OAK RIDGE National Laboratory | SPALLATION NEUTRON SOURCE

Phase space painting into non-planar modes for space charge mitigation

Austin Hoover (with Nick Evans, Kelly Anderson, Vasiliy Morozov)

7th ICFA Workshop on Space Charge | Oxford, UK

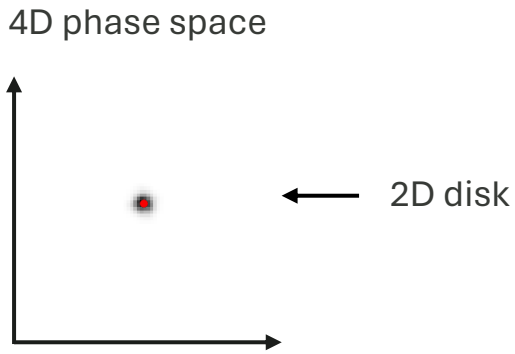
October 01, 2026

U.S. DEPARTMENT of ENERGY

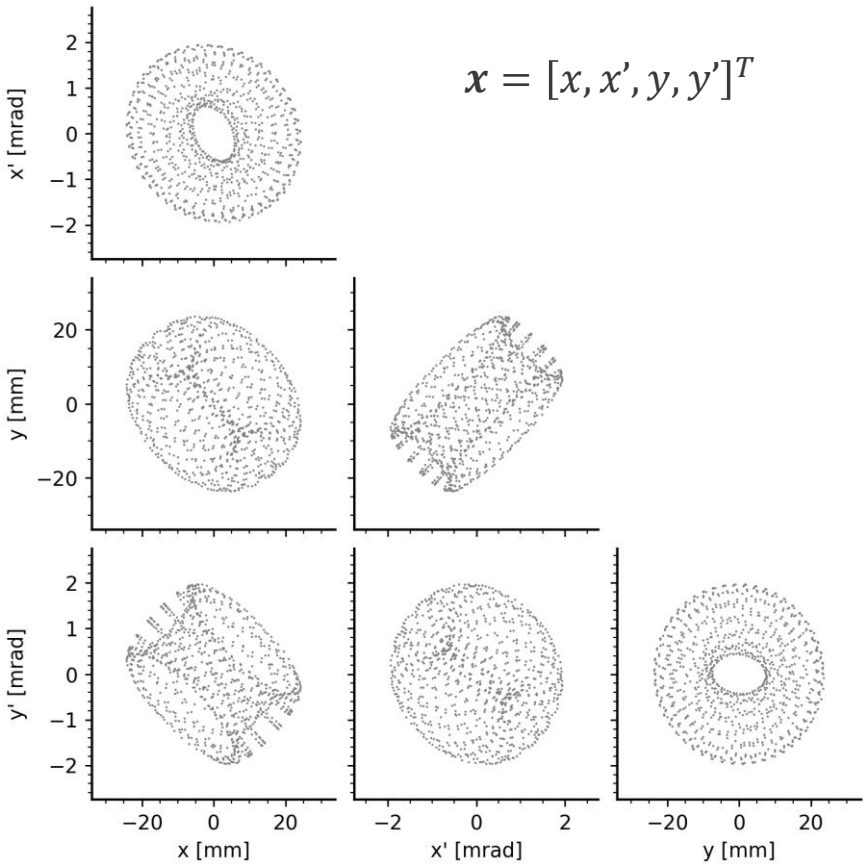
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Eigenpainting is a method to generate intense beam distributions with small 4D emittance

- Originally proposed by Danilov in 2003.
- Experimental program at SNS going (slowly) since 2018.
- I will explain the method and initial experiments + planned improvements and open questions.

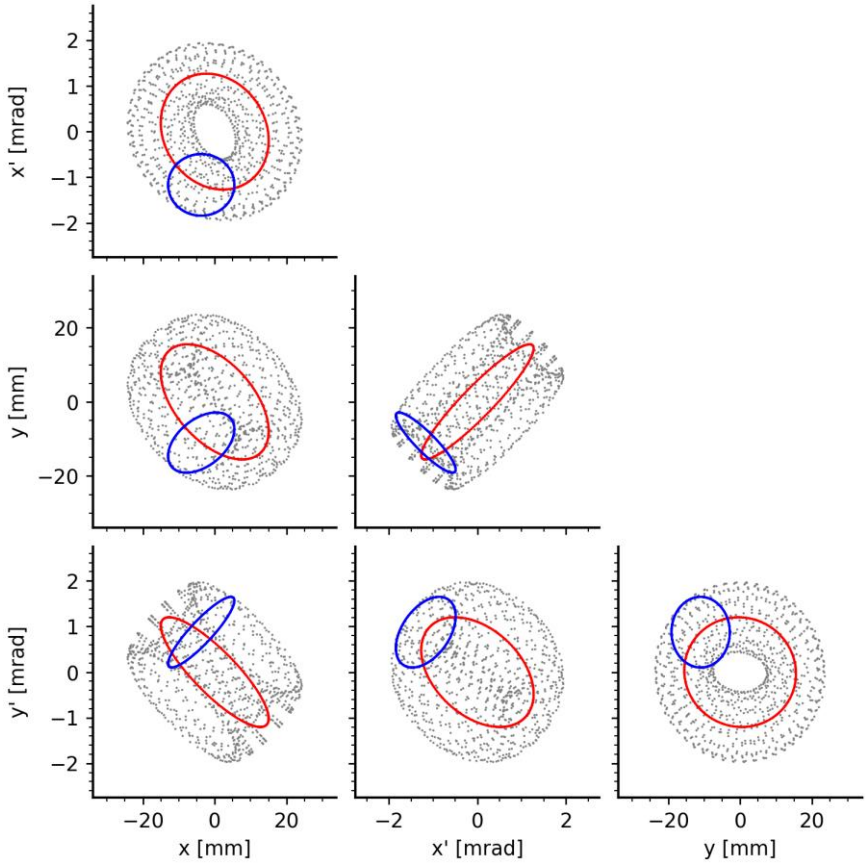


Start with turn-by-turn coordinates in a linear lattice



$$\boldsymbol{x} = [x, x', y, y']^T$$

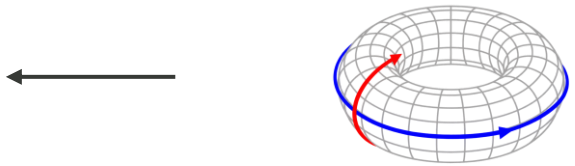
Express coordinates as linear combination of transfer matrix eigenvectors (modes)



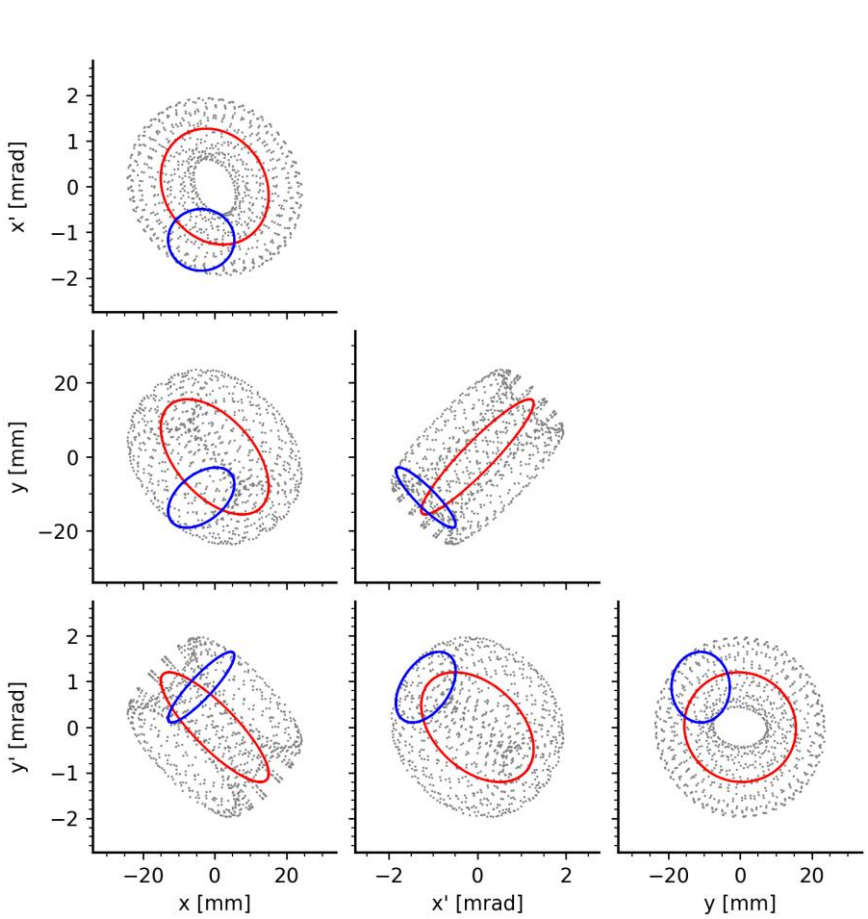
$$\boldsymbol{x} = \Re\{\sqrt{2J_1}\boldsymbol{v}_1 e^{-i\psi_1} + \sqrt{2J_2}\boldsymbol{v}_2 e^{-i\psi_2}\}$$

$$\boldsymbol{Mx} = \Re\{\sqrt{2J_1}\boldsymbol{v}_1 e^{-i(\psi_1+\mu_1)} + \sqrt{2J_2}\boldsymbol{v}_2 e^{-i(\psi_2+\mu_2)}\}$$

