Inclusive vs exclusive $b \to s\mu\mu$: probing nonperturbative effects

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Based on:
Huber, Hurth, Jenkins, EL, Qin, Vos, 2007.04191
Huber, Hurth, Jenkins, EL, Qin, Vos, 2306.03134
Huber, Hurth, Jenkins, EL, Qin, Vos, 2404.03517
Capdevila, Alvarez, EL, Matias, to appear

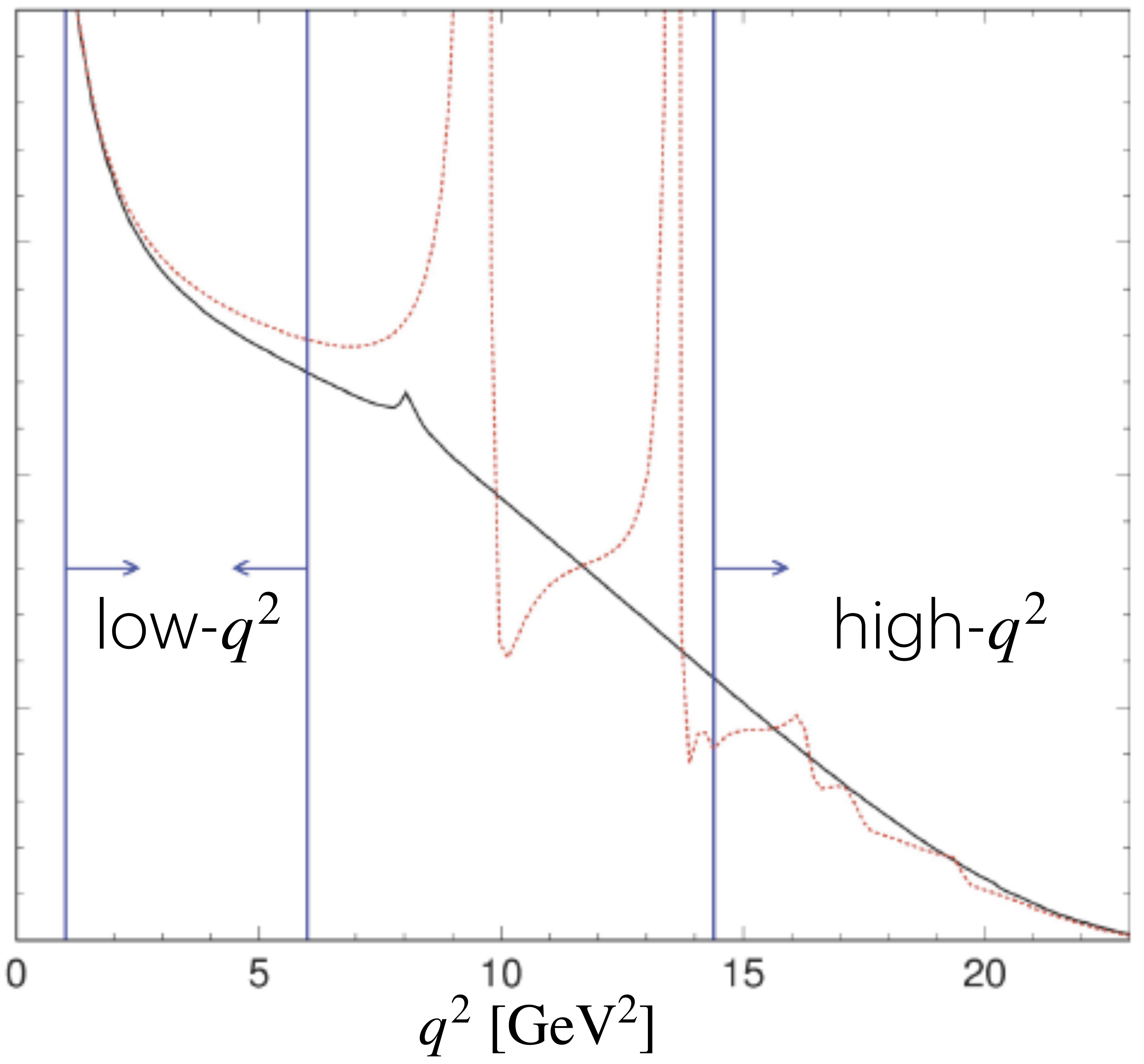
Theoryland: *Where Dreams Come True*

Exclusive at low-q²

- Soft-Collinear factorization at leading power (SCET-II)
- Inputs: Form Factors, LCDA
- Leading uncertainties: **non-local power corrections**, form factors (K*)

Inclusive at low-q²

- OPE
- Parametric inputs under control
- Leading uncertainties: **resolved photon contributions** (SCET-I)



Exclusive at high-q²

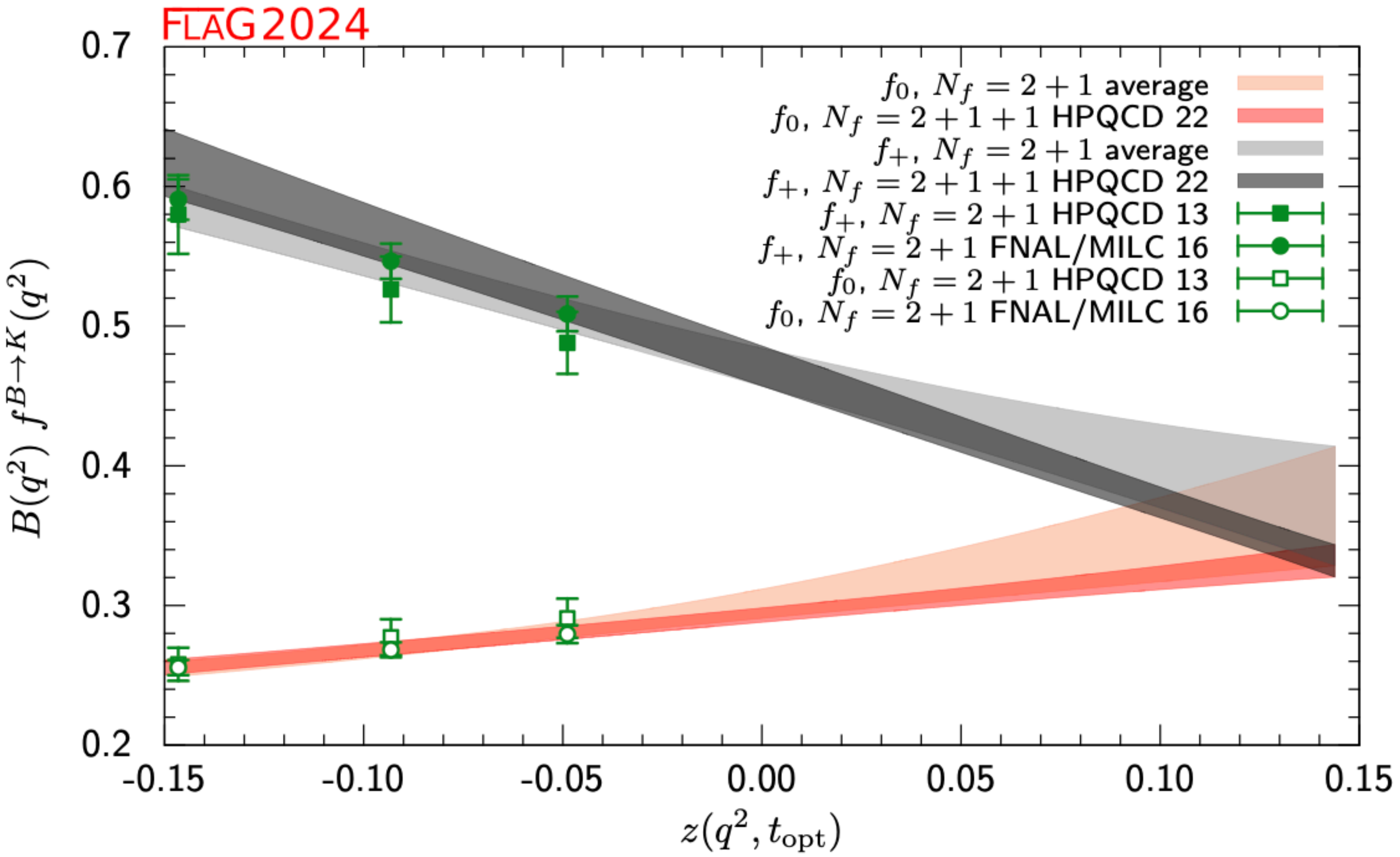
- $1/q^2$ expansion ("OPE" in q^2?): only integrated spectrum is predicted
- Inputs: **Form Factors, HQET matrix elements**
- if OPE == True:
  nonlocal_PC = False

Inclusive at high-q²

- OPE breaks down but the ratio $b \to s\ell\ell / b \to u\ell\nu$ **with the same q² cut** is well behaved
- Leading uncertainties: $1/m_b^3$ HQET matrix elements (Weak annihilation)

Exclusive: form factors

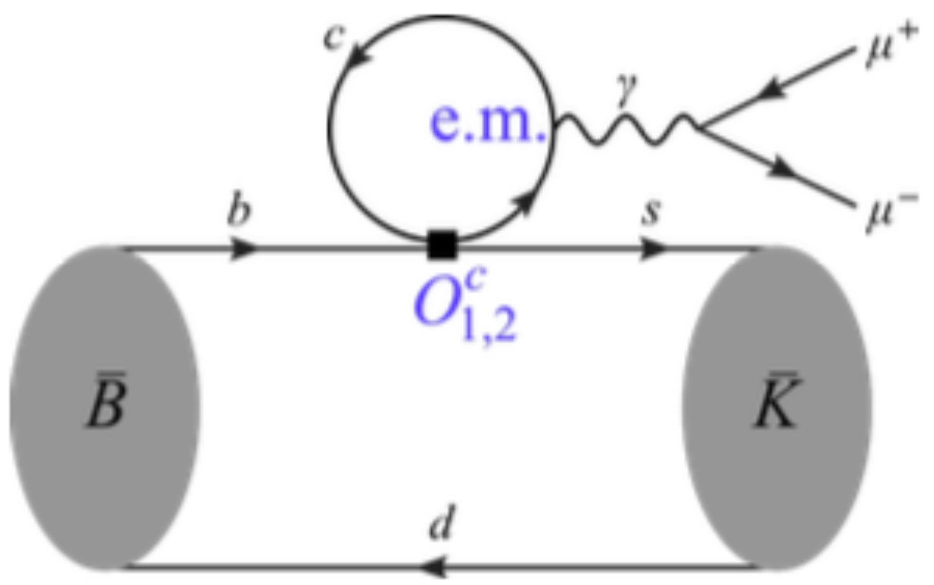
- **Light-Cone Sum Rules** (low-q^2): some uncertainties have to be ball-parked but allow access to all form factors
  [Bharucha, Straub, Zwicky, 1503.05534]
- **Lattice QCD** (high-q^2)
  - $B \to K$: multiple calculations
    [see FLAG 2024 review, 2411.04268]



