# Parametric instabilities in actual high-intensity linear accelerators

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# Space-charge halo mechanisms

- There are largely two families of space-charge mechanisms: particle resonances and parametric instabilities.

- Their differences were well pointed out in references:

Jeon et al, NIM A 832, 43 (2016);
Jeon, J Korean Phys Soc 72, 1523 (2018)

1. (particle) resonances:
   a.k.a. single particle resonances, resonances, incoherent resonances …

2. (parametric) instabilities:
   a.k.a. parametric resonances, structure resonances, coherent resonances, coherent instabilities, mode instabilities, instabilities …

- This talk focuses on space charge of linear accelerators.

# Particle resonances
## in high-intensity linacs

- In 2009, $4\sigma = 360°$ particle resonance was discovered in high-intensity linacs.
  [Jeon et al, PRST-AB 12, 054204 (2009)]
  - Confirmed by two experiments.

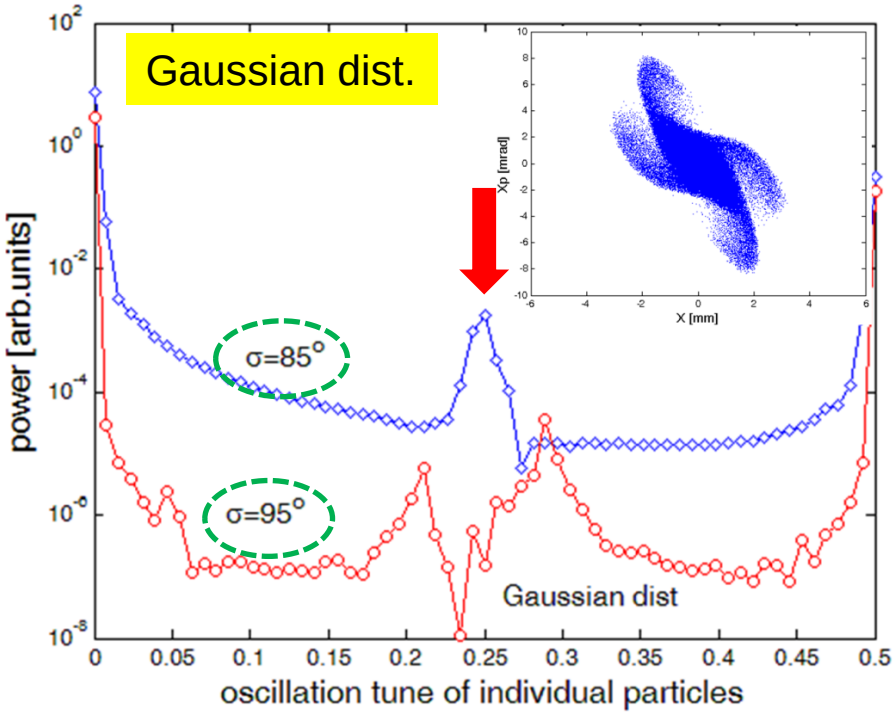
- In 2015, $6\sigma = 720°$ particle resonance was discovered.
  [Jeon et al, PRL 114, 184802 (2015)]

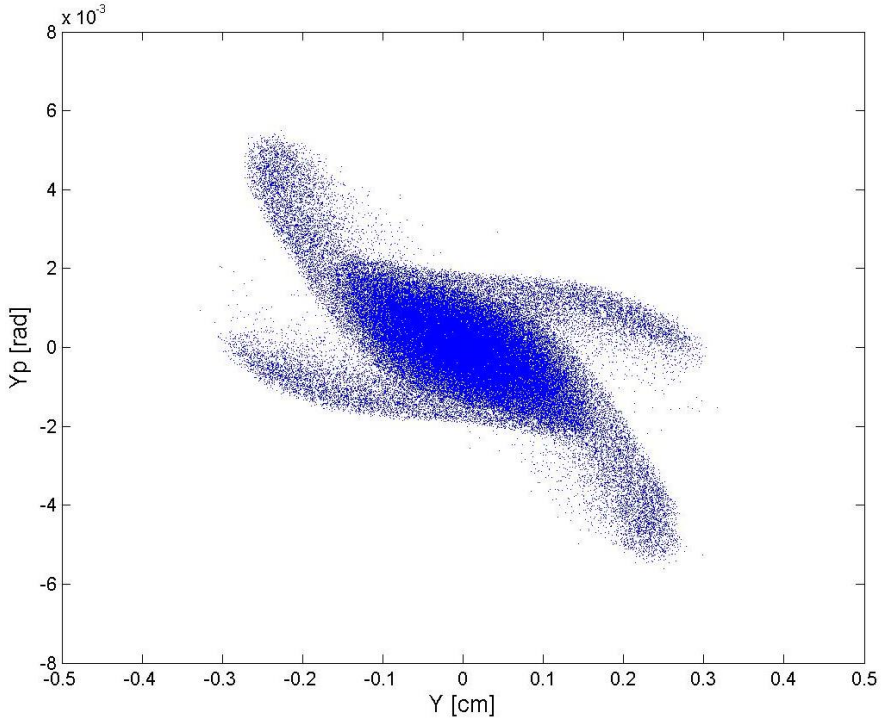
- In 2016, higher order particle resonances were discovered:
  $8\sigma = 1080°$ resonance, $10\sigma = 1440°$ resonance [Hofmann, HB2016]

$\sigma$ : depressed phase advance per period
$\sigma_0$ : zero current phase advance

# 4th order particle resonance
## in high-intensity linacs

Jeon et al, PRST-AB **12**, 054204 (2009)

