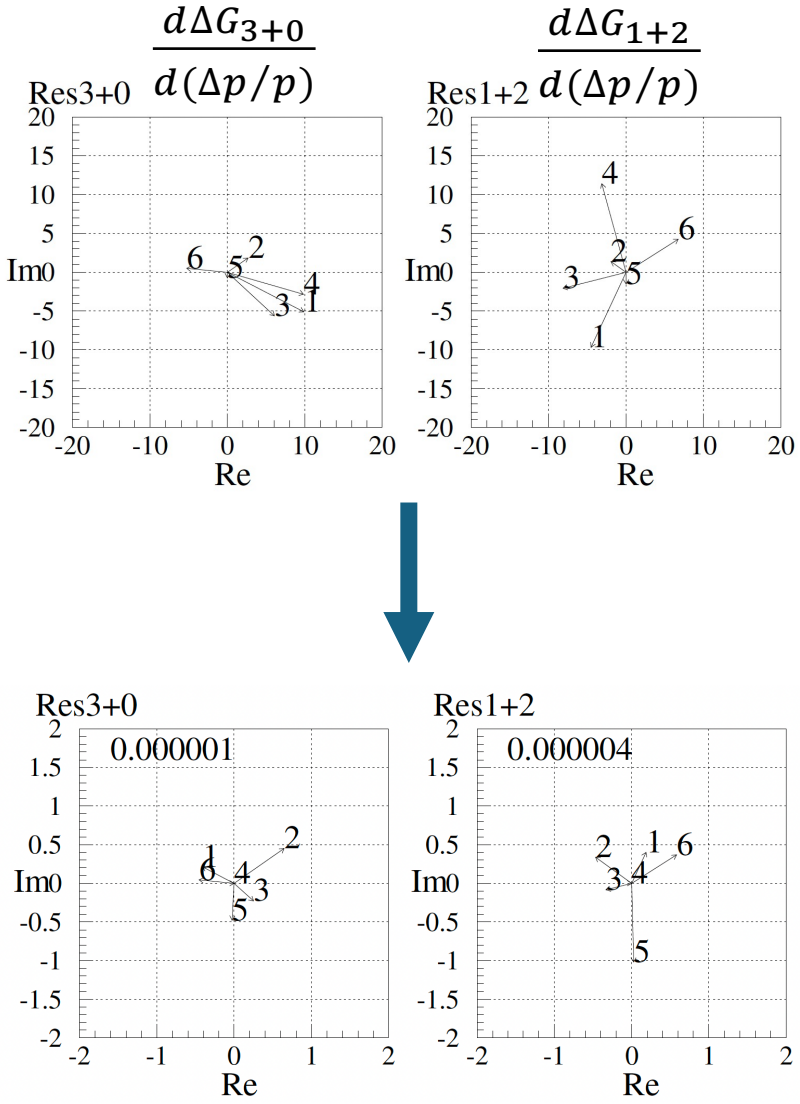
$$\sum_{i=1}^{5} f_i \times Re\{\frac{d\Delta G_{3+0}}{d\,(\Delta p/p)}\,(group\ i)\} = 0 \qquad \sum_{i=1}^{5} f_i \times Im\{\frac{d\Delta G_{3+0}}{d(\Delta p/p)}\,(group\ i)\} = 0$$

$$\sum_{i=1}^{5} f_i \times Re\{\frac{d\Delta G_{1+2}}{d\,(\Delta p/p)}\,(group\ i)\} = 0 \qquad \sum_{i=1}^{5} f_i \times Im\{\frac{d\Delta G_{1+2}}{d(\Delta p/p)}\,(group\ i)\} = 0$$

$$\sum_{i=1}^{5} f_i = 1$$

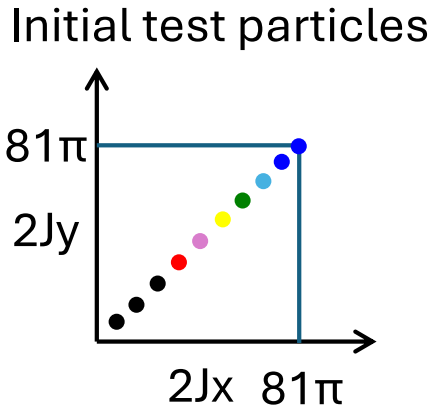
$$\begin{cases} \sum_{i=1}^{5} f_i \times \Delta G_{3+0}(group\ i)_{cor} = -\Delta G_{3+0\ source} \\ \sum_{i=1}^{5} f_i \times \Delta G_{1+2}(group\ i)_{cor} = -\Delta G_{1+2\ source} \end{cases}$$