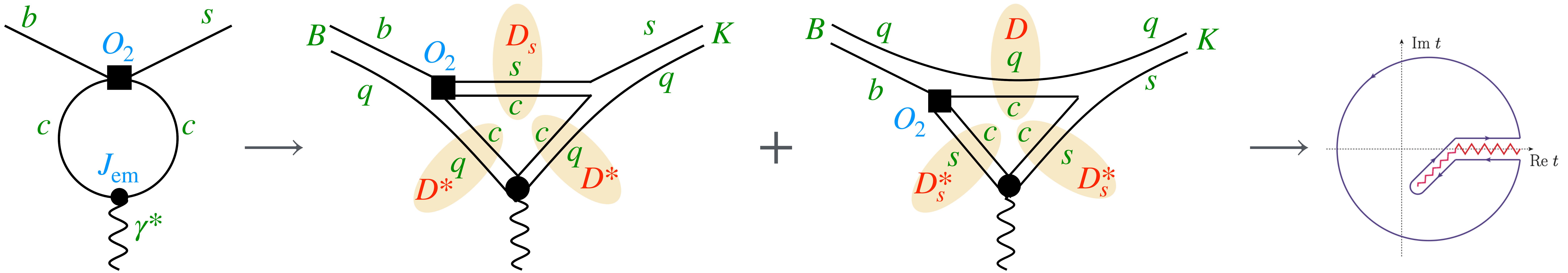
- $B \to K^*$ (also $B \to \rho$ and $B_s \to \phi, K^*$) are complicated by the resonant nature of the final state
  - Calculations with stable resonances in the narrow width approximation
    [Horgan, Liu, Meinel, Wingate, 1310.3722 and 1501.00367]
    [RBC/UKQCD 2016 - preliminary results]
  - Calculations based on the Lüscher formalism (volume dependence of energy levels)
    [Rendon et al, 2006.14035 - $K\pi$ scattering amplitudes]
    [Leskovec et al, 2403.19543, 2501.00903 - $B \to \rho$ form factors
    [Boyle et al, 2406.19194 - mass and width of K^* and ρ]
    [Li et al, 2603.16266]
    [Di Carlo, Erben, Tsang et al, ongoing]
  - **Currently a combination of Lattice and LCSR is used**
    [Gubernari, Reboud, Dyk, Virto, 2305.06301]

Exclusive: non-local power corrections

- **Power corrections at low-q^2**



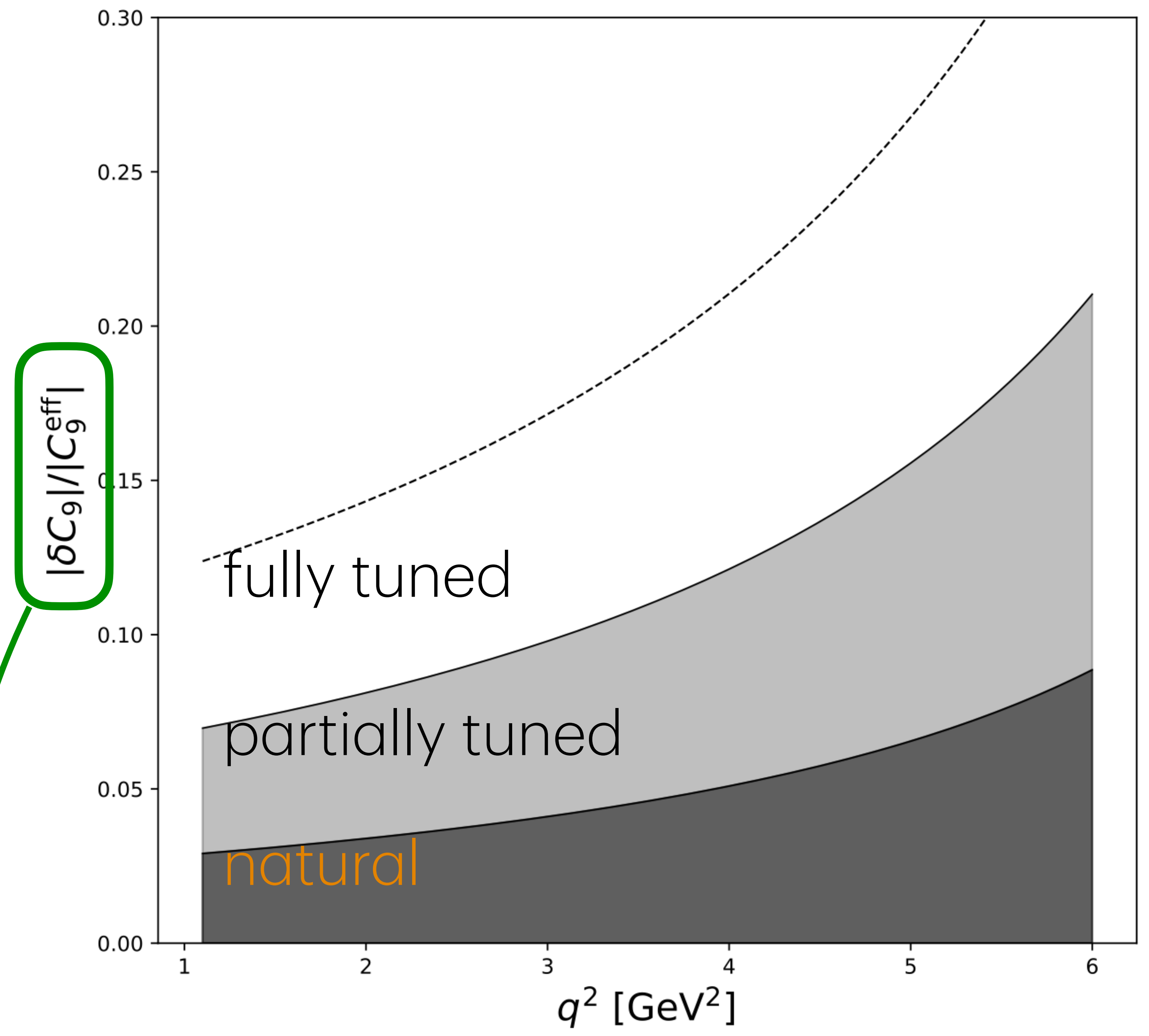
  - Within **SCET-II**, nonlocal matrix elements, like $\langle K^{(*)} | T J^{\text{em}}_\mu O_2 | B \rangle$, are factorized up to <u>unknown subleading</u> non-perturbative functions
  - These power corrections are usually estimated at the $O(10\% - 20\%)$ of the leading power amplitude
  - The presence of anomalous thresholds (associated to triangle diagrams) could possibly lead to large effects:

