Charming Penguins on the Lattice

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- Introduction
- The Spectral Function Reconstruction method
- SFR of charming penguins in $B \to K \ell^+ \ell^-$
- An exploratory study
- Outlook

Based on R. Frezzotti, G. Gagliardi, V. Lubicz, G. Martinelli, C. Sachrajda, F. Sanfilippo, S. Simula, N. Tantalo & LS, PRD '26.
Many slides courtesy of G. Gagliardi

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Introduction

- $b \to s\ell^+\ell^-$ transitions in the SM mediated by Z-penguins, electroweak boxes and photonic penguins
- Z-penguins and electroweak boxes hard-GIM suppressed, i.e. $\propto \frac{m_{u_i}^2}{M_W^2}$; charm and up negligible
- photonic penguins soft-GIM suppressed, i.e. $\propto \log \frac{m_{u_i}^2}{M_W^2}$; logarithmic IR sensitivity: $\log \frac{\mu^2}{M_W^2}$ from mixing with charm current-current operators

Introduction

- Two semileptonic operators at the hadronic scale, C_{9V} and C_{10A}:
  - C_{10A} purely generated at the electroweak scale, $C_{10A}(m_b) \sim C_{10A}(M_W)$
    - C_{10A} is a short distance quantity
  - C_{9V} dominated by RG log, with sizable scale dependence:

$$\frac{C_{9V}(m_b/2) - C_{9V}(2m_b)}{C_{9V}(m_b)} \sim 15\%$$

    - EW scale contribution not dominant, sensitive to scales $\leq m_b$, C_{9V} not a short distance quantity
    - scale dependence canceled by matrix element of four-quark operators
    - it is already evident in perturbation theory that the hadronic matrix element provides a contribution to C_{9V} that cannot be separated from the short-distance one
    - Calculation of charming penguins from first principles necessary to probe possible NP contributions to C_{9V}!

The Spectral Function Reconstruction

- Lattice QCD provides a first-principles framework for the numerical calculation of correlation functions in Euclidean spacetime
- Analytic continuation from Euclidean to Minkowski problematic for amplitudes containing imaginary parts
- Maiani-Testa no-go theorem: for more than one incoming or outgoing hadron, correlation function always dominated by lowest-lying state

Maiani & Testa, PLB '90

The Spectral Function Reconstruction

- Hadronic amplitudes with zero or one incoming and outgoing hadron can still develop imaginary parts if intermediate states have energy smaller than external states
- However, the problem of analytic continuation can be solved by writing the hadronic amplitude H as

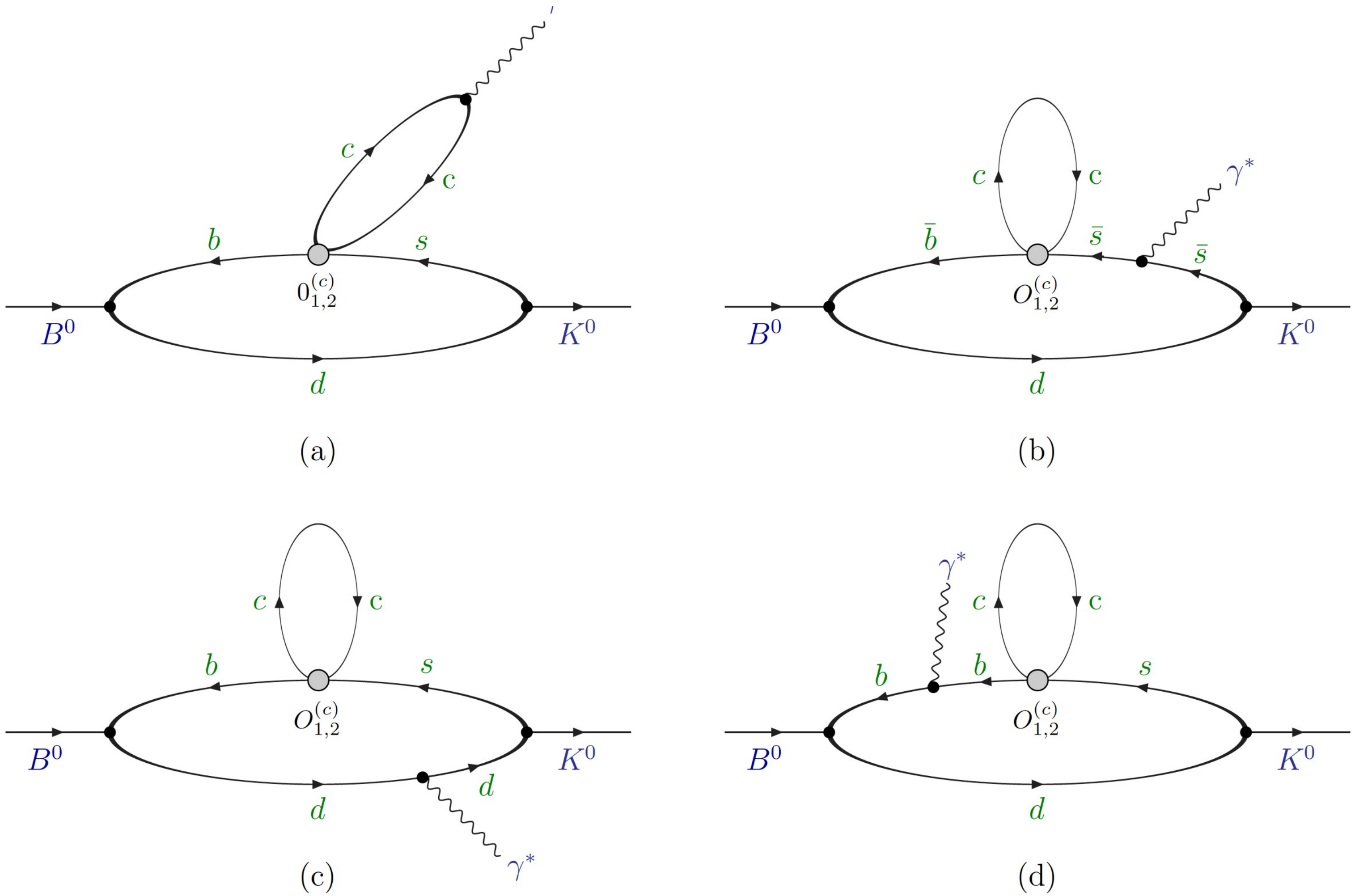
$$H(E) = \lim_{\varepsilon \to 0} \int_{E^*}^{\infty} \frac{dE'}{2\pi} \frac{\rho(E')}{E' - E - i\varepsilon}$$

which is independent of time, and thus equal in Euclidean and Minkowski.

- If $E^*>E$, the $\varepsilon \to 0$ limit is trivial; if $E^*<E$, the limit is highly nontrivial and can only be taken at the end of the calculation

Barata & Fredenhagen CMP '91; Bulava & Hansen PRD '19; Frezzotti et al. PRD '23

Charming Penguins in B→Kγ*



The $O_{1,2}^{(c)}$ contribution to the $B \to K\ell^+\ell^-$ amplitude is given by

$$\mathcal{A}_{1,2} \propto \left\langle K\ell^+\ell^- | O_{1,2}^{(c)} | B \right\rangle_{\mathrm{QCD+QED}} \overset{LO}{=} \frac{\alpha_{\mathrm{em}}}{4\pi} L_V^{\mu} \underbrace{D_{\mu\nu}(q)}_{photon\,propagator} H_{1,2}^{\nu}(\boldsymbol{q})$$

where the hadronic part $H_{1,2}^{\nu}(q)$ is given by

$$H_{1,2}^{\nu}(q) = \int d^4x\, e^{iqx} \left\langle K(p_K) \right| \mathrm{T}\left[J_{\mathrm{em}}^{\nu}(x) O_{1,2}^{(c)}(0) \right] \left| B(p_B) \right\rangle_{\mathrm{QCD}}$$