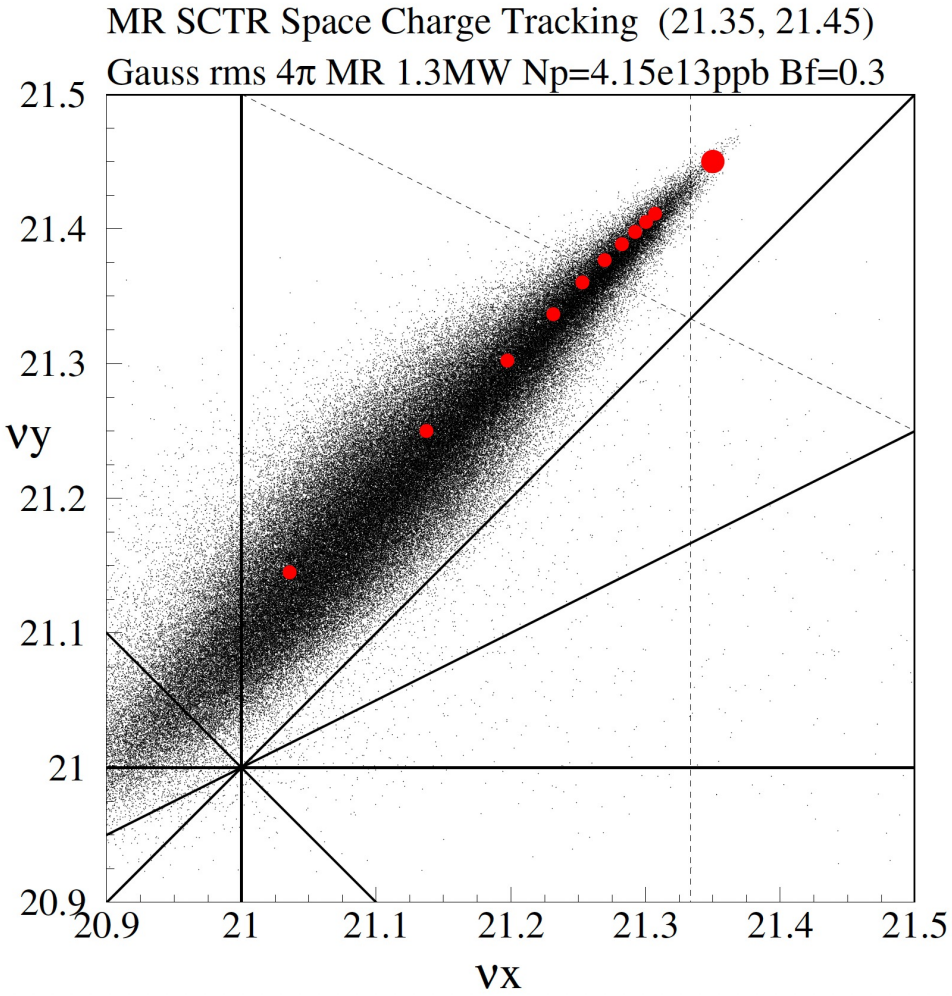
SFA	ΔG_{3+0} / (Δp/p)	ΔG_{1+2} / (Δp/p)	factor: f
group 1	11.09	10.64	-0.041718
group 2	3.25	2.36	0.242718
group 3	8.26	8.39	0.040839
group 4	10.22	11.83	0
group 5	0.71	1.52	0.671723
group 6	5.30	7.96	0.086440



Optics modulated by the space charge force

- Optics is modulated by the space charge force.
- It should be considered for the resonance corrections.