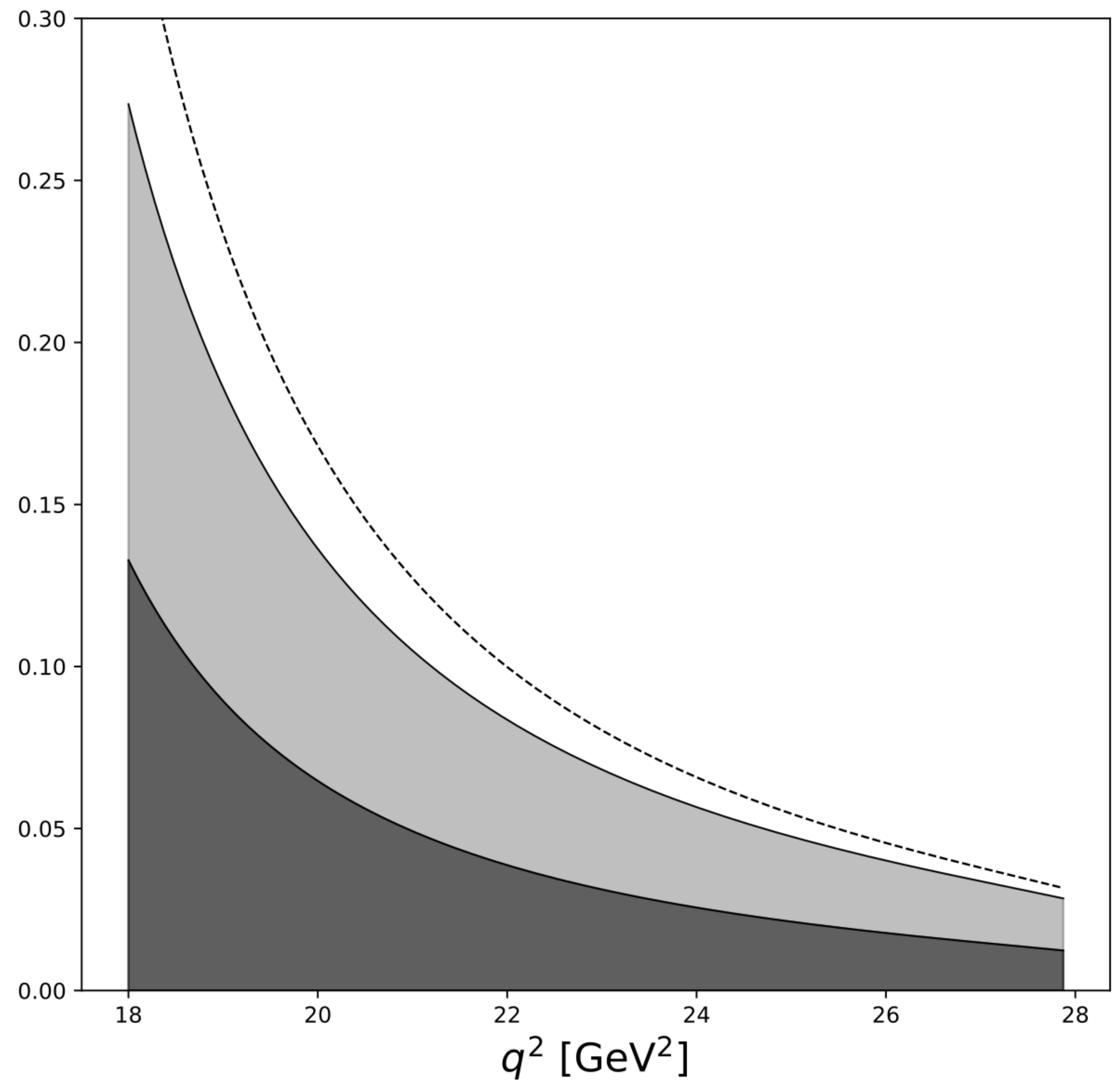


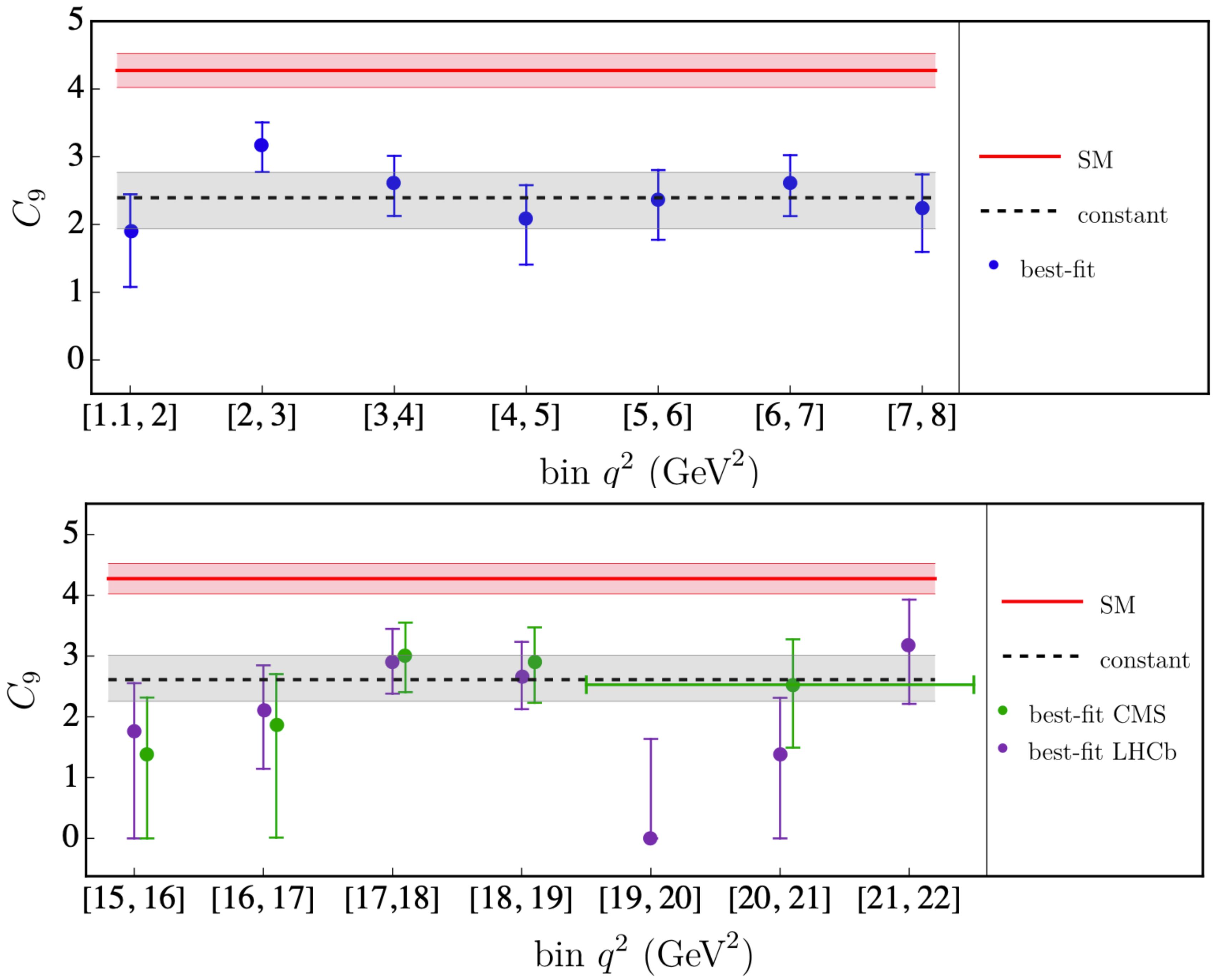
    - Use parameterizations, dispersion relations, understand the analytical structure [See S. Mutke's talk]
      [Mutke et al 2406.14608; Gubernari, Reboud, van Dyk, Virto 2305.06301; Gopal, Gubernari 2412.04388; Balz et al, 2510.25584; Hoferichter et al, 2604.01284]
    - Direct calculation using lattice-QCD (in O(5) years) [See L. Silvestrini's talk]
      [Frezzotti, Gagliardi, Lubicz, Martinelli, Sachrajda, Sanfilippo, Simula and Silvestrini, in progress]

Exclusive: non-local power corrections

- q^2 dependence of DD_s^* and D_sD^* rescattering [Isidori, Polonsky, Tinari 2405.17551, 2507.17824] vs fit results (assuming NP in C_9) [Bordone, Isidori, Mächler, Tinari 2401.18007]





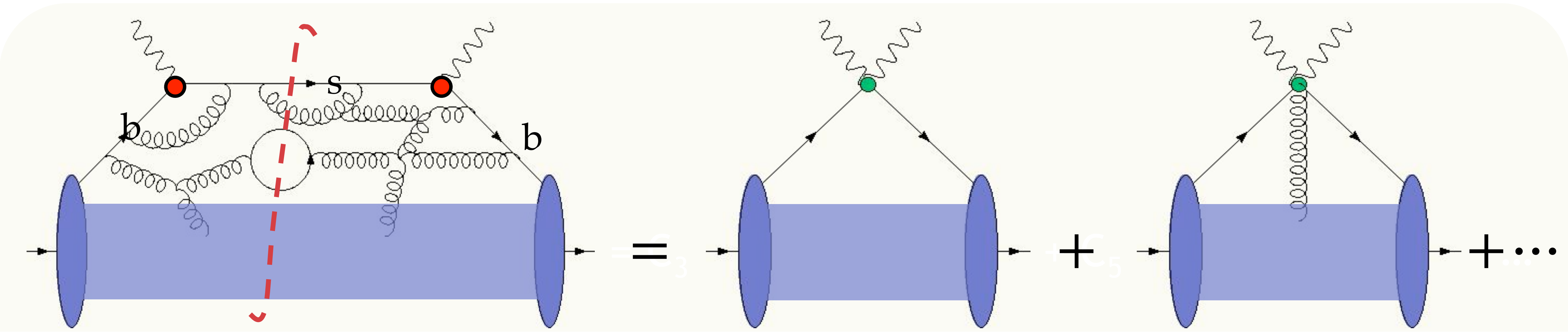


- Global fits: $|\delta C_9/C_9| \sim 0.25$
- Monopole (dipole) contributions have opposite (same) sign at low and high q^2.
  ⇒ Simultaneous large and same sign effects at low and large-q^2 are difficult to arrange.

Inclusive: OPE

- Up to power corrections the inclusive rate is free of hadronic uncertainties:

$$\Gamma[B \to X_s \ell\ell] = \Gamma[b \to X_s \ell\ell] + O\left(\frac{\Lambda^2_{QCD}}{m_b^2}, \frac{\Lambda^3_{QCD}}{m_b^3}, \frac{\Lambda^2_{QCD}}{m_c^2} \cdots\right)$$

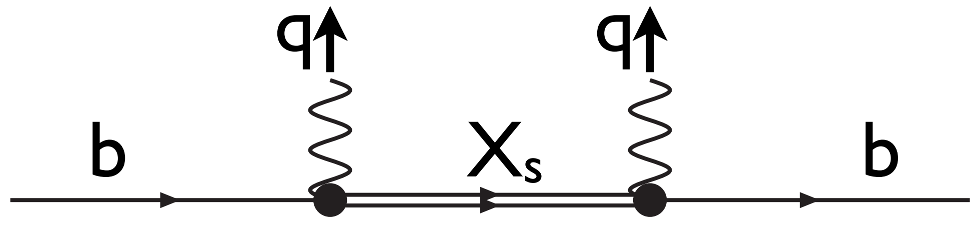


$\langle B | \bar{b} b | B \rangle = 1$

$\langle B | \bar{b} D^2 b | B \rangle \sim \lambda_1$ $\quad \langle B | \bar{b} \sigma_{\mu\nu} G^{\mu\nu} b | B \rangle \sim \lambda_2$

The leading power contribution is most expressed as a series in α_s and $\kappa = \alpha_{em}/\alpha_s$ and is known (almost) up to and including $\alpha_s^3 \kappa^3$

- The OPE breaks down at high-q^2:



$$p^2_{X_s} = (p_b - q)^2 = m_b^2 + q^2 - 2 m_b q_0$$

$$< m_b^2 + q^2 - 2 m_b \sqrt{q^2} = \left(m_b - \sqrt{q^2}\right)^2$$

expansion in $\dfrac{\Lambda_{\rm QCD}}{m_b - \sqrt{q^2}}$

- This breakdown manifests as very large power corrections

Inclusive: workaround at high-q2

- Power corrections proportional to $|C_{9,10}|^2$ are identical to those which appear in $\bar{B}^0 \to X_u \ell \nu$:
  [Lee, Ligeti, Stewart, Tackmann]

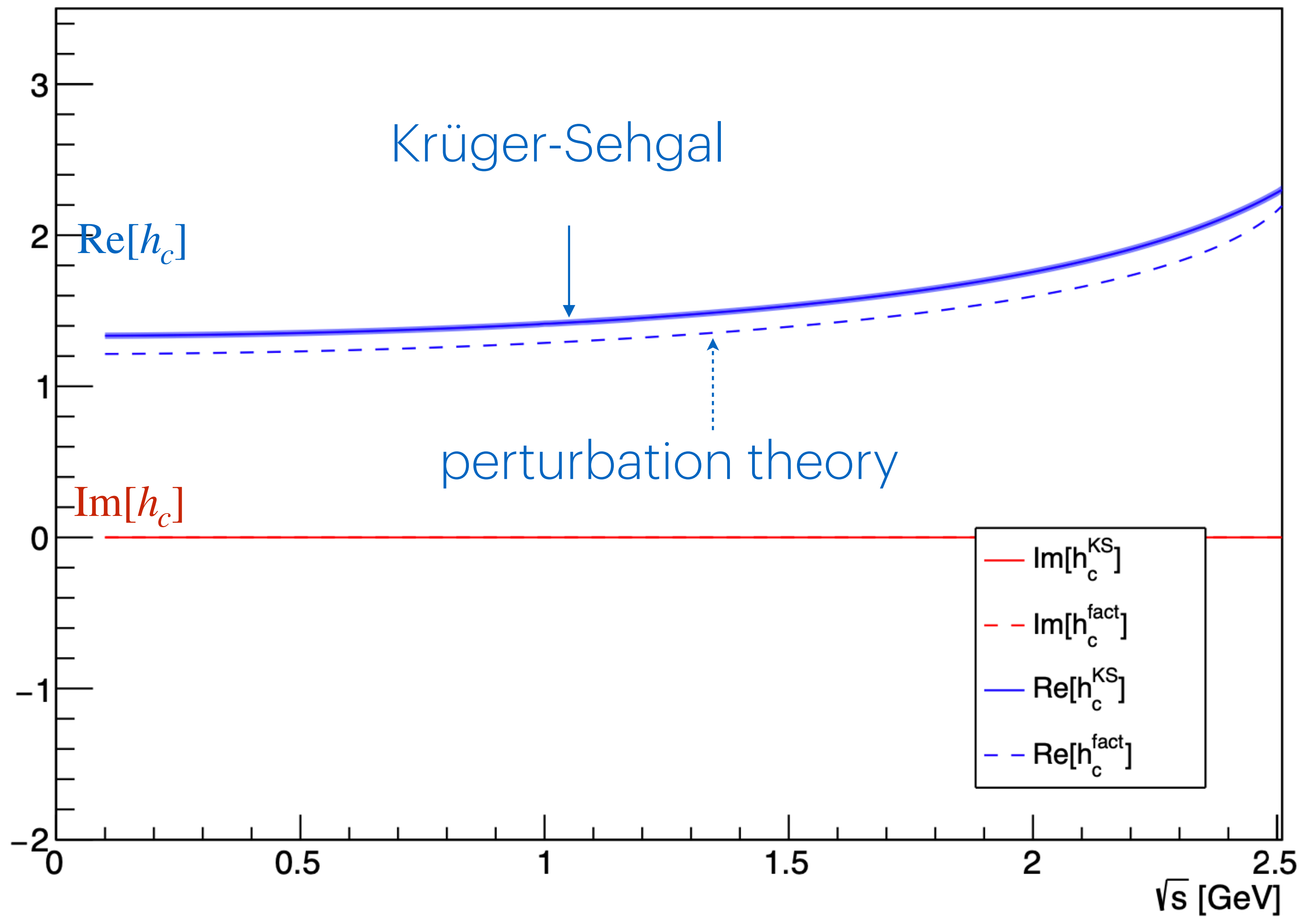
$$\mathcal{R}(q_0^2) = \frac{\int_{q_0^2}^{m_b^2} dq^2 \frac{d\Gamma(\bar{B} \to X_s \ell^+ \ell^-)}{dq^2}}{\int_{q_0^2}^{m_b^2} dq^2 \frac{d\Gamma(\bar{B}^0 \to X_u \ell \nu)}{dq^2}}$$

  - Using the **neutral B⁰** semileptonic rate the (poorly known) weak annihilation matrix elements cancel in the ratio
  - Currently the differential $B \to X_u \ell \nu$ spectrum has been presented only for the isospin average channel [Belle 2107.13855 (711 fb^{-1}), Belle II 2512.08056 (365 fb^{-1})]
  - Non-perturbative effects associated to other operators ($|C_7|^2$, $C_7 C_9$) do not necessarily cancel