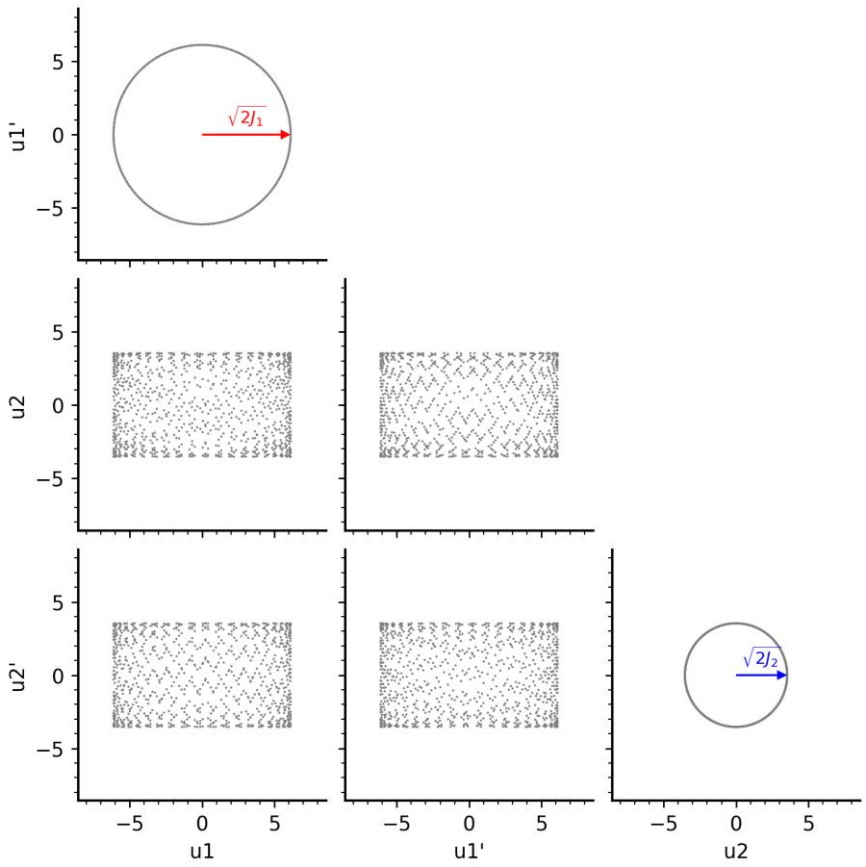
4D torus



Define normalized coordinates through symplectic decoupling transformation

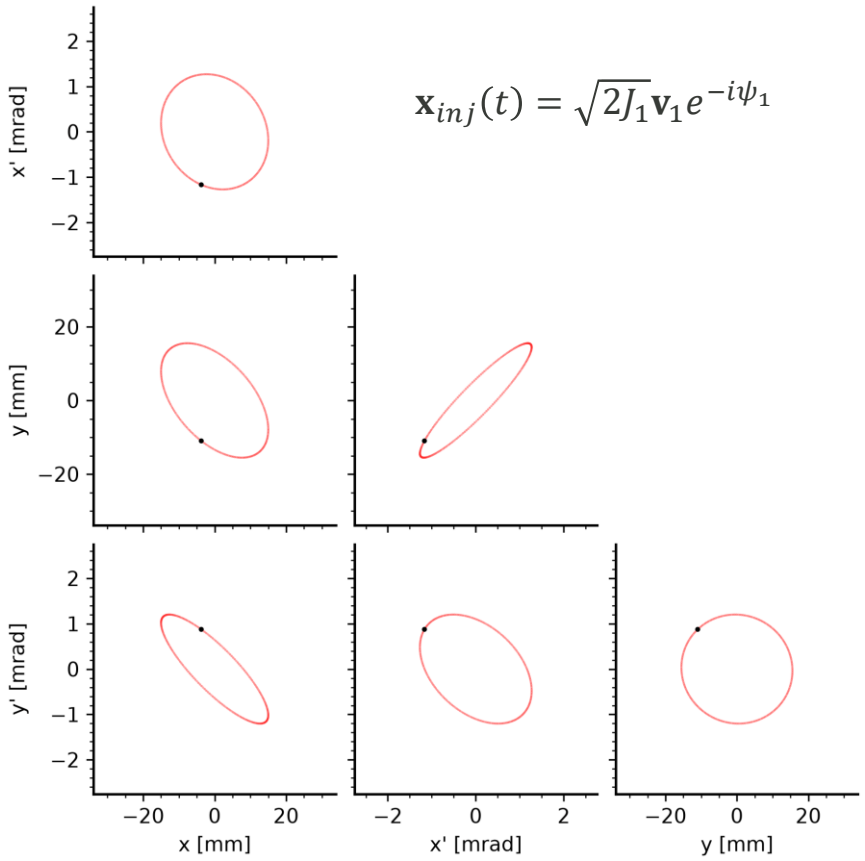


Normalized

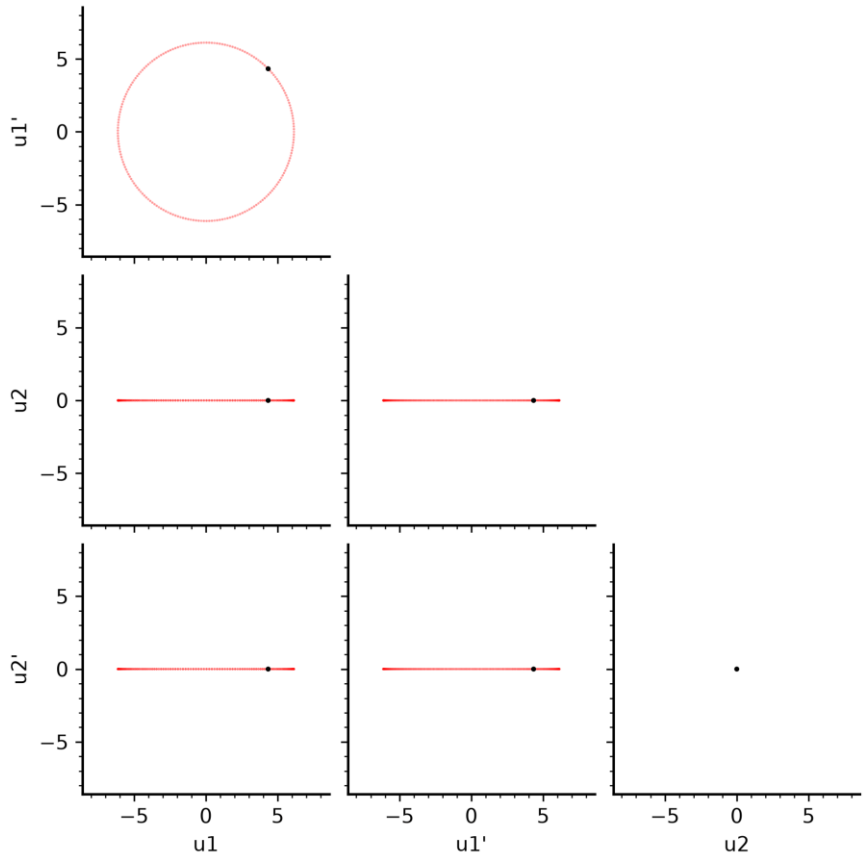


Create elliptical paintbrush by injecting particles into one mode

$$\mathbf{x}_{inj}(t) = \sqrt{2J_1}\mathbf{v}_1 e^{-i\psi_1}$$

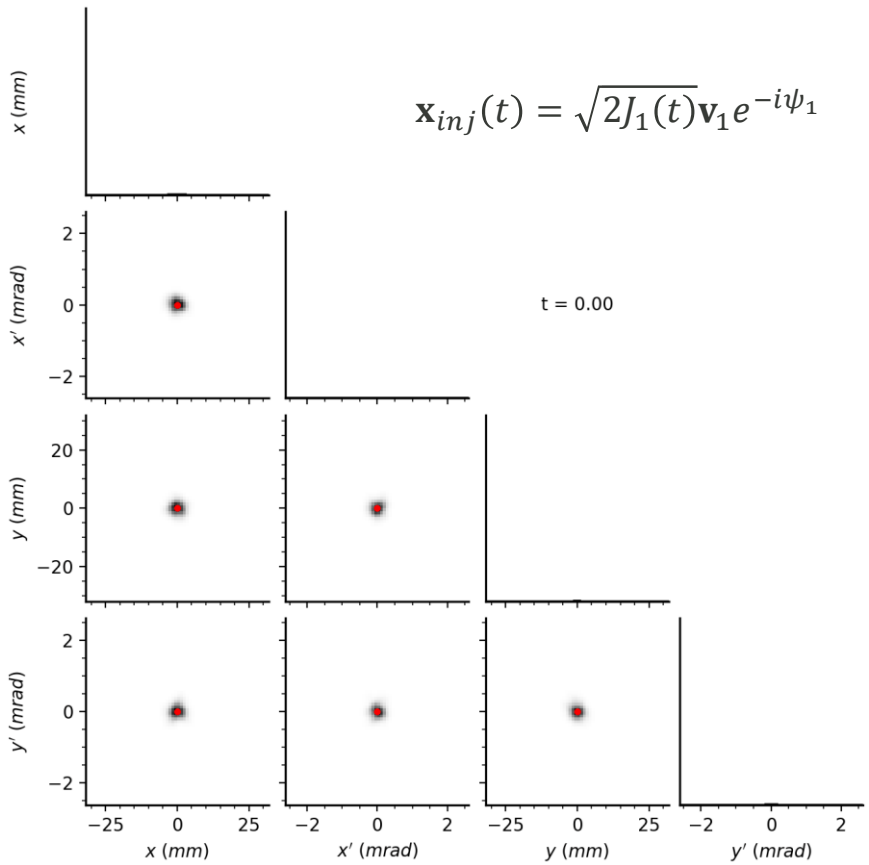


Normalized

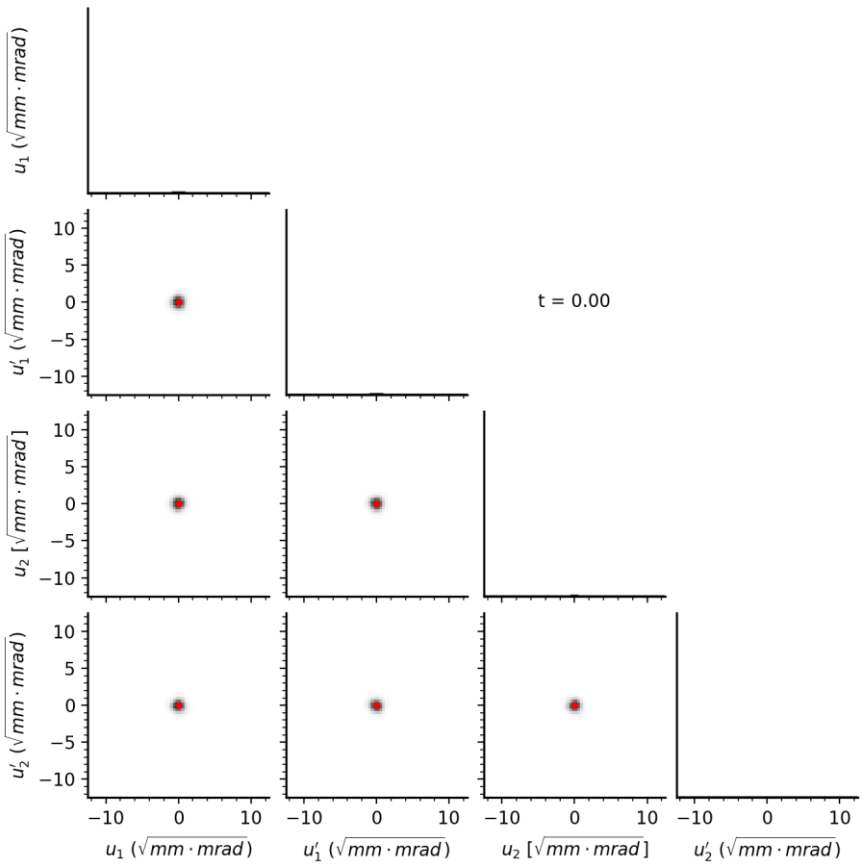


Scale injection amplitude to create any elliptically symmetric distribution

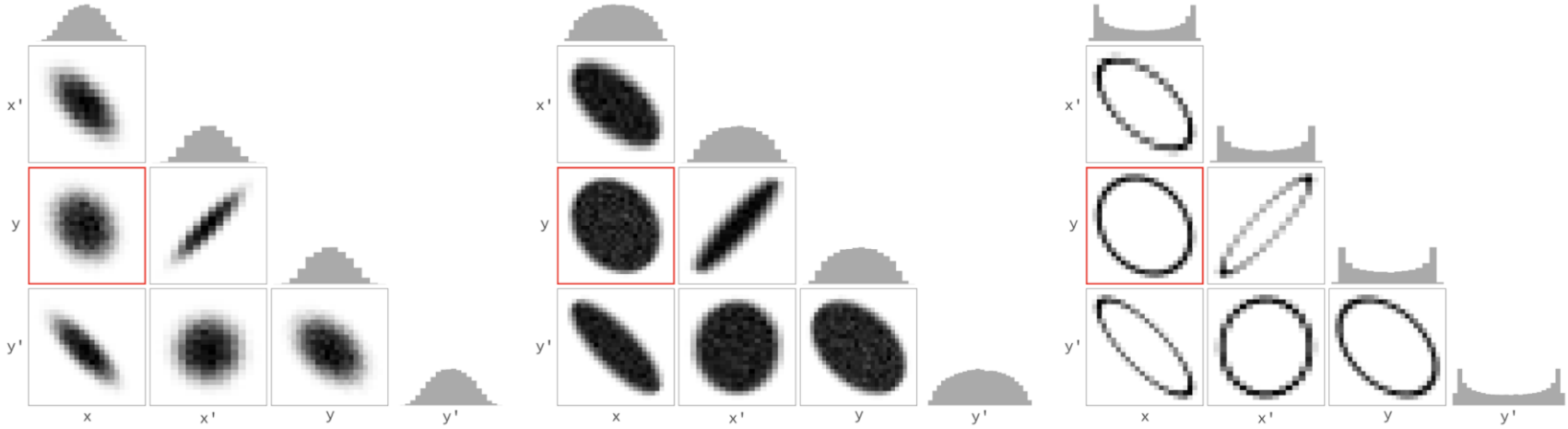
$$\mathbf{x}_{inj}(t) = \sqrt{2J_1(t)}\mathbf{v}_1 e^{-i\psi_1}$$



