Study of beam redistribution on approach to the half integer resonance with space charge

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Contents

1. Introduction
2. Frozen Waterbag Space Charge Model (Review)
3. Details of Motion in and around the Stop Band
4. Frozen Gaussian Space Charge Model; 2D considerations
5. Coherent Picture ~ Overlap with Frozen ~ Predicted Behaviour
6. Simulations Confirming Model (Review) ~ “Compensating” Space Charge effect
7. Next Steps: simulations, experiments … relation to RCS operations
8. Summary

Introduction

- The half integer resonance is a potential intensity limitation on many machines
- Here we study beam redistribution as the resonance is approached from above
- Look at simpler 2D coasting beams (no RF, storage ring mode)
  - This is the basis for understanding half integer losses on the ISIS RCS in operations
- Give a summary of some simple models to help understand the redistribution process
- Outline plans to test these with more detailed simulation and experiment

1 Frozen Waterbag space charge model of half integer resonance (HB2016 [1])

- Use Hamiltonian of frozen WB, phase averaging to find *H(J)*, *Q(J)*, add driving term: find approx. motion
- Approximations: many!
  - Circular beam, on axis (1D), …
  - Non-self consistent ~ changes with distribution
- But gives useful predictions for redistribution
  - SFP indicate location of tails or lobes
  - Can predict central SFP and UFP …

$$n(r) = n_0(1 - \tfrac{r^2}{a^2}) \text{ for } r \le a, \text{ zero otherwise) with } r^2 = x^2 + y^2$$

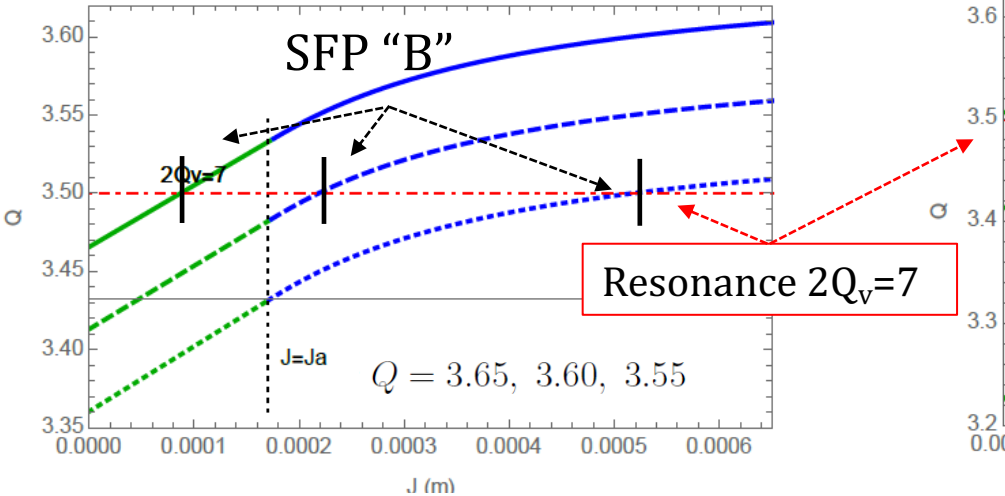
$$H(x, P_x, s) = \tfrac{1}{2}P_x^2 + \tfrac{1}{2}\omega^2 x^2 + K_d(s)x^2 + V(x)$$

$$V(x) = \begin{cases} V_i = -k(\frac{x^2}{a^2} - \frac{x^4}{4a^4}), & \text{if } x \le a \\ V_o = -k(\frac{3}{4} + \log\frac{|x|}{a}), & \text{if } x > a \end{cases} \qquad K_d(s) = k_l \cos l\theta$$

$$x = \sqrt{\frac{2J}{\omega}}\sin\phi; \quad P_x = \sqrt{2J\omega}\cos\phi \qquad \overline{H(\phi, J, s)} = \tfrac{1}{2\pi}\int_0^{2\pi} H(\phi, J, s)d\phi$$