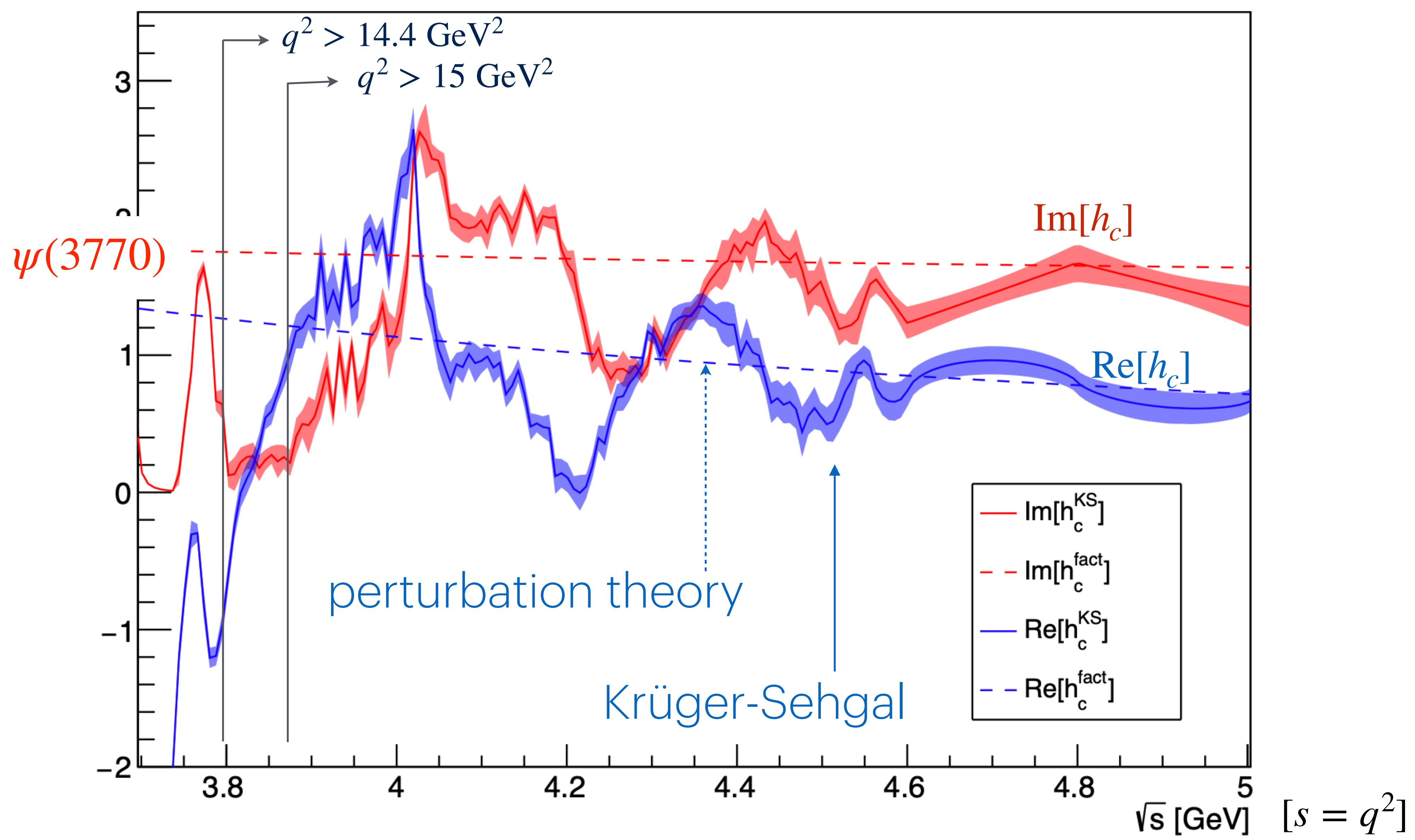
Inclusive: resonances

- We use $e^+e^- \to$ **hadrons** data [BESII, BaBar, ALEPH; Keshavarzi, Nomura, Teubner] and perturbation theory for asymptotically large q^2 [Harlander, Steinhauser: **`rhad`**]

Low-q^2



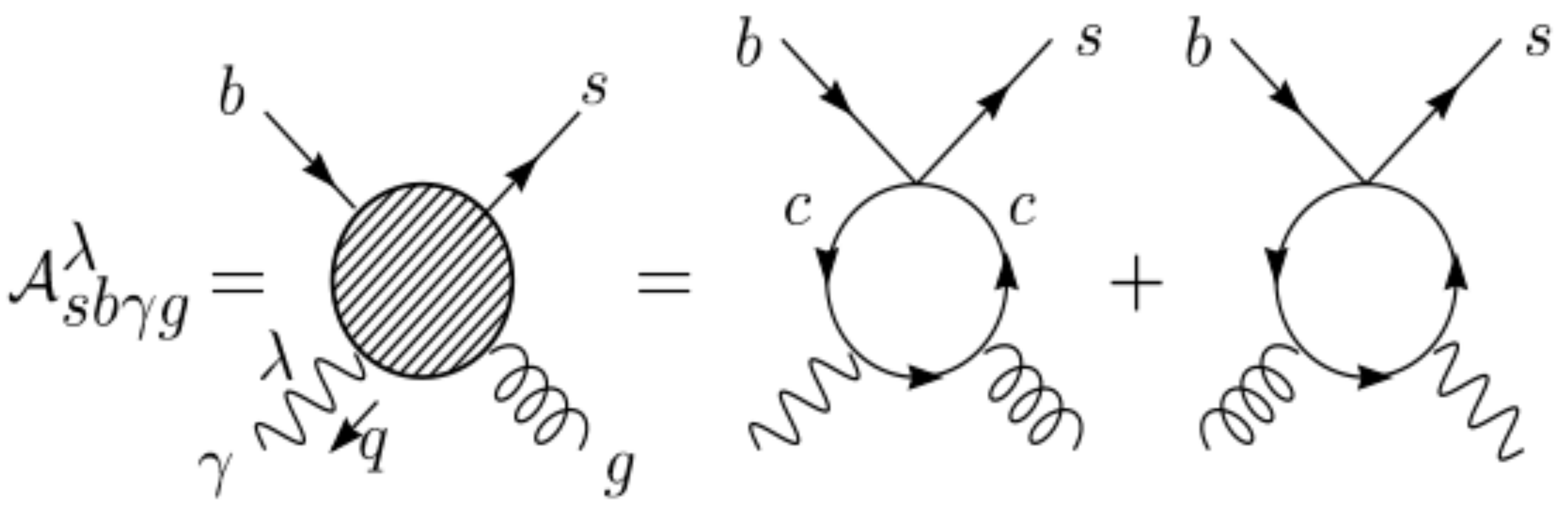
High-q^2



- Impact at low-q^2 is small ($\simeq 2\,\%$)
  Perturbation theory and dispersive approaches agree because below threshold we are mostly sensitive to the total integral over R_{had} which is well described in perturbation theory
- Impact at high-q^2 region is larger ($\simeq -10\,\%$)

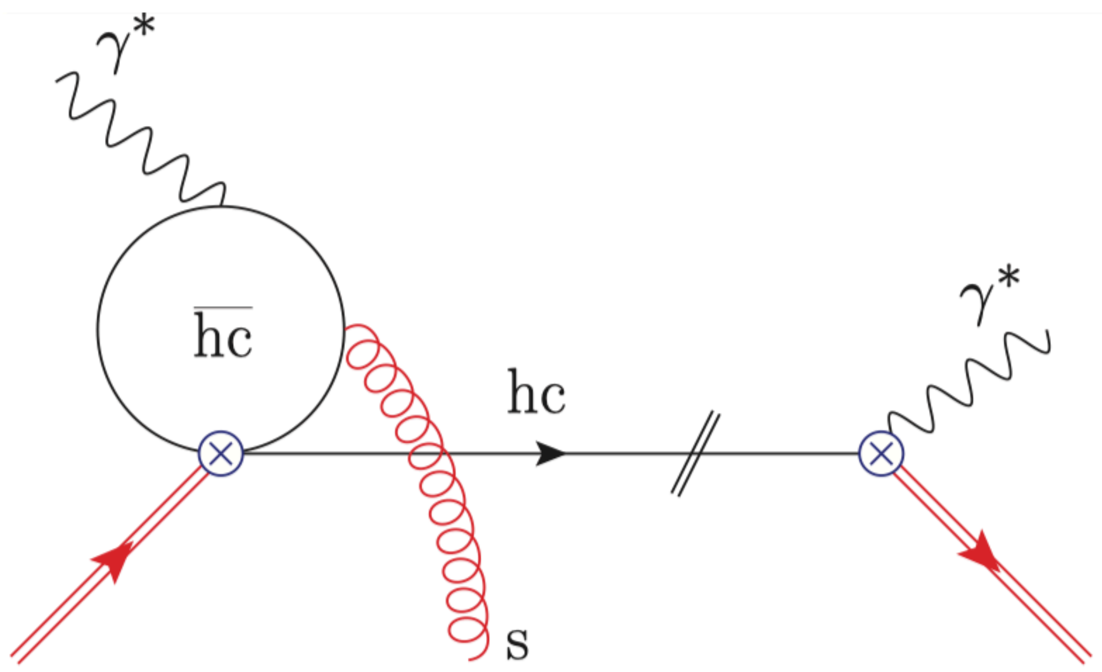
Inclusive: resonances

- Non-resonant color octet effects at **high-q²** can be calculated in perturbation theory for the integrated rate, they are **local** and scale as $\Lambda^2_{\rm QCD}/q^2$ [Buchalla, Isidori, Rey]:



$$\Longrightarrow \frac{\langle B|\bar b \sigma_{\mu\nu} G^{\mu\nu} b|B\rangle}{m_c^2}\left(-6\frac{m_c^2}{q^2}\right) \sim \frac{\lambda_2}{q^2}$$

- Non-resonant color octet effects at **low-q²** and with a cut on m_X, are **nonlocal** and can treated using SCET [Hurth, Benzke, Fickinger, Turczyk]:



  - Power corrections remain non-local after m_X cut is released ⇒ so-called resolved contributions
  - Depend on mostly unknown subleading B shape functions
  - A rough estimate yields an irreducible uncertainty of about 5%

- Effect of resonant non-factorizable charmonium production at low and high q² is being investigated

Inclusive: resonances

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$$\Longrightarrow \frac{\langle B\,|\,\bar b \sigma_{\mu\nu} G^{\mu\nu} b\,|\,B\rangle}{m_c^2}\left(-6\frac{m_c^2}{q^2}\right) \sim \frac{\lambda_2}{q^2}$$

- Non-resonant color octet effects at **low-q²** and with a cut on m_X are **nonlocal** and can be treated using SCET [Hurth, Benzke, Fickinger, Turczyk]:

These effects correspond to the problematic nonlocal power corrections in exclusive decays

  - Power corrections remain non-local after m_X cut is released $\Rightarrow$ so-called resolved contributions
  - Depend on mostly unknown subleading B shape functions
  - A rough estimate yields an irreducible uncertainty of about 5%

- Effect of resonant non-factorizable charmonium production at low and high q² is being investigated

Inclusive: theory summary

issues

- **NNLO$_{\rm QCD}$ + NLO$_{\rm QED}$**
- $c\bar{c}$ **resonances**: included using e^+e^- data via a dispersion relation (Krüger-Sehgal mechansim)
- **QED radiation**: soft/collinear photons treatment at B-factories and LHCb needs to be taken into account
- m_X **cuts**: at low-q² removal of double semileptonic background, $b \to (c \to s\ell\nu)\ell\nu$, requires extrapolation in m_X

errors

- Dominant uncertainties:
  - **Low-q²** ⇒ **resolved photon contributions** (irreducible 5%) **and scale** (N³LO)
  - **High-q² (BR)** ⇒ **power corrections** (OPE breakdown)
  - **High-q² ($b \to s\ell\ell / b \to u\ell\nu$ ratio)** ⇒ **parametric** (CKM) and **power corrections** (5x smaller than in BR!)