Normalized



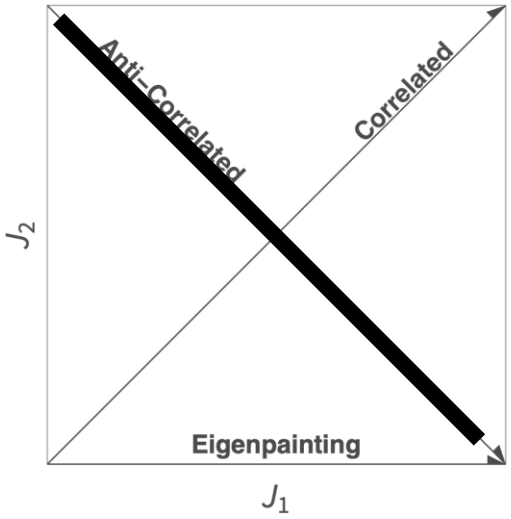
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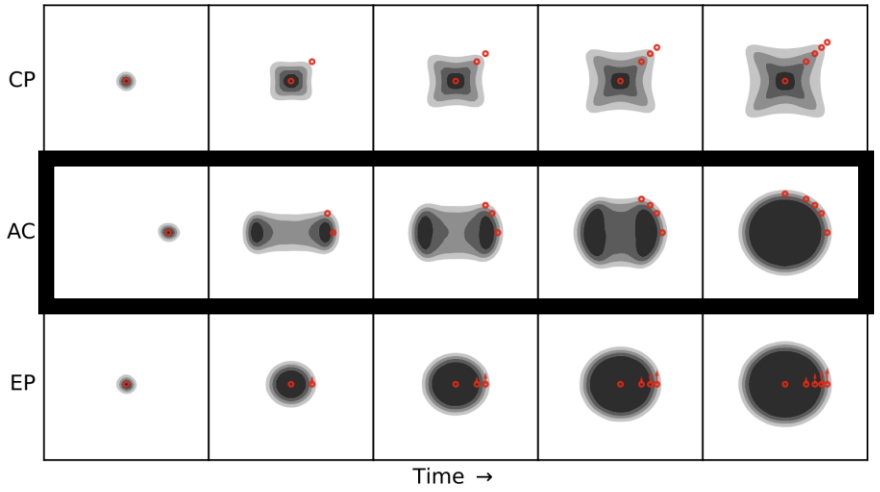
(a) Gaussian (b) Danilov (c) Hollow

Anti-correlated painting generates equilibrium distribution after painting, ignoring space charge

$$f(\mathbf{x},\mathbf{x}') = \delta\left(1 - \frac{J_1(\mathbf{x},\mathbf{x}')}{\tilde{J}_1} - \frac{J_2(\mathbf{x},\mathbf{x}')}{\tilde{J}_2}\right)$$



(a) Schematic trajectories for several painting schemes.



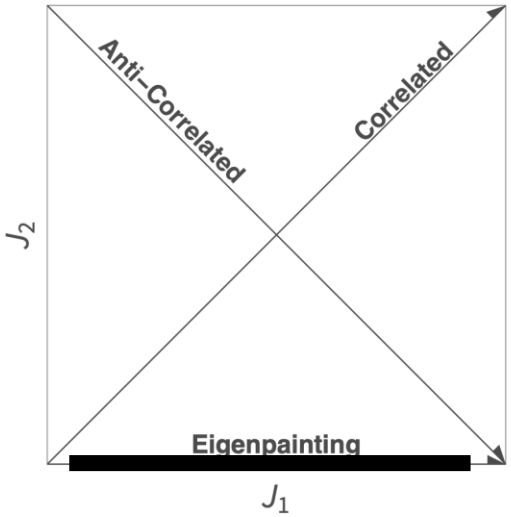
(b) x-y distributions.

Uniform eigenpainting maintains equilibrium distribution throughout painting (with space charge)!

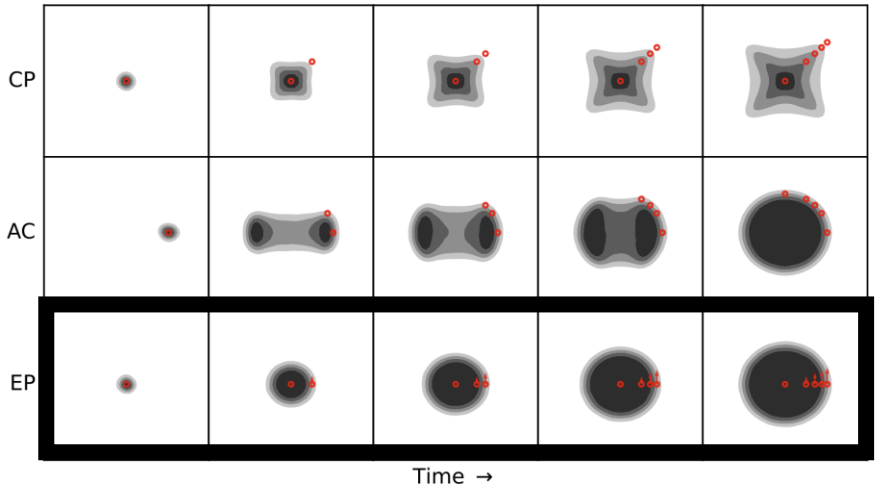
$$f(\mathbf{x},\mathbf{x}') \to \theta\left(1 - \frac{J_1(\mathbf{x},\mathbf{x}')}{\tilde{J}_1}\right)\delta\big(J_2(\mathbf{x},\mathbf{x}')\big)$$

(KV in limit $\tilde{J}_2 \to 0$)

$\tilde{J}_2 \to 0$



(a) Schematic trajectories for several painting schemes.

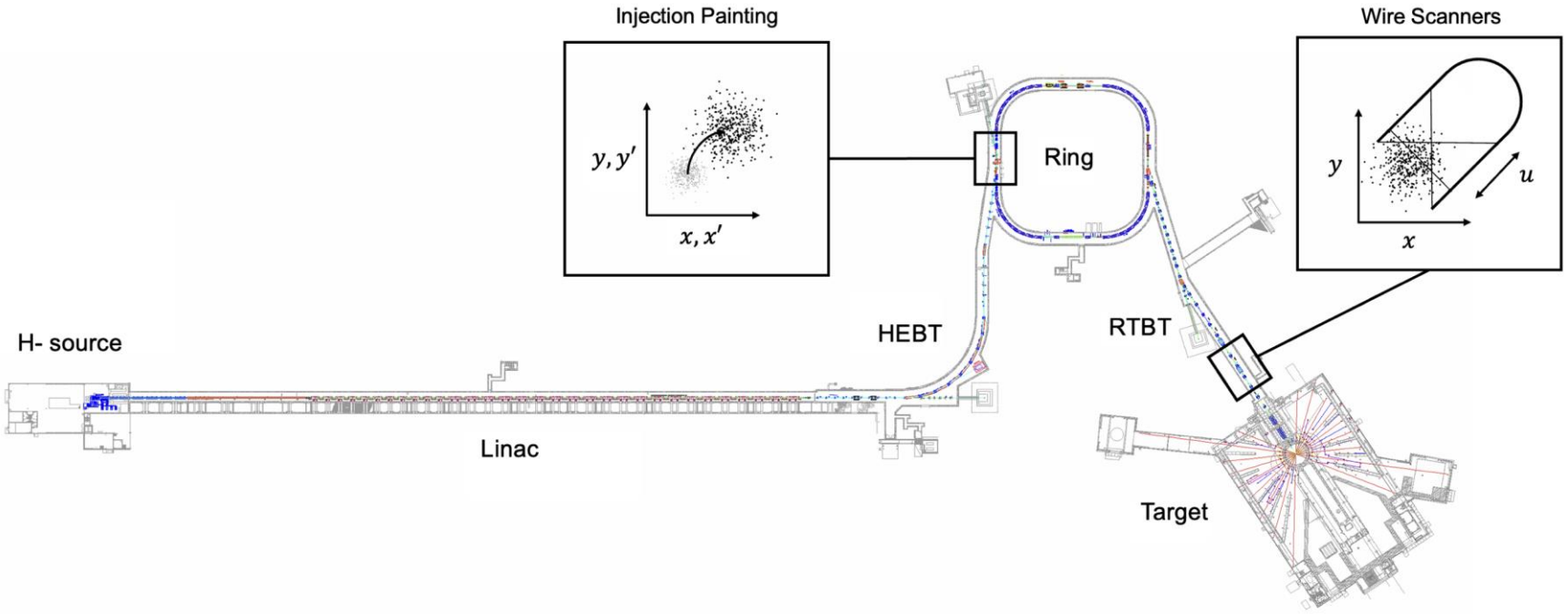


(b) x-y distributions.