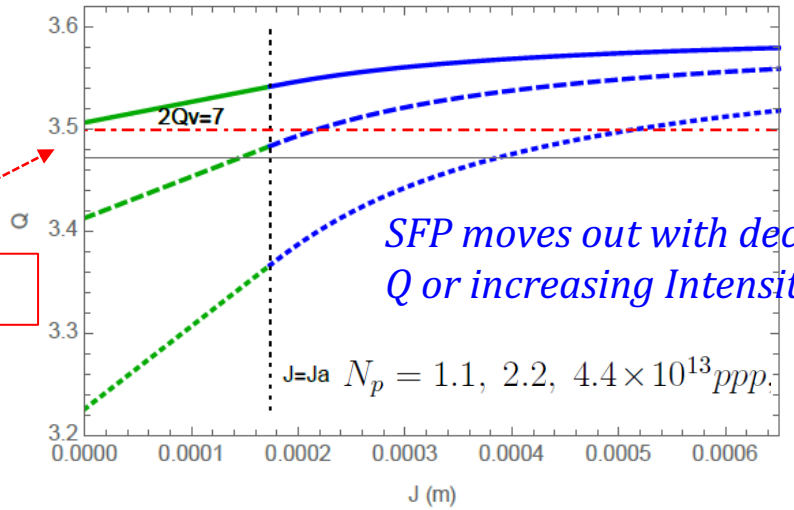
$$\overline{H} = \overline{H(J, \bar{\phi})} = \delta J + SJ\cos 2\bar{\phi} + \overline{V(J)}$$

Tune vs Action: vary lattice Q_0

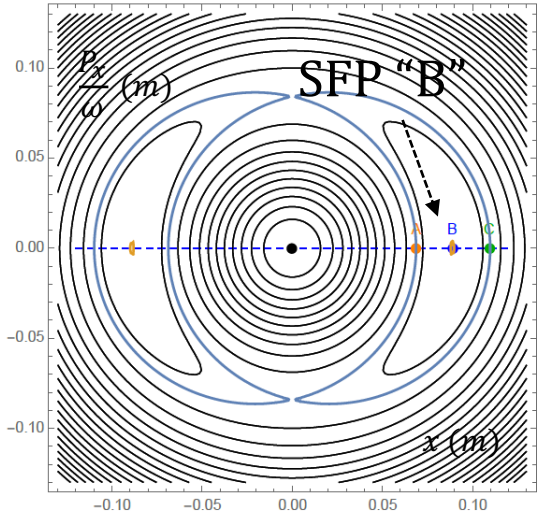


Resonance $2Q_v=7$

Tune vs Action: vary Intensity



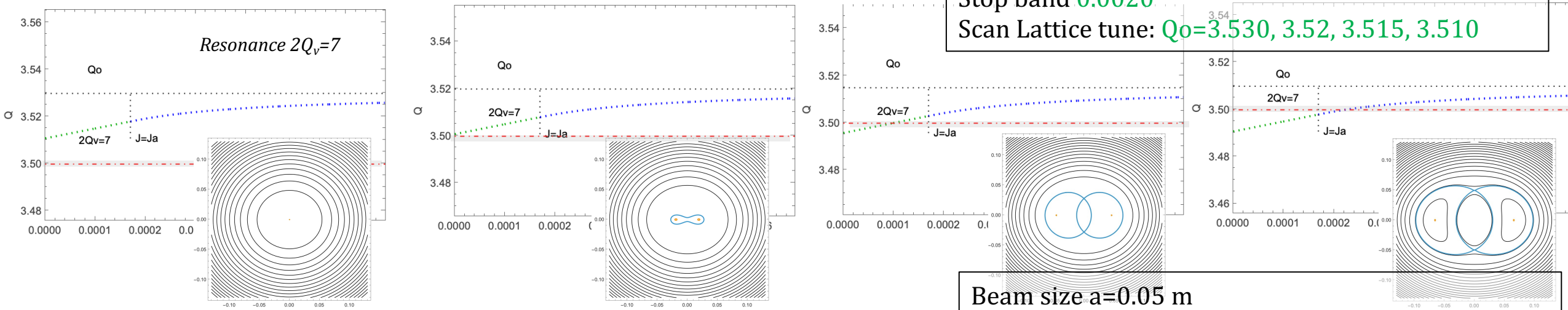
SFP moves out with decreasing Q or increasing Intensity