results

$$\mathcal{B}[1,6]_{\mu\mu} = 17.29\,(1 \pm 4.4\%_{\rm scale} \pm 1.1\%_{m_t} \pm 2.3\%_{C,m_c} \pm 1.2\%_{m_b} \pm 0.5\%_{\alpha_s} \pm 0.1\%_{\rm CKM} \pm 1.5\%_{\rm BR_{sl}} \pm \underbrace{0.7\%_{\lambda_2} \pm 5\%_{\rm resolved}}_{5.1\,\%}) \times 10^{-7}$$
$$= (17.29 \pm 1.28) \times 10^{-7} \quad [7.4\%]$$

$$\mathcal{B}[>15]_{\rm no\ QED} = 2.59\,(1 \pm 8.1\%_{\rm scale} \pm 1.2\%_{m_t} \pm 1.9\%_{C,m_c} \pm 7.3\%_{m_b} \pm 0.2\%_{\alpha_s} \pm 0.08\%_{\rm CKM} \pm 1.5\%_{\rm BR_{sl}} \pm \underbrace{3.9\%_{\lambda_2} \pm 10\%_{\rho_1} \pm 21\%_{f_{u,s}}}_{23.5\,\%}) \times 10^{-7}$$
$$= (2.59 \pm 0.68) \times 10^{-7} \quad [26\%]$$

$$\mathcal{R}(15)_{\rm no\ QED} = 27.00\,(1 \pm 0.93\%_{\rm scale} \pm 1.1\%_{m_t} \pm 0.41\%_{C,m_c} \pm 0.63\%_{m_b} \pm 0.56\%_{\alpha_s} \pm 4.3\%_{\rm CKM} \pm \underbrace{0.26\%_{\lambda_2} \pm 1.4\%_{\rho_1} \pm 5.3\%_{f_{u,s}}}_{5.5\,\%}) \times 10^{-4}$$
$$= (27.0 \pm 1.9) \times 10^{-4} \quad [7.2\%]$$

4x reduction

Inclusive: theory summary

- The inclusive calculations that we need for current phenomenology are:

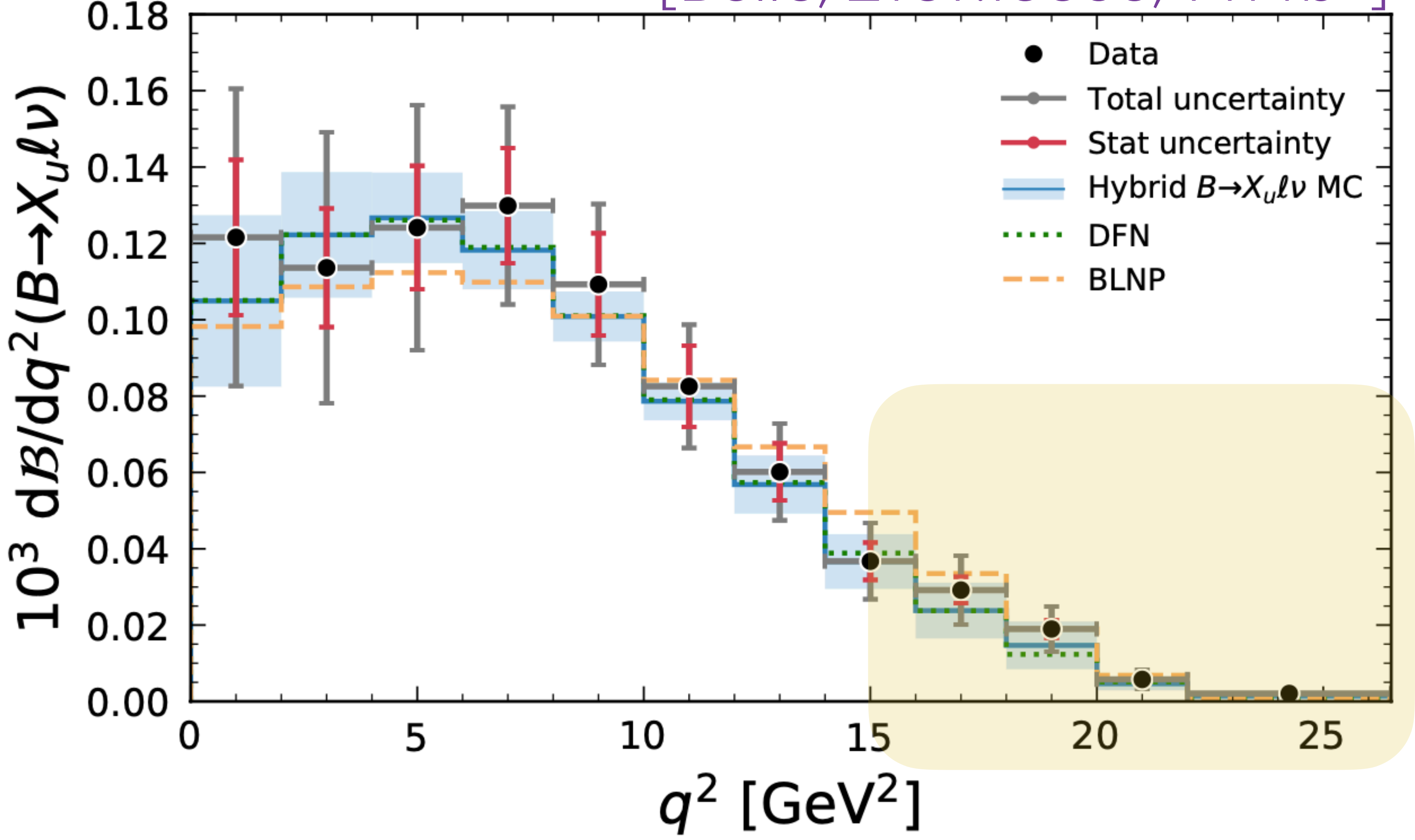
$$\mathscr{B}[1,6]_{\ell\ell} = (17.4 \pm 1.3) \times 10^{-7} \qquad [7.4\%]$$

$$\mathscr{B}[>15]_{\text{no QED}} = (2.59 \pm 0.68) \times 10^{-7} \qquad [26\%]$$

$$\mathscr{R}(15)_{\text{no QED}} = (17.00 \pm 1.9) \times 10^{-4} \qquad [7.2\%]$$

- In order to use $\mathscr{R}(15)$ we need the $B \to X_u \ell \nu$ normalization:

[Belle, 2107.13855, 711 fb^{-1}]



$$\mathscr{B}(\bar{B} \to X_u \ell \bar{\nu})[>15]_{\text{exp}} = (1.52 \pm 0.28) \times 10^{-4} \quad [18.4\%]$$

$$\Downarrow$$

$$\mathscr{B}[>15]_{\text{SM},\mathscr{R}} = \mathscr{R}(15)_{\text{SM}} \times \mathscr{B}(B \to X_u \ell \bar{\nu})[>15]_{\text{exp}}$$
$$= (4.10 \pm 0.81) \times 10^{-7} \quad [19.8\%]$$

The total uncertainty is dominated by the $B \to X_u \ell \nu$ partial rate

[see also the recent the Belle-II analysis, 2512.08056]

Inclusive at high-q^2 as sum of exclusive

- $B \to \left[K + K^* + (K\pi)_{\text{S-wave}} + K\pi\pi\right] \mu^+\mu^-$ saturates $B \to X_s\mu^+\mu^-$ for $q^2 > 15\ \text{GeV}^2$:
  This allows for a measurement of the inclusive at high-q^2 using LHCb measurements
  [Isidori, Polonsky, Tinari, 2305.03076; Huber, Hurth, Jenkins, EL, Qin, Vos, 2404.03517]

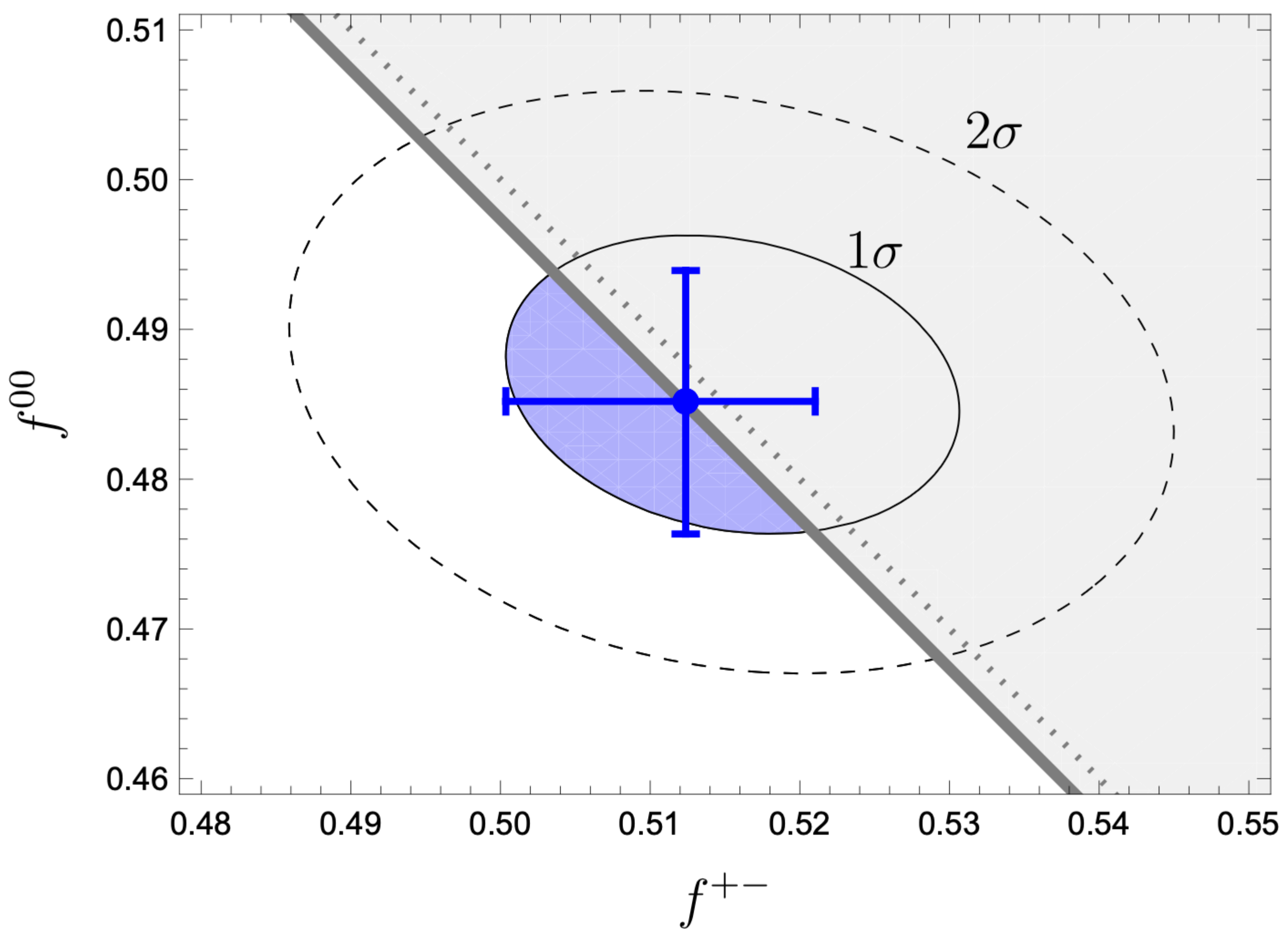
- Limitations to phenomenological uses of this measurement are
  - branching ratios of charged an neutral $B \to J/\psi K^{(*)}$ (used as normalizations by LHCb)
  - $B^0 \to X_u \ell\nu$ branching ratio for $q^2 > 15\ \text{GeV}^2$ (required for a clean theoretical prediction for $B \to X_s\mu^+\mu^-$)

  $\Rightarrow$ both normalizations will be measured with improved accuracy at Belle II

