

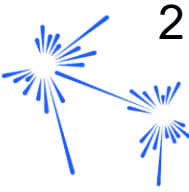
Impacts of Tune Asymmetry on the Space Charge Limit

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High Intensity Machines Often Operate Near-Equal Tunes

- High Intensity Working Points:

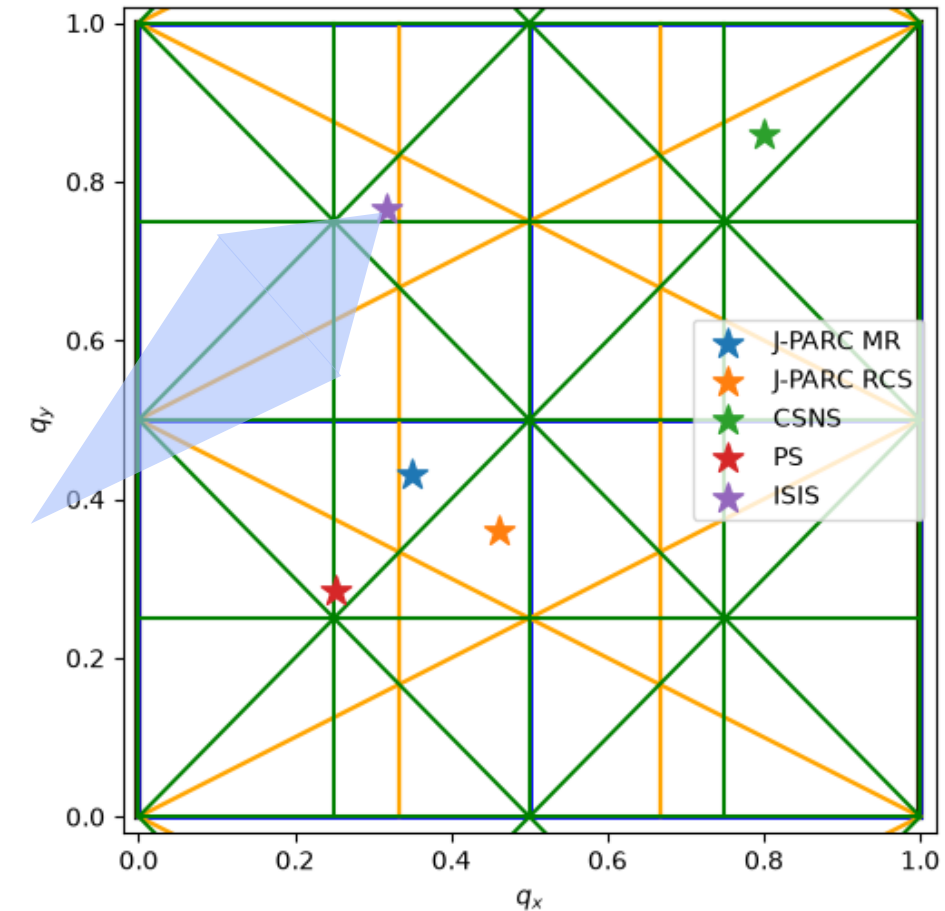
Facility	Intensity	Working Point
J-PARC MR	3.3e14	(21.35, 21.43)
J-PARC RCS	8.3e13	(6.46, 6.36)
ISIS	3e13	(4.316, 3.765)
CSNS	1.5e13	(4.8, 4.86)
CERN PS	8e12	(6.252, 6.285)

- Choice of Working Point

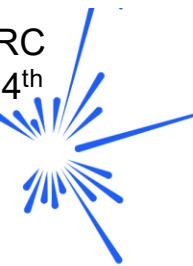
- Hardware limitations
- Location of resonances
- Tune Footprint

- Resonance Constraints

- Montague Resonance, $2Q_x - 2Q_y = 0$, 4th order coupled resonance
- To avoid impact of the Montague resonance, beam types are constrained to be in equilibrium with the Montague resonance, ie round beams



Fractional working points for J-PARC MR, CSNS, PS & ISIS, with up to 4th order resonance lines.



Does Tune Asymmetry Change the High-Intensity Limit?