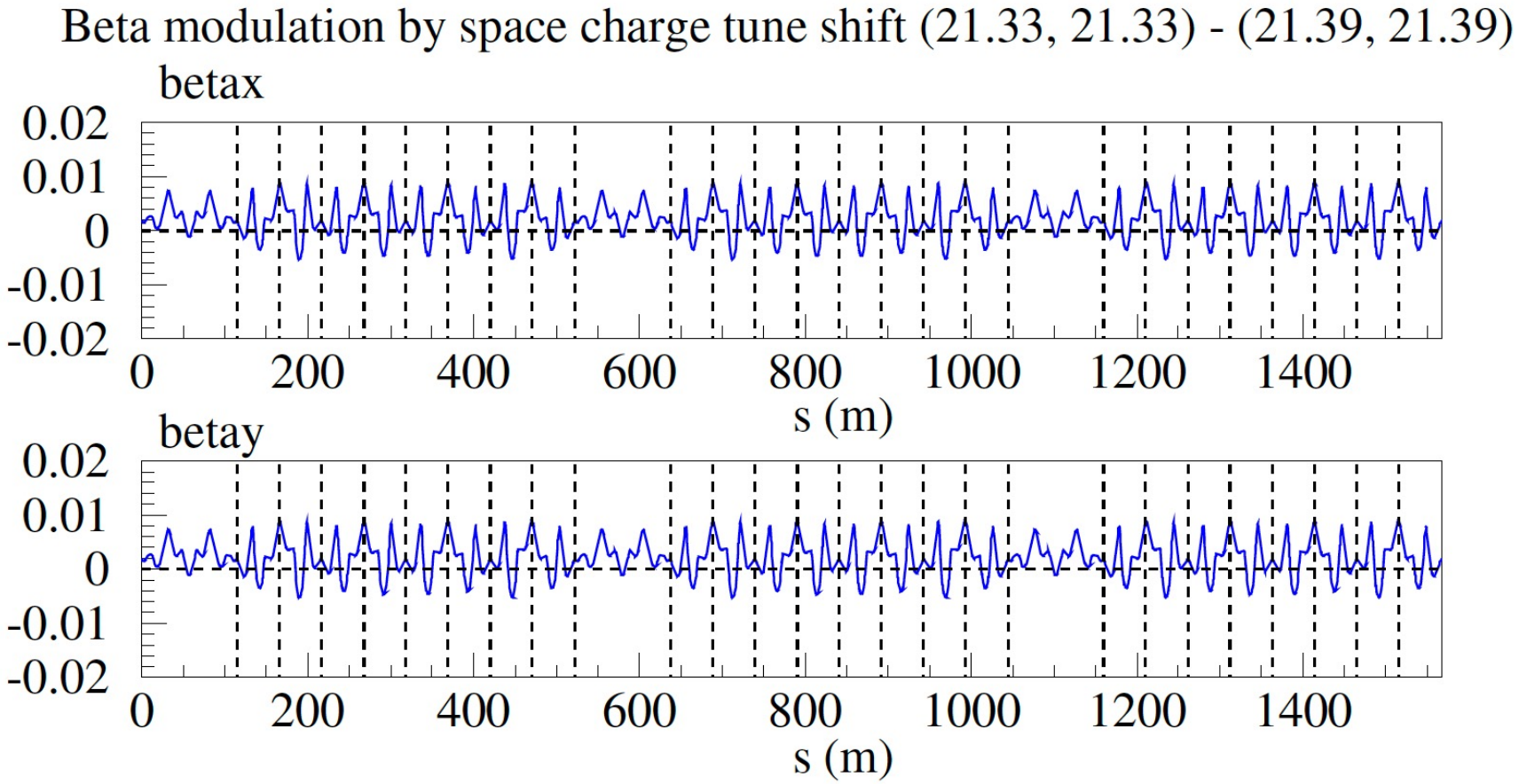




- Optics modulation by space charge force was approximately calculated.
- Space charge tune shift is 0.06 for the 60π mm mrad particles.

Beam optics modulated by the space charge force

- For dp=0%, space charge tune shift = −0.06
  - Start with the optics of (21.39, 21.39).
  - Multiply the scale factor to K1 values for the tune (21.33, 21.33).
- Same methods for the chromatic tune shifts of dp/p = ±0.2%, ±0.4%.