In terms of the Minkowskian correlator(s)

$$C_{1,2}^{\nu -}(t < 0, \boldsymbol{q}) = \int d^3x\, e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} \left\langle K(p_K) \middle| O_{1,2}^{(c)}(0)\, J_{\mathrm{em}}^{\nu}(t, \vec{x}) \middle| B(p_B) \right\rangle$$

$$C_{1,2}^{\nu +}(t > 0, \boldsymbol{q}) = \int d^3x\, e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} \left\langle K(p_K) \middle| J_{\mathrm{em}}^{\nu}(t, \vec{x})\, O_{1,2}^{(c)}(0) \middle| B(p_B) \right\rangle$$

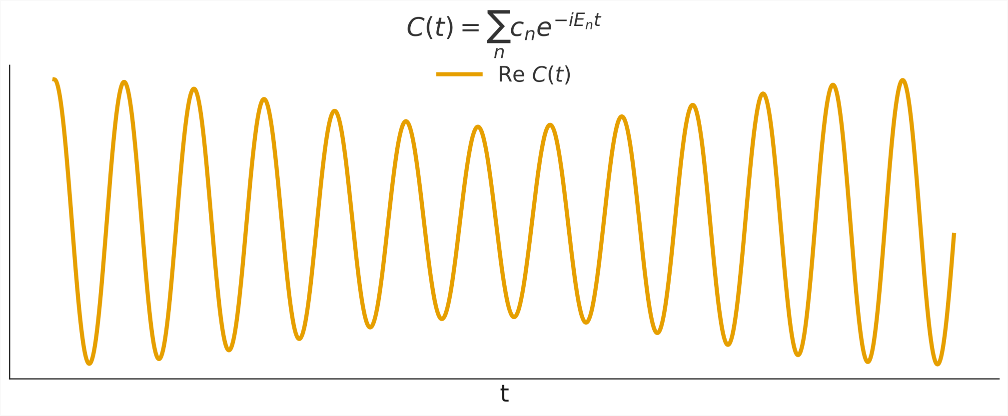
The hadronic part is simply given by

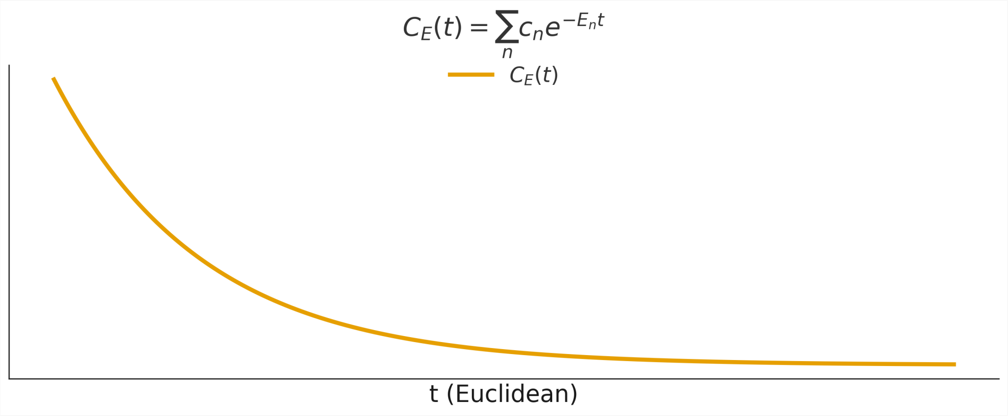
$$H_{1,2}^{\nu}(q) = \int_{-\infty}^{0} dt\, e^{iq_0 t}\, C_{1,2}^{\nu -}(t, \boldsymbol{q}) + \int_{0}^{\infty} dt\, e^{iq_0 t}\, C_{1,2}^{\nu +}(t, \boldsymbol{q}) \; .$$

Through lattice QCD simulations we do not have direct access to Minkowskian correlators...

The quantities that we can compute non-perturbatively on the lattice are instead Euclidean lattice correlators:

$$C^{\nu\pm}_{1,2}(t,\boldsymbol{q}) \to C^{\nu\pm}_{1,2}(-it,\boldsymbol{q}) \equiv C^{\nu\pm}_{1,2;E}(t,\boldsymbol{q})$$





While the Minkowskian correlator is a sum of oscillating exponential phases, the Euclidean correlator is a sum of decaying exponentials.

The question is then: can we evaluate $H^{\nu}_{1,2}(q)$ from the only knowledge of $C^{\nu\pm}_{1,2;E}(t,\boldsymbol{q})$?

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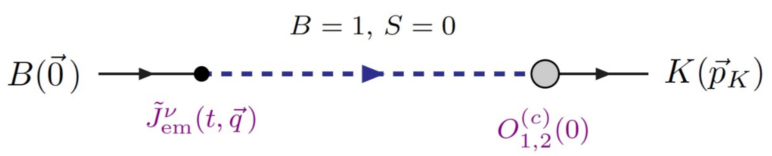
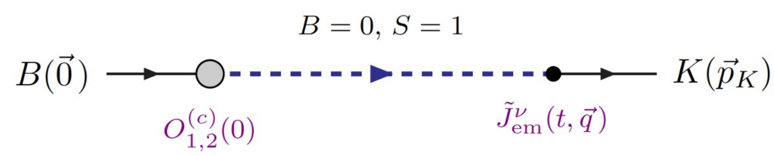
Charming Penguins in the SFR

- The two time orderings correspond to different intermediate states:





E* > E　　　E* < E

- In terms of spectral densities:

$$H^{\nu}_{1,2}(q) = \underbrace{\int_{E^-_{th}}^{\infty} \frac{dE}{2\pi} \frac{\rho^{\nu -}_{1,2}(E,\boldsymbol{q})}{E - m_B + q_0 - i\epsilon}}_{H^{\nu -}_{1,2}(q)} + \underbrace{\int_{E_{th}}^{\infty} \frac{dE}{2\pi} \frac{\rho^{\nu +}_{1,2}(E,\boldsymbol{q})}{E - m_B - i\epsilon}}_{H^{\nu +}_{1,2}(q)}$$

The spectral densities $\rho^{\pm}_{1,2}$ are related to the Euclidean correlators through (from now on I remove the suffix E in $C_E(t)$)

$$C^{\nu-}_{1,2}(t,\boldsymbol{q}) = \int_{E^-_{th}}^{\infty} \frac{dE}{2\pi} e^{-E|t|} \rho^{\nu-}_{1,2}(E,\boldsymbol{q}) \;, \qquad C^{\nu+}_{1,2}(t,\boldsymbol{q}) = \int_{E_{th}}^{\infty} \frac{dE}{2\pi} e^{-(E-E_K)t} \rho^{\nu+}_{1,2}(E,\boldsymbol{q})$$

- The term $H^{\nu-}_{1,2}(q)$ does not need any special treatment, in terms of $C^{\nu-}_{1,2}(t,\boldsymbol{q})$ after setting $\epsilon = 0$ it is simply

$$H^{\nu-}_{1,2}(q) = \int_{-\infty}^{0} dt\, e^{q_0 t}\, C^{\nu-}_{1,2}(t,\boldsymbol{q})$$

- The second term is problematic: the integrand develops singularities for $\varepsilon \to 0^+$. In the SFR method one promotes ε to a non-zero energy scale (which regularizes the singularities). This defines the smeared amplitude

$$H^{\nu+}_{1,2}(q,\varepsilon) \equiv \int_{E_{th}}^{\infty} \frac{dE}{2\pi}\, \frac{\rho^{\nu+}_{1,2}(E,\boldsymbol{q})}{E - m_B - i\epsilon} \;, \qquad \rho^{+}_{1,2}(E,\boldsymbol{q}) \propto \langle K(p_K)| J^{\nu}_{\rm em}(\boldsymbol{q}) \delta(\mathcal{H}-E) O^{(c)}_{1,2}|B(0)\rangle$$

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$H^{\nu}_{1,2}(q,\varepsilon)$, at non-zero ε, can be targeted using the Hansen-Lupo-Tantalo method [PRD 99 '19], which allows to compute any observable of the form

$$O = \int_{E_0}^{\infty} dE\, \rho(E)\, K(E)\ ,$$

where $K(E)$ is a C^{∞} kernel, from the only knowledge of the Euclidean correlator , associated to the spectral density $\rho(E)$.

- The HLT finds the optimal representation of the kernel in terms of exponentials

$$K(E) \simeq \sum_t g_t e^{-tE} \implies O \simeq \sum_t g_t C(t)$$

- Striking the right balance between systematic errors associated to imperfect kernel reconstructions and statistical errors originating from those of $C(t)$ is highly non-trivial. Requiring an overly precise kernel reconstruction lead to strongly oscillating coefficients g_t.