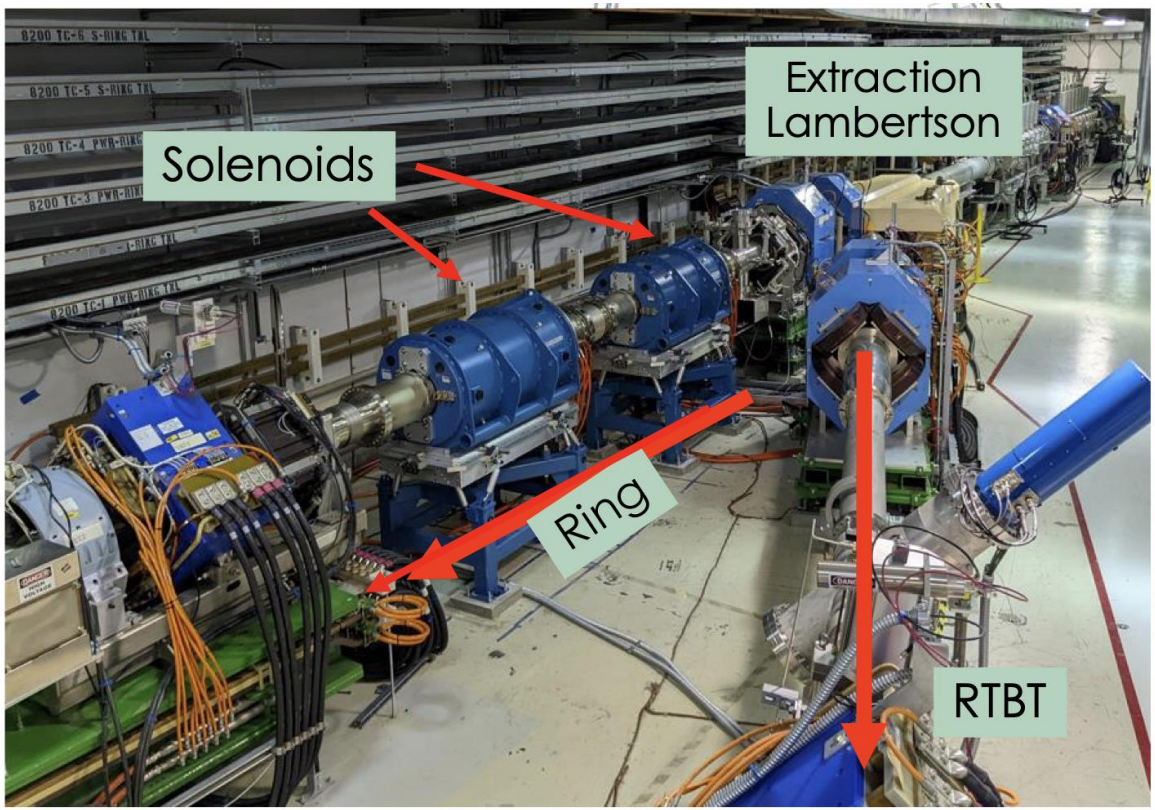
Why would we want to do this?

- Space charge: reduced tune spread, suppressed particle-core effects
- Flat beams for colliders [Phys. Rev. ST Accel. Beams 16, 061002]
- Low-energy hadron cooling [Phys. Rev. E 66, 016503]
- Academic studies: approach “unrealistic” distribution in the lab

Initial experiments tried to implement eigenpainting in the SNS AR

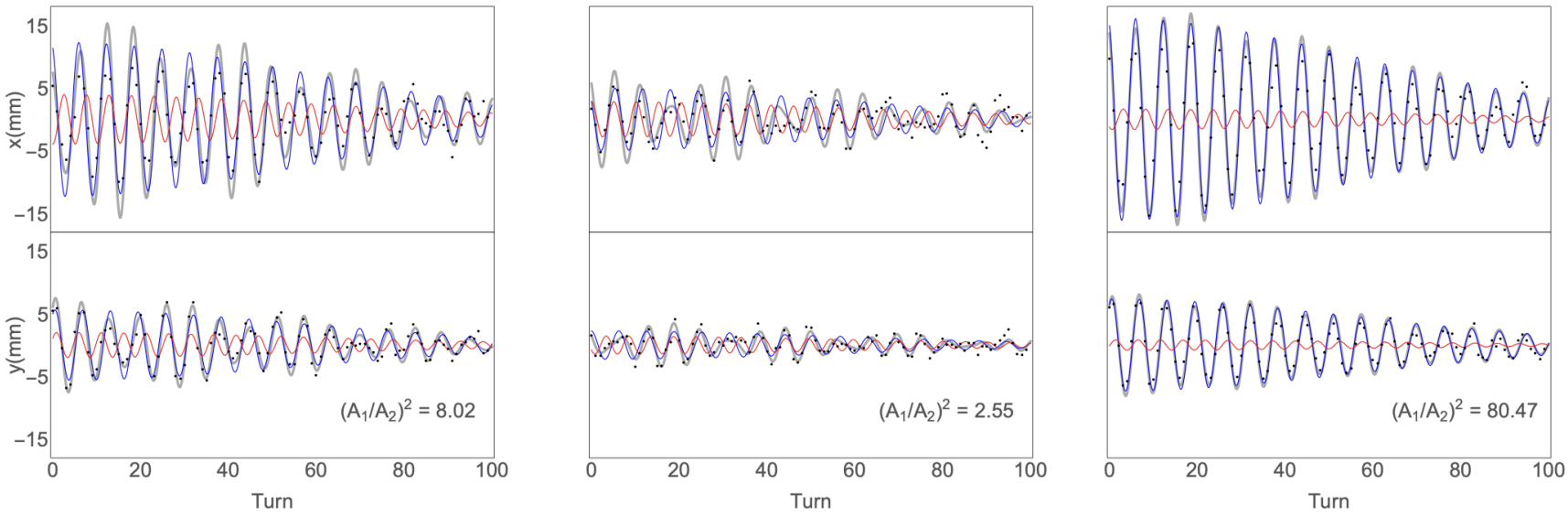


Two solenoids provide linear coupling



BPM data show injection mostly into one mode… couldn’t get all the way to closed orbit

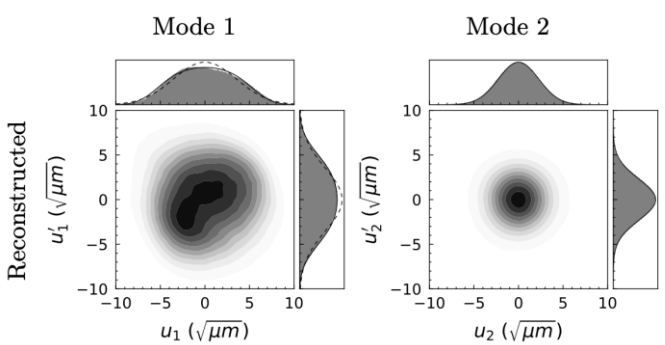
$$\mathbf{x}^j(n) = \sum_{k=1,2} A_k e^{-\gamma_k n^2 - i(2\pi n \nu_k + \Psi_k)} \mathbf{v}_k^j(n) + \mathbf{c}^j,$$



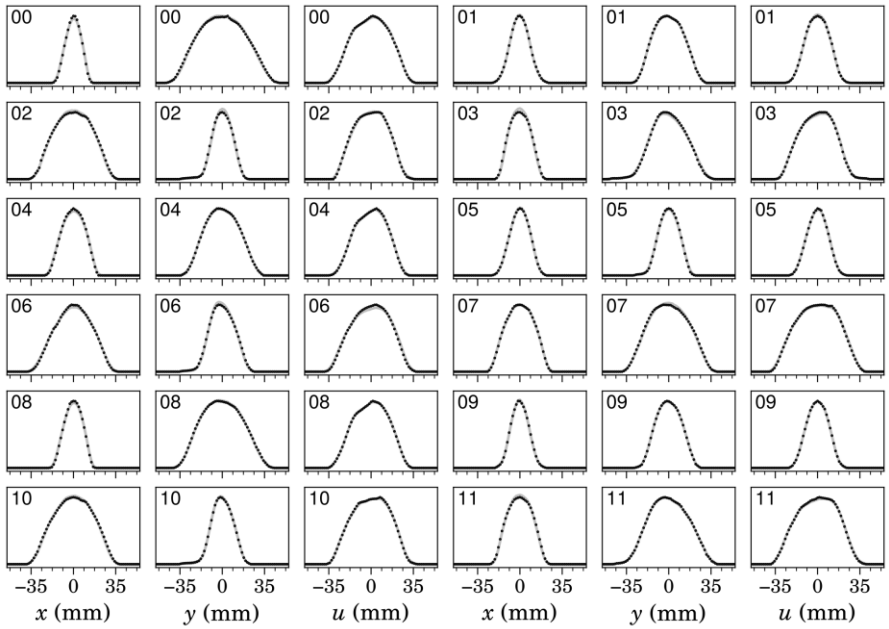
(a) Calibration Data (b) Injection at $t = 0$ (c) Injection at $t = T_f$

Connect with sqrt(t) waveform

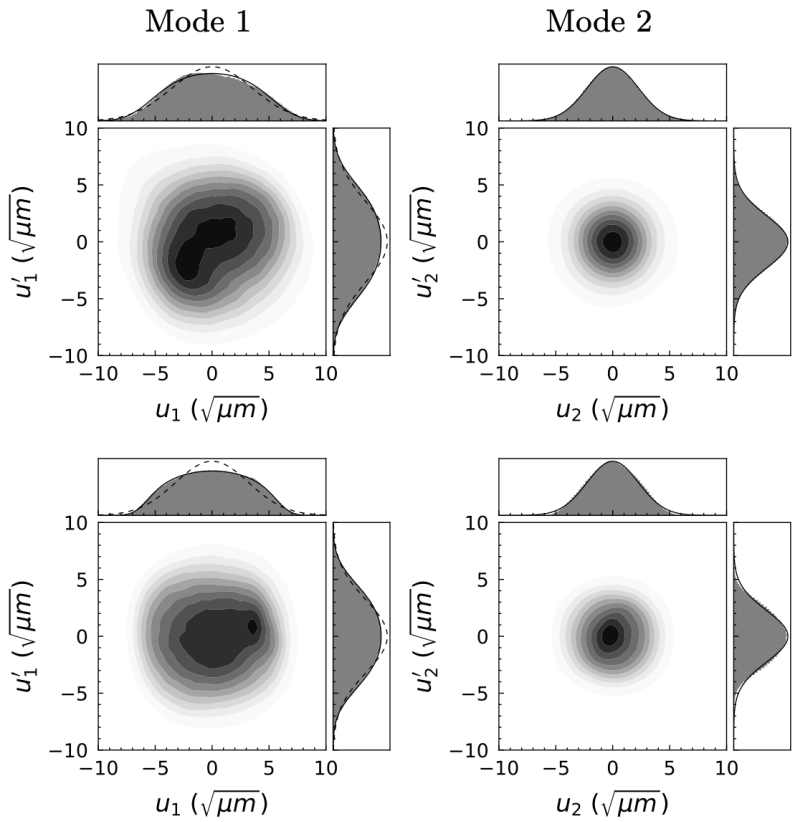
4D MENT reconstruction performed using 1D profiles of painted beam