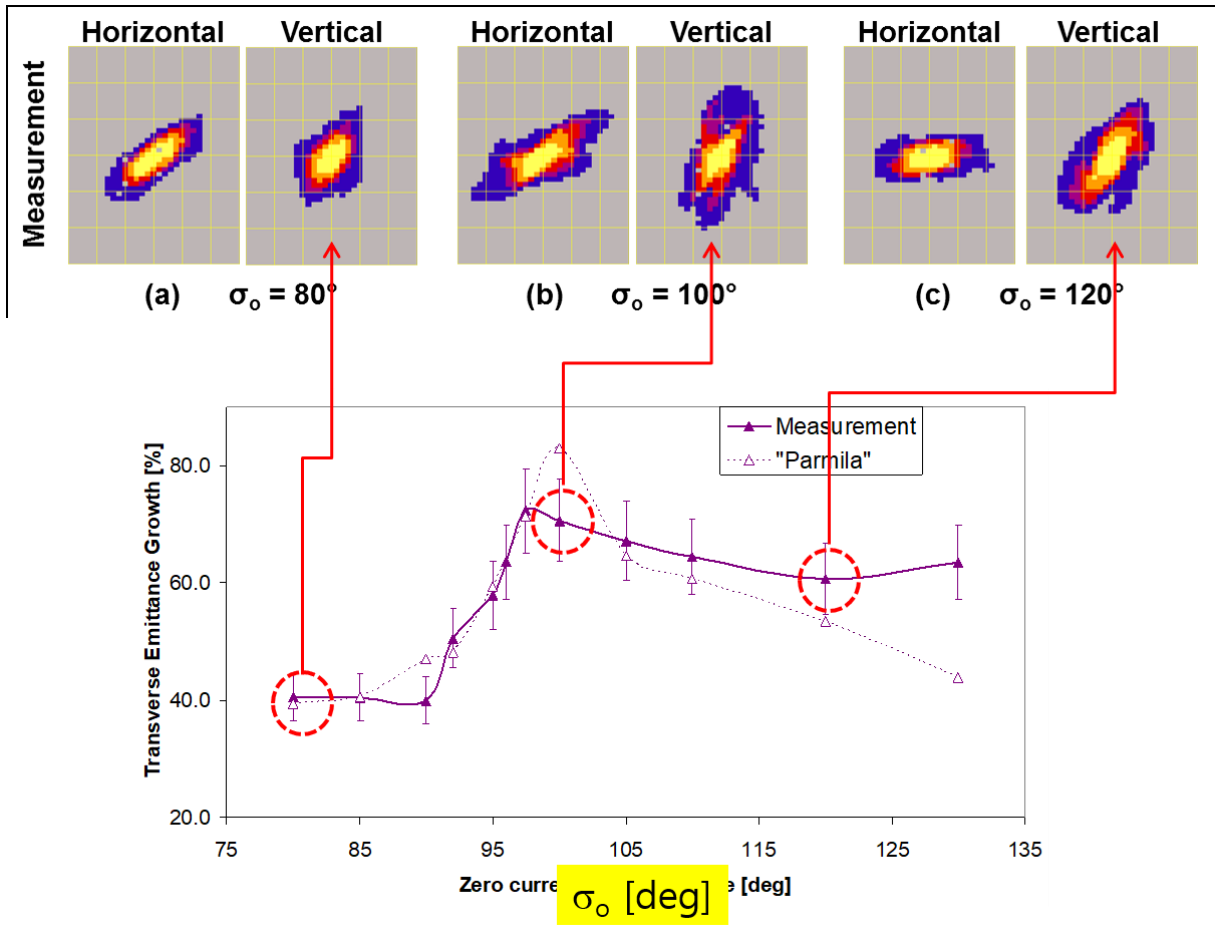


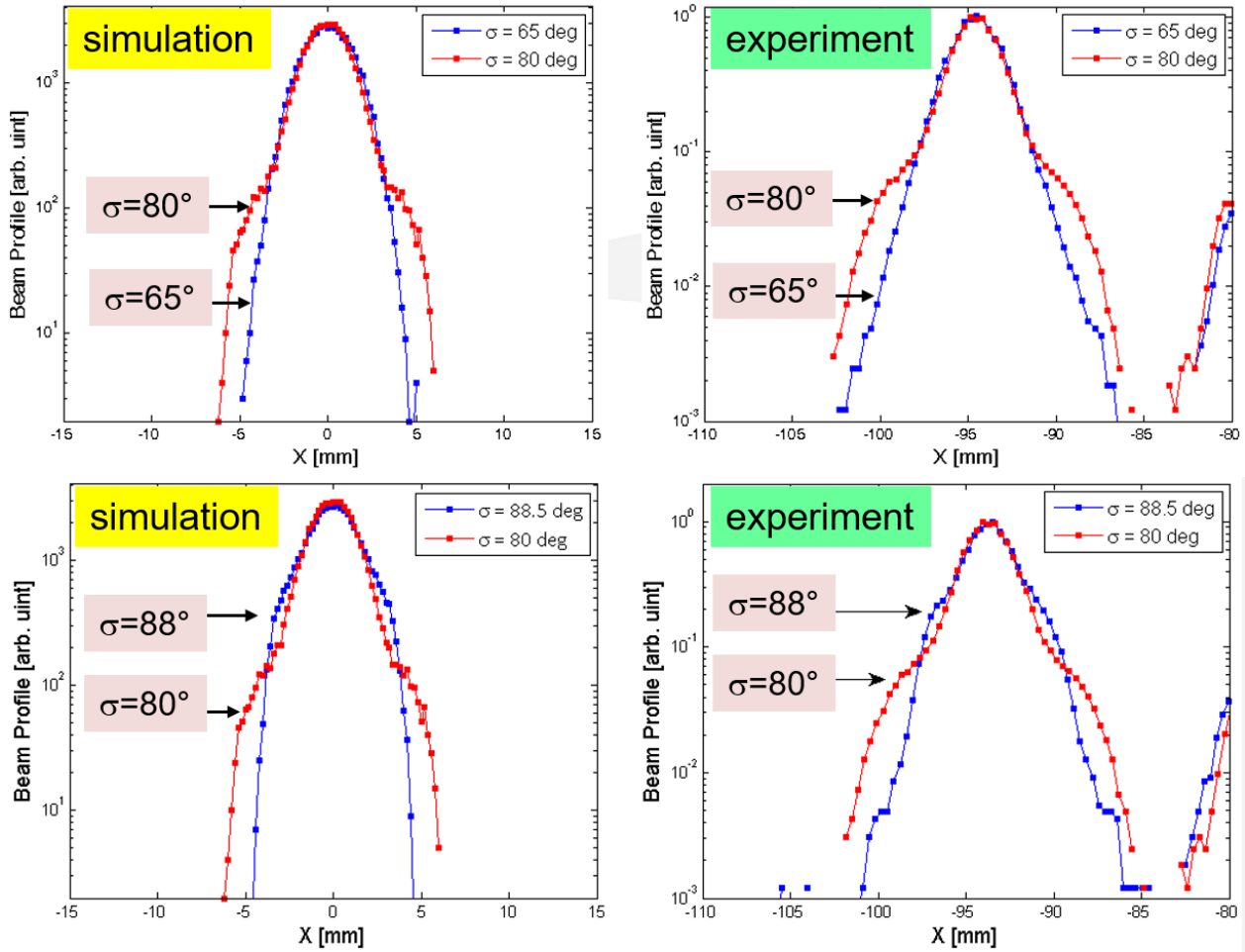

- $4\sigma = 360°$ particle resonance in high-intensity linacs discovered in simulations.
- Resonance effect when $\sigma < 90°$ and $\sigma_0 > 90°$.
- FFT resonance frequency peak at 1/4.
- Confirmed by two experiments.

# Experiment 1 & 2: 4th order resonance

**Exp 1 @ GSI**
Groening et al, PRL 102, 234801 (2009)




**Exp 2 @ SNS**
Jeon, PRAB 19, 010101 (2016)



# Tiefenback's experiment (1986)

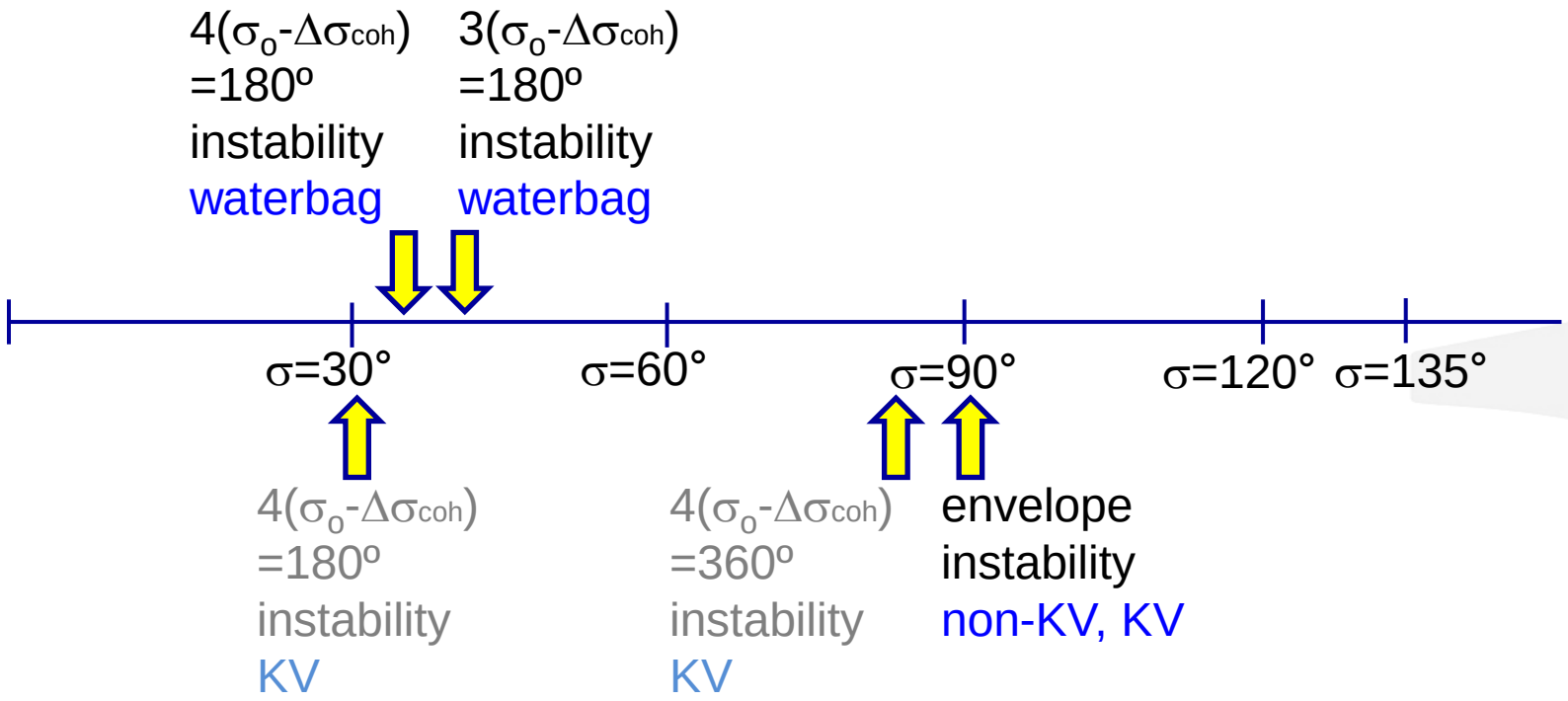
- Tiefenback's PhD thesis (1986) reported an experiment observation of "four-pointed structure" for the $\sigma_o$=98° lattice,
- But, misidentified it as the 4$^{th}$ order parametric instability.
- Rather, it was the fourth-order particle resonance.



