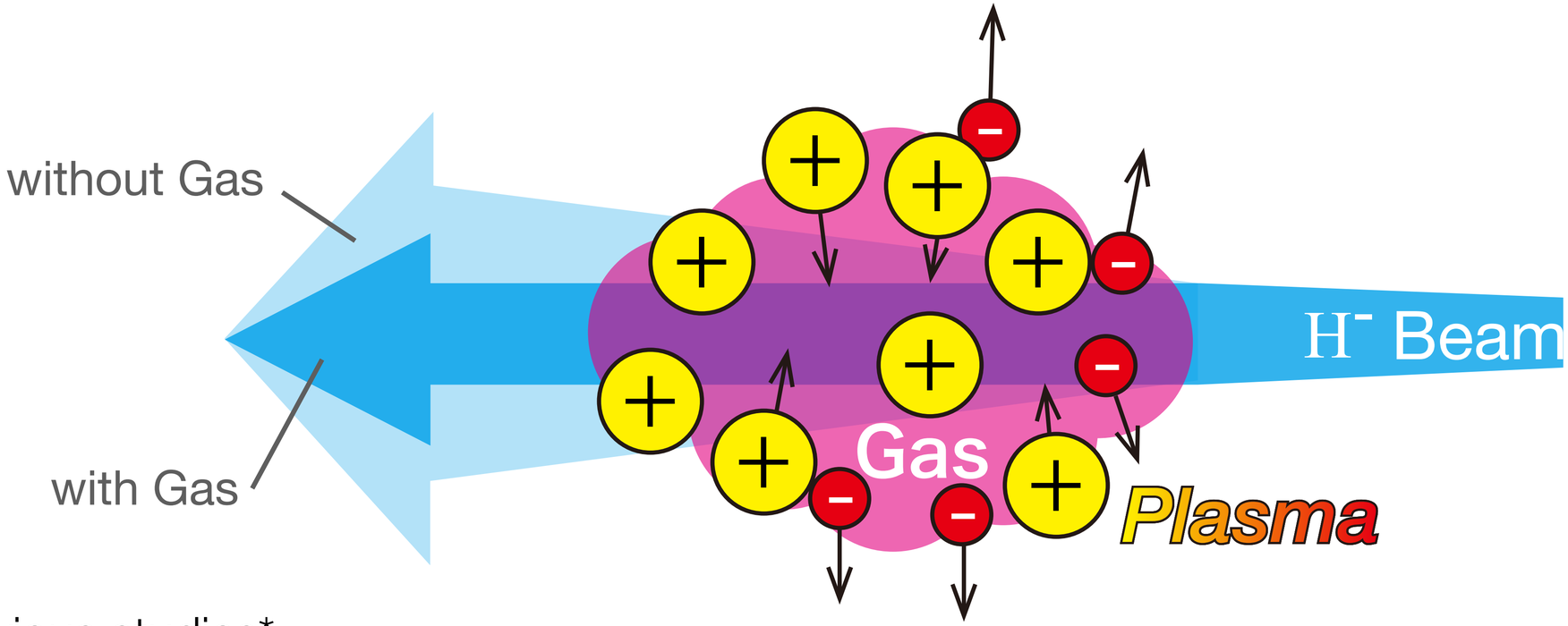
7th ICFA Mini-Workshop on Space Charge
30 Sep. - 2 Oct. 2026

Emittance Growth under Space Charge Neutralization Condition in LEBT
— Beam-Plasma Coupling Oscillation —

Ippei YAMADA, Kunihiro KOJIMA, Motoki CHIMURA
J-PARC Center, Japan Atomic Energy Agency

Introduction: Background of SCN/SCC

❖ **Space charge neutralization/compensation is essential for beam transport in LEBT**
but, still not completely under control or understood…



In some previous studies*,
plasma-like wave is observed in PIC simulation
However, details and its direct effect on beam quality are not well discussed (just on plasma loss)