- In the following I'll focus on two issues:
  - How to combine all existing $B \to J/\psi K^{(*)}$ measurements taking into account
    1. the correct branching ratios f^{+-} and f^{00} of $\Upsilon(4S)$ into B^+B^- and $B^0\bar{B}^0$
    2. uncertainties associated to isospin invariance assumptions used in the branching ratios measurements
  - The construction of new observables that can help disentangle Non Perturbative charm contributions from New Physics ones

Impact of $\mathscr{B}(\Upsilon(4S) \to B^+B^-) \neq \mathscr{B}(\Upsilon(4S) \to B^0\bar{B}^0) \neq 1/2$

- Almost all but few of the most recent Belle measurements assume $f^{+-} = f^{00} = 1/2$
- This issues have been recently addressed in:
  [Bernlochner, Jung, Khan, Landsberg, Ligeti 2306.04686]
  [Jung, Schacht 2604.08391]
  [Belle II, Michele Mantovano @ Challenges in Semileptonic B decays 2026, Bad Honnef]
- Currently PDG and HFLAV ignore this problem and simply average all measurements together (irrespectively to whether the $f^{+-} = f^{00} = 1/2$ assumption was or was not made (!)



$$\begin{cases} f^{+-} = 0.511 \pm 0.010 \\ f^{00} = 0.4851 \pm 0.0088 \end{cases}$$

[Symmetrized averages]

$$\mathcal{B}(B^0 \to K^{*0} J/\psi)_{\rm avg} = (13.23 \pm 0.54) \times 10^{-4}.$$
$$\mathcal{B}(B^0 \to K^{*0} J/\psi)_{\rm PDG} = (12.65 \pm 0.46) \times 10^{-4}.$$
$$\mathcal{B}(B^0 \to K^{*0} J/\psi)_{\rm HFLAV} = (12.78 \pm 0.44) \times 10^{-4}.$$

- Similarly for all other normalization modes [Capdevila, Alvarez, EL, Matias, to appear]

$\mathcal{B}(B \to K^{(*)}\mu\mu)$ averages

- For each measurement, reconstruct what is truly measured: $\mathcal{R}(K^{(*)}) \equiv \mathcal{B}(B \to K^{(*)}\mu\mu)/\mathcal{B}(B \to K^{(*)}J/\psi)$
- Average (if more than one measurement exist, i.e. $B^+ \to K^+\mu\mu$ we have LHCb and CMS)
- Reintroduce the normalization calculated as detailed above

For instance for $q^2 \in [1,6]\ \mathrm{GeV}^2$ we find:

K^+

$$\mathcal{R}(K^+)^{\text{low}}_{\text{LHCb}} = (1.188 \pm 0.046) \times 10^{-4}$$
$$\mathcal{R}(K^+)^{\text{low}}_{\text{CMS}} = (1.228 \pm 0.051) \times 10^{-4}$$
$$\mathcal{R}(K^+)^{\text{low}}_{\text{avg}} = (1.206 \pm 0.034) \times 10^{-4}$$
$$\mathcal{B}(B^+ \to K^+\mu\mu)^{\text{low}}_{\text{avg}} = (1.228 \pm 0.048) \times 10^{-7}$$

K^0

$$\mathcal{R}(K^0)^{\text{low}}_{\text{LHCb}} = (0.995 \pm 0.18) \times 10^{-4}$$
$$\mathcal{B}(B^0 \to K^0\mu\mu)^{\text{low}}_{\text{avg}} = (0.90 \pm 0.16) \times 10^{-7}$$

$$\mathcal{B}(B \to K\mu\mu)^{\text{low}}_{\text{avg}} = (1.202 \pm 0.046) \times 10^{-7}$$

K^{*+}

$$\mathcal{R}(K^{*+})^{\text{low}}_{\text{LHCb}} = (1.265 \pm 0.27) \times 10^{-4}$$
$$\mathcal{B}(B^+ \to K^{*+}\mu\mu)^{\text{low}}_{\text{avg}} = (1.77 \pm 0.40) \times 10^{-7}$$

K^{*0}

$$\mathcal{B}(B^{*0} \to K^{*0}\mu\mu)^{\text{low}}_{\text{avg}} = (1.578 \pm 0.081) \times 10^{-7}$$

For $B \to K^{*0}\mu\mu$, LHCb adopts a normalization which is adjusted to $m_{K^+\pi^-} \in [0.7459, 1.0959]$ GeV.
This quantity can only be reconstructed from the most recent Belle measurement which adopts the HFLAV values for f^{+-} and f^{00}

$$\mathcal{B}(B \to K^*\mu\mu)^{\text{low}}_{\text{avg}} = (1.585 \pm 0.079) \times 10^{-7}$$

Inclusive at high-q² from LHCb

- For $q^2 > 15\ \mathrm{GeV}^2$ the inclusive rate is saturated by $X_s = K, K^*, (K\pi)_{\text{S-wave}}, K\pi\pi$ modes
- Putting together the averages above we get:

$K + K^*$: $(2.628 \pm 0.094) \times 10^{-7}$ [LHCb, 1403.8044 (3 fb^{-1}), 2512.18053 (8.4 fb^{-1}), CMS 2401.07090]

$(K\pi)_{\text{S-wave}}$: $(0.11 \pm 0.04) \times 10^{-7}$ [2512.18053 (8.4 fb^{-1})]

$K\pi\pi$: $(0.06 \pm 0.04) \times 10^{-7}$ (using isospin) [LHCb, 1408.1137 (3 fb^{-1})]

$K(n\pi)_{n>2}$: $(0.00 \pm 0.04) \times 10^{-7}$ (estimate)

total non-resonant:

$\mathscr{B}(\bar{B} \to K(n\pi)\mu\mu)[>15] = (0.176 \pm 0.070) \times 10^{-7}$

Combining K, K^* and $K(n\pi)$ modes:

$\mathscr{B}[>15]_{\text{exp}} = (2.80 \pm 0.12) \times 10^{-7}$ ⟷ 1.6σ ⟷ $\mathscr{B}[>15]_{\text{SM},\mathscr{R}} = (4.10 \pm 0.81) \times 10^{-7}$

[Capdevila, Alvarez, EL, Matias, to appear]

[Uncertainty is dominated by the $B \to X_u \ell\nu$ normalization]

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[compared to the SCET-II calculation $(3.75 \pm 0.28) \times 10^{-7}$ [Alguero et al. 2304.07330]

$(K\pi)_{\text{S-wave}}$: $(0.11 \pm 0.04) \times 10^{-7}$ [2512.18053 (8.4 fb^{-1})]

[compared to the χ_{PT} estimate $0.55\ (1 \pm 0.5) \times 10^{-7}$ [Isidori et al, 2305.03076]

$K\pi\pi$: $(0.06 \pm 0.04) \times 10^{-7}$ (using isospin) [LHCb, 1408.1137 (3 fb^{-1})]

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total non-resonant:

$\mathcal{B}(\bar{B} \to K(n\pi)\mu\mu)[>15] = (0.176 \pm 0.070) \times 10^{-7}$

$\mathcal{B}[>15]_{\text{SM},\sum\text{excl}} = (4.30 \pm 0.39) \times 10^{-7}$

3.7σ

Combining K, K^* and $K(n\pi)$ modes:

$\mathcal{B}[>15]_{\text{exp}} = (2.80 \pm 0.12) \times 10^{-7}$

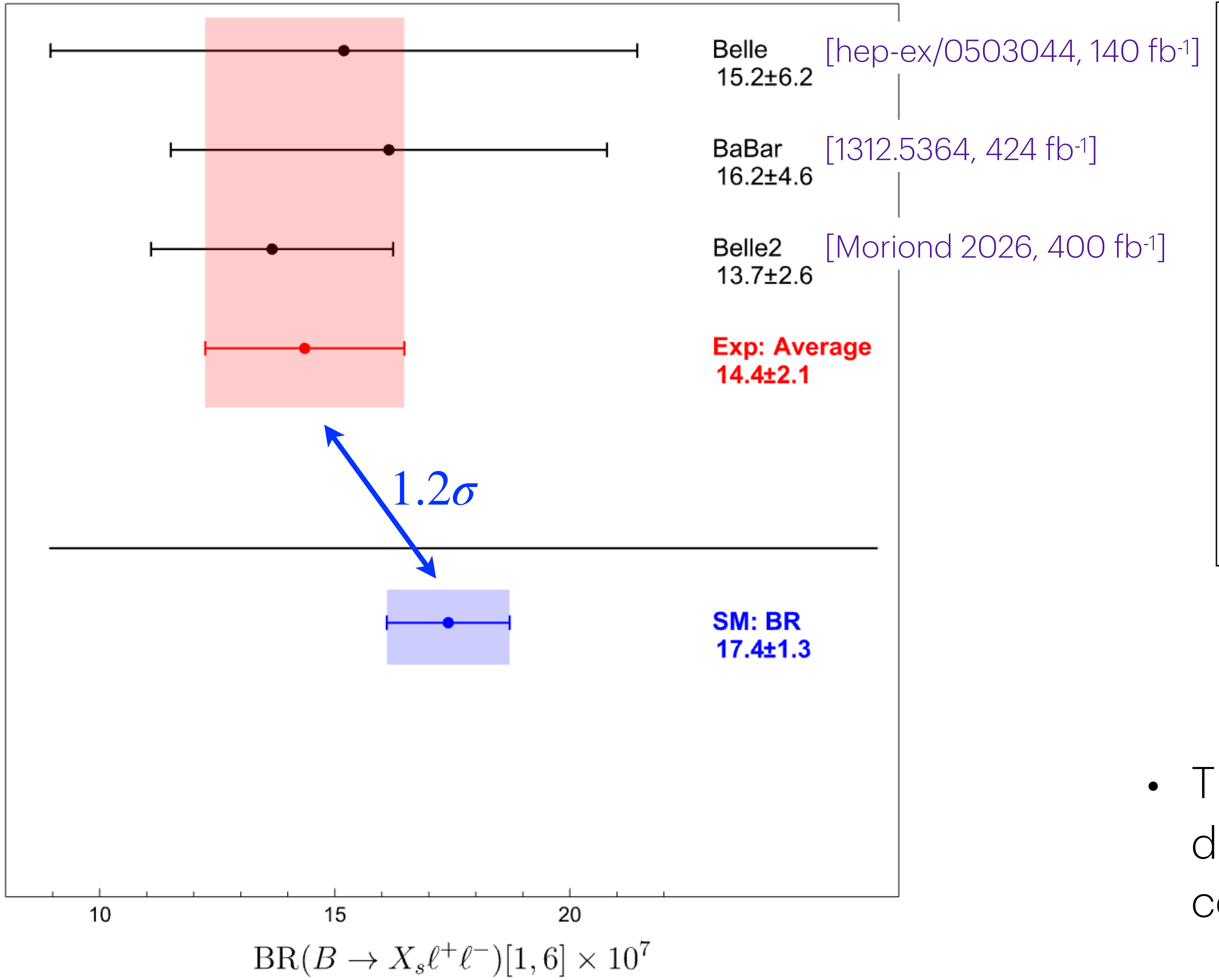
1.6σ

$\mathcal{B}[>15]_{\text{SM},\mathcal{R}} = (4.10 \pm 0.81) \times 10^{-7}$

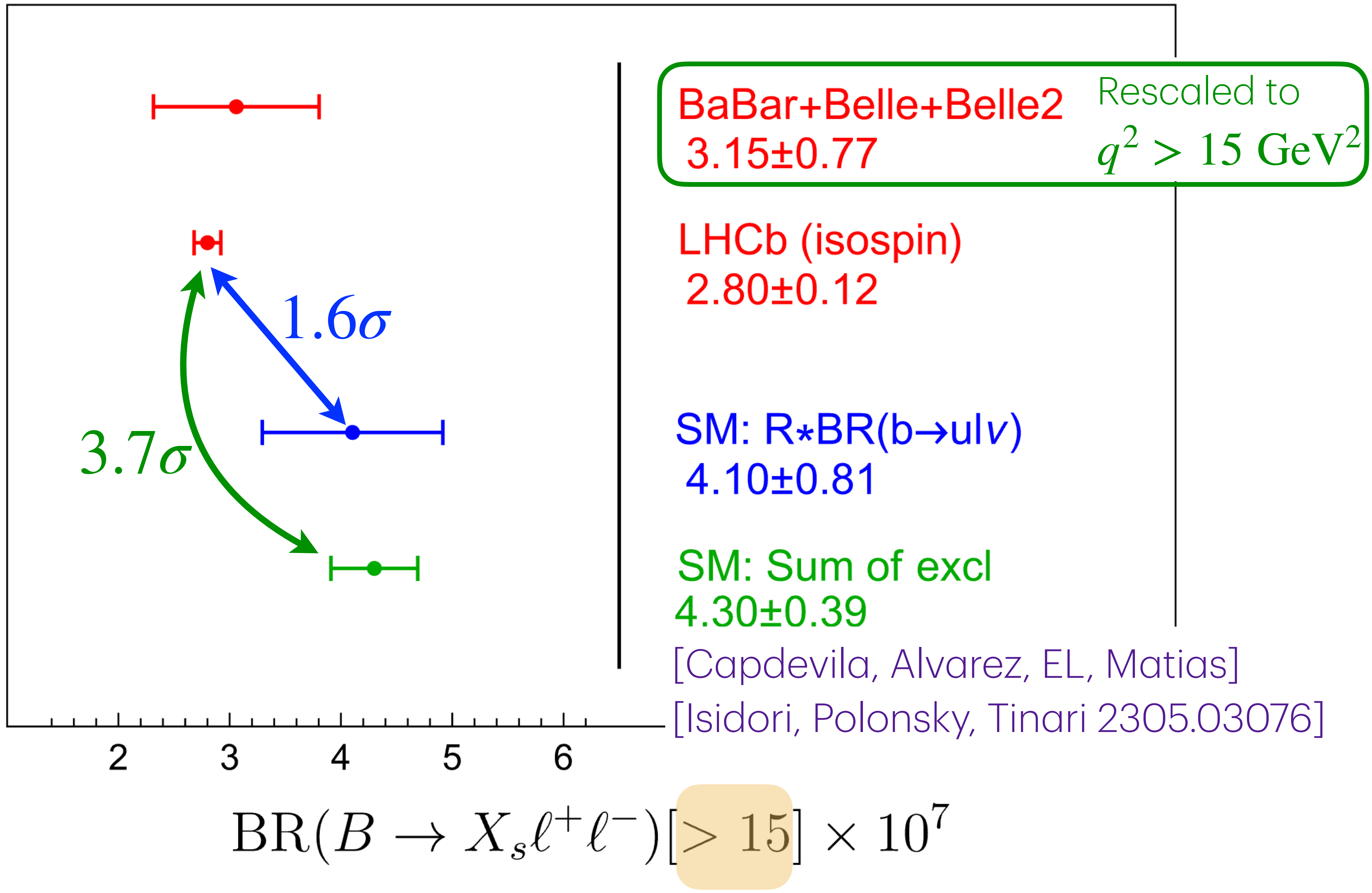
[Capdevila, Alvarez, EL, Matias, to appear]

[Uncertainty is dominated by the $B \to X_u \ell \nu$ normalization]

Current status: experiment vs SM



Including recent $B \to K^{*0}\mu\mu$ update



- The inclusive at both low and high q^2 show a slight deficit with respect to the SM predictions [which are completely independent from exclusive calculations]

Testing unknown rescattering effects

- While low and high-q^2 exclusive BRs are conjectured to be affected by potentially large rescattering effects, this is not the case for the inclusive high-q^2 branching ratios
- This suggests the introduction of the following "exclusive fraction" observables:

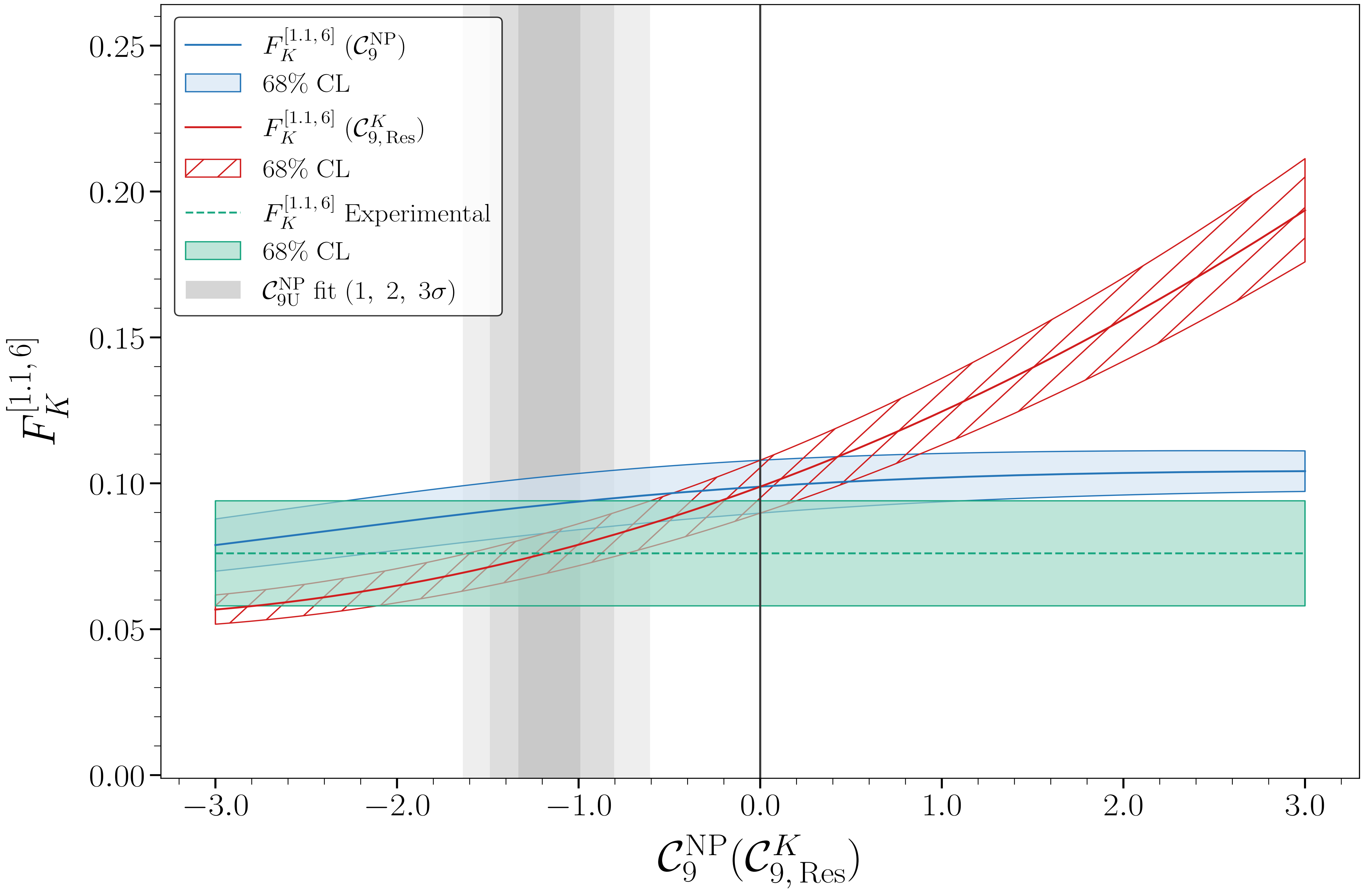
$$\mathcal{F}_{K^{(*)}}^{[1.1,6]} = \frac{\mathcal{B}(B \to K^{(*)}\mu\mu)_{[1.1,6]}}{\mathcal{B}(B \to X_s\ell\ell)_{[1,6]}} \qquad \mathcal{F}_{K^{(*)}}^{>15} = \frac{\mathcal{B}(B \to K^{(*)}\mu\mu)_{>15}}{\mathcal{B}(B \to X_s\ell\ell)_{>15}}$$

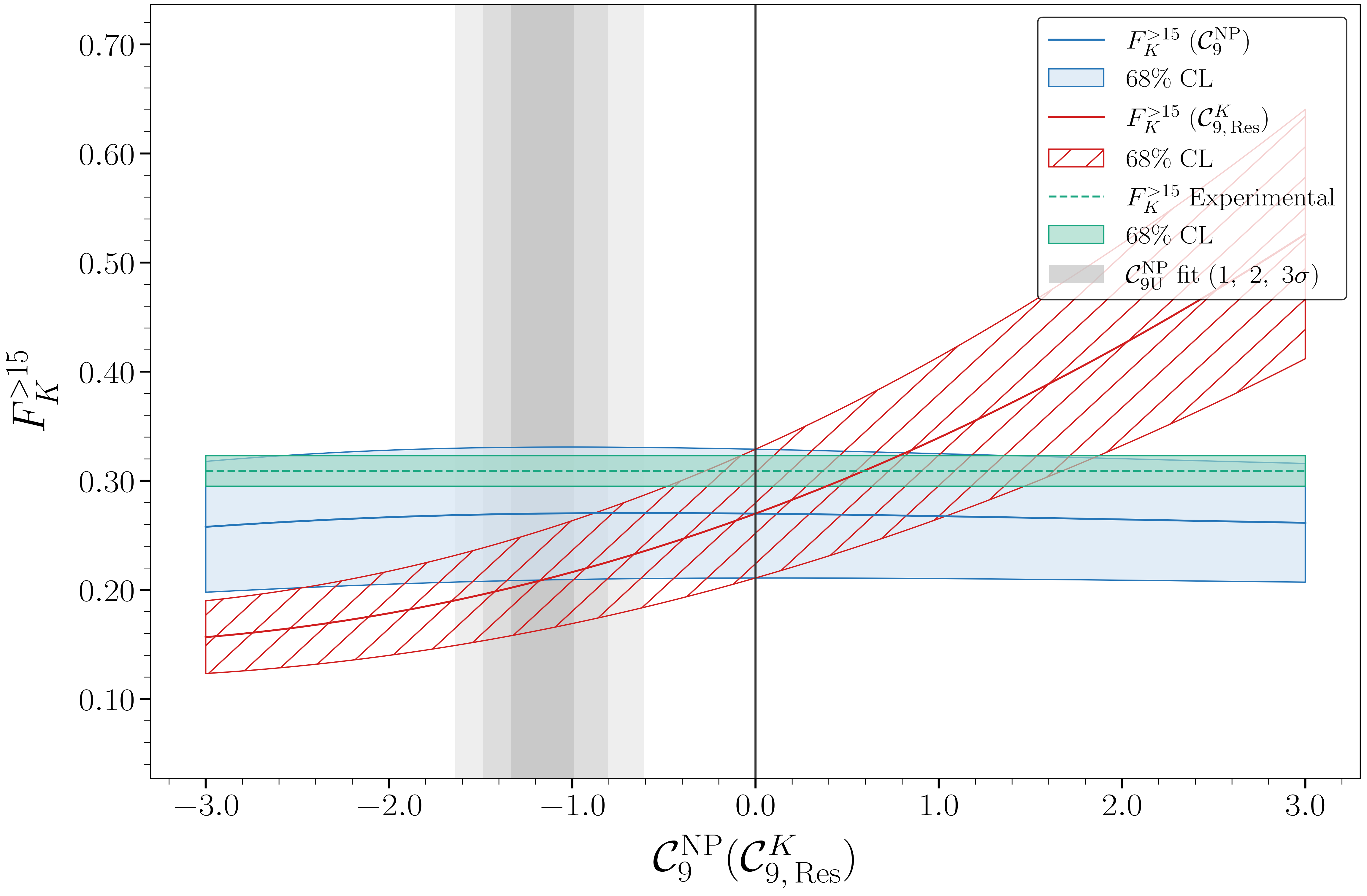
- Properties of these observables:
  - **Reduced** sensitivity to $C_9^{\rm NP}$
  - **Enhanced** sensitivity to $C_{9,\rm res}$

$$\mathcal{F}_K^{>15} = (0.270 \pm 0.065) \cdot \frac{1 + 0.228\,(C_9^{\rm NP} + C_{9,\rm res}^K) + 0.0325\,(C_9^{\rm NP} + C_{9,\rm res}^K)^2}{1 + 0.234\,C_9^{\rm NP} + 0.0293\,C_9^{\rm NP2}}$$