- The HLT method is a well established tool in lattice QCD. It has been successfully applied to study the $e^+e^- \to \mathrm{hadr.}$ annihilation cross section, inclusive hadronic $\tau-$decays, inclusive semileptonic decays, ...

See also Andreas' talk

The idea is to evaluate the smeared amplitude $H^{\nu+}_{1,2}(q,\varepsilon)$ using the HLT method, for several non-zero values of ε...

- trying to obtain the physical amplitude extrapolating the results to $\varepsilon \to 0^+$, making use of the asymptotic relations valid at small ε:

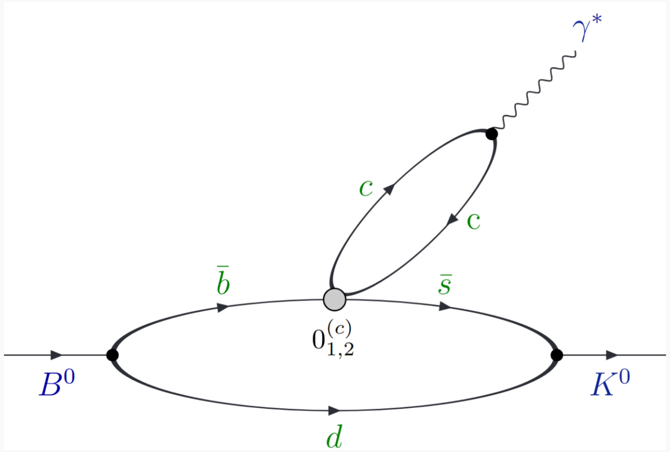
$$H^{\nu+}_{1,2}(q,\varepsilon) = H^{\nu+}_{1,2}(q) + A\varepsilon + O(\varepsilon^2) \ .$$

- The onset of the asymptotic regime occurs when $\varepsilon \ll \Delta(E)$, where $\Delta(E)$ is the typical size of the energy-interval around $E = q_0 + E_K = m_B$ where the spectral density $\rho(E, \boldsymbol{q})$ is significantly varying.

- The infinite-volume limit must be performed before $\varepsilon \to 0$.

- Note: as ε decreases, the uncertainties grow. The range of ε that can be employed is codetermined by the number N of Euclidean times accessible and by the statistical noise of $C^{\nu+}_{1,2}(t, \boldsymbol{q})$ (which is systematically improvable).

An Exploratory Study



- We focus on the charming penguin diagram in the figure.
- We have investigated the possibility of computing this diagram on a single Extended Twisted Mass (ETM) gauge ensemble ($a \simeq 0.08$ fm).

- We considered a single heavy quark mass $m_h = 2m_c < m_b$, and single photon momentum $\boldsymbol{q} \simeq 250$ MeV in the decaying meson rest frame.
- A full calculation requires handling both the IR part (through the SFR/HLT method) and the UV part (renormalization of the relevant matrix elements).
- For now we only performed a proof-of-principles calculation to show that the IR part (the previously-considered limiting factor) can be controlled.
- Additionally we considered the case $B_s \to \eta_s \ell^+ \ell^-$ (i.e. with the spectator light quark replaced by a strange quark), whose correlation functions have much reduced stat. uncertainties.

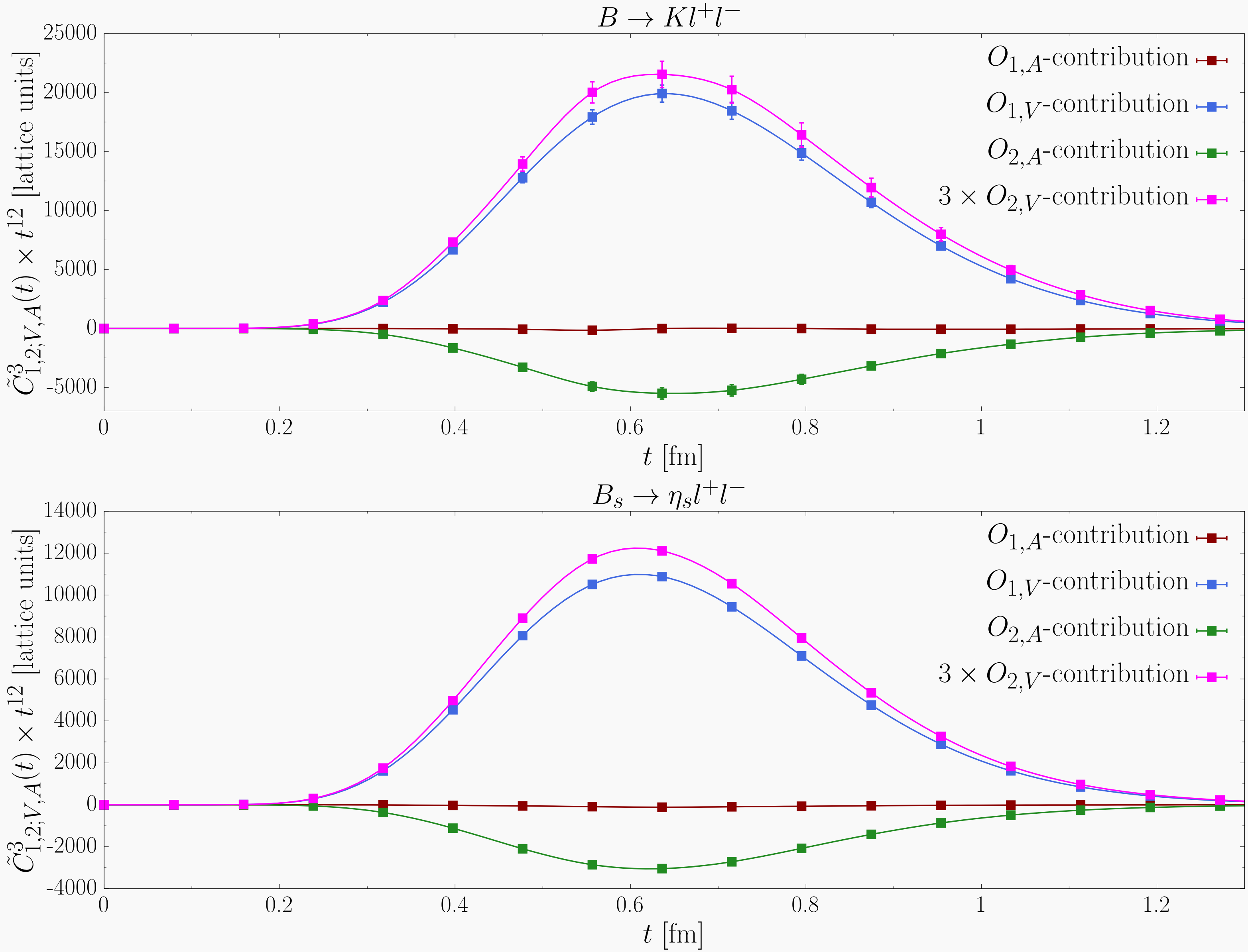
Performing a complete calculation of the long-distance contributions is a very complex task, which requires performing several non-trivial calculation-steps

Todo list:

- Compute the smeared amplitude for several values of ε for all Feynman diagrams
- Perform the non-perturbative renormalization of the effective weak Hamiltonian to remove power and logarithmic divergences from bare quantities.
- Extrapolate the results to the physical B-meson mass (direct simulation is currently not feasible due to $m_B > 1/a$).
- Perform the continuum-limit and infinite-volume extrapolation.
- Extrapolate the smeared amplitude to the physical limit of vanishing ε.
- Explore both high and low q^2 (optional).

While performing all the steps above is very time consuming, IN PRINCIPLE it can be done!