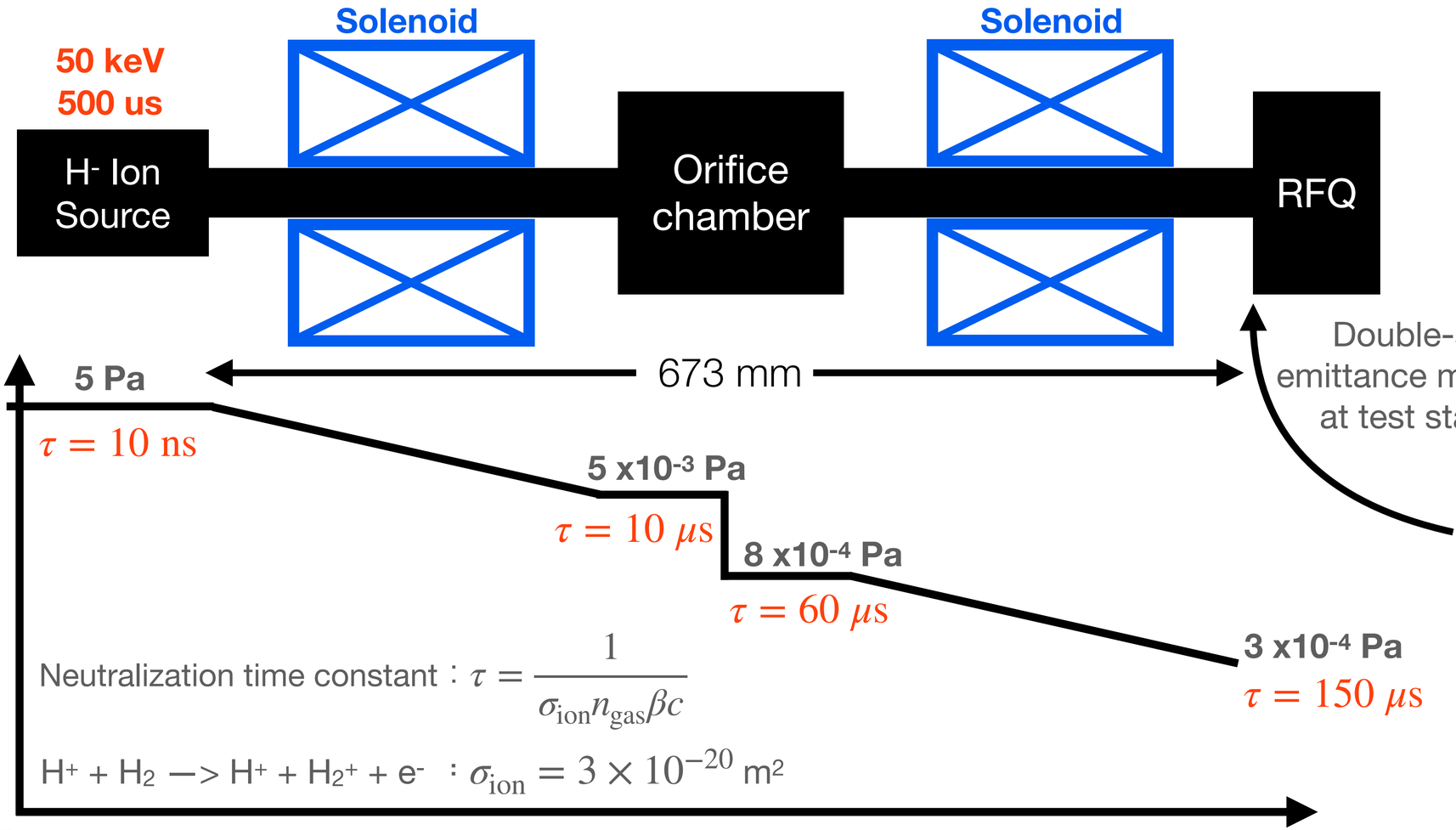
*B. John et al., JINST 20, C05024 (2025)
C. A. Valerio-Lizarraga et al., PR-ST AB 18, 080101 (2015)

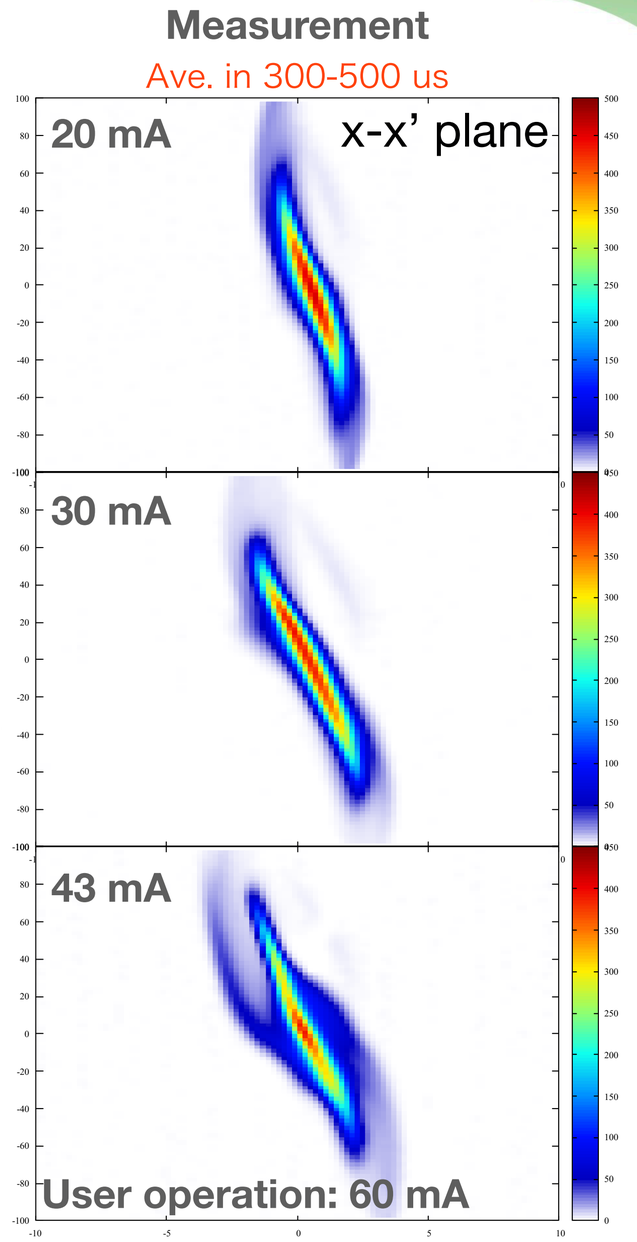
>> We investigate "***Beam-Plasma Coupling Oscillation***" with numerical and analytical approaches

Introduction: Another Motivation

❖ **There still exists emittance growth even at enough neutralization**

Dose coupling oscillation contribute to emittance growth?