  - It can be measured by LHCb **without need of external normalizations**

Testing unknown rescattering effects





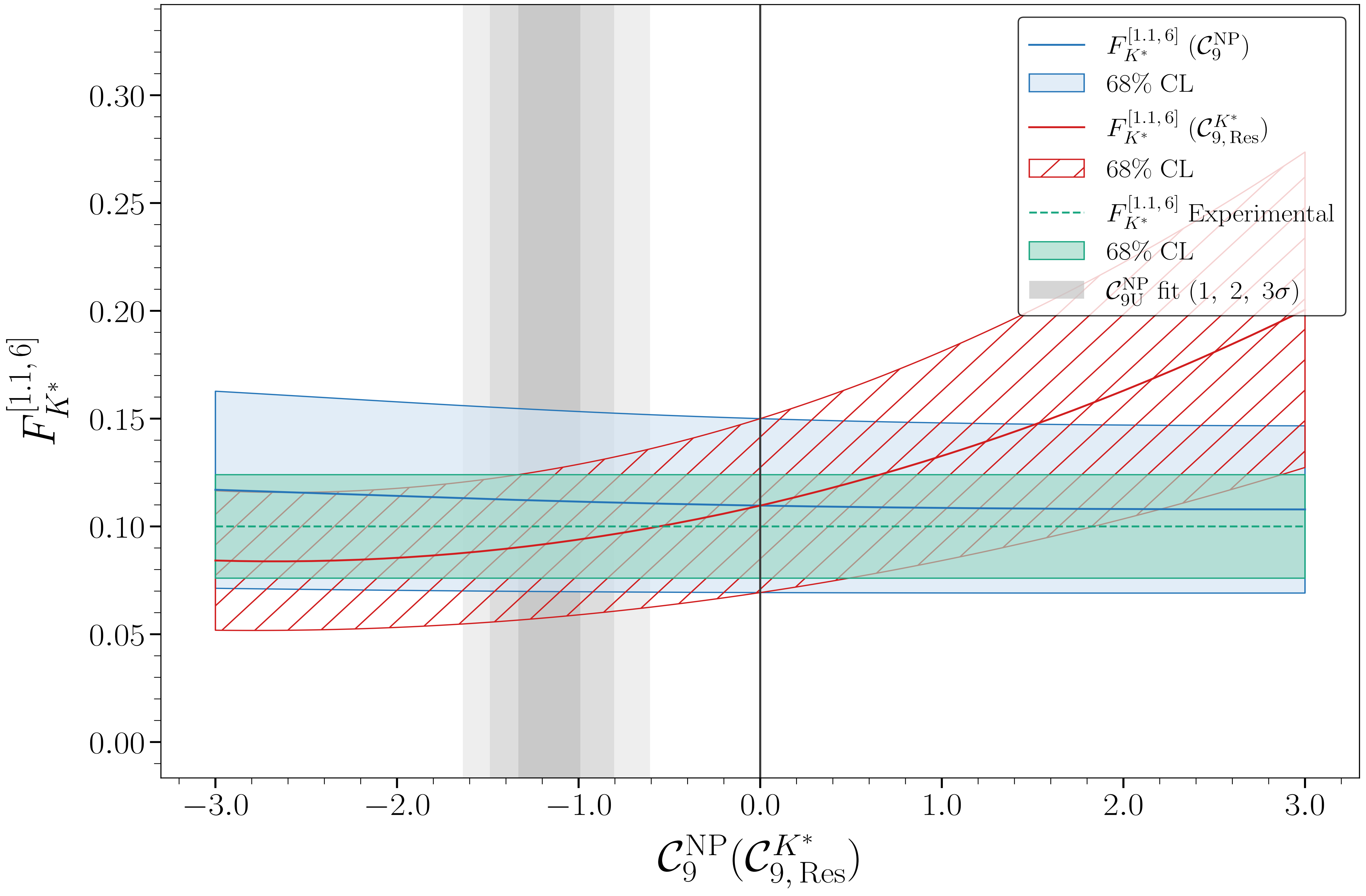
$$\left[\mathcal{F}_K^{[1.1,6]}\right]_{\rm exp} = 0.076 \pm 0.018$$

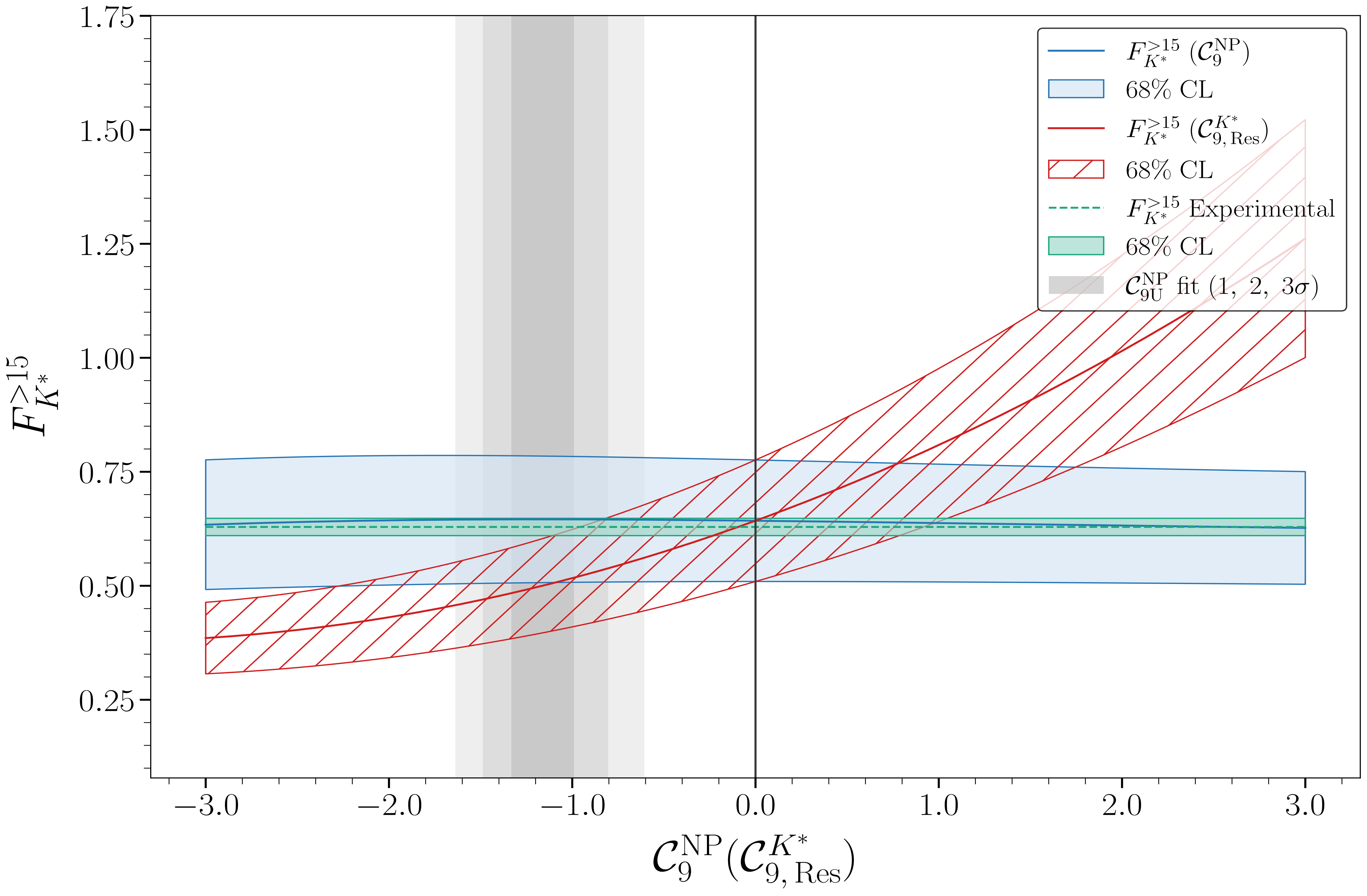
[LHCb,CMS, Babar, Belle, Belle2]

$$\left[\mathcal{F}_K^{>15}\right]_{\rm exp} = \left[1 + \frac{\mathcal{B}(K^*\mu\mu) + \mathcal{B}((K\pi)_S\mu\mu) + \mathcal{B}(B \to K\pi\pi\mu\mu)}{\mathcal{B}(K\mu\mu)}\right]^{-1} = 0.309 \pm 0.015$$

[LHCb, CMS]

Testing unknown rescattering effects





$$\left[\mathscr{F}_{K^*}^{[1.1,6]}\right]_{\rm exp} = 0.100 \pm 0.024$$

[LHCb,CMS, Babar, Belle, Belle2]

$$\left[\mathscr{F}_{K^*}^{>15}\right]_{\rm exp} = \left[1 + \frac{\mathscr{B}(K\mu\mu) + \mathscr{B}((K\pi)_S\mu\mu) + \mathscr{B}(B \to K\pi\pi\mu\mu)}{\mathscr{B}(K^*\mu\mu)}\right]^{-1}$$

$$= 0.629 \pm 0.021$$

[LHCb, CMS]

Future expectations

- Without updated $B \to J/\psi K^{(*)}$ measurements from Belle II, all exclusive branching ratios will be eventually dominated by the normalizations. In this scenario we find:

$$\delta\mathcal{F}_K^{>15} = 4.9\,\% \longrightarrow 3.1\,\%$$

$$\delta\mathcal{F}_{K^*}^{>15} = 3.3\,\% \longrightarrow 2.1\,\%$$

- LHCb can determine these ratios **without making use of normalizations!**

  For instance, for the $\mathcal{B}(K^*\mu\mu)/\mathcal{B}(K\mu\mu)$ ratio very naively and schematically one gets:

normalizations

$$\frac{K^*}{K} = \begin{cases} \dfrac{\langle K^{*0}, K^{*+}\rangle}{\langle K^0, K^+\rangle} \sim \dfrac{K^{*0}}{K^+} = \dfrac{R_{*0}}{R_+}\,\dfrac{\mathcal{B}_{*0}}{\mathcal{B}^+} \to \sqrt{2}\delta_{\rm small} \oplus^2 \delta\mathcal{B} \\ \langle \dfrac{K^{*0}}{K^0}, \dfrac{K^{*+}}{K^+}\rangle \sim \dfrac{\delta_{\rm large}}{\sqrt{2}} \sim \dfrac{4\delta_{\rm small}}{\sqrt{2}} = 2\sqrt{2}\delta_{\rm small} \end{cases} \Longrightarrow \begin{cases} \sqrt{2}\delta_+ \to 2\sqrt{2}\delta_+ \\ \delta\mathcal{B} \quad \to 0 \end{cases}$$

- In both instances, these results will benefit immensely from the upcoming $B \to X_u \ell \nu$ differential branching ratio measurement at Belle II (which currently dominates the theoretical prediction)