Results for $\tilde{C}^{\nu+}_{1,2}(t,\boldsymbol{q}) = e^{-E_{K(\eta_s)}t} C^{\nu+}_{1,2}(t,\boldsymbol{q})$

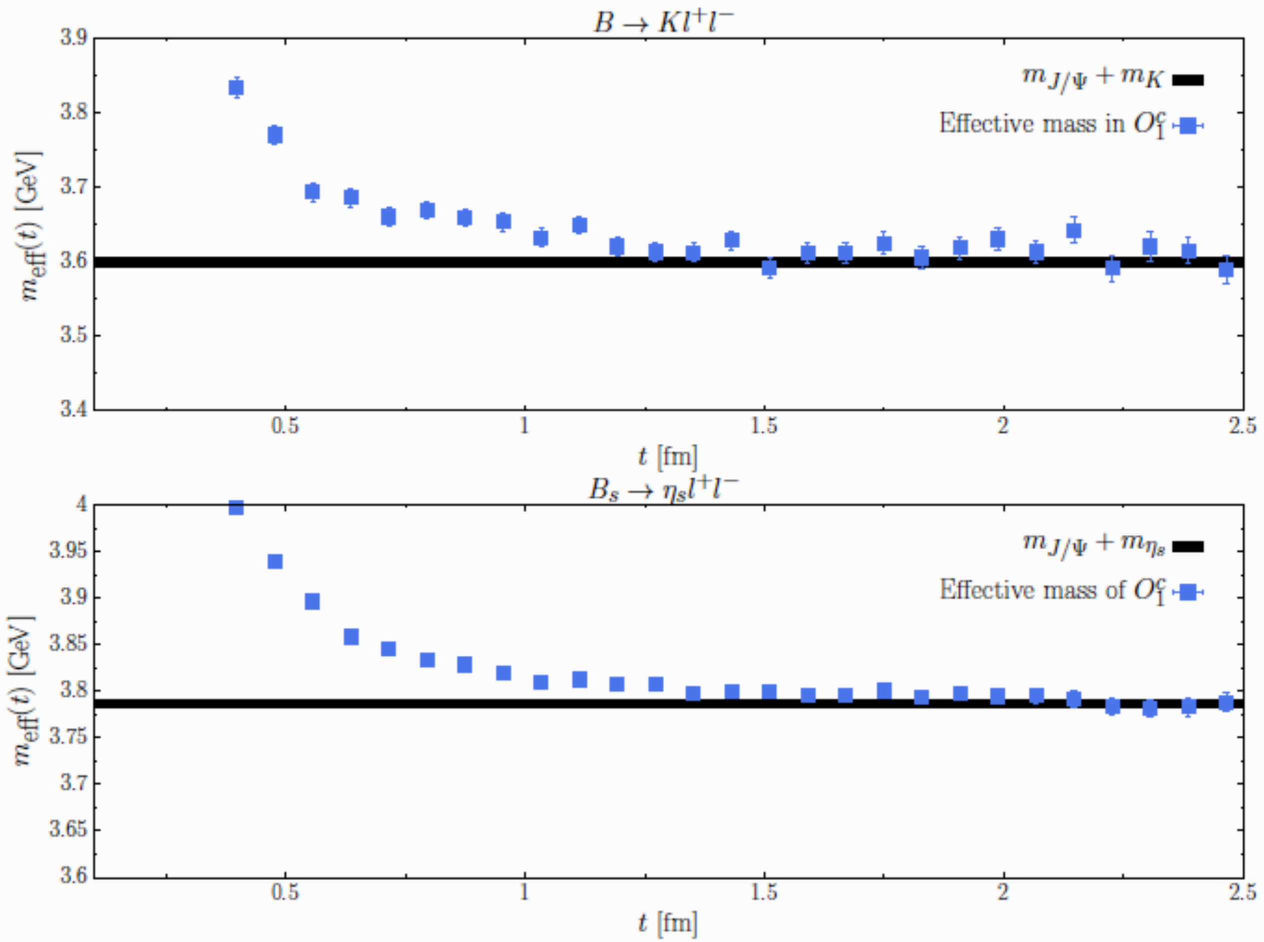


- We compute separately the axial (A) and vector (V) components of $C^{\nu}_{1,2}(t)$.
- In the factorization approximation: $C^{\nu+}_{1,V} = 3C^{\nu+}_{2,V}$, $C^{\nu+}_{1,2,A} = 0$.
- We find a very small $C^{\nu+}_{1,A}$, but $C^{\nu+}_{2,A}$ (purely non-fact.) is sizable.

Lowest Lying Intermediate States

- Correlators $\tilde{C}^{\nu}_{1,2}(t,q)$ are of the form: $\tilde{C}^{\nu}_{1,2}(t,q) = \sum_n A^{\nu(n)}_{1,2} e^{-E_n t}$.
- Lowest expected energy E_0 is:

$$E_0 = m_{J/\Psi} + m_K \ \ (B \to K\ell^+\ell^-), \qquad E_0 = m_{J/\Psi} + m_{\eta_s} \ \ (B_s \to \eta_s \ell^+\ell^-)$$

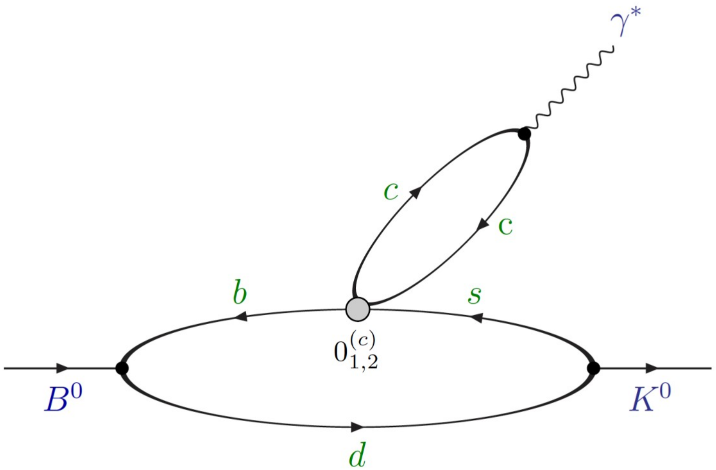


Renormalization

- Lattice bare operators must be matched to the effective Hamiltonian which is perturbatively computed
- Nonperturbative renormalization methods exist: RI-MOM, RI-SMOM, Schrödinger functional, Gradient Flow
- Lattice power divergences ($1/a^{3,2,1}$) must be subtracted nonperturbatively
- GIM does not help reducing divergences in this case

Renormalization

- By gauge invariance this diagram is only logarithmically divergent, mixing $O_{1,2}^{(c)}$ with $O_{7,9}$
- The two time orderings are individually quadratically divergent
- We are interested in the SFR part, so add and subtract an SFR-trivial quadratic divergent part



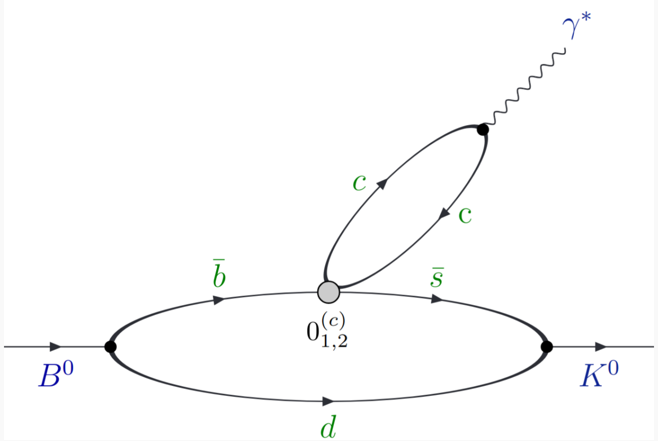
(a)

$$\frac{1}{E-m_B-i\epsilon}-\frac{3}{E-i\epsilon}+\frac{3}{E+m_B-i\epsilon}-\frac{1}{E+2m_B-i\epsilon}=\frac{6m_B^3}{(E-m_B-i\epsilon)(E-i\epsilon)(E+m_B-i\epsilon)(E+2m_B-i\epsilon)}\,.$$

$$H_{1,2}^{\nu+;\,3\mathrm{subs}}(\vec{q},\epsilon)\;\equiv\;\int_{E_+^*}^{\infty}\frac{dE}{2\pi}\left\{\frac{1}{E-m_B-i\epsilon}-\frac{3}{E-i\epsilon}+\frac{3}{E+m_B-i\epsilon}-\frac{1}{E+2m_B-i\epsilon}\right\}\rho_{1,2}^{\nu+}(E,\vec{q})$$