Numerical Approach: Validity of PIC code

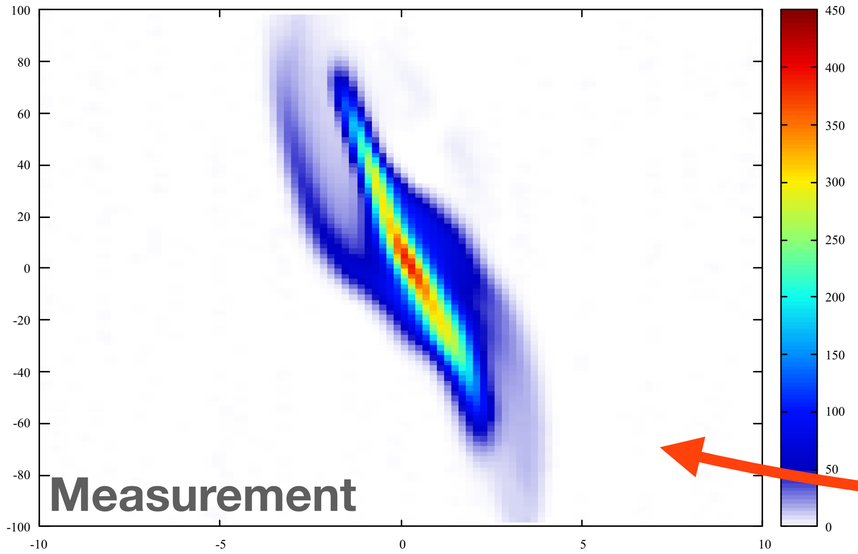
❖ PIC simulation reproduces measured distribution