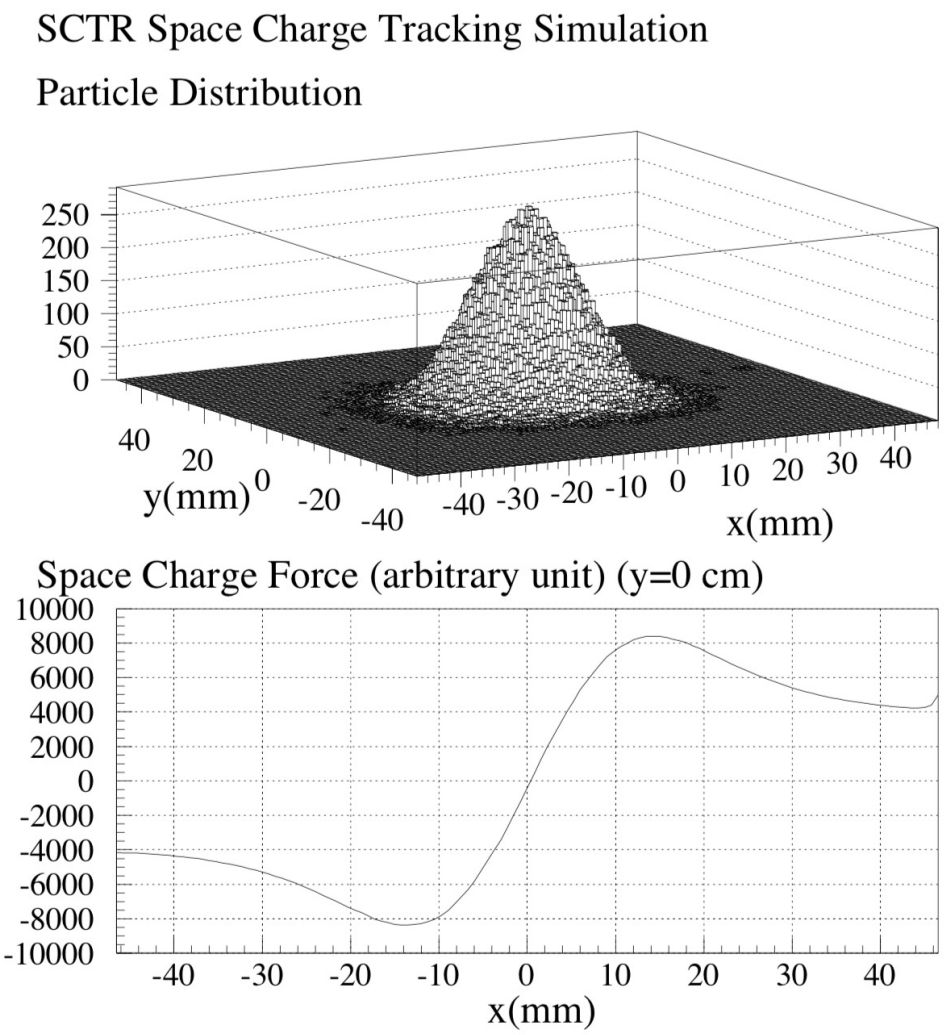


Space Charge Tracking Simulation

- Space charge tracking simulation program SCTR was made for J-PARC MR by Ohmi san.

- Particle in cell calculation
- Space charge potentials and kicks are calculated about 1000 elements in the circumference.
- Number of grid : $128 \times 128 \times 128$
- Grid size : 0.78 mm $\times$ 0.78 mm $\times$ 1 m
- Number of Macro Particles : 200,000