It is convenient to organize the calculation in such a way that the (difficult) part treated with SFR/HLT does not have contact divergences:

$$H^{\nu}_{1,2}(q) = \underbrace{\int_{-\infty}^{\infty} dt\, C^{\nu}_{1,2}(t,\boldsymbol{q})\, f(t)}_{\text{easy-part, contact-log-divergence}} + \lim_{\varepsilon\to 0+} H^{\nu+;3-\text{subs}}_{1,2}(q,\varepsilon)$$



$$H^{\nu+;3-\text{subs}}_{1,2}(q,\varepsilon) = \int_{E_{th}}^{\infty} \frac{dE}{2\pi} \rho^{\nu+}(E,\boldsymbol{q}) K^{3-\text{subs}}(E,m_B;\varepsilon)$$

where the three-times-subtracted kernel is given by

$$K^{3-\text{subs}}(E,m;\varepsilon) = \frac{6m^3}{(E-i\varepsilon)((E-i\varepsilon)^2-m^2)(E+2m-i\varepsilon)}$$

By making explicit the dependence of the spectral density on the heavy meson (the wannabe $B(B_s)$) mass employed we have done the exercise of computing

$$H^{\nu+;3-\text{subs}}_{1,2}(\boldsymbol{q},\varepsilon;m_H,m) = \int_{E_{th}}^{\infty} \frac{dE}{2\pi} \rho^{\nu+}(E,\boldsymbol{q},m_H) K^{3-\text{subs}}(E,m;\varepsilon)$$

as a function of m, for different ε.

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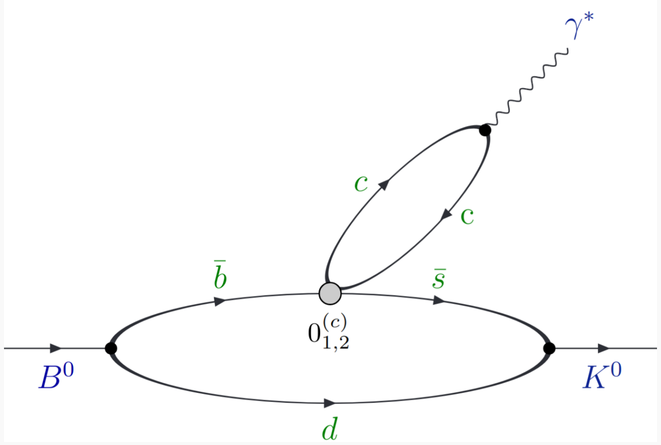

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The physical B amplitude is then obtained in the triple limit:

$$\lim_{\varepsilon \to 0^+} \lim_{m \to m_B} \lim_{m_H \to m_B} H^{\nu+;3-\text{subs}}_{1,2}(\boldsymbol{q}, \varepsilon; m_H, m)$$

Setting $m = m_H$ might seem a good idea, actually it is not as it would render the mass-extrapolation very non-smooth.

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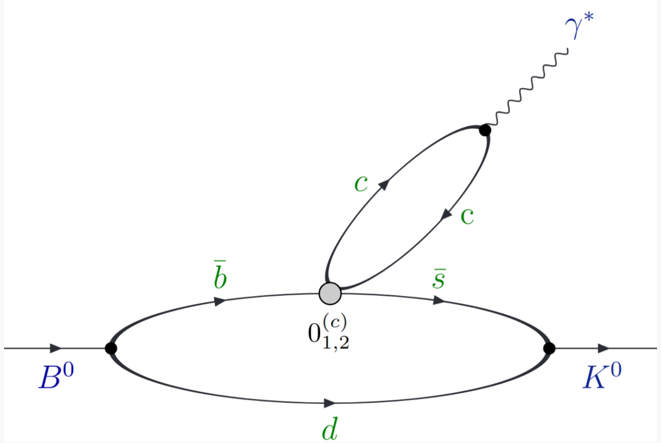

In principle one could just keep m=m_B fixed but this would lead to large discretization errors

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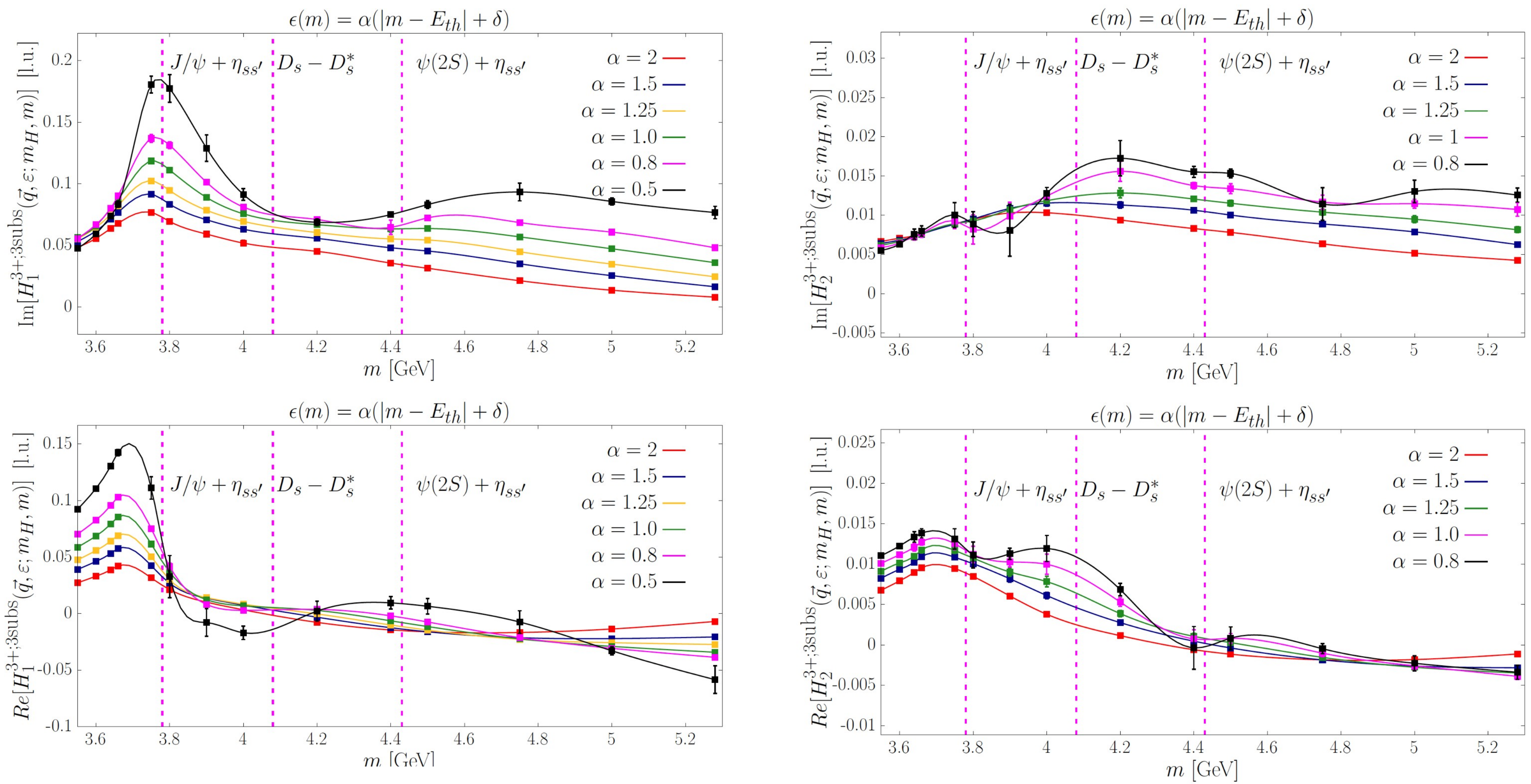


- Negative interference between VV and AA contributions in O_2, giving an order-of-magnitude suppression wrt O_1
- O_2 seems to "feel" D_s-$\overline{D}_s$ more than O_1

Let's try to compare with a rough model for the spectral density $\rho^{\nu+}_{1,2}$, assuming the factorization approximation ($\rho^{\nu+}_1 = 3\rho^{\nu+}_2$)

$$\rho_1^{\nu+;\mathrm{FA}}(E,\boldsymbol{q},m_H) = \langle \eta_s(-\boldsymbol{q})|\bar{s}\gamma^\mu b|B_s(\boldsymbol{0})\rangle \times \langle 0|V_\mu(0)\,\delta(E-\mathbb{H})\,V^\nu(0,\boldsymbol{q})|0\rangle$$