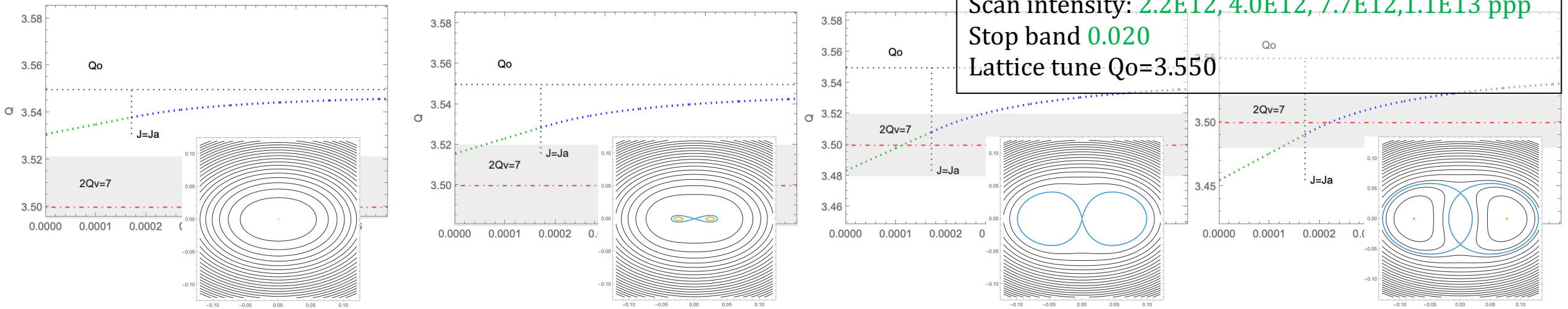
2 Frozen Waterbag space charge model of half integer ~ details around stopband

- More detailed calculations of structure in core wrt stop band
  - *SFP at origin disappears in stopband*
- Motion could be much more complicated! (Batygin et al. [2])

Beam size a=0.05 m
Intensity: 2.2E12 ppp
Stop band 0.0020
Scan Lattice tune: Qo=3.530, 3.52, 3.515, 3.510



Beam size a=0.05 m
Scan intensity: 2.2E12, 4.0E12, 7.7E12,1.1E13 ppp
Stop band 0.020
Lattice tune Qo=3.550



3 Frozen Gaussian space charge model of half integer and 2D effects

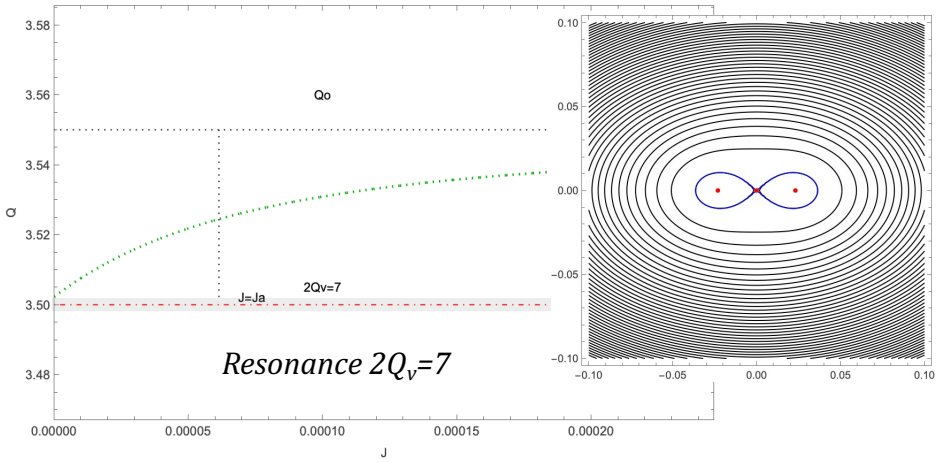
- Have redone phase averaging calculation for Gaussian distribution
  - As WB: Circular, on-axis (1D)
  - Very similar results to WB if take *a=2 σ*
  - Same basic behaviour, but now *Q(J)* non-linear in core

$$H(x, P_x, s) = \tfrac{1}{2}P_x^2 + \tfrac{1}{2}\omega^2 x^2 + K_d(s)x^2 - \frac{k}{2}(\mathrm{Ei}(-\frac{x^2}{2a^2}) + \mathrm{Ei}(-\frac{1}{2}) + \log(\frac{x^2}{a^2}))$$

same phase averaging …

$$\overline{H} = \overline{H(J,\bar{\phi})} = \delta J + SJ\cos 2\bar{\phi} + \overline{V(J)}$$