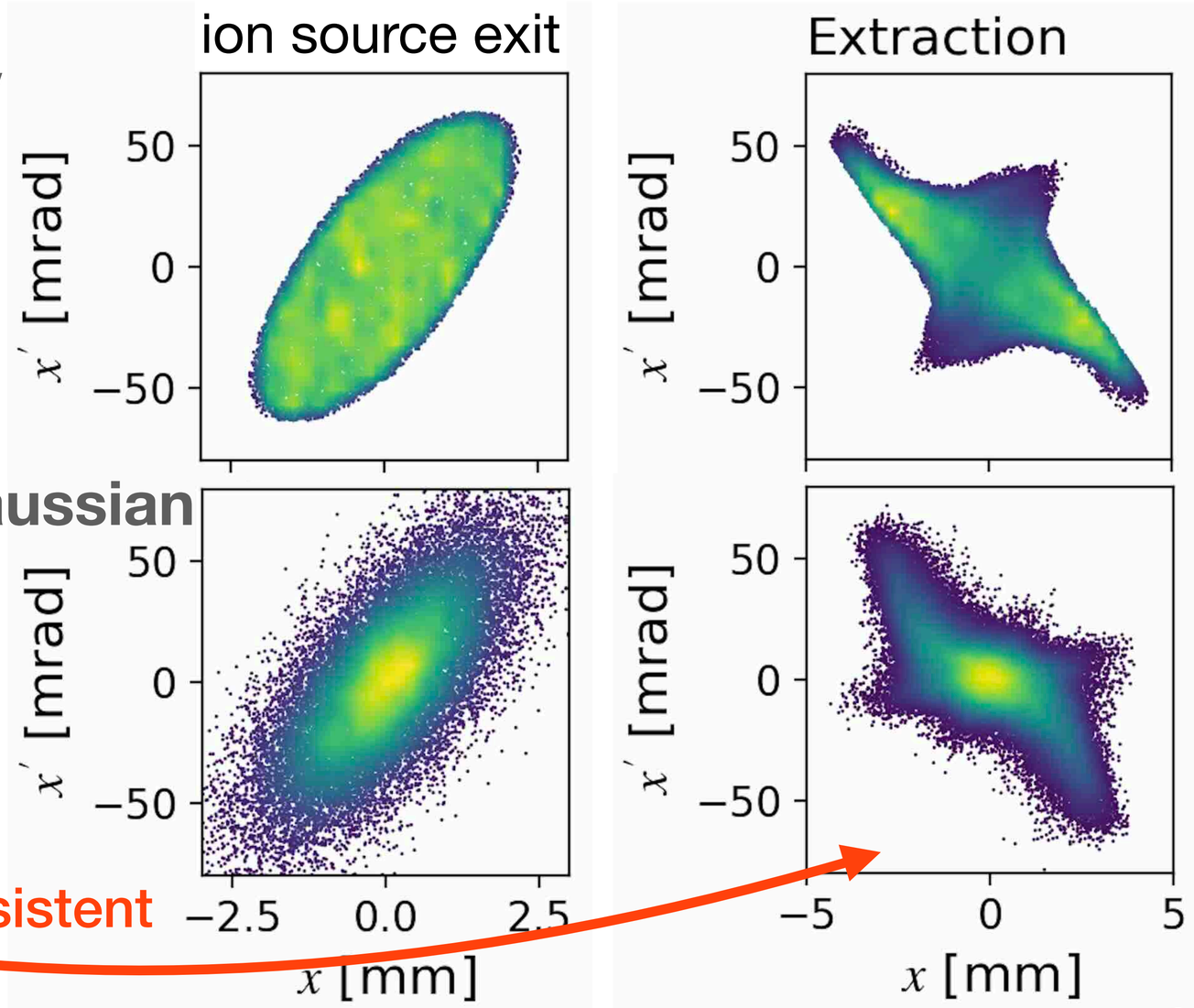
Code: WARP developed at LBNL

Plasma: H_2^+ in every step, ignore e-

Geometry: same as the reality

Initial condition: based on simulation in I.S.
ion source exit





Numerical Approach: Simplification of Conditions