The first term parametrized by local FF f_0 and f_+

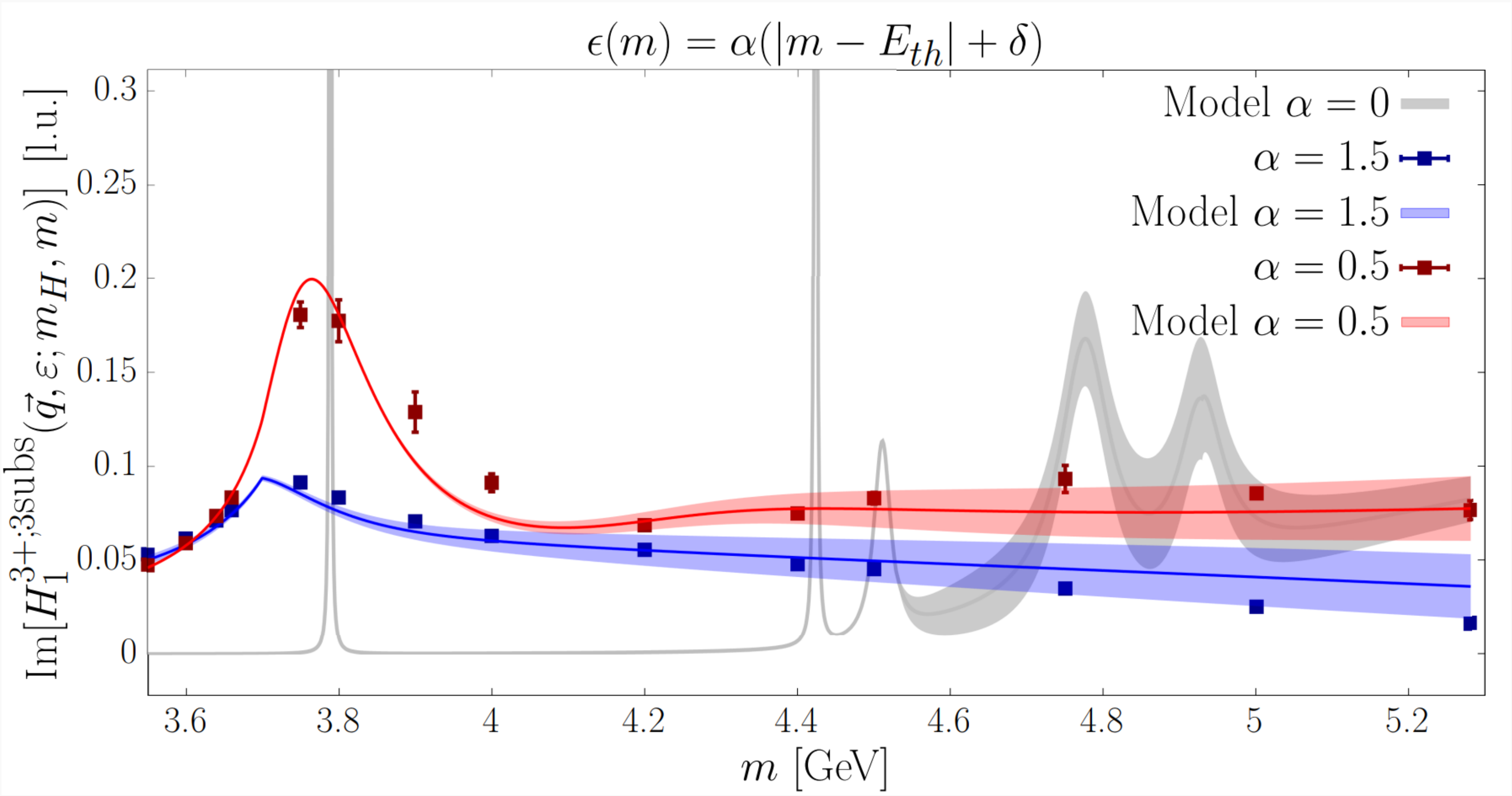
$$\langle \eta_s(-\boldsymbol{q})|\bar{s}\gamma^\mu b(0)|B_s(\vec{0})\rangle = f_+(q^2)\left(p_H^\mu + p_\eta^\mu - \frac{M_H^2 - M_{\eta_s}^2}{q^2} q^\mu\right) + f_0(q^2)\frac{M_H^2 - M_{\eta_s}^2}{q^2} q^\mu$$

- $f_+(q^2)$, $f_0(q^2)$ directly evaluated on the lattice.
- The $\bar{c}c$ part modeled as a sum of charmonium resonances (V) as:

$$\langle 0|V_\mu(0)\,\delta(E-\mathbb{H})\,V^\nu(0,\boldsymbol{q})|0\rangle = \sum_V \frac{f_V^2 m_V^2}{2E_V}\left(\delta_\mu^\nu - \frac{1}{m_V^2} k_{V\mu} k_V^\nu\right)\delta(E-E_V)$$

- $E_V = \sqrt{m_V^2 + |\boldsymbol{q}|^2}$. Masses and decay constants taken from PDG for the first few $c\bar{c}$ resonances.
- We added to $\rho_1^{\nu+;\mathrm{FA}}$ a perturbative high-energy part evaluated at tree-level.

The comparison between model and lattice data is qualitative. Overall scale (due to the missing renormalization in our results) adjusted to make model and data agree at low values of m (turns out to be $O(1)$). Discretization effects are present in the data (especially relevant at large m).



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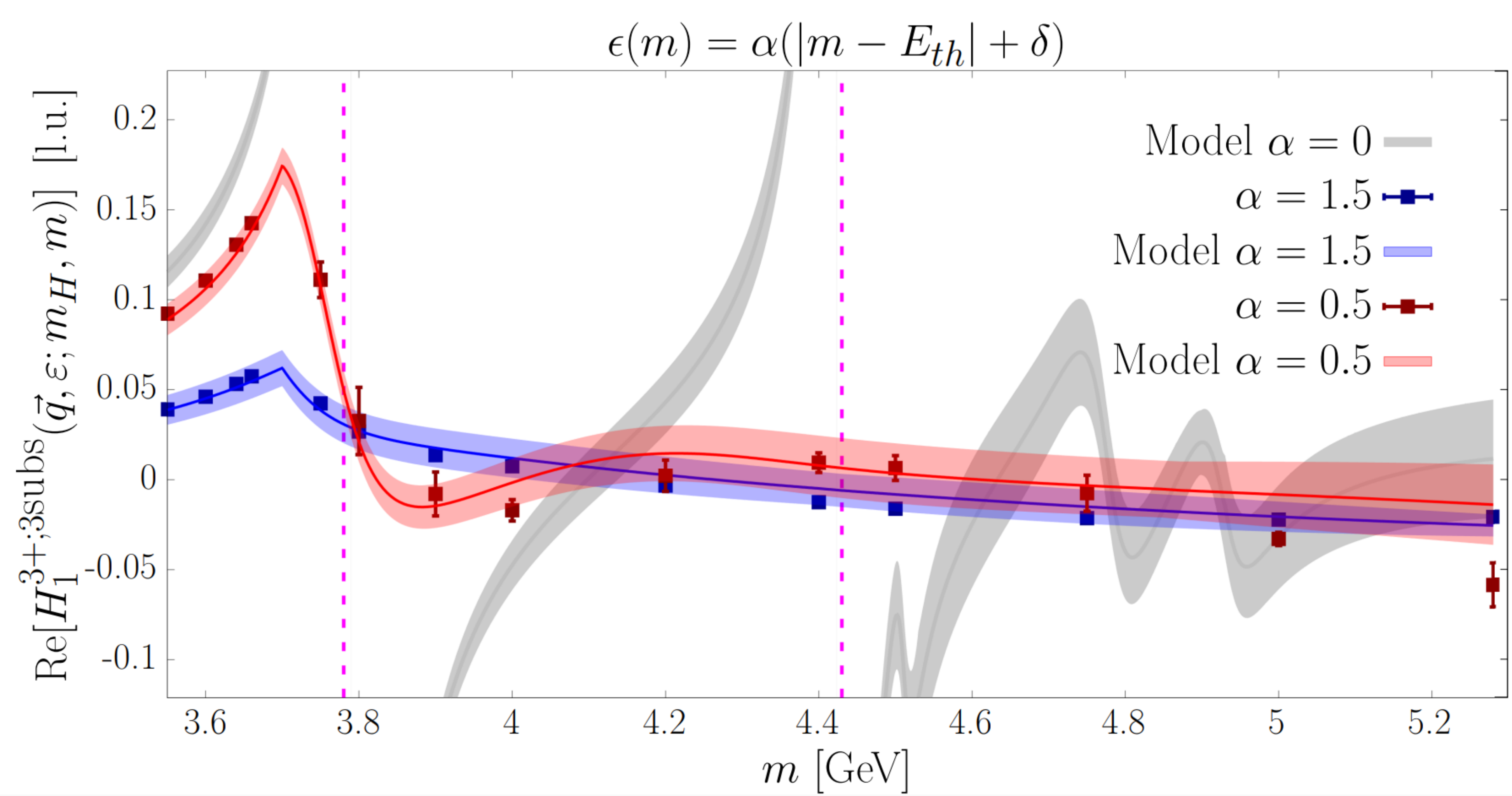