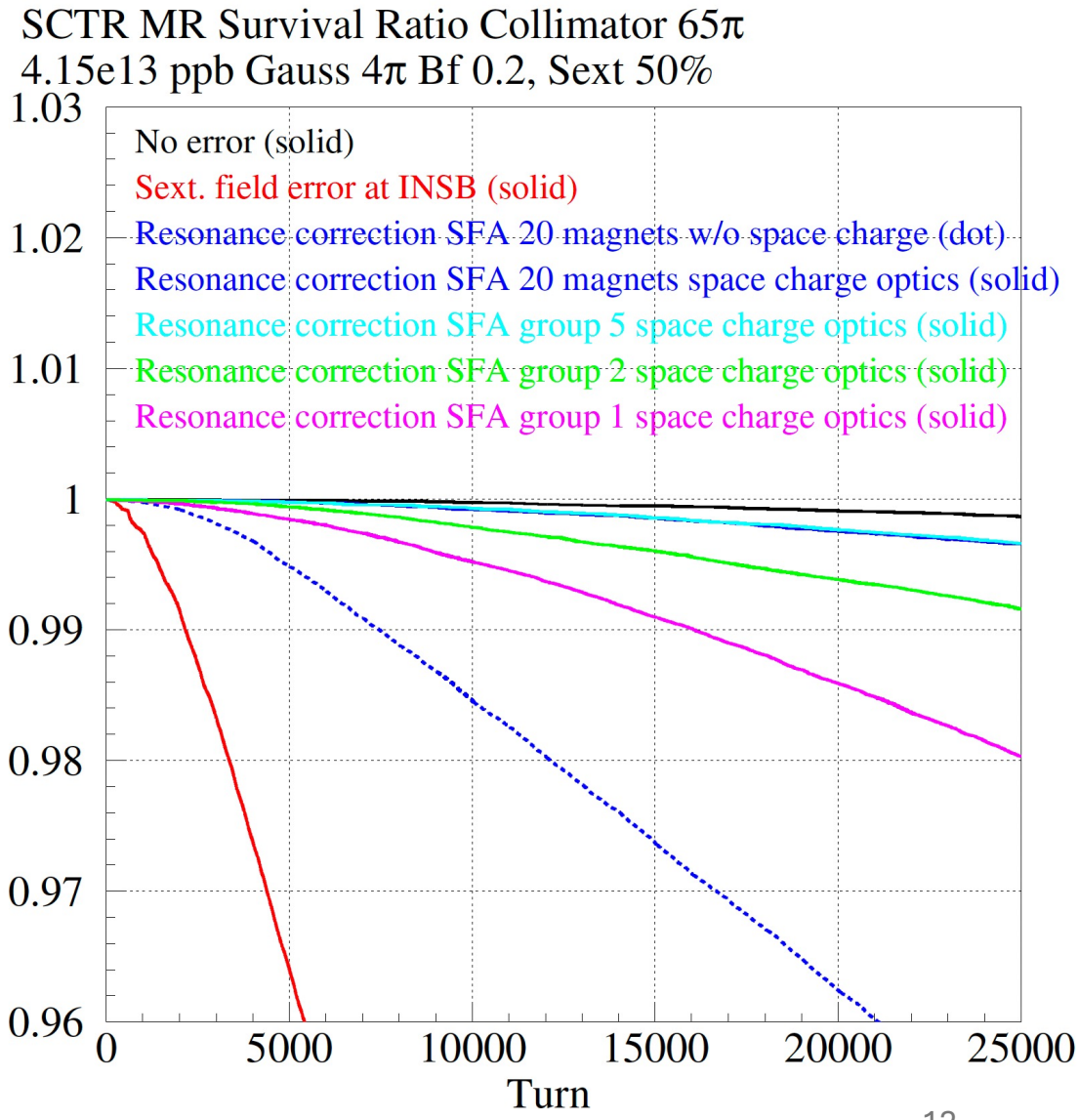
- Initial beam distribution
  - Transverse Gaussian rms emittance : 4π mm mrad
  - Longitudinal Bunching factor : 0.2



Simulation results

- Beam survival

SFA	ΔG_{3+0} / ($\Delta p/p$)	ΔG_{1+2} / ($\Delta p/p$)	Beam Survival
group 1	11.09	10.64	0.9803
group 2	3.25	2.36	0.9916
group 3	8.26	8.39	
group 4	10.22	11.83	
group 5	0.71	1.52	0.9966
group 6	5.30	7.96	
20 mags	0	0	0.9966



Summary

- Simulation study indicated that the beam loss was reduced by correcting third order resonances for both on-momentum and off-momentum particles.
- The correction can be done with 5 groups of 4 sextupole magnets; total 20 sextupole magnets.
- We must consider the beam optics modulated by the space charge effect for the resonance correction.
- In this study we assume the error source. But…
  - We should be able to apply this method even if the error source is not known.
  - We should be able to measure the momentum deviation dependence of the resonance driving terms with the real beam.