$\sigma_o$=98° lattice

Figure 5.15 of Tiefenback's thesis

# Parametric instabilities
## observed in multiparticle simulations




- 3rd and 4th order parametric instabilities have been observed for waterbag distribution, other than KV distribution.

# Parametric instabilities in actual linacs?

- 3rd and 4th parametric instabilities are observed in multiparticle simulations for waterbag distribution beside KV;
- But, not observed for Gaussian distribution.
  → indicating that these parametric instabilities are not observable in actual linacs.

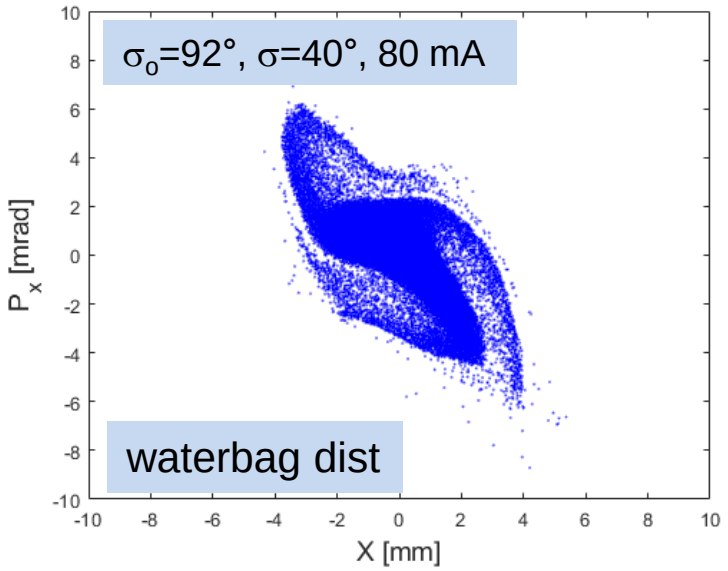
- We have claimed that high order instabilities are not observable in actual linear accelerators since 2018.

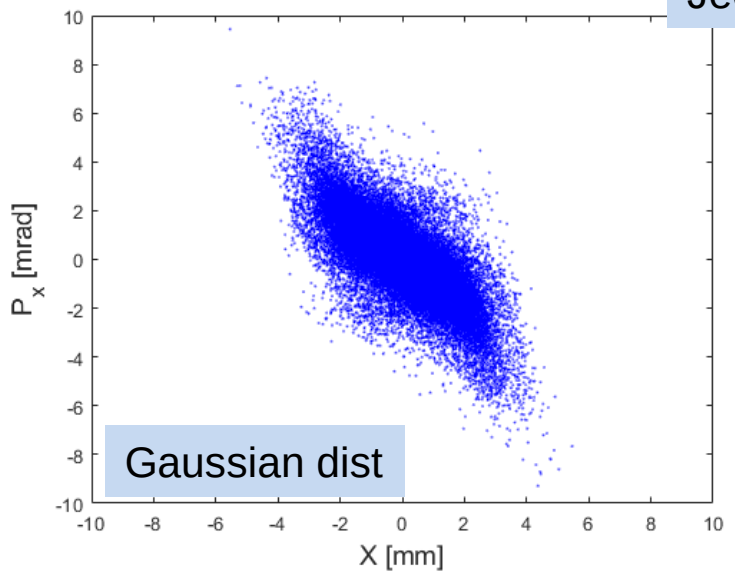
Jeon, J. Korean Phys. Soc. 72, 1523 (2018);
HB2018 and Space Charge 2019

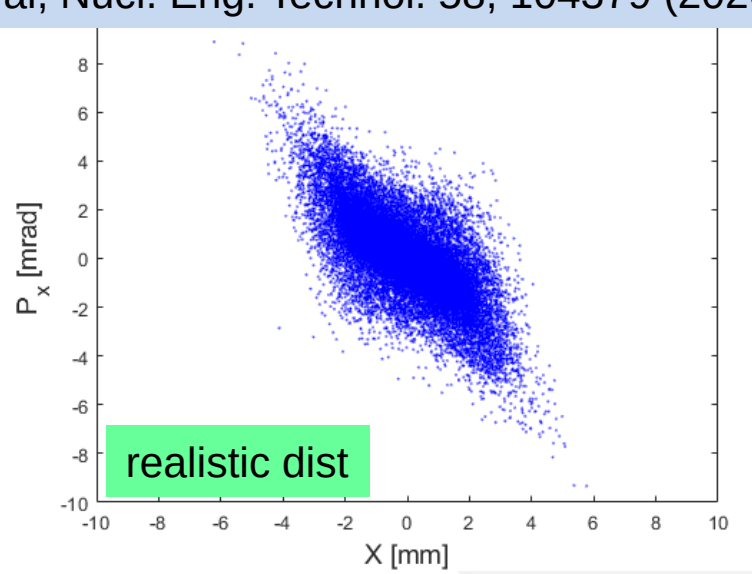
| **Parametric instability** | **KV** | **waterbag** | **Gaussian** |
|---|---|---|---|
| 2nd order | Observed | Observed | Observed |
| 3rd order, 4th order | Observed | Observed | Not observed |
| higher order | Observed | Not observed | Not observed |

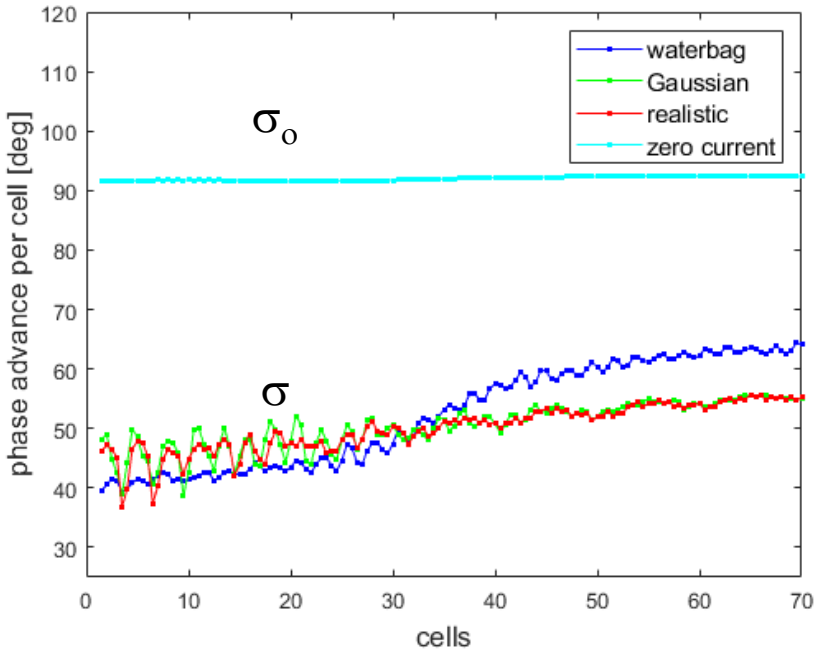
# No 3rd order parametric instability for realistic beam