1. Resonance Dynamics

- How does changing the integer or fractional working point change how a space-charge dominated beam interacts with a resonance?
 - Resonance trapping, action exchange, tune modulation

2. Observable Consequences

- How does this dynamic appear and manifest?
 - Emittance growth, emittance exchange, halo formation and beam loss

3. Intensity Limit

- Can tune asymmetry alter beam behaviour with resonances or increase the usable tune space, such that the beam intensity can be increased?
 - Tune-space area and space-charge-driven tune spread

Are some accelerator types better suited for high intensity operations due to the tune space they naturally operate in?

- **Synchrotrons:** Coupled transverse focusing favours similar tunes, so large tune separation can require more extreme optics
- **Fixed-Field Accelerators:** Greater focusing flexibility allows lattices to be designed with strongly unequal tunes.



Comparisons with an identical fractional tune

Fractionally Similar

For example,

$$Q_x = 3.66 \text{ and } Q_y = 4.66$$

With analogous horizontal third-order resonance conditions,

$$3Q_x = 11 \text{ and } 3Q_y = 14$$

But not dynamically identical

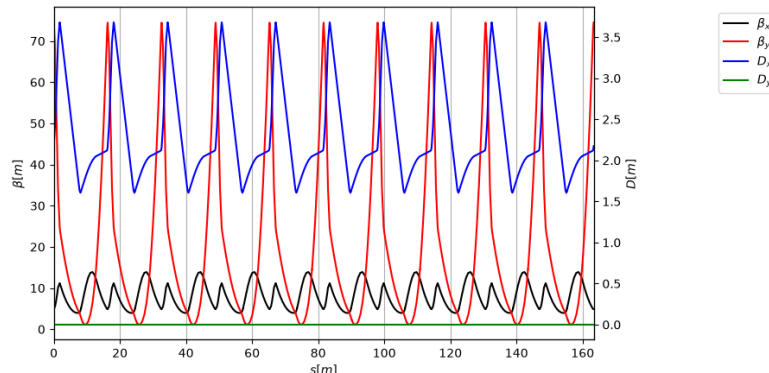
Changing the integer tune changes the optics of the machine:

$$Q = \frac{1}{2\pi} \int \frac{ds}{\beta(s)}$$

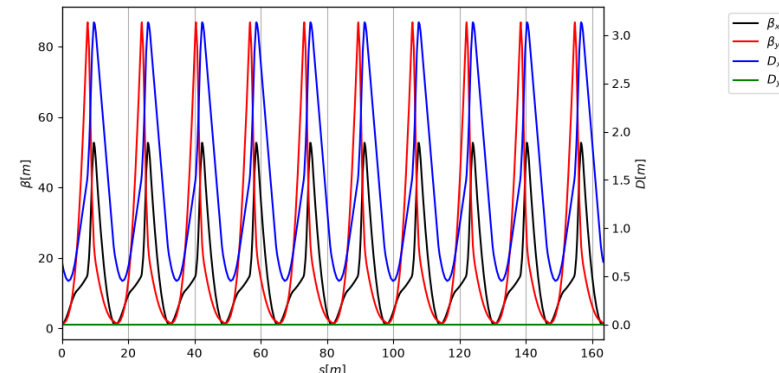
- Different $\beta(s)_{x,y}$ and beam size
- Different space charge forces
- Different incoherent tune spreads
- Different proximity to resonances

Therefore, a different integer tune may not produce the same beam response to resonances