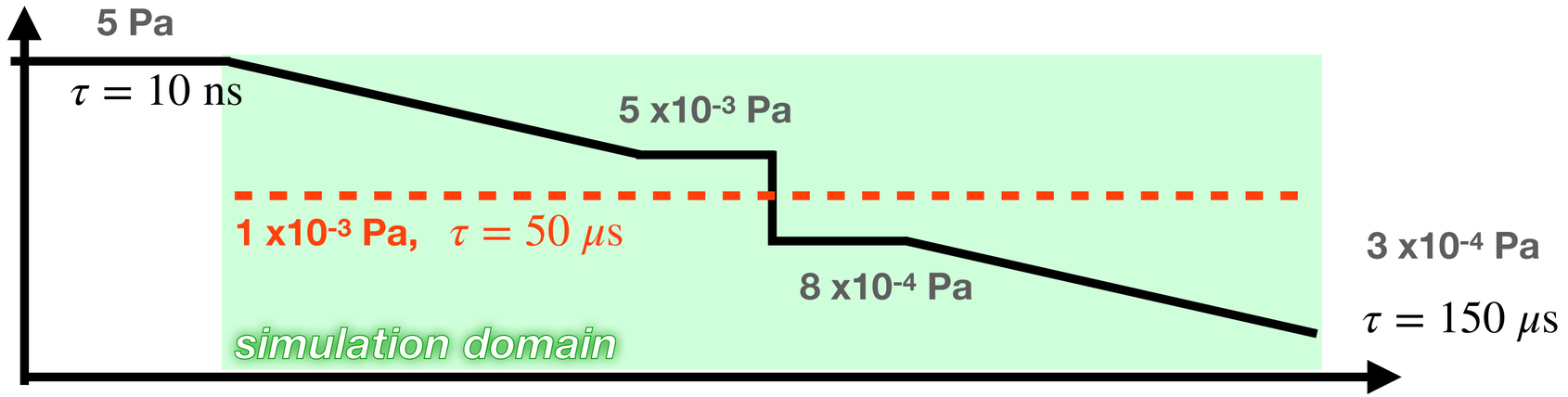
❖ **Simplify conditions to compare PIC with analytical approach**

Geometry:

- Cylindrical symmetry

Beam:

- KV distribution



Plasma:

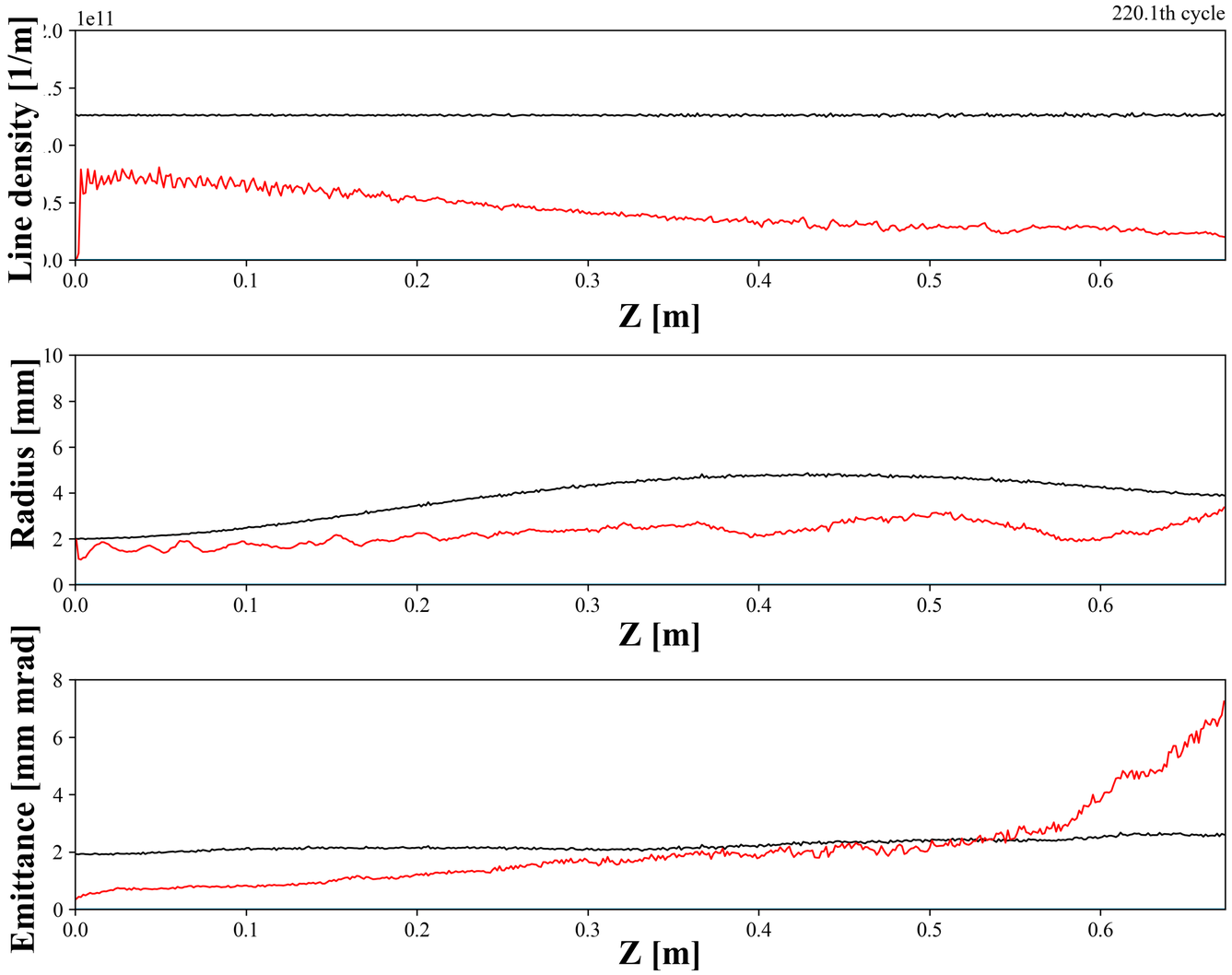
- uniform gas distribution (and no orifice), H^+
- w/o electrons