Jeon et al, Nucl. Eng. Technol. 58, 104379 (2026)



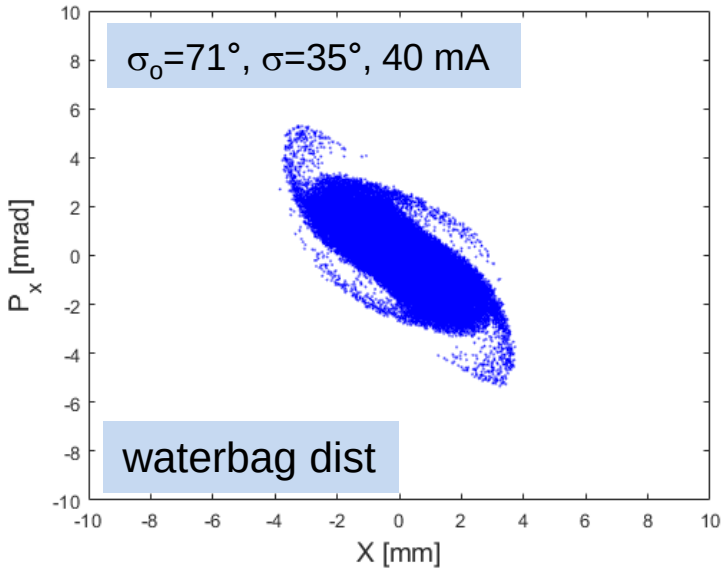



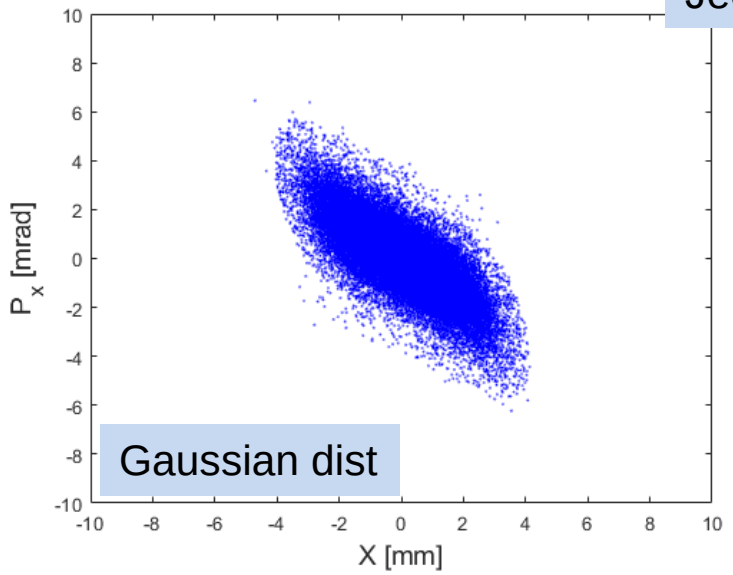



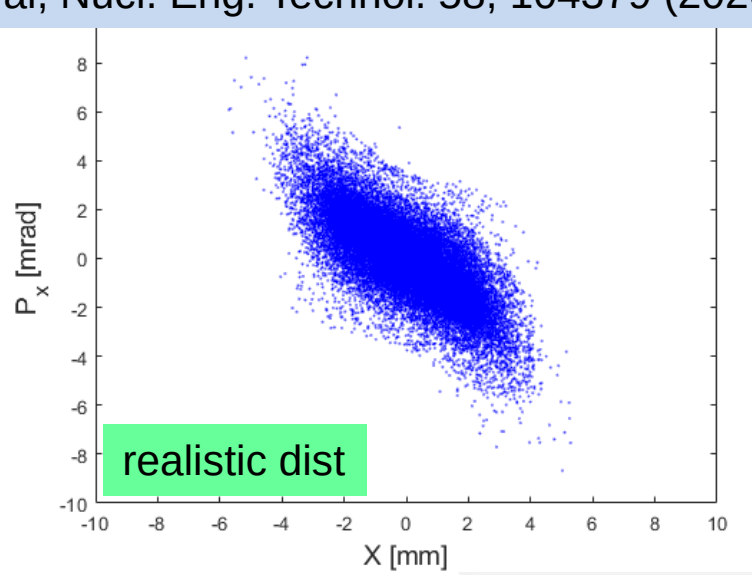

- 3rd order parametric instability is not manifested for Gaussian or realistic distributions.
- Realistic distribution from the emittance measurement of 35 mA SNS beam

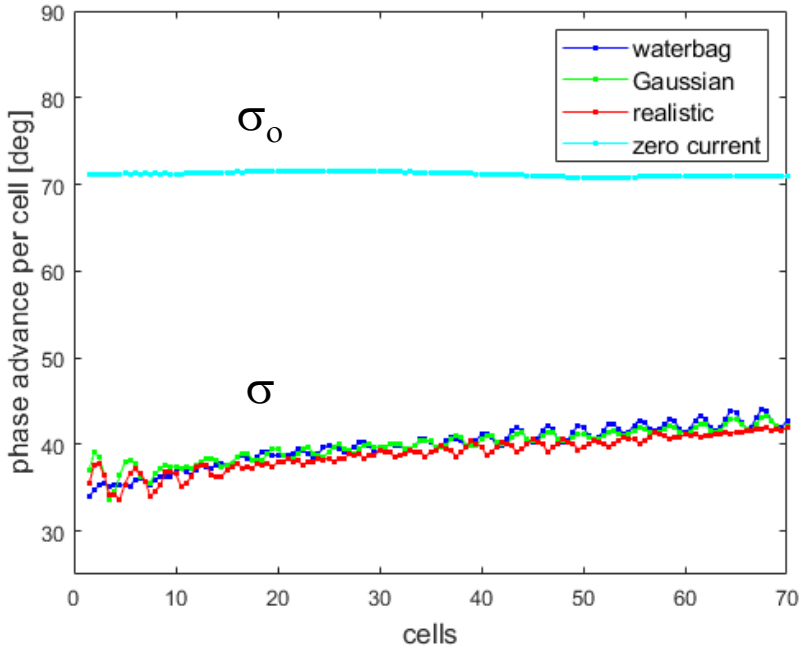
# No 4th order parametric instability for realistic beam

Jeon et al, Nucl. Eng. Technol. 58, 104379 (2026)