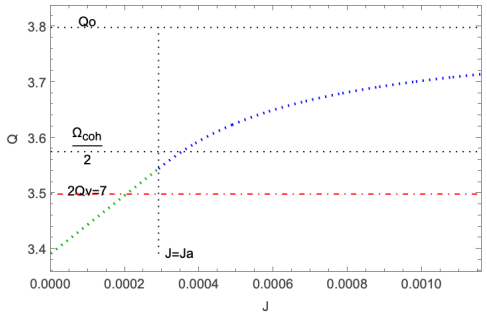
Example of Q(J) and phase space for Gaussian beam



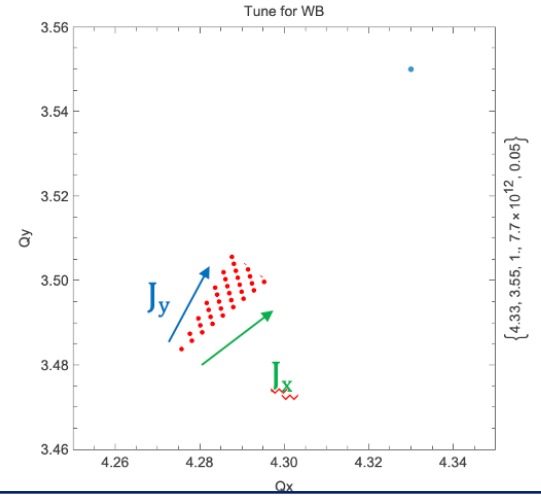
Resonance $2Q_v$=7

Beam size a=0.03 m
Intensity: 2.0E12 ppp
Lattice tune Q_0=3.550
Stopband 0.0002

Q(J) for WB beam



Q footprint of 2D WB beam



Relation to 2D beam

- Calc above for *J*=0 in perp plane
- If $J_x, J_y \neq 0$, smaller tune shift
- But same behaviour …

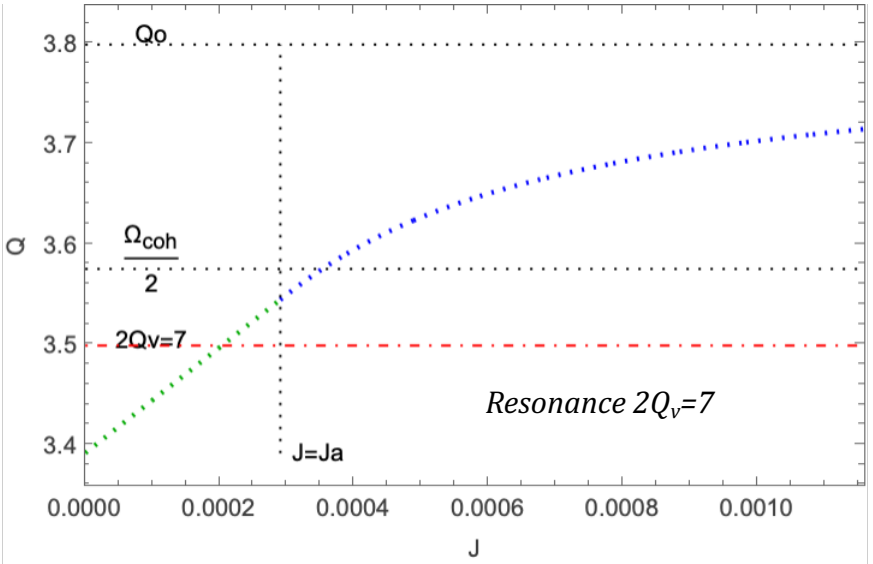
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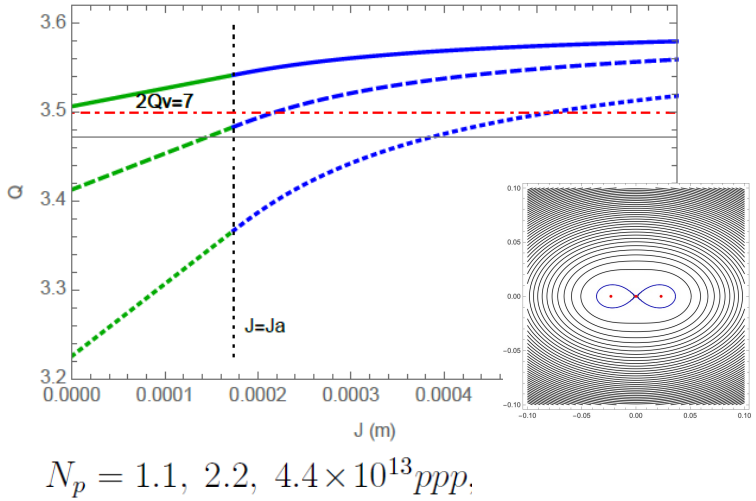
References: (Sacherer [3, 4], Bartman[5], Hofmann [6])