$Q = (3.666, 4.666)$



$Q = (4.666, 4.666)$



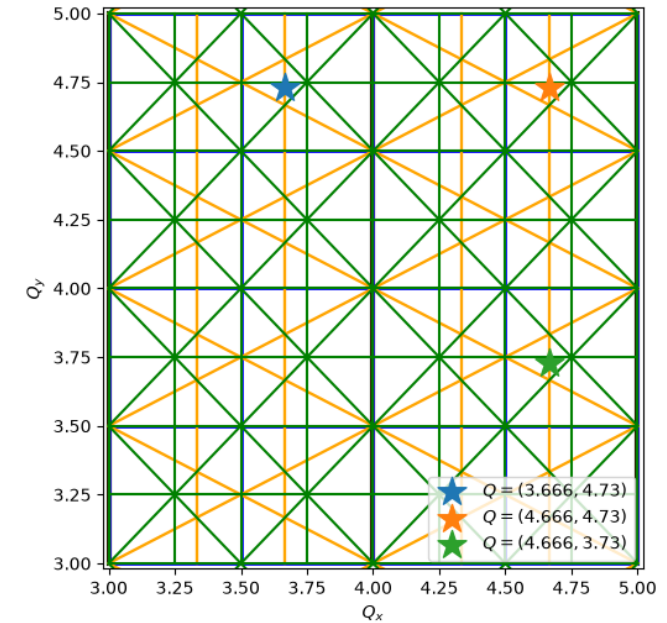
Investigating Integer Tunes Using Third-Order Resonances

Setup

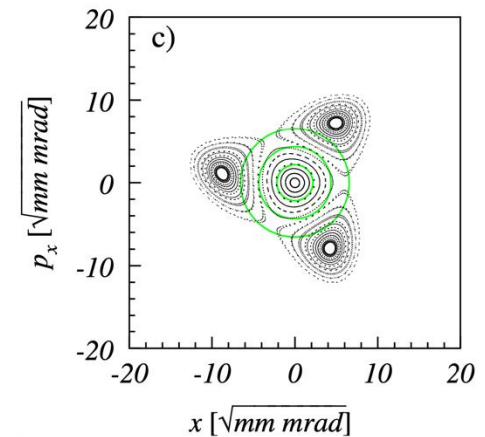
- Simulated on the ISIS Lattice using Xsuite with space charge modelled with self-consistent Particle-in-Cell
- Using a single sextupole to drive $3Q_x = n$ with resonance strength $G = 0.05$ for a 3σ gaussian beam with $\Delta Q_{max}^{SC} = 0.2$

Expectation

- In normalised phase space, we expect to see three-fold bounded symmetric motion when inside the separatrix, with three resonance islands outside
- These should be clear traces that outline a shape
- When near the Montague resonance, we expect coupling between transverse planes, but the overall action should be constant in this case



Target interactions for ISIS, with up to 4th order resonance lines.



Third-Order Phase Space Portrait
Credit: G. Franchetti, et al.

Initial Observations (Unequal Tunes)

Observation:

A strong second order modulation is seen in the vertical action, most clearly at high Q_y

Hypothesis:

Beam-lattice mismatch with space charge at large Q

Explanation:

Zero-Intensity Matching:

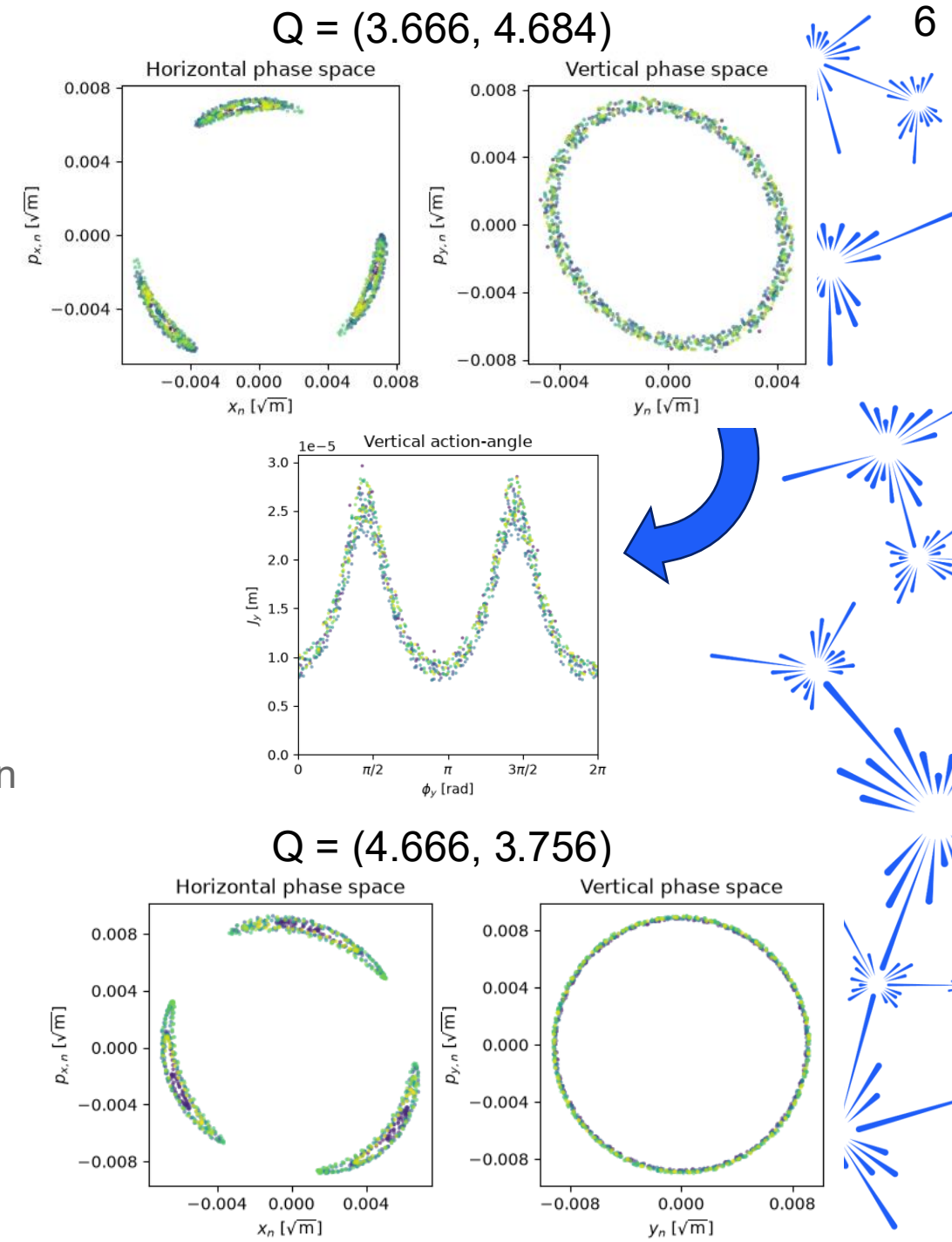
- Match beam to the zero-intensity lattice optics

High-Intensity Matching:

- Space charge adds defocusing to the beam optics
- When mismatch occurs, second order oscillations are driven

Matching at Extreme Q :

- As ISIS has 10-fold periodicity, $2Q = 10$ is a systematic half-integer resonance
- As $Q \rightarrow 5$, the lattice becomes increasingly sensitive to 2nd order perturbations, magnifying any beam-lattice mismatch



Initial Observations (Equal Tunes)

Observation:

Resonance islands are “fuzzy” when near equal tunes.

Hypothesis:

Transverse action exchange near the Montague resonance

Explanation:

Montague Resonance:

- Approaching $2Q_x - 2Q_y = 0$
- Space charge couples transverse planes, so action can exchange