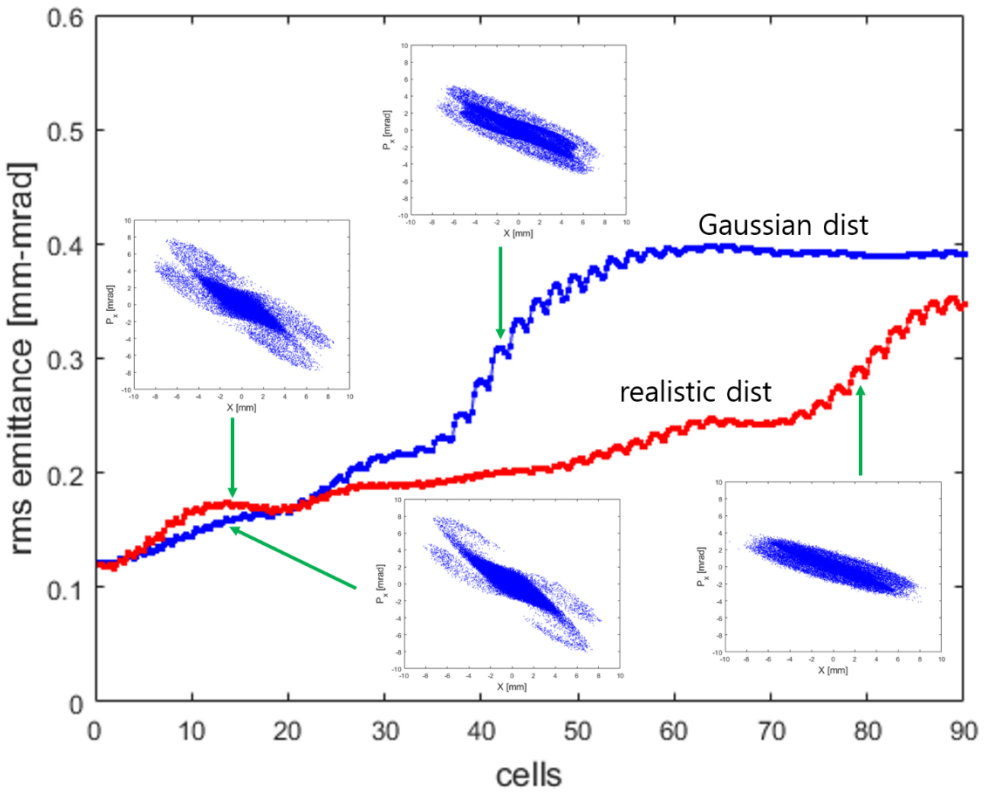



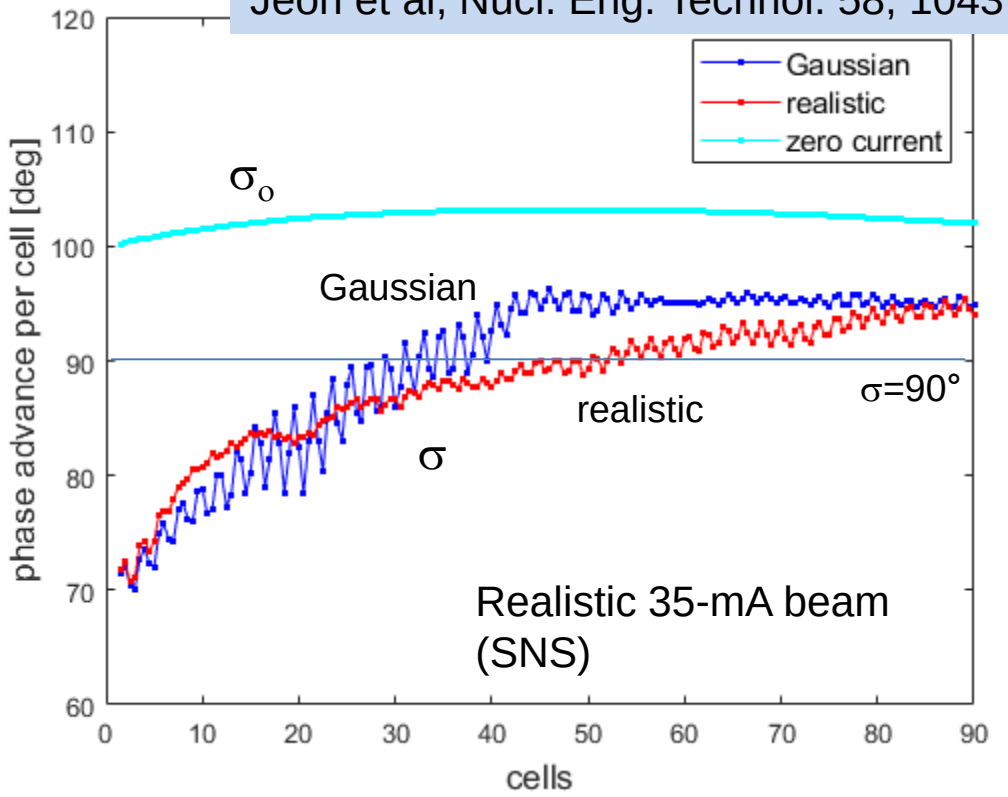




- 4th order parametric instability is not manifested for Gaussian or realistic distributions.
- Realistic distribution from the emittance measurement of 35 mA SNS beam

# Envelope instability effect reduced
## for realistic distribution

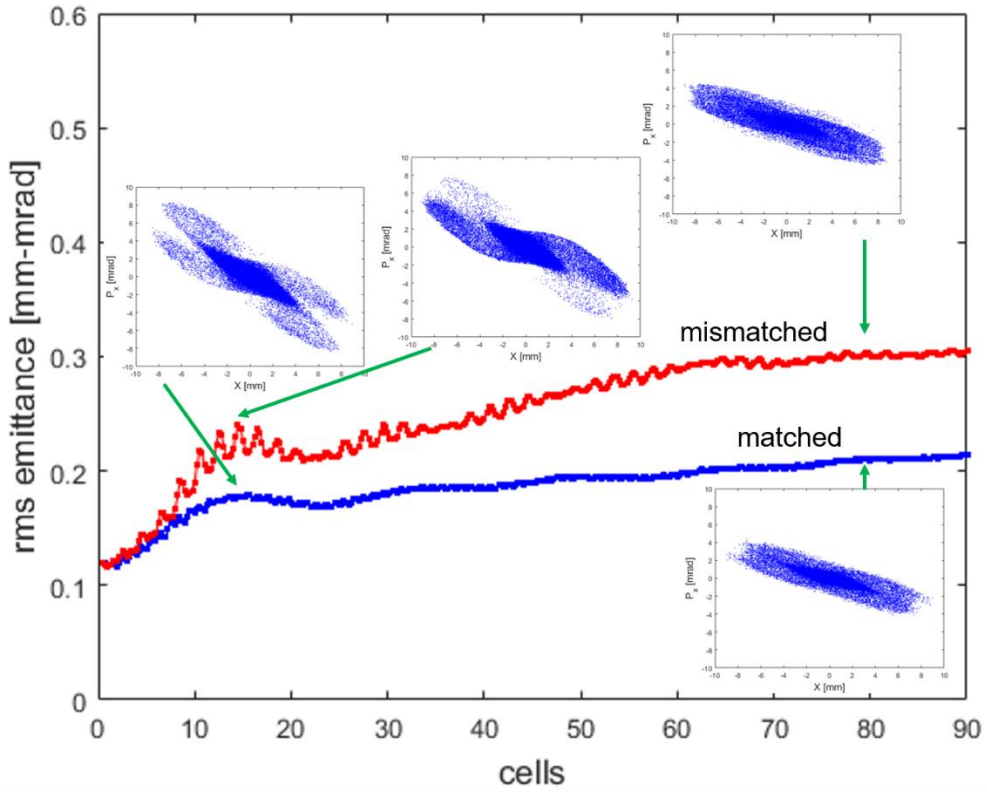
Jeon et al, Nucl. Eng. Technol. 58, 104379 (2026)