4 Coherent effects: combined behaviour

- Can calculate coherent envelope motion for RMS equivalent beam (if ε_{rms} conserved)
  - IE solve KV equation and apply results to RMS equiv. of real distribution (Sacherer [3])
- For KV beam find coherent resonance occurs at higher intensity than incoherent prediction
  - The self consistent solution includes resulting coherent envelope motion (cancels driving term for KV)
  - **Coherent advantage**: C=5/8 (large split); C=1/2, 3/4 (small split); (C=1 incoherent)) ~ Envelope amplitude increases as approach coherent resonance: beam lost at resonance
- The above coherent advantage applies to the incoherent tune shift of a KV beam
  - Non-uniform (WB, Gaussian) have larger tune shift (~2x) so additional coh. advantage for equiv. beams
- We can picture this by plotting $\Omega_{coh}/2$ on the *Q(J)* plot

$$Q_0 - C \cdot \Delta Q_{sc} = \frac{n}{2}$$





$N_p = 1.1,\ 2.2,\ 4.4 \times 10^{13} ppp.$

Think of resonance as descent of Q(J) line

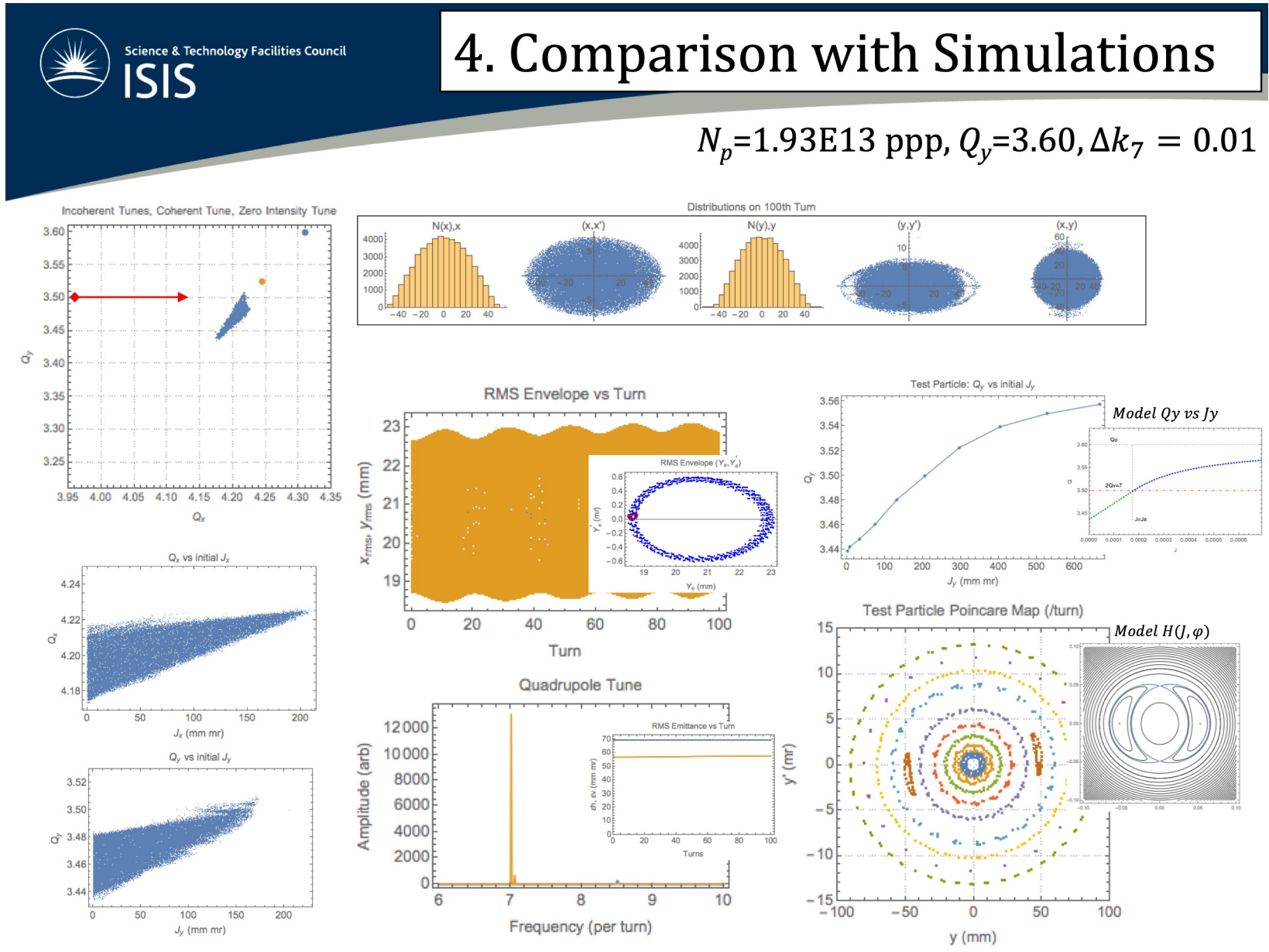
- If $\Omega_{coh}/2 >> l/2$, small env. motion
- As push Q_0 down, or increase I
- *Q(J)* line moves down
- Intersection with $2Q_v=l$ gives SFP
- These move out as *Q(J)* moves down
- As $\Omega_{coh}/2 \rightarrow l/2$ larger env. motion
- $\Omega_{coh}/2 \sim l/2$ beam unstable