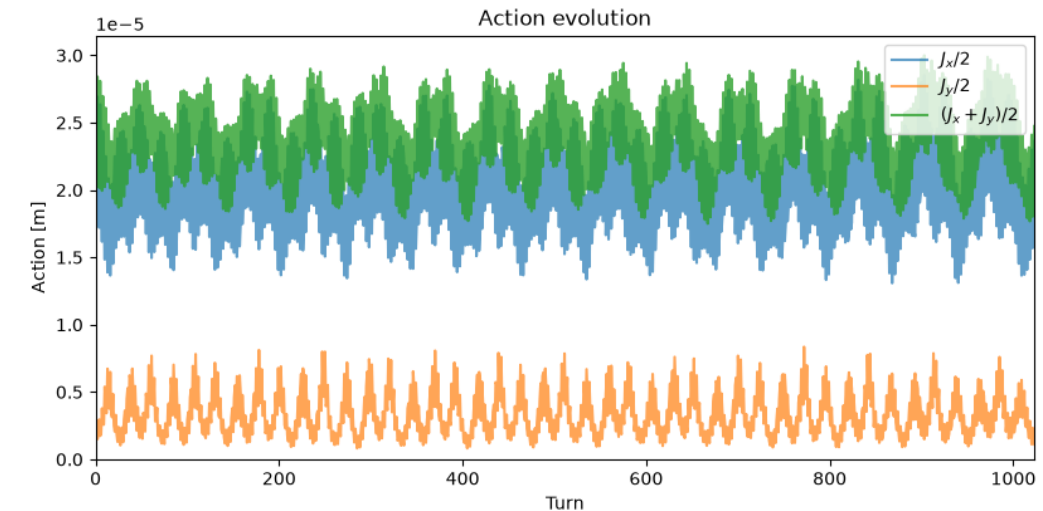
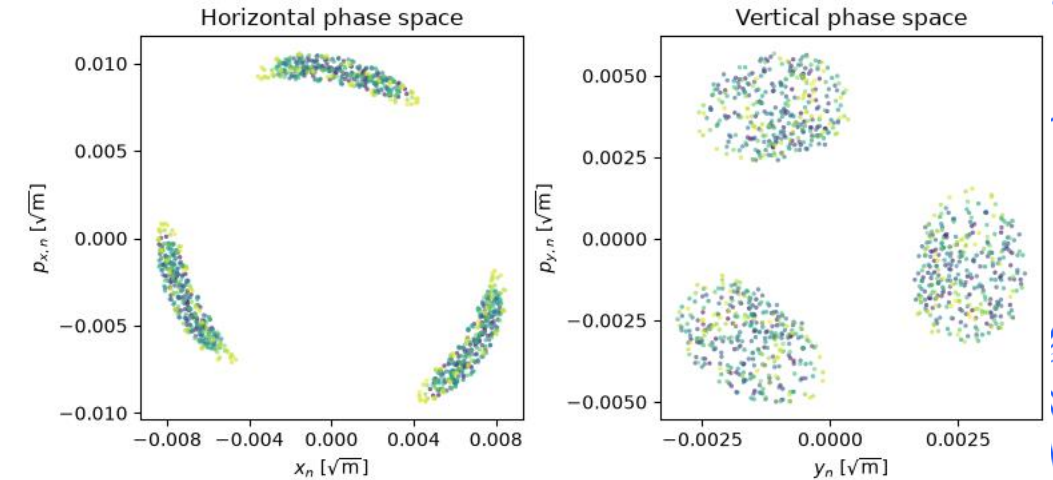
Expectation:

- Anti-correlated exchange between J_x (blue) and J_y (orange)
- Total action (green) remaining constant.

Observation:

- Action exchange cannot be isolated yet
- Action shows significant modulation
- May be masked by the mismatch

$$Q = (4.666, 4.666)$$



From Resonance Dynamics to the Space Charge Limit

1. Establishing the Underlying Motion
 - Remove mismatch with beam matching
 - Minimise structural optics contributions
2. Identify Coupling Mechanism
 - Quantify action exchange near equal tunes
3. Compare Resonance Response
 - Trapping, emittance growth, halo formation, beam loss
4. Compare the Space Charge Limits at different tunes
 - Equal vs Unequal tunes

Are near-equal tunes intrinsically favourable at high intensity?



Thank you for listening

Any Questions?

Appendix A: Xsuite Simulation Model

Particle Distribution:

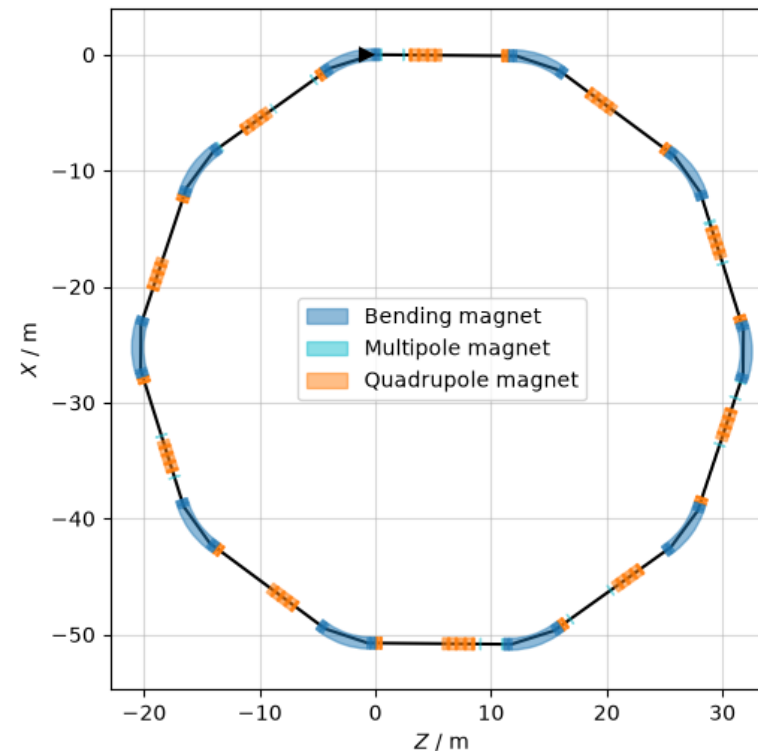
- 3σ truncated gaussian
- $5e12$ protons ($1e6$ macroparticles)
- Kinetic energy: $70.44e6$ eV
- $\epsilon_{n(x,y)}^{rms} = 7.19e6$ m rad
- $\frac{\Delta p}{p} = 0$ (uniform coasting, 4d beam)
- $\Delta Q_{max}^{sc} = 0.2$

Simulation Settings:

- 1024 Turns

PIC Settings:

- Xsuite "FFTSolver2p5D"
- Evaluated 250 / turn
- $128 \times 128 \times 2$ grids (9 grids)
- 6σ minimum extent + aperture
- (Custom: Uses next largest PIC grid)



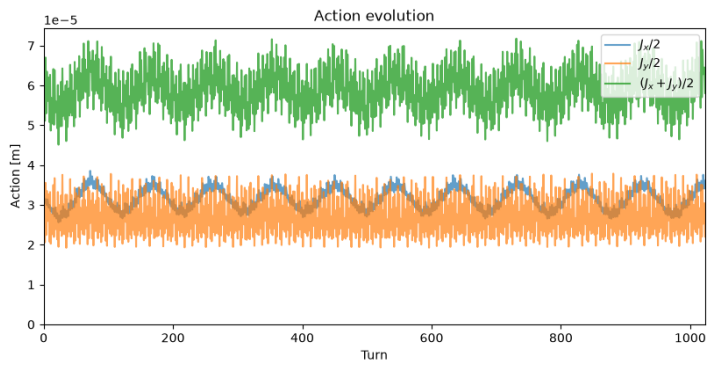
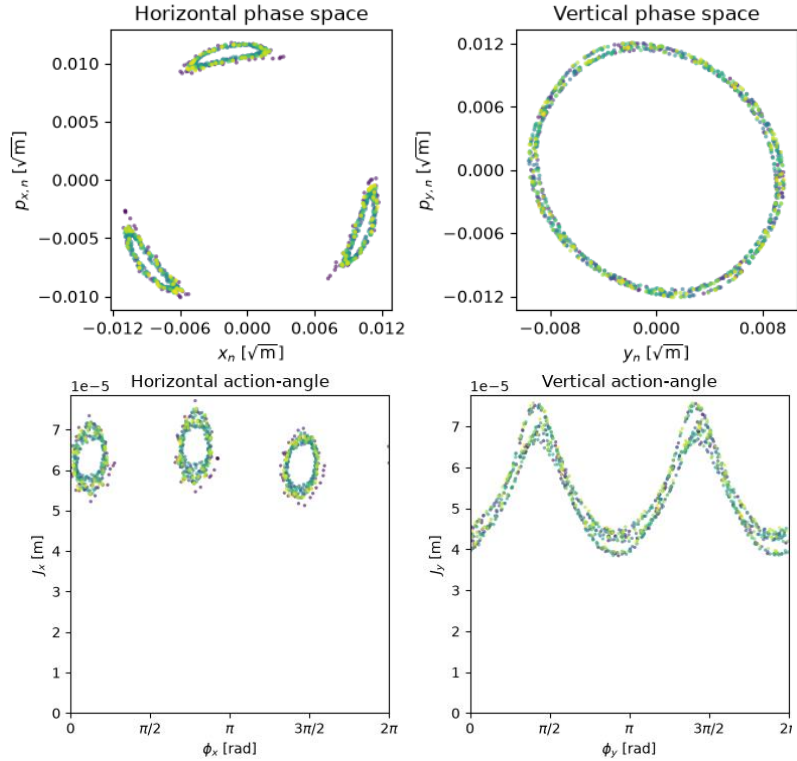
Appendix B: Equilibration Process