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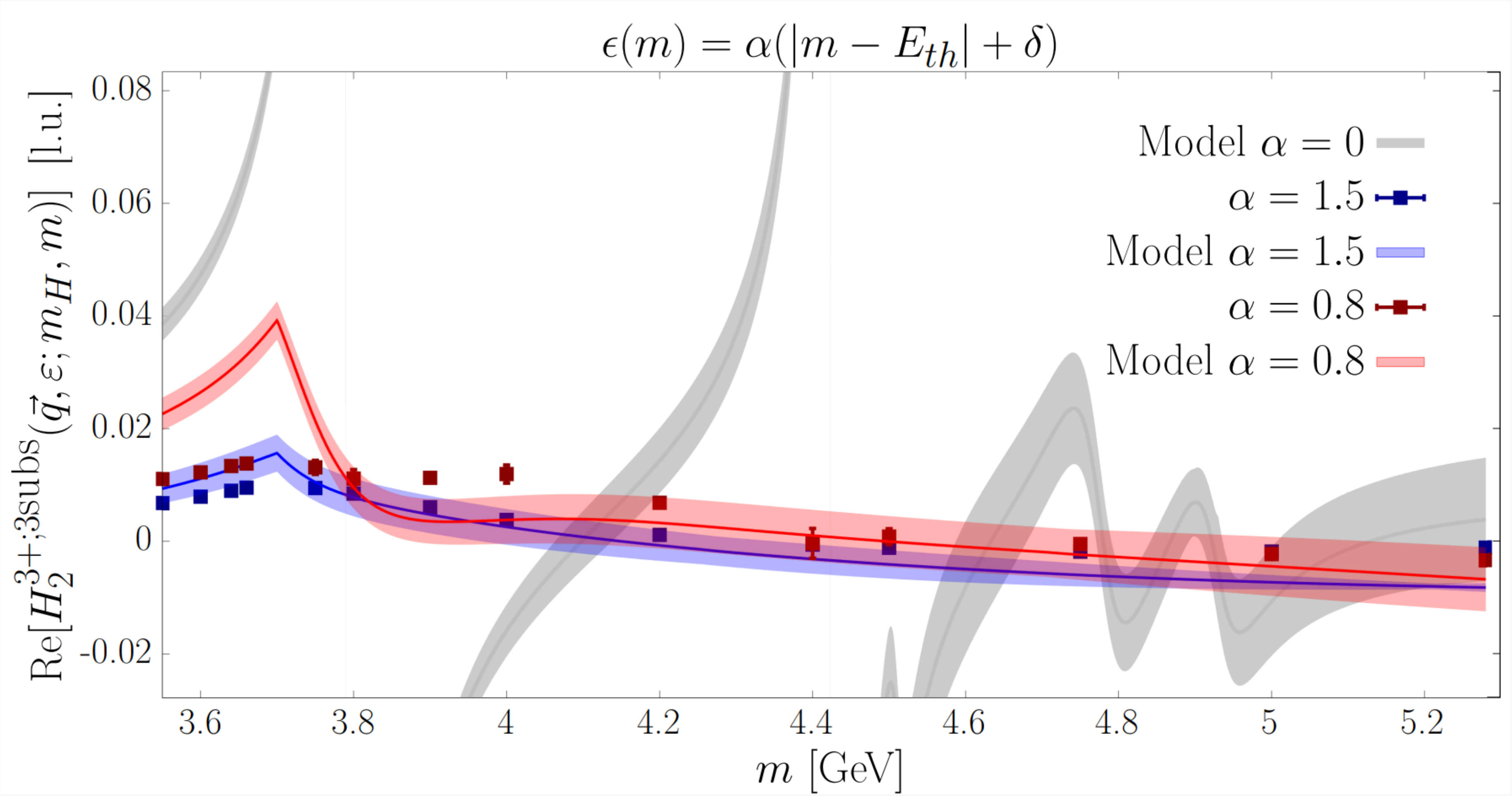

Model results do not describe well our data for the $O_2^{(c)}$ contribution!

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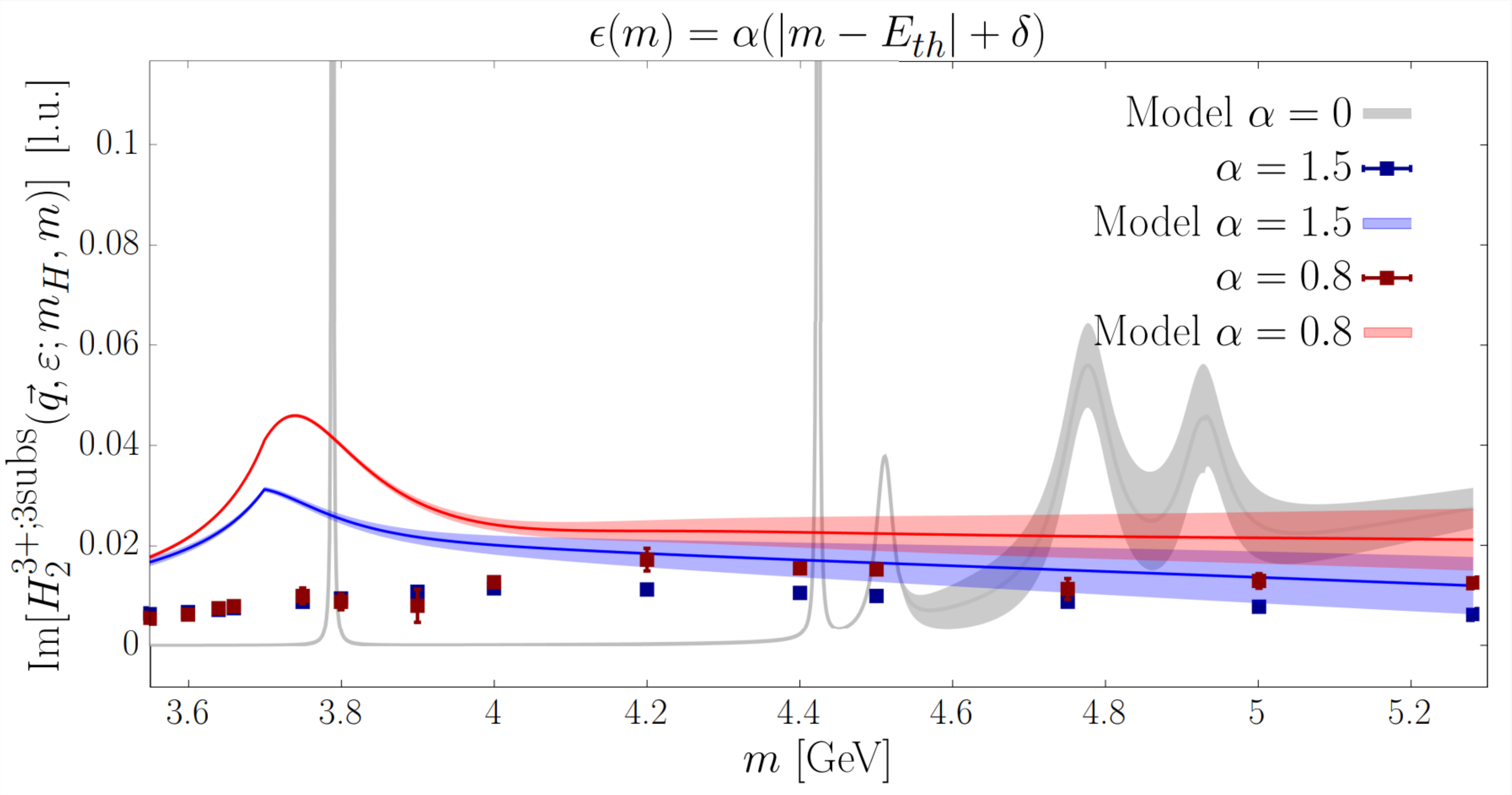