$$\max_f S[f] \text{ s.t. } G_k[f] = 0$$



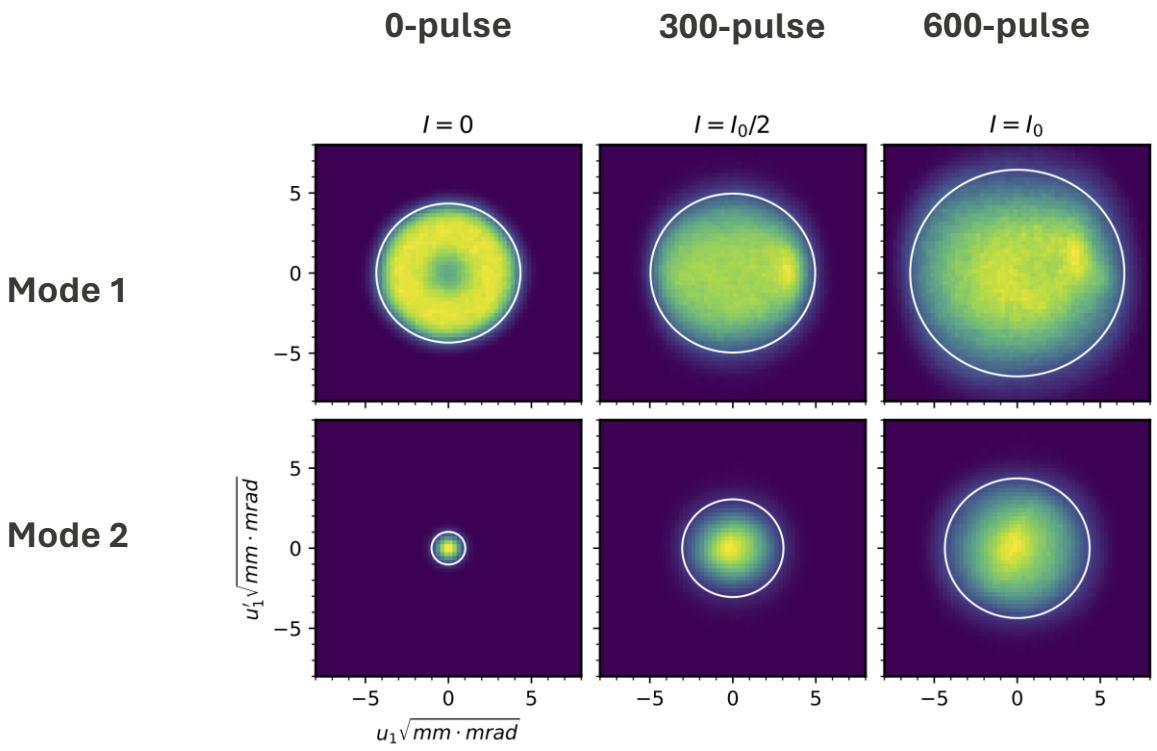
Simulations produce correct emittances and qualitative agreement with MENT reconstruction



$\varepsilon_1/\varepsilon_2 \approx 2.4$

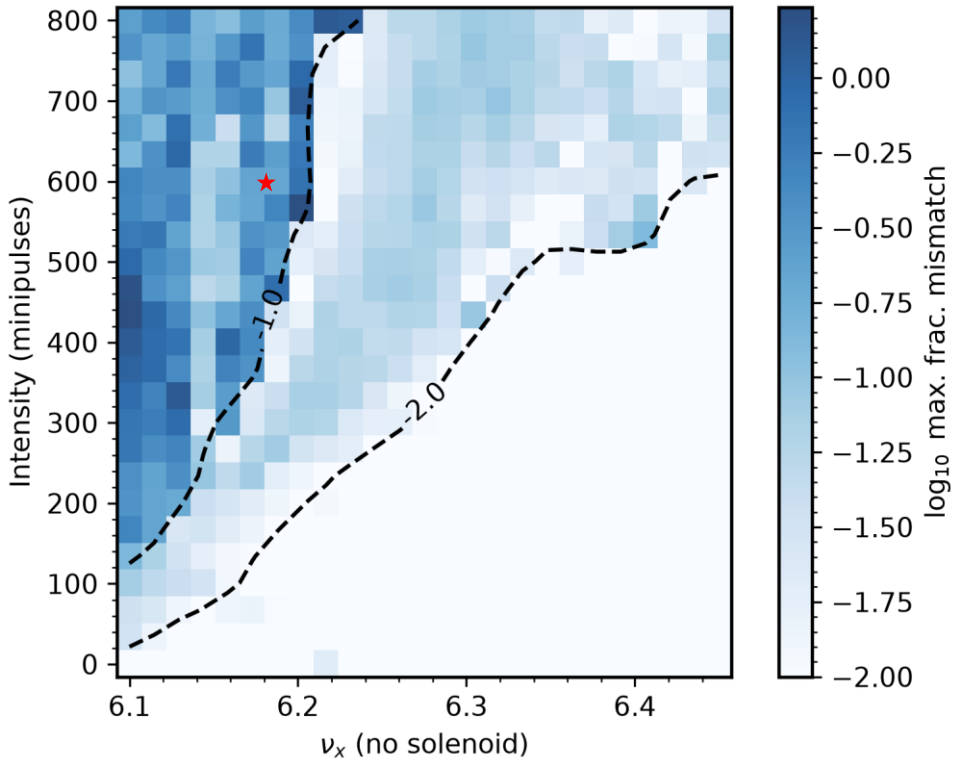
$\varepsilon_1/\varepsilon_2 \approx 2.3$

Simulations suggest 4D emittance growth was driven by nonlinear space charge forces

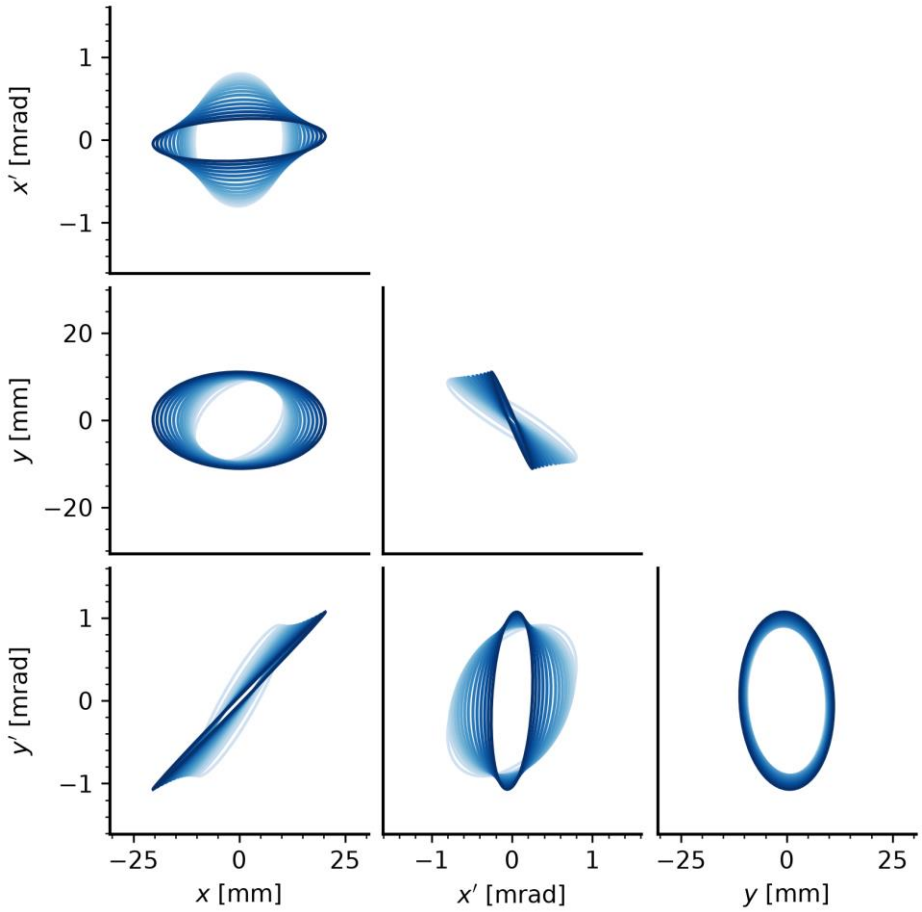


Emittance growth may have been caused by envelope instability during accumulation

Can’t find periodic solution

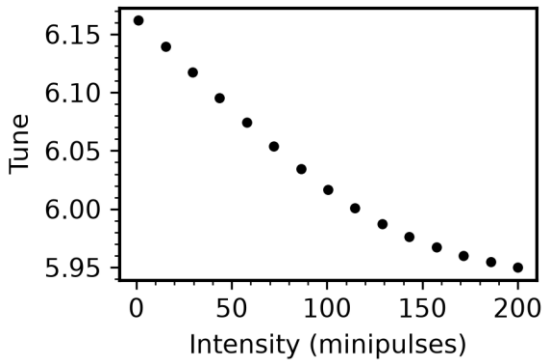


Space charge has significant effect on eigenvectors

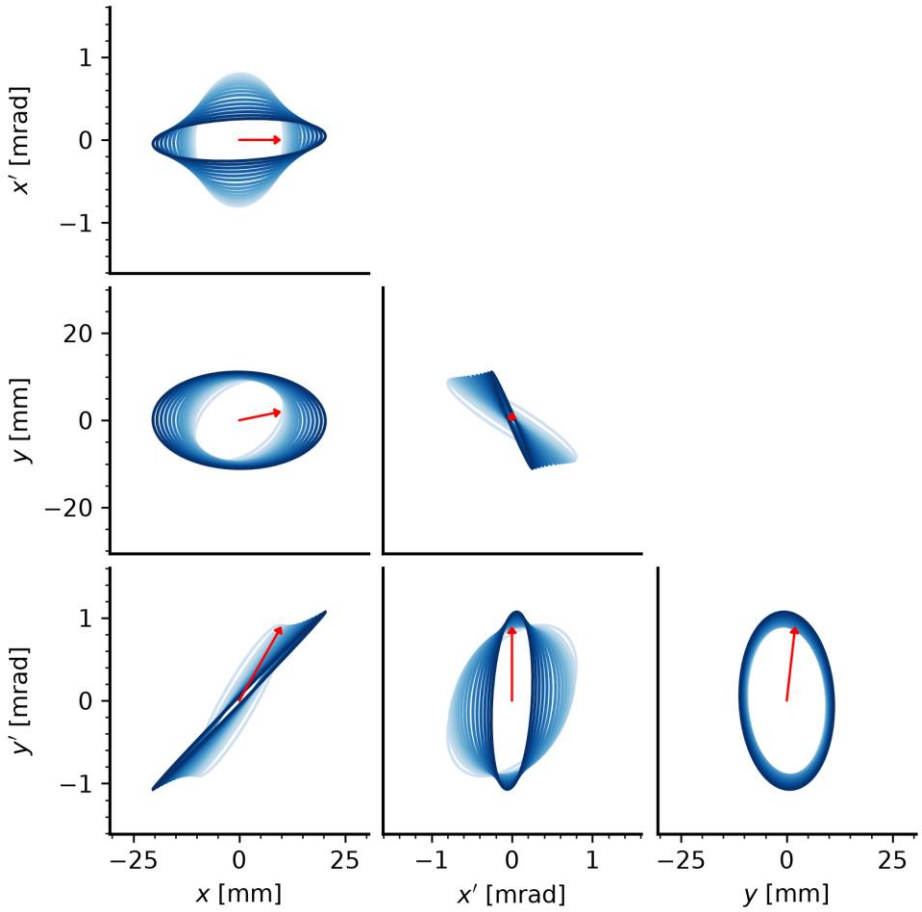


Modified eigenvectors calculated from periodic beam envelope.

$$\boldsymbol{\Sigma} = \mathbf{M}\boldsymbol{\Sigma}\mathbf{M}^{T}$$
$$\boldsymbol{\Sigma}\mathbf{U}\hat{\mathbf{v}} = i\epsilon\hat{\mathbf{v}}$$