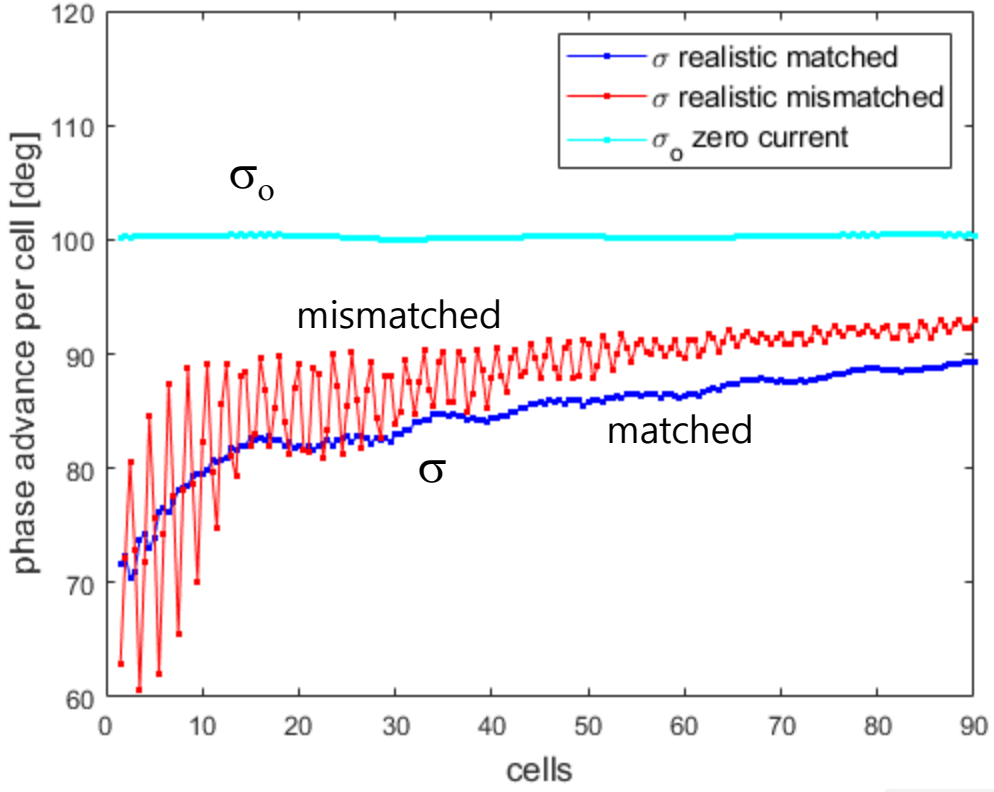



- For realistic distribution, envelope instability onset is delayed, emittance growth reduced, but stronger 4th order particle resonance effect.
- The envelope instability manifests in a linac when the 4th order resonance disappears (as σ goes above 90°).

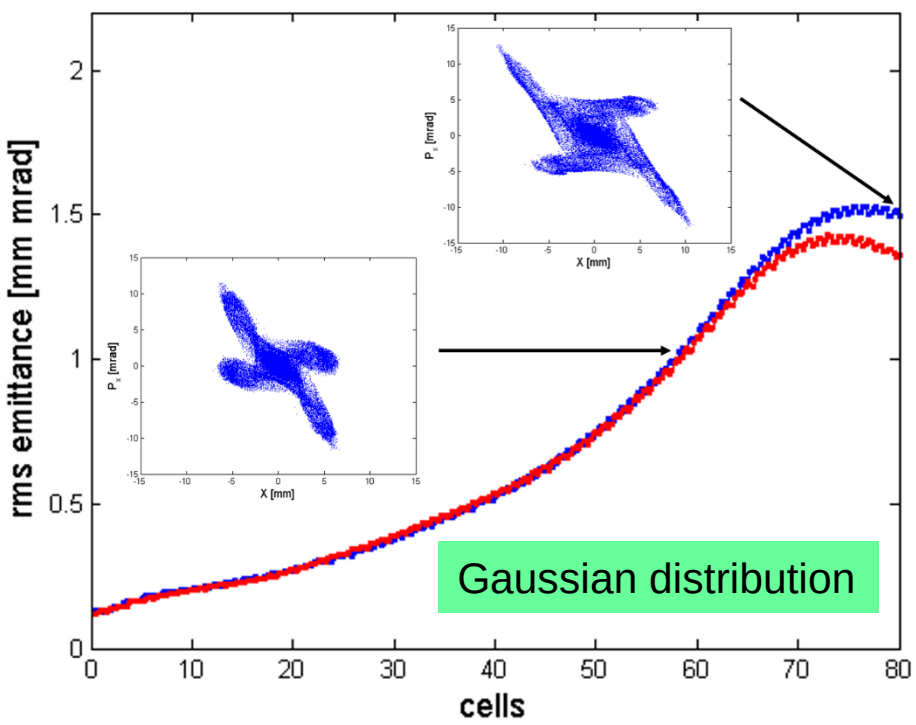
# Mismatch effect
## for realistic distribution

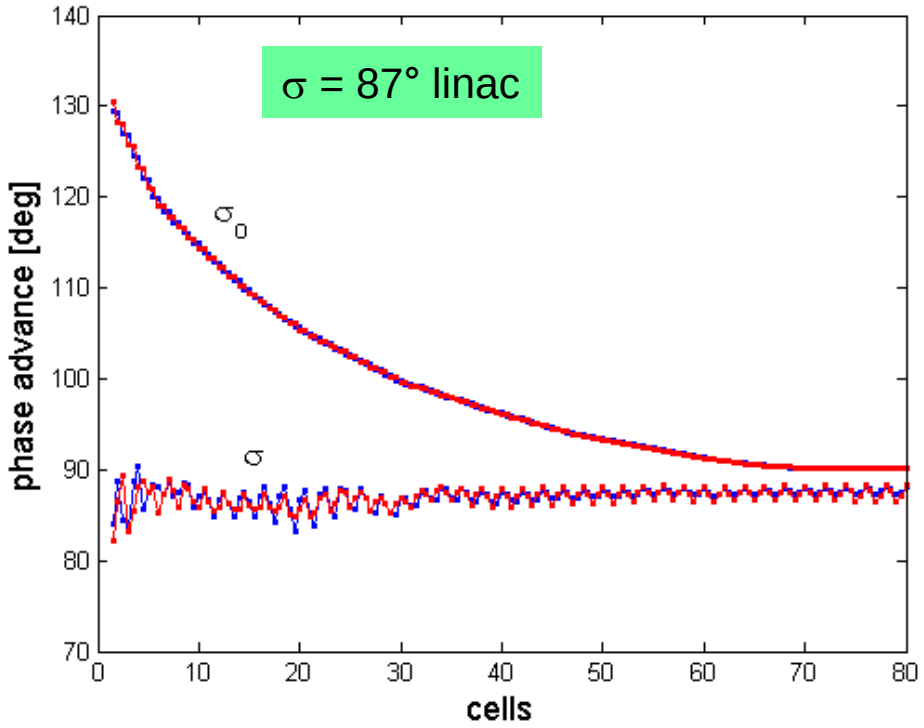



- Initial mismatch does not excite the envelope instability, when σ stays below 90° ($4^{th}$ order resonance is active).
- Mismatch by increasing $\beta_x$ and $\beta_y$ by 20%.