External field:

- uniform linear focusing force: $B_\theta(r) = Cr$ for compensating divergence due to emittance

Numerical Result 1:

❖ **PIC simulation showed oscillation on both beam and plasma >> coupling may occur**



1 cycle = 0.673 us

Beam

Plasma

Analytical Approach: Linear Vlasov-Poisson System

❖ Linear Vlasov-Poisson analysis to clarify if coupling oscillation can be occur

Conditions:

- Distribution function: cylindrically symmetric uniform $f(r, z, v_r, v_z) = \frac{1}{\pi a^2 v_\perp} \Theta(-r + a)\ \Theta(-v_r + v_\perp)\ \delta(v_z - \beta c)$ (step func.)

- EB-field: with or without unperturbed field => linear focusing E-field $E_r = -qKr$

Vlasov-Poisson system reduces to an eigenvalue problem (dispersion relation):

Normalized formula

$$\frac{\partial f_\mathrm{s}}{\partial t} + \boldsymbol{v} \cdot \nabla f_\mathrm{s} - \eta_\mathrm{s} \frac{Z_\mathrm{s}}{A_\mathrm{s}} \nabla \phi \cdot \nabla_v f_\mathrm{s} = 0$$

$$-\nabla^2 \phi = \sum_s \eta_\mathrm{s} Z_\mathrm{s} \rho_\mathrm{s}$$

s: particle species, beam/plasma

$$\Rightarrow \begin{pmatrix} \chi_\mathrm{b} - 1 & \chi_\mathrm{b} \\ \chi_\mathrm{p} & \chi_\mathrm{p} - 1 \end{pmatrix} \begin{pmatrix} \rho_\mathrm{b} \\ \rho_\mathrm{p} \end{pmatrix} = 0$$

χ_s: electric susceptibility
ρ_s: charge density
η_s: density ratio
Z_s: charge number
A_s: mass number
$\boldsymbol{k}$: wave number
J_l : Bessel function

without unperturbed field: $\chi_\mathrm{s} = \eta_\mathrm{s} \frac{Z_\mathrm{s}^2}{A_\mathrm{s}} \frac{1}{(\omega - k_z \beta_\mathrm{s} c)^2 - (k_r v_{\perp,s})^2}$

$$\omega_{0,\mathrm{s}} = \sqrt{\frac{qK}{m_\mathrm{s}}}$$

with unperturbed field: $\chi_\mathrm{s} = \eta_\mathrm{s} \frac{Z_\mathrm{s}^2}{A_\mathrm{s}} \frac{1}{k_r^2 + k_z^2} \sum_{l=-\infty}^{\infty} \left\{ -\frac{k_r J_l \left(J_{l+1} + J_{l-1}\right)}{\omega - l\omega_0} + \frac{k_z^2 v_{\perp,s} \left(J_l^2 - J_{l+1} J_{l-1}\right)}{(\omega - l\omega_0)^2} \right\}$

Analytical Result 1: without Unperturbed Field

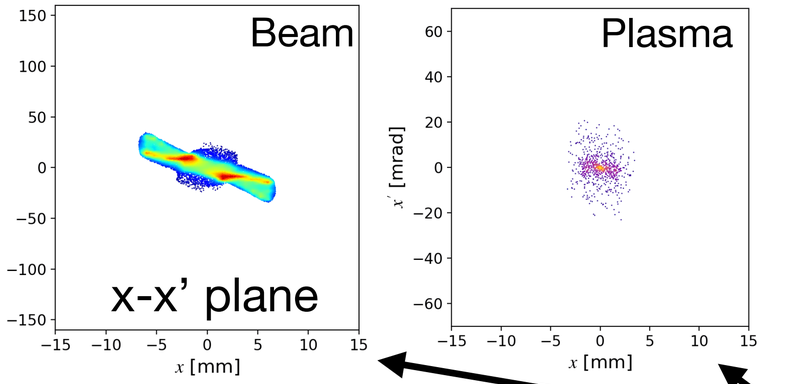
❖ Vlasov-Poisson analysis is consistent with PIC and supports existence of coupling oscillation

Coupling degree is roughly evaluated from eigenvector ratio:

$\rho_b/\rho_p \sim 1$ => coupled

$\rho_b/\rho_p \ll 1,\ \rho_b/\rho_p \gg 1$ => uncoupled

$$\begin{pmatrix} \chi_b - 1 & \chi_b \\ \chi_p & \chi_p - 1 \end{pmatrix} \begin{pmatrix} \rho_b \\ \rho_p \end{pmatrix} = 0$$