But on approach, incoherent picture works …?
(if limited mismatch, and no other effects …)

5 Combined behaviour: 2D PIC simulations (Space Charge 2017)

- On approach to resonance
  - Expected incoherent motion
    - Clear islands
  - Gentle, regular envelop motion
- Fully self consistent PIC simulation
  - B G Pine's SET code

(Warsop [7])

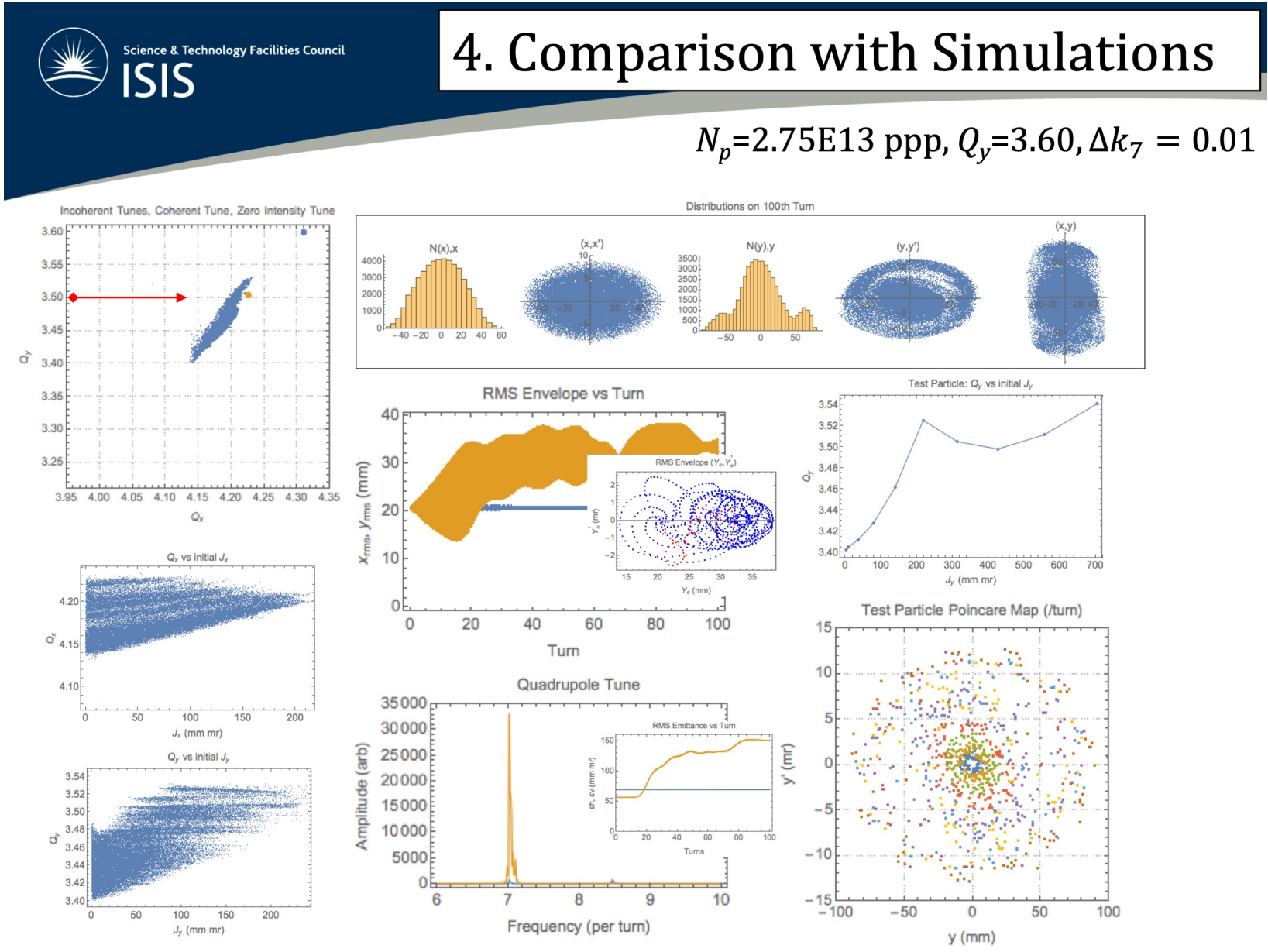


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5 Combined behaviour: 2D PIC simulations [Space Charge 2017]

- On resonance
  - Beam lost!

(Warsop [7])



5 Combined behaviour: 2D PIC simulations [Space Charge 2017]

- These results show:
  - For a well-behaved* WB get qualitative agreement with model (*minimal mismatch, coupling, … etc.)
  - Poincare map corresponds to expected structure until near coherent resonance
- Expect coherent envelope response "compensates" for driving term (KV case does perfectly: Venturini [8])