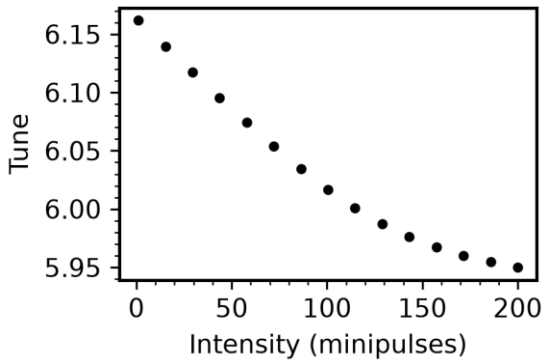


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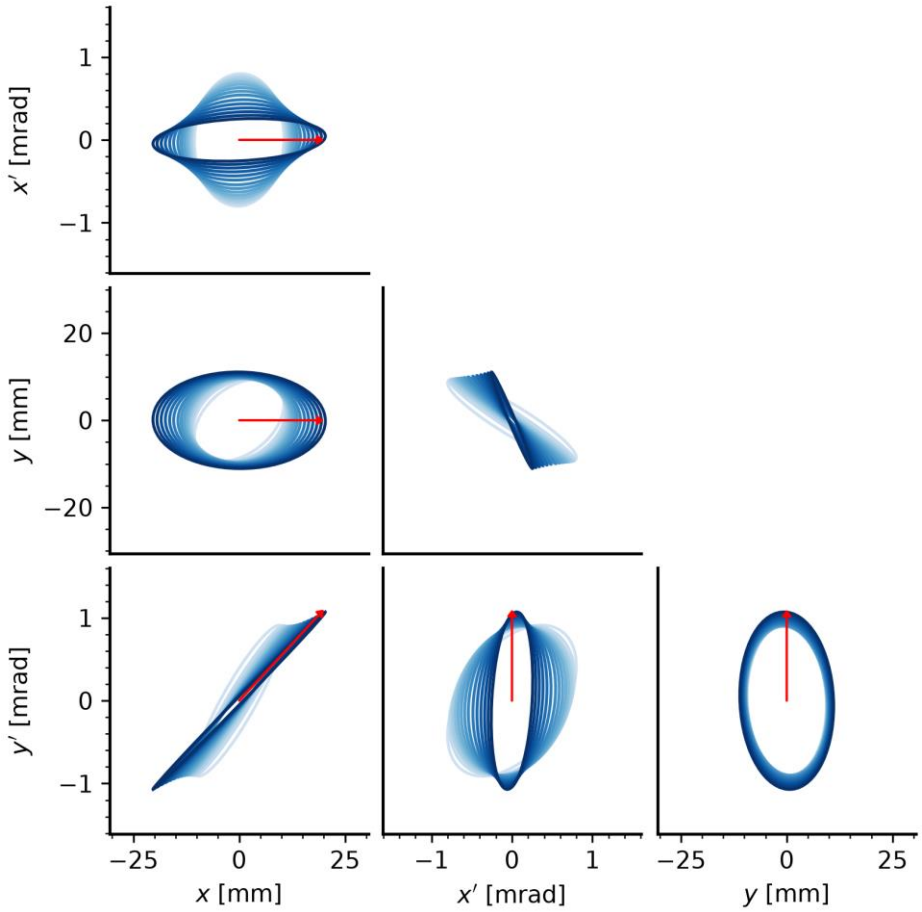


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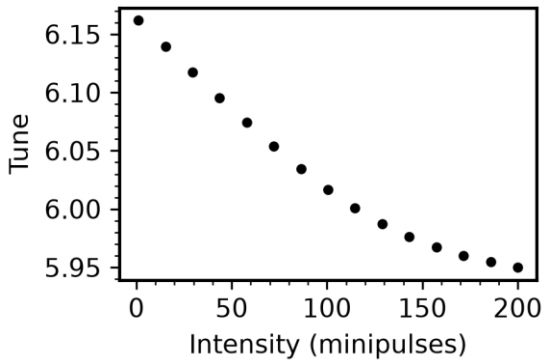


Space charge has significant effect on eigenvectors



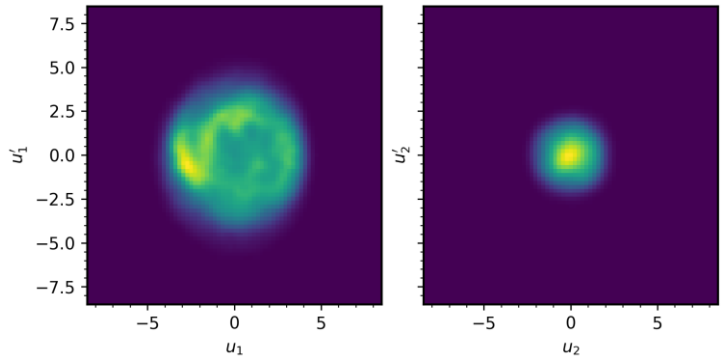
Modified eigenvectors calculated from periodic beam envelope.

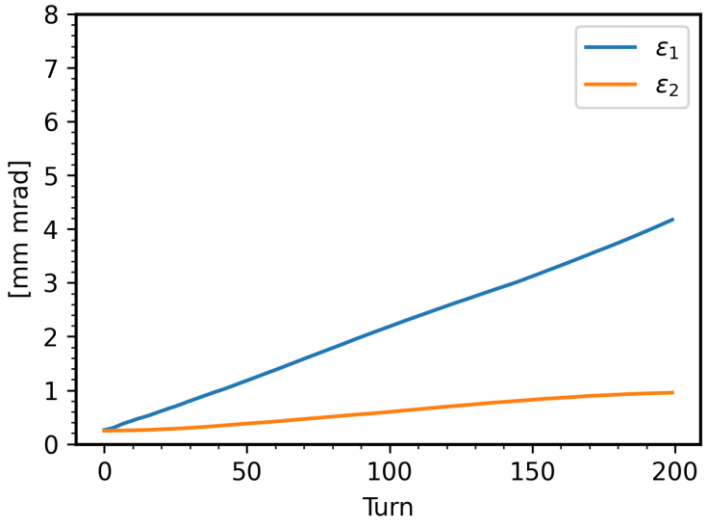
$$\boldsymbol{\Sigma} = \mathbf{M}\boldsymbol{\Sigma}\mathbf{M}^T$$
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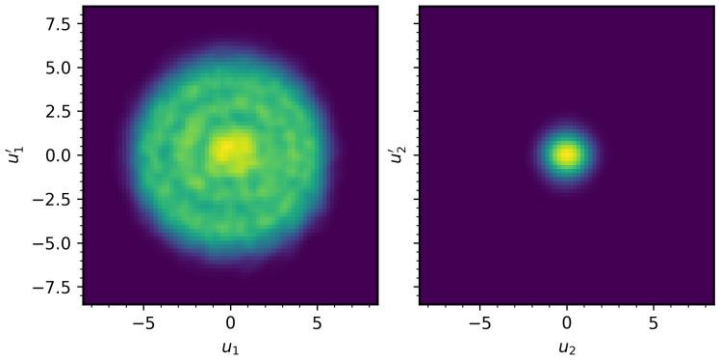
SC-modified eigenvectors could generate larger emittance ratio (> 10)

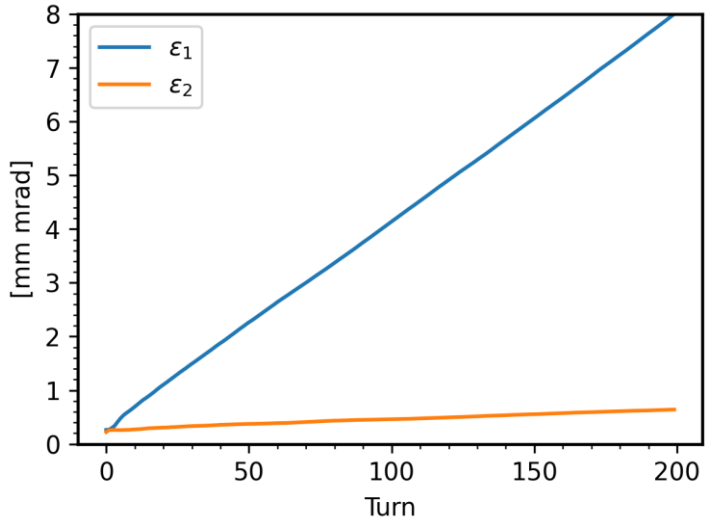
$\mathbf{x} \propto \mathbf{v}$





$\mathbf{x} \propto \hat{\mathbf{v}}$



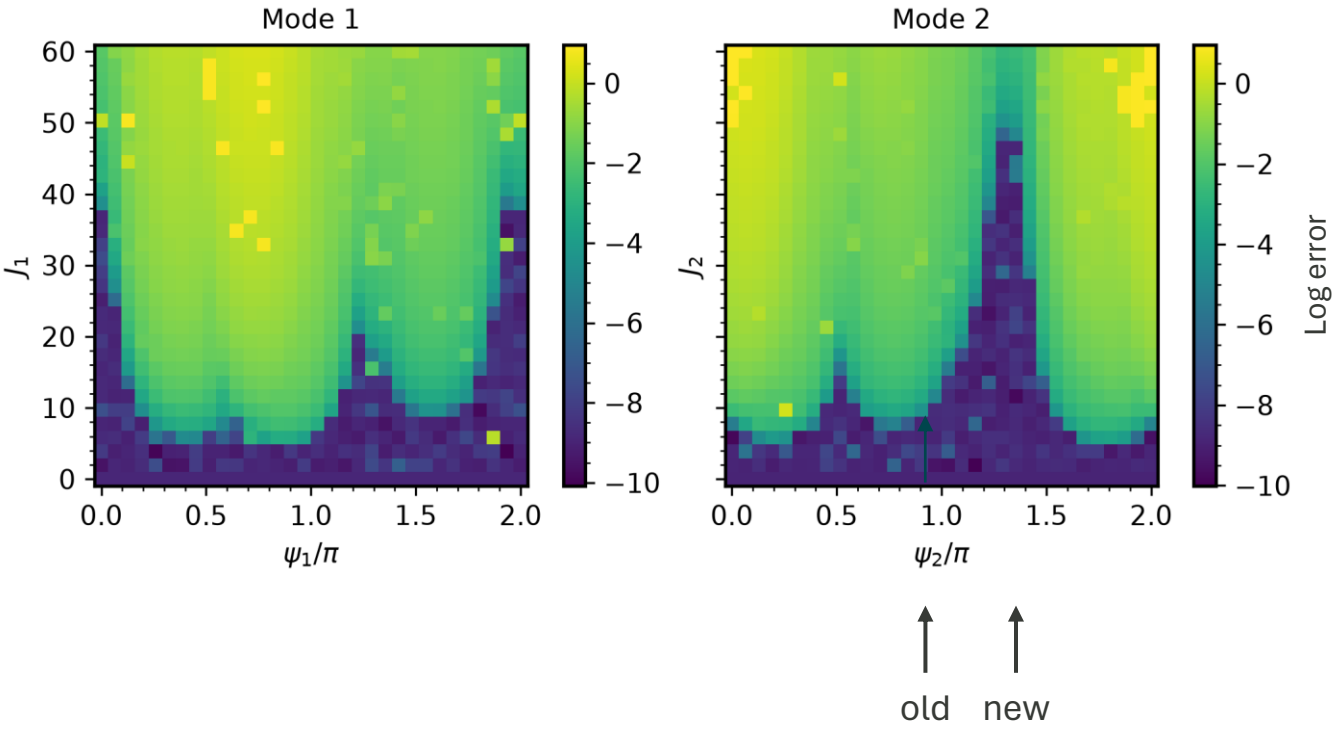


Conclusion

- Paper: Phys. Rev. Lett. **136**, 075002
- Experimental goals:
  - Automate closed-orbit measurement and kicker optimization.
  - Directly optimize kicker waveforms using electron scanner profiles.
  - Demonstrate significant (10?) emittance ratio using modified painting path.
  - Study beam dynamics before/after painting.
- Research topics/questions:
  - Space-charge-driven halo, losses, and coherent instabilities (“Gluckstern modes”)
  - 4D emittance preservation during long-term storage and acceleration
  - Do smooth equilibria exist and, if so, can they be painted in a ring?