Freq [MHz]	ρ_b/ρ_p	Mode
$6.0 \pm j5.0$	0.64	Coupled
13	0.22	~Plasma
31	4.1	Beam

Conditions assumed from simulation result

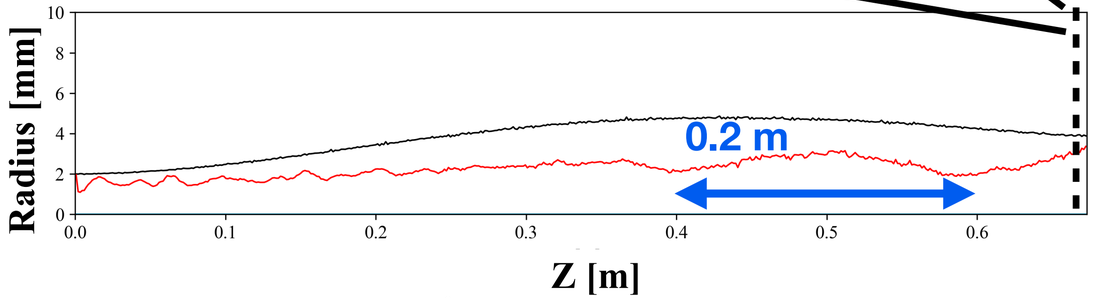
λ_z =0.2 m

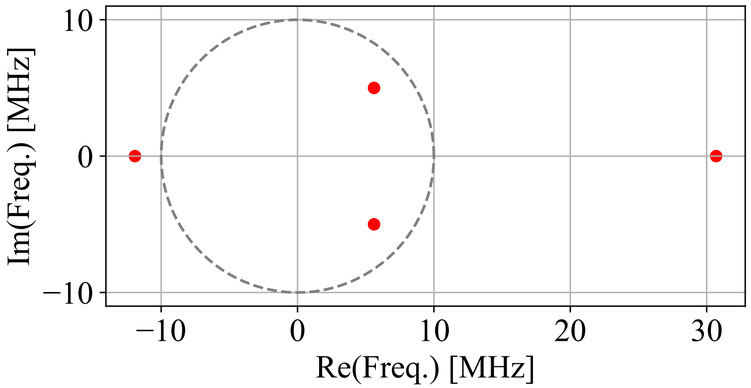
SCN degree = 0.6

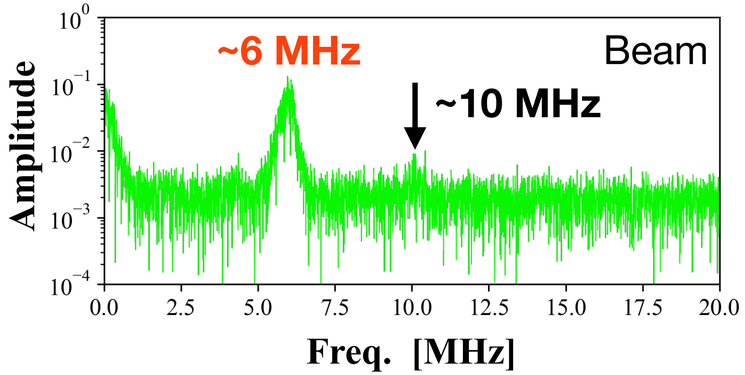
Beam radius = 3 mm

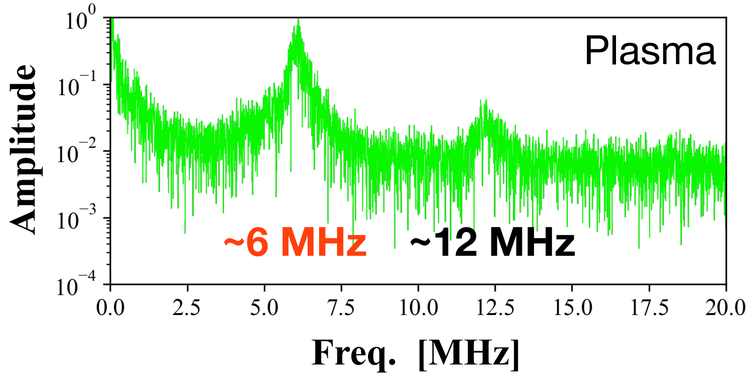
$v_{\perp,b}$ = 30 mrad

$v_{\perp,p}$ = 5 mrad







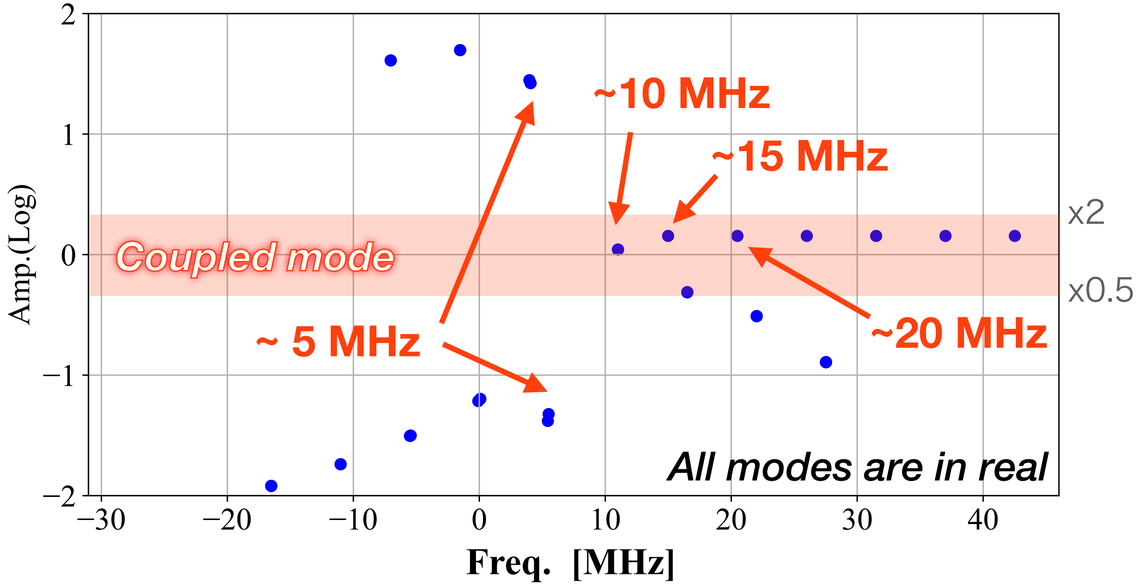


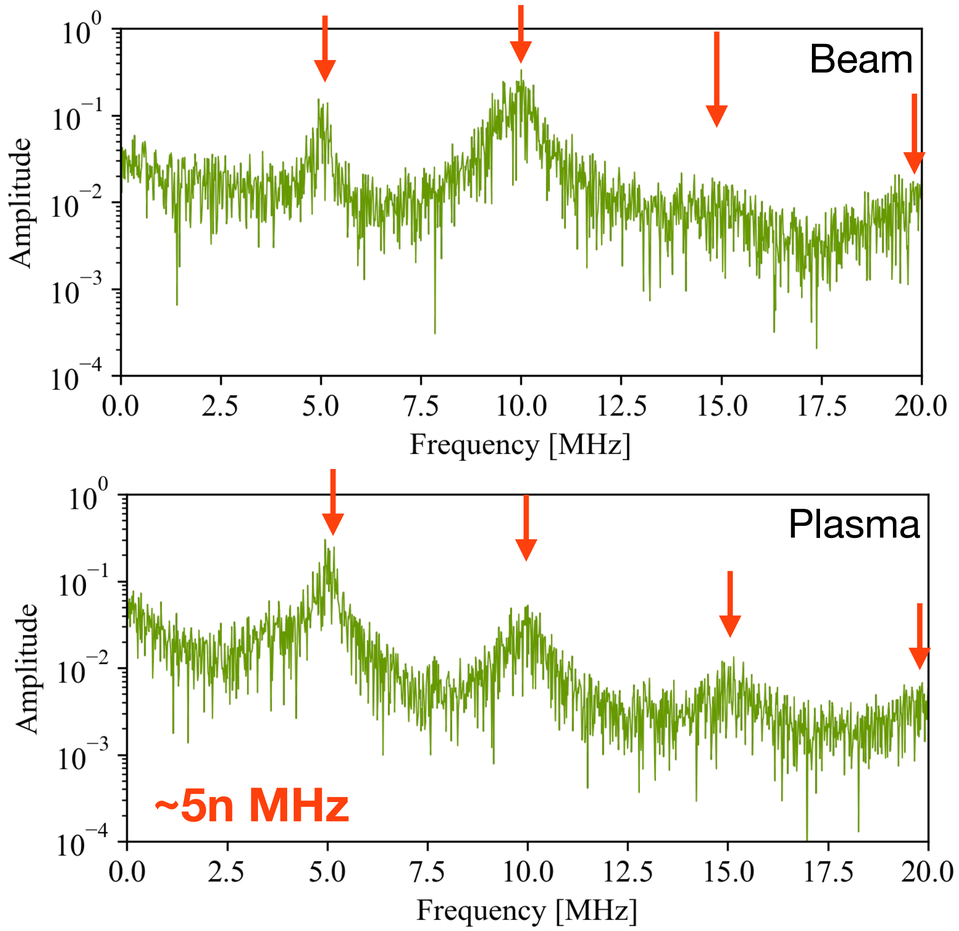
Analytical Result 2: with Unperturbed Field

❖ **Vlasov-Poisson analysis is consistent with PIC and supports existence of coupling oscillation**

At enough high gas pressure (10^{-1} Pa, x100 as previous), the number of eigen-modes increased
=> it implies unperturbed field becomes important

$$\chi_s = \eta_s \frac{1}{k_r^2 + k_z^2} \frac{Z_s^2}{A_s} \sum_{l=-\infty}^{\infty} \left\{ -\frac{k_r J_l \left(J_{l+1} + J_{l-1}\right)}{\omega - l\omega_0} + \frac{k_z^2 v_{\perp,s} \left(J_l^2 - J_{l+1} J_{l-1}\right)}{(\omega - l\omega_0)^2} \right\}$$