# 4th order particle resonance dominates over the envelope instability

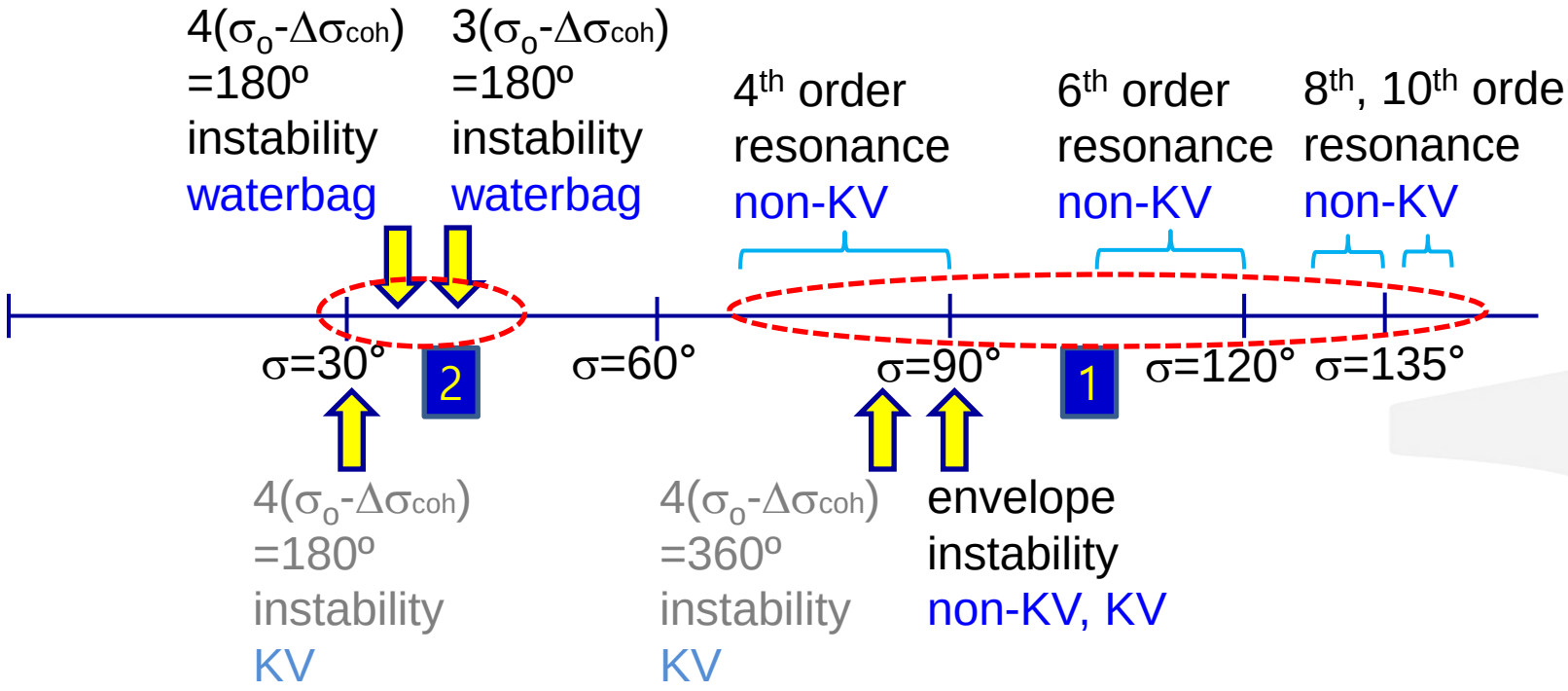



- The envelope instability is suppressed when 4th order resonance is active.

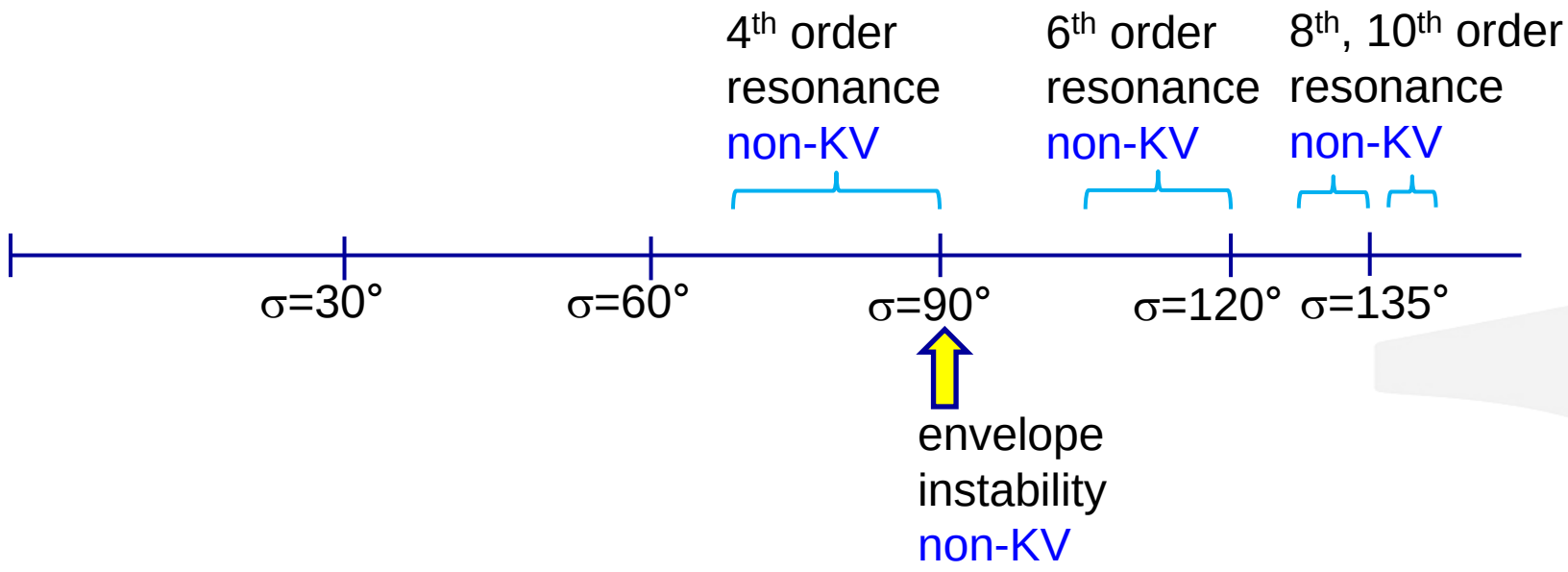
- In circular accelerators, there are a web of resonances (lattice + space charge).
- Is the envelope instability observable in 4ν=m resonance stopband?

# In multi-particle simulations




- In region 1, parametric instabilities of higher than 2nd order are not manifested for non-KV beams.
- In region 2, parametric instabilities are manifested for waterbag beam, but not for Gaussian.
- Envelope instability manifests when 4th order resonance disappears.

# In actual linear accelerators




- Only particle resonances and the envelope instability are observable in actual linear accelerators.

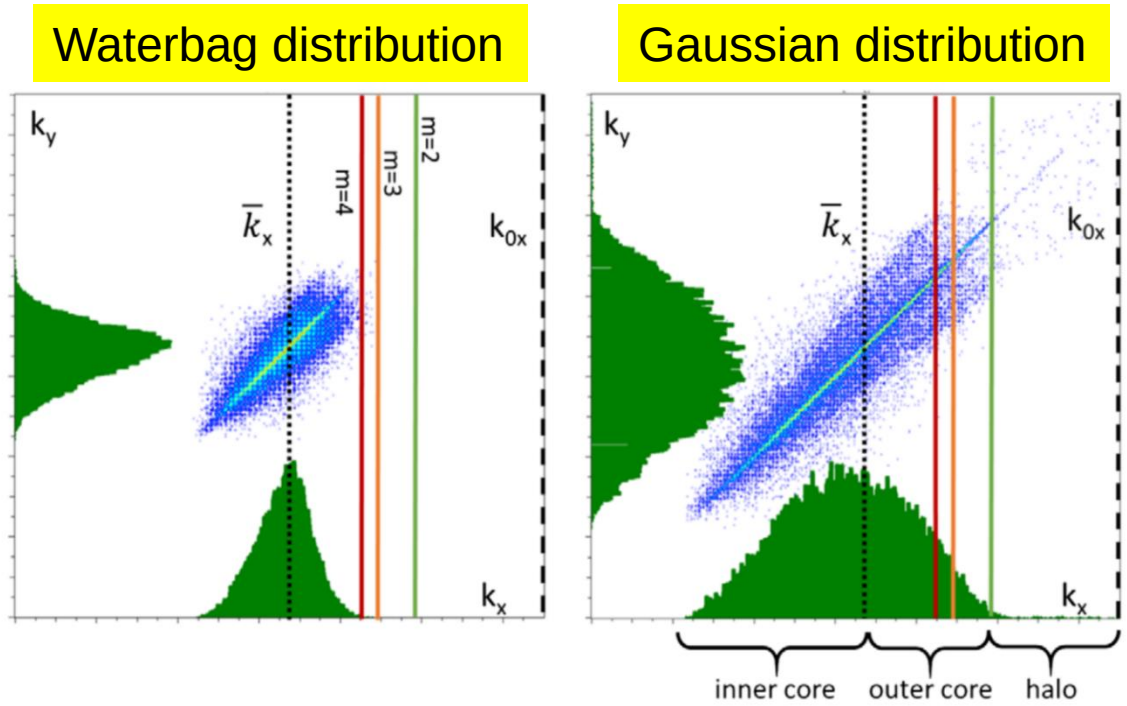
- High order parametric instabilities are like dragons in an imaginary world; they are not seen in real world.


Thank you for your attention!

# Parametric instabilities in actual linacs?

- Landau damping suppresses most of parametric instabilities except the envelope instability for Gaussian distribution.