  - Coherent envelope motion modifies frozen model ~ *but to first order compensates or reduces driving term*?
- Perhaps we can put rms predictions back into frozen model: *slightly defrosted model*?

Hamiltonian in waterbag core

$$H(x,P_x,s)=\tfrac{1}{2}P_x^2+\tfrac{1}{2}\omega^2x^2+k_l\cos(l\theta)x^2-k\left(\frac{x^2}{a^2}-\frac{x^4}{4a^4}\right)$$

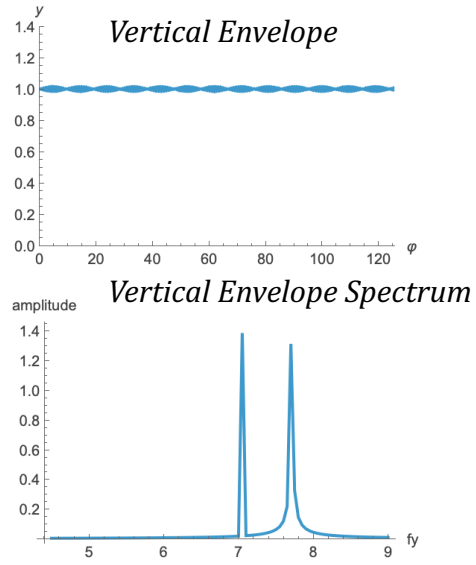
V_{sc}

- Assume. $a\to a(s)=a_0(1+\frac{\Delta a}{a_0}\cos(l\theta))$ then after expanding

$$V_{sc}=-k\left(\frac{x^2}{a_0^2}\left((1-2\frac{\Delta a}{a_0}\cos(l\theta))+\ldots\right)-\frac{x^4}{4a_0^4}(1-4\frac{\Delta a}{a_0}\cos(l\theta))+\ldots\right)$$

  - … cos terms may "compensate" – but also add non-linear terms …?
- Next steps: will test these ideas with
  - Simulations: compare coh. freq., ampl. with RMS eqns.; Poincare maps vs theory etc.
  - Experiments …