1. Match the particle distribution from the zero-intensity lattice twiss
2. Track over $6n$ turns, recalculating the beam distribution twiss each turn.
3. Calculate the mean twiss in each n sample window
4. Estimate the residual variation from double the final 4 windows
5. Equilibrium is reached when 6 consecutive windows are stable in both transverse planes
6. If not stable, track another n turns and repeat
7. If stable, average the twiss parameters over the full stable region
8. Match the particle distribution with these new high-intensity twiss values

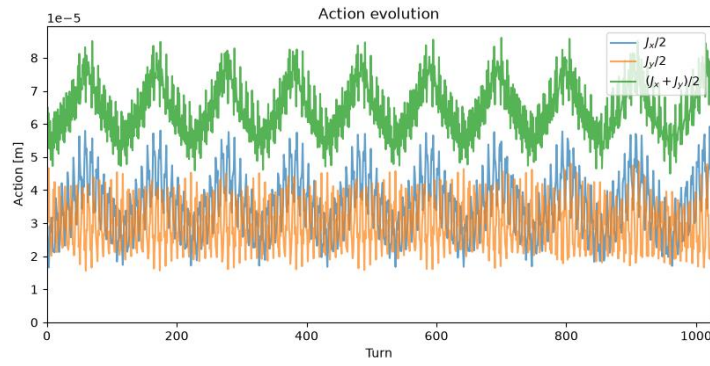
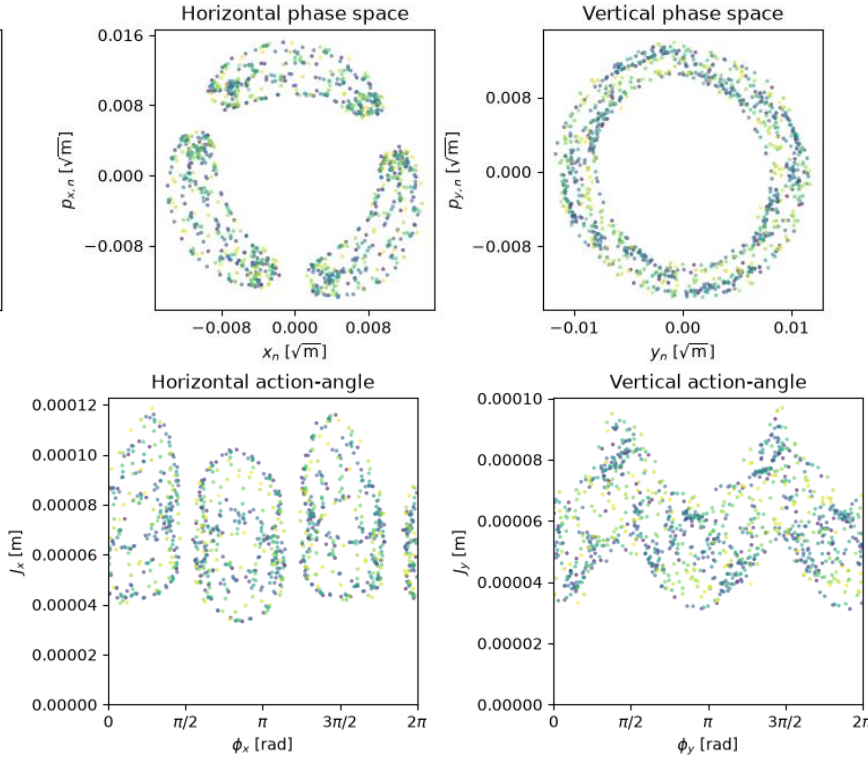
Appendix C: Additional Montague Coupling Examples

Additional examples as to show this isn't numerical noise/clarify coupling evidence

$Q = (3.666, 4.713)$



$Q = (4.666, 4.724)$



$Q = (4.663, 4.719)$