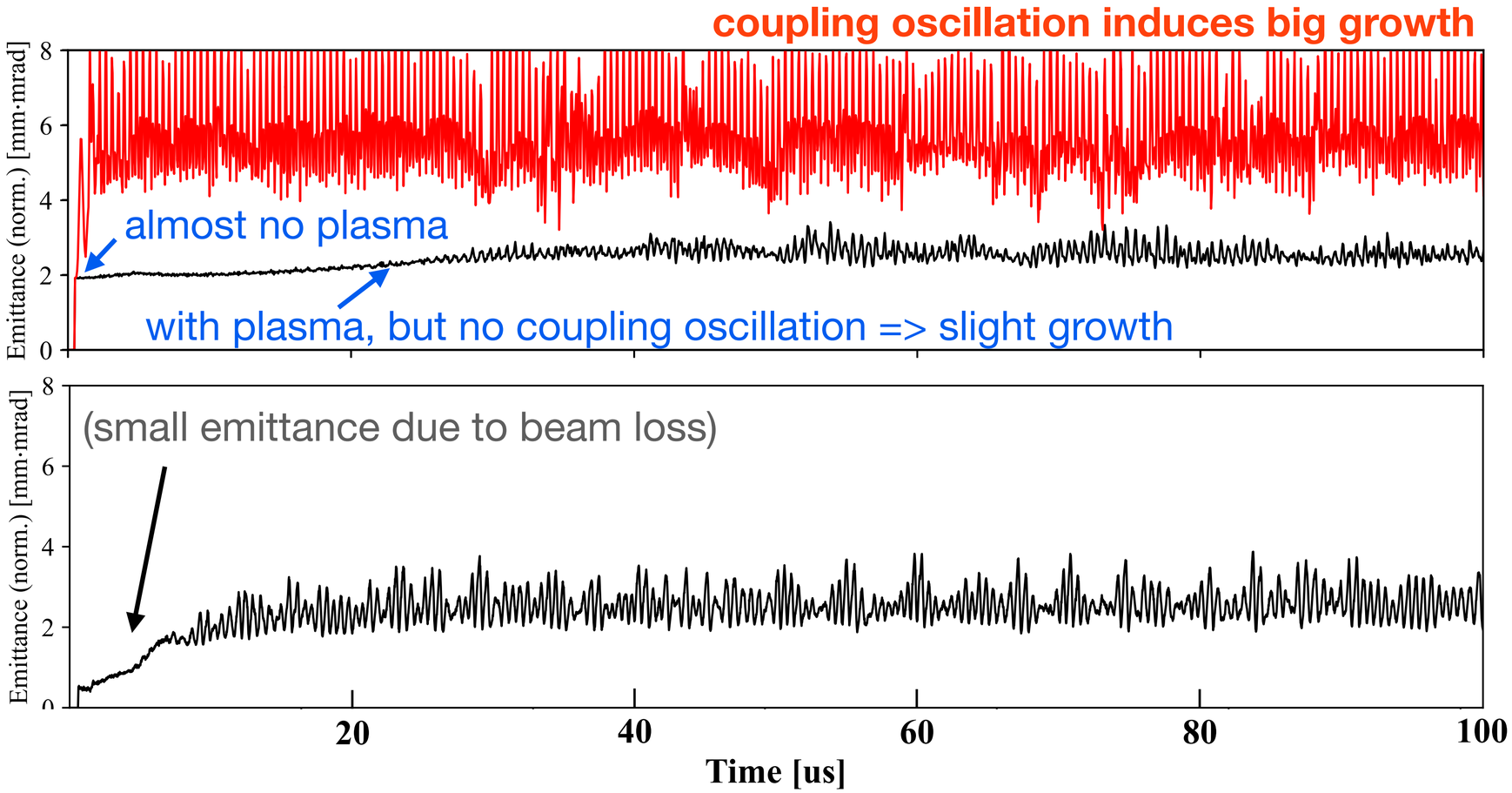
Numerical Result 2: Effect of Coupling on Beam Emittance

❖ Beam-plasma coupling oscillation induces emittance growth

Strong coupling oscillation could potentially occur in actual systems

Simplified model
KV
H_2 gas, uniform at **10^{-3} / 10^{-1} Pa**
uniform linear focusing

Actual J-PARC LEBT model
Gaussian
H_2 gas, strong z-dependance
in range of 1 - 10^{-4} Pa
2 solenoids focusing



SUMMARY

- Beam-Plasma coupling oscillation was studied with numerical and analytical approaches
  >> Both approaches are consistent and support coupling oscillation
  >> It was found coupling could potentially occur in actual systems

- Coupling oscillation could increase beam emittance
  but it has not been confirmed that coupling contributes the measured growth

- Higher gas pressure induces stronger coupling and emittance growth
  >> Coupling should be taken into consideration for gas injection

- We will try to measure the coupling effect

$$\begin{pmatrix} \chi_b - 1 & \chi_b \\ \chi_p & \chi_p - 1 \end{pmatrix} \begin{pmatrix} \rho_b \\ \rho_p \end{pmatrix} = 0$$

w/o unperturbed field: $\chi_s = \eta_s \frac{Z_s^2}{A_s} \frac{1}{(\omega - k_z \beta_s c)^2 - (k_r v_{\perp,s})^2}$

with unperturbed field: $\chi_s = \eta_s \frac{1}{k_r^2 + k_z^2} \frac{Z_s^2}{A_s} \sum_{l=-\infty}^{\infty} \left\{ -\frac{k_r J_l \left(J_{l+1} + J_{l-1}\right)}{\omega - l\omega_0} + \frac{k_z^2 v_{\perp,s} \left(J_l^2 - J_{l+1} J_{l-1}\right)}{(\omega - l\omega_0)^2} \right\}$

Beam
Plasma

high pressure: 0.1 Pa