Example of solution of KV Eqn showing beating of envelope

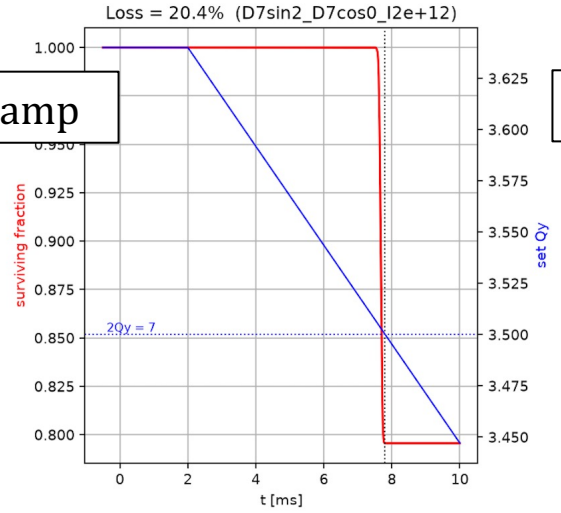


6 Experiments …

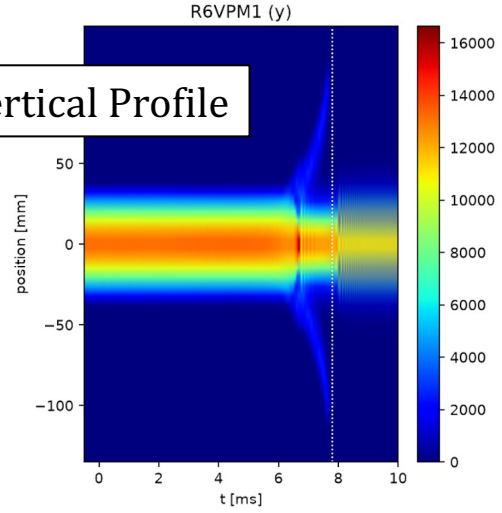
- Have done low intensity half integer measurements: are well understood (HB2025 Warsop [9])
  - Run ISIS in SRM, coasting beams, slow vertical ramp, apply driving term ….

- Now working on high intensity … early Xsuite high intensity simulations
  - Still analyzing half integer motion … seems expected behaviour

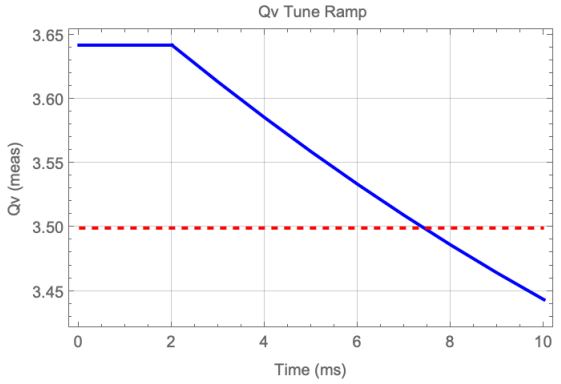
Tune Ramp

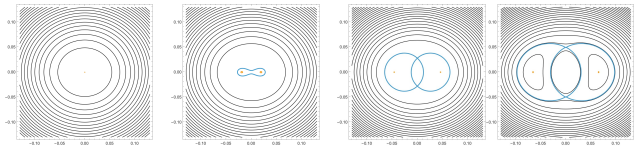


Vertical Profile



Nominal Parameters
Storing ring mode
T=70 MeV, I=2E12 ppp, WB
$\varepsilon_{x100\%}$= $\varepsilon_{y100\%}$=100 π mm mr
SB=0.002, dp/p=0.2%





Esmaeil Ahmadi

- Presently working on corresponding experiments!

Summary

- We have further developed a frozen space charge model of the half integer resonance …
- Together with standard coherent resonance theory, it may give a useful picture of behaviour on approach to half integer, at high intensity
- We will test in detail with simulations …
- Hopefully use to predict experimental behaviour

- This coasting beam model is basis for understanding fast, half integer crossing in ISIS RCS ops mode
- Eventually it should help us to understand, minimize losses … ☺

- This is part of a more general R&D program to measure and improve modelling and control of the ISIS beams …

References

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[5] R Baartman, Betatron Resonances with space charge, AIP Conf. Proc. CP448, 1998 (Shelter Island Workshop)

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[8] M. Venturini, R. L. Gluckstern, Resonance analysis for a space charge dominated beam in a circular lattice, PRSTAB, V3, p. 034203, March 2000

[9] C M Warsop et al., Measurements of linear resonances and stopbands on the ISIS Ring,, HB2025