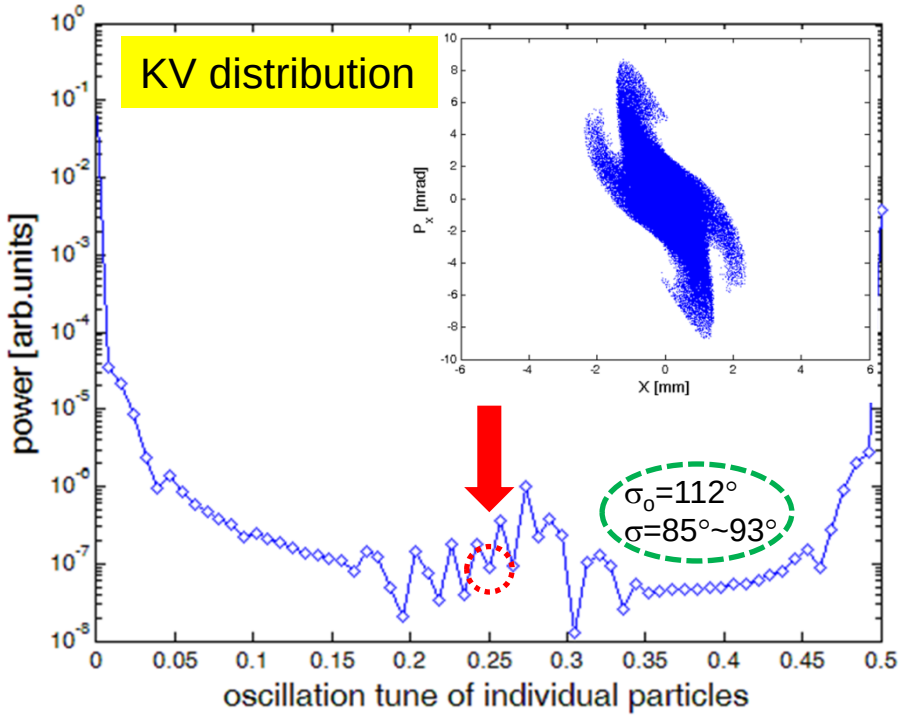
Hofmann et al, PRAB 24, 024201 (2021)

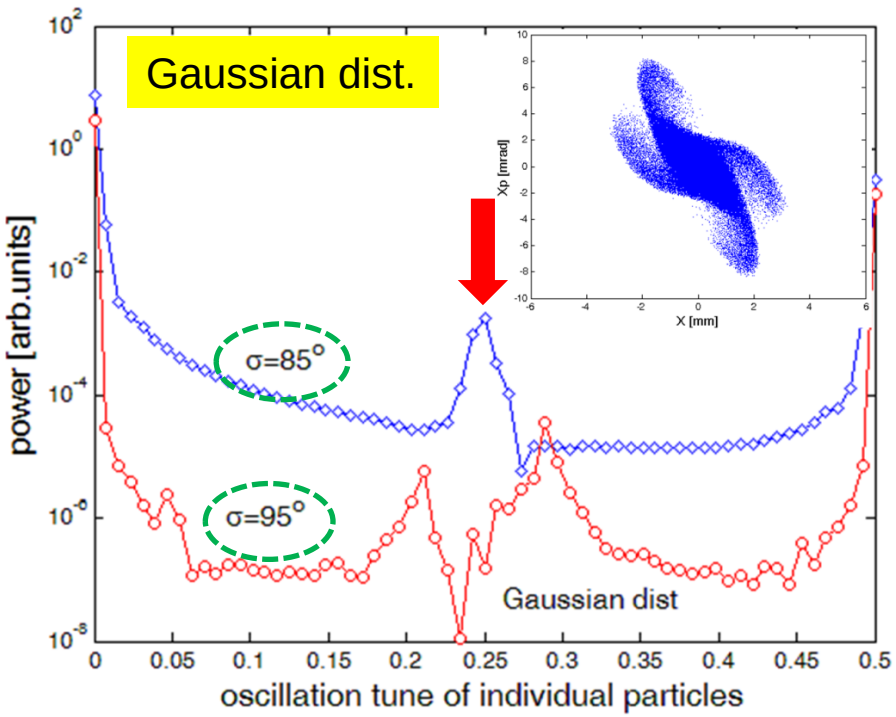



- Only the envelope instability is observable in actual linacs.

# Parametric Instability vs Particle Resonance

Jeon, J Korean Phys Soc 72, 1523 (2018)



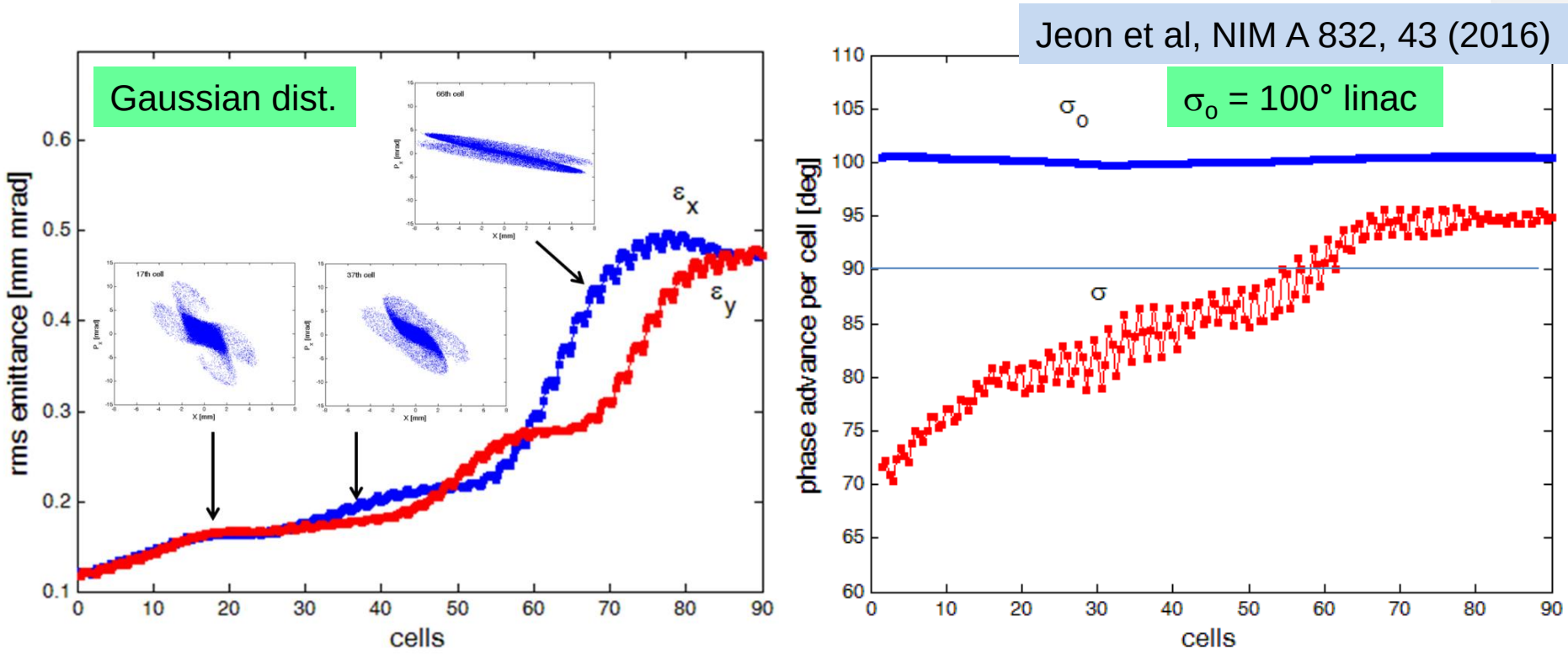




$4(\sigma_o-\Delta\sigma_{coh})=360^o$
**parametric instability**

$4\sigma=360^o$
**particle resonance**

- FFT resonance frequency for the particle resonance.
- No FFT resonance frequency for the parametric instability.

# When does the envelope instability develop?




- The envelope instability manifests in a linac when the 4th order resonance disappears (as σ goes near and above 90°).
- The 4th order particle resonance is dominant over the envelope instability.