Extra slides

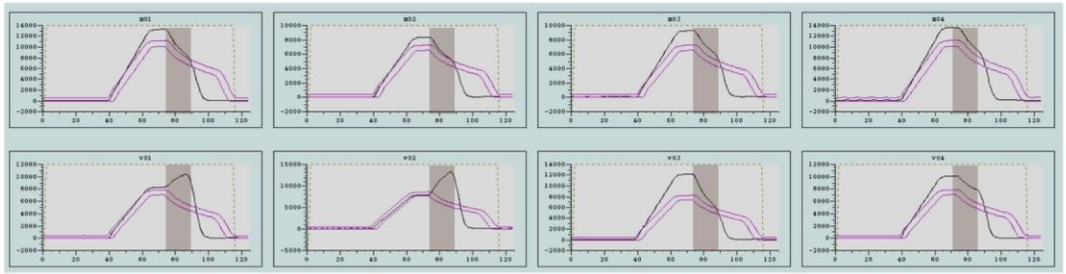
Also identified phase which allows larger amplitude within kicker constraints (but requires nonzero x’)



Kicker waveforms and closed-orbit trajectory

Horizontal Kickers

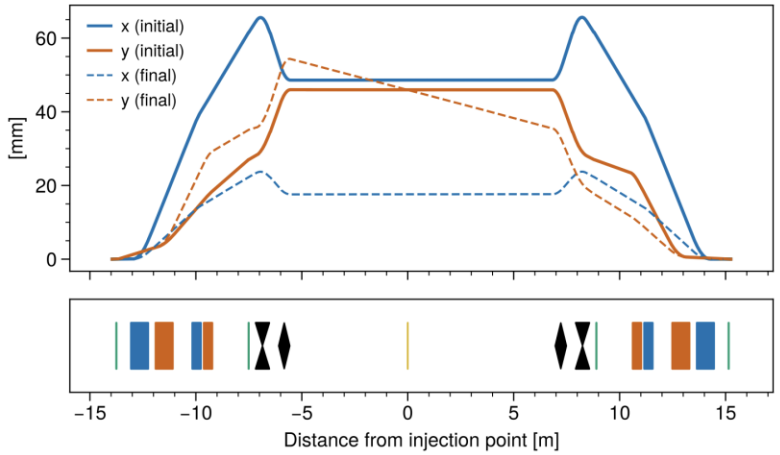
Vertical Kickers



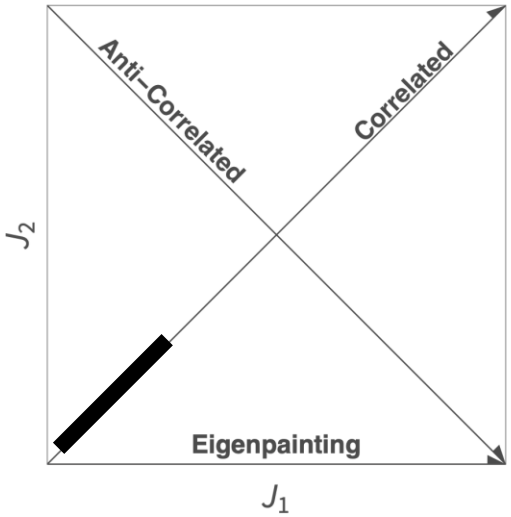
Experiment WF

Production WF

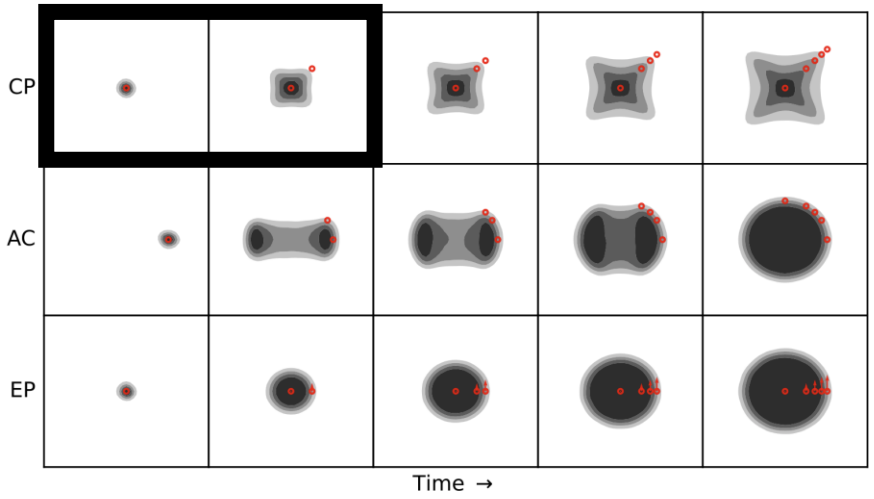
Example trajectory in kicker region



Eigenpainting is equivalent to correlated painting with zero amplitude in one plane

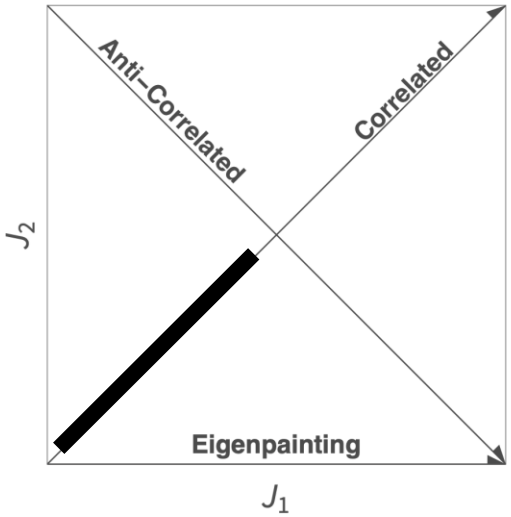


(a) Schematic trajectories for several painting schemes.

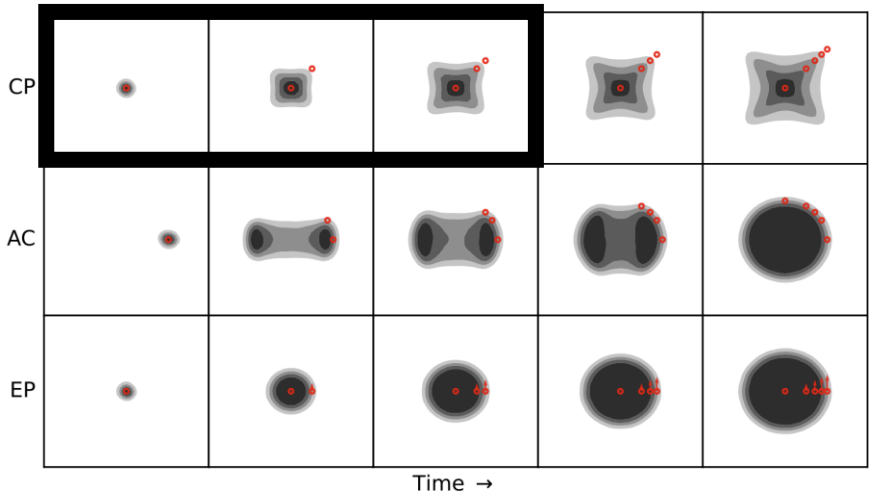


(b) x-y distributions.

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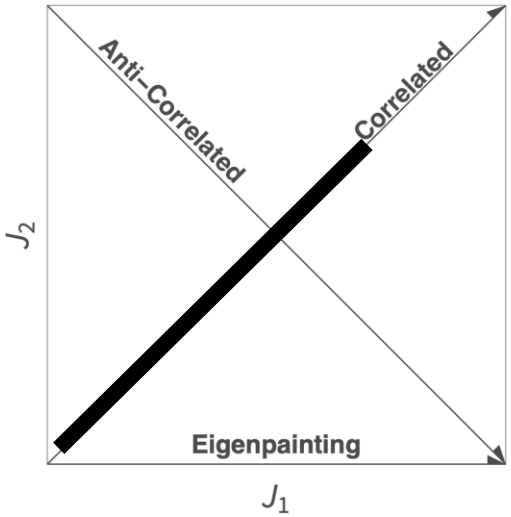


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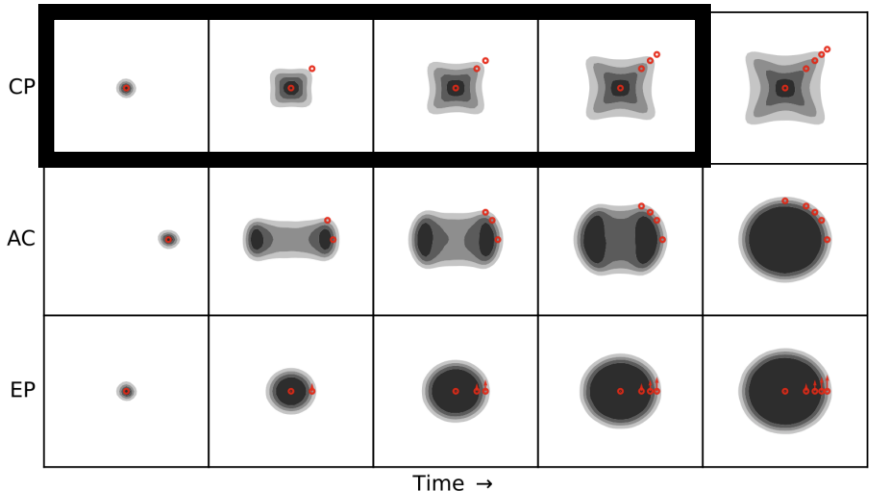


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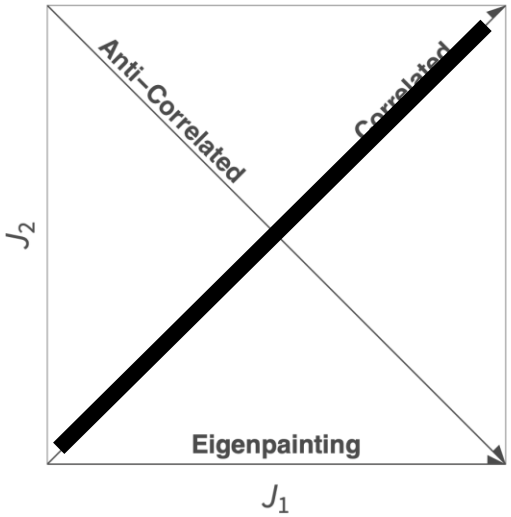


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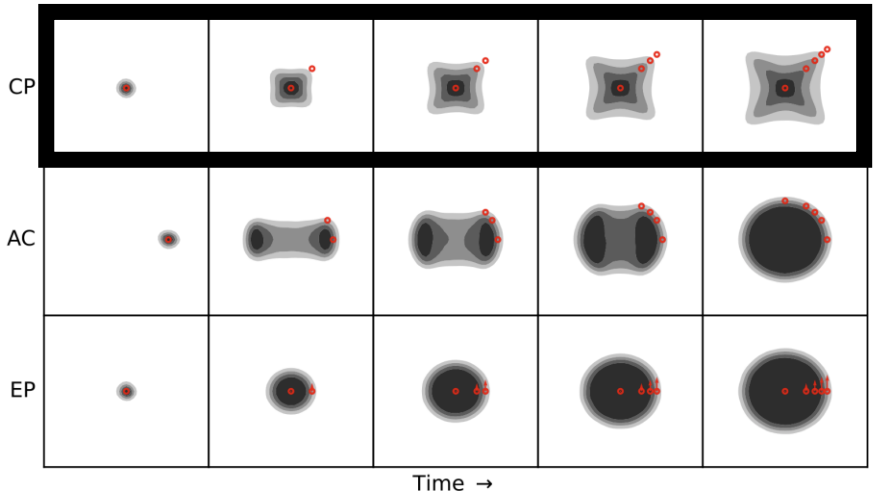


(b) x-y distributions.

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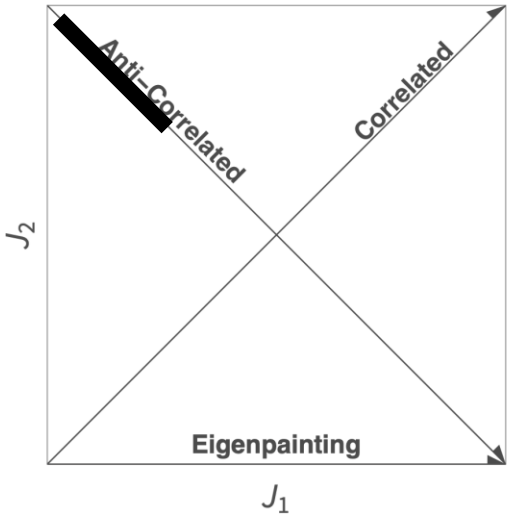


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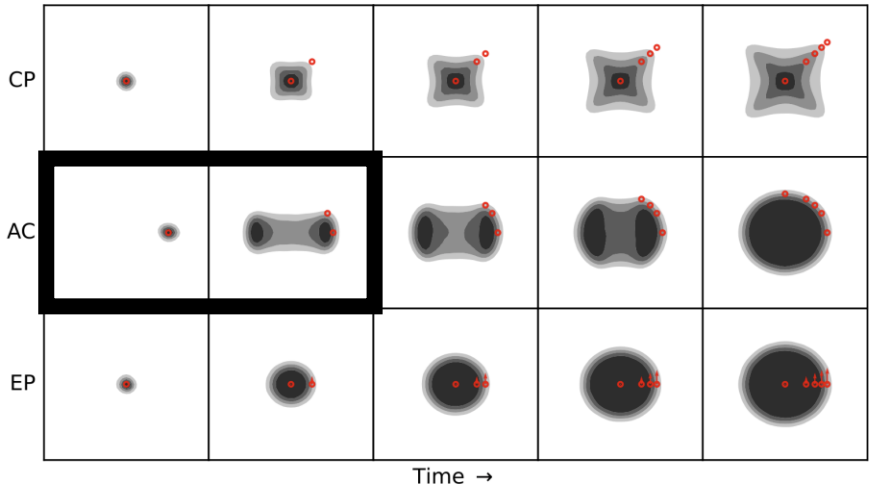


(b) x-y distributions.

Eigenpainting is equivalent to anti-correlated painting with zero amplitude in one plane

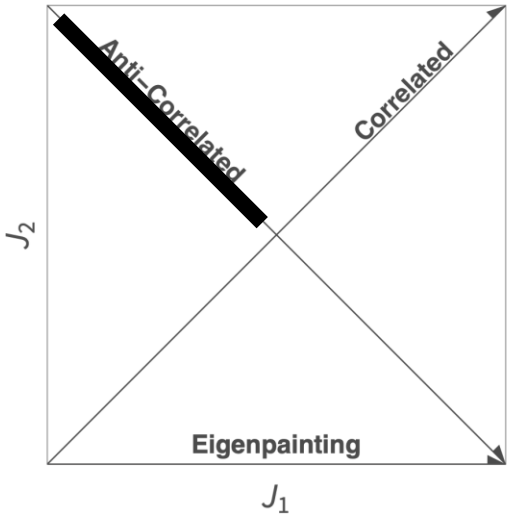


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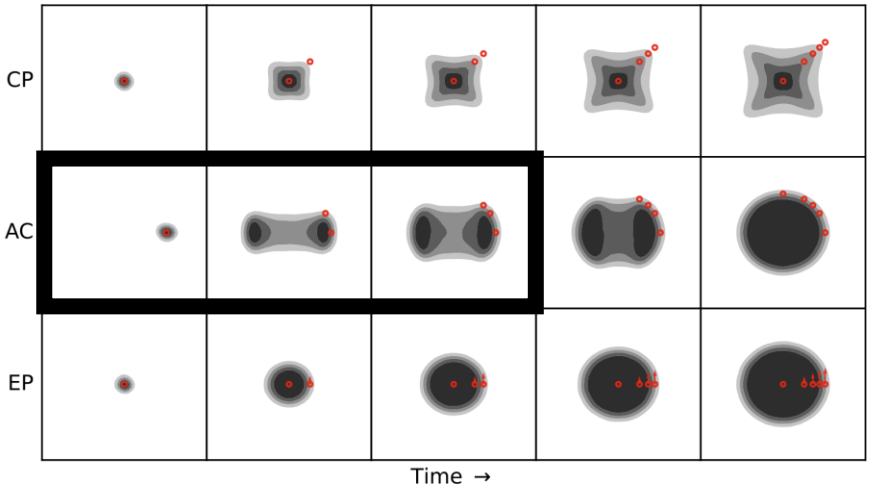


(b) x-y distributions.

Eigenpainting is equivalent to anti-correlated painting with zero amplitude in one plane

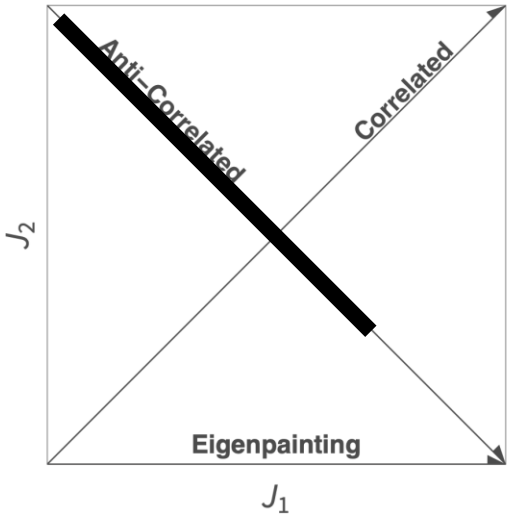


(a) Schematic trajectories for several painting schemes.

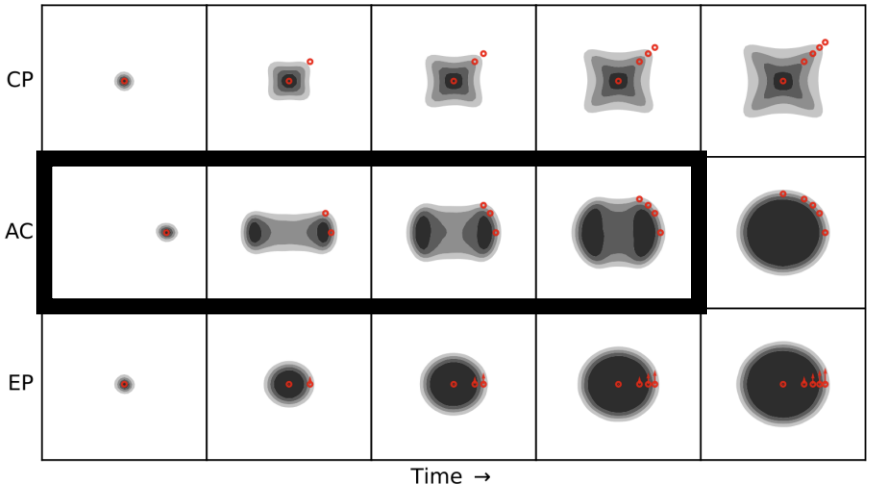


(b) x-y distributions.

Eigenpainting is equivalent to anti-correlated painting with zero amplitude in one plane

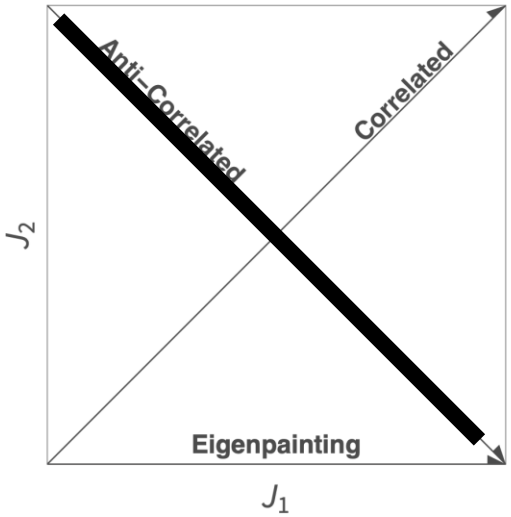


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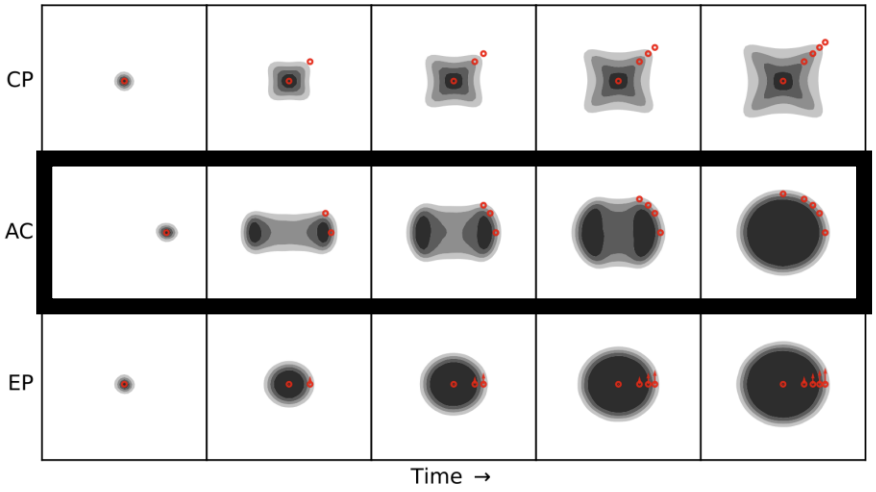


(b) x-y distributions.

Eigenpainting is equivalent to anti-correlated painting with zero amplitude in one plane



(a) Schematic trajectories for several painting schemes.



(b) x-y distributions.