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Outlook

- We now have a theoretical framework, based on spectral-density reconstruction techniques, that allows, in principle, to evaluate on the lattice long-distance contributions to FCNC $B_{(s)}$ techniques
- The exercise of applying the new method to the calculation of a single, wisely chosen, charming-penguin diagram, in an unphysical setup (lower-than-physical m_B, single $\mathbf{p}_K$, single a, no non-perturbative renormalisation), gives encouraging results in the large-q^2 region
- The road to a full-fledged calculation is however still very long. It requires:
  - computing the smeared amplitude for all diagrams and letting $\varepsilon \to 0$
  - performing non-perturbative renormalization
  - performing the continuum and infinite volume extrapolations
  - extrapolating to physical m_B
  - extending the q^2 range

Backup: HLT optimization

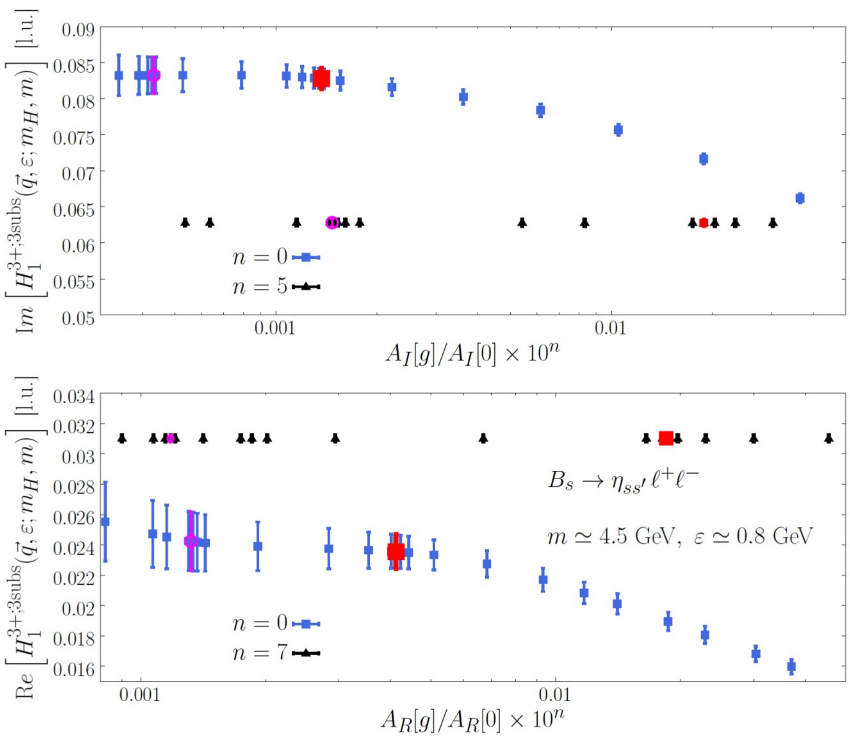

FIG. 11. Real (bottom panel) and imaginary (top panel) parts of $H_1^{3+;3\mathrm{subs}}(\vec{q},\varepsilon;m_H,m)$ for $m=4.5$ GeV and $\varepsilon=0.8$ GeV, shown in lattice units as a function of the ratio $A_{R,I}[\boldsymbol{g}]/A_{R,I}[\mathbf{0}]$, which is a measure of the goodness of the kernel reconstruction. The plot illustrates a representative example of the stability analysis. Blue and black points correspond respectively to the contributions from the first term and the remaining three terms of the kernel function in the first line of Eq. (134). As described in the text, the sign of the latter has been inverted and its imaginary part scaled by a factor of six for better visualization. Moreover, the values of $A_W[\boldsymbol{g}]/A_W[\mathbf{0}]$ corresponding to the black points in the figure have been rescaled by factors of 10^7 and 10^5 for $W=R$ and $W=I$, respectively. Filled squares and empty circles indicate reconstructions performed with $\lambda=\lambda^{\mathrm{opt}}$ and $\lambda=\lambda^{\mathrm{syst}}$, respectively; see text for details.

Backup: HLT optimization

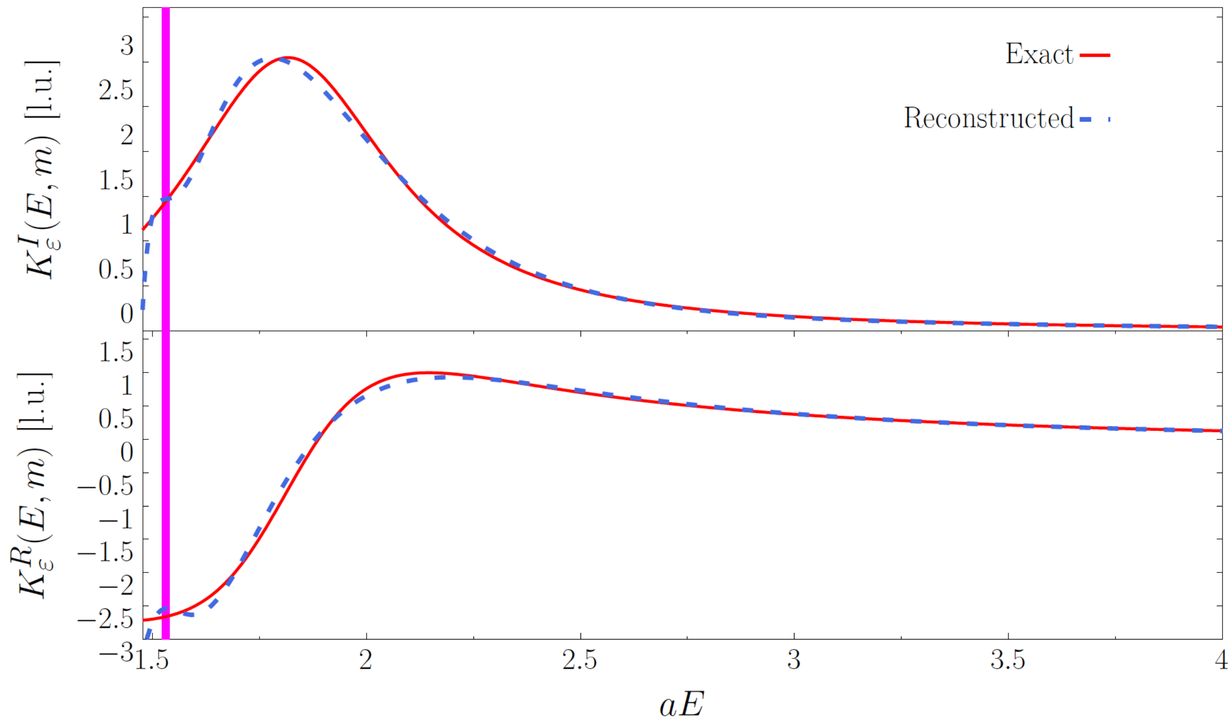

FIG. 12. The reconstructed kernels $K_\varepsilon^{R/I}(E,m)$ obtained in the HLT reconstruction with $\lambda = \lambda^{\rm opt}$ shown in Fig. 11 (dashed lines), are compared to the exact ones (solid lines). The vertical line corresponds to the value of the algorithmic parameter $E_{\rm th} \leq E^*$ in Eq. (139). The results are given in lattice units.

Backup: $\varepsilon \to 0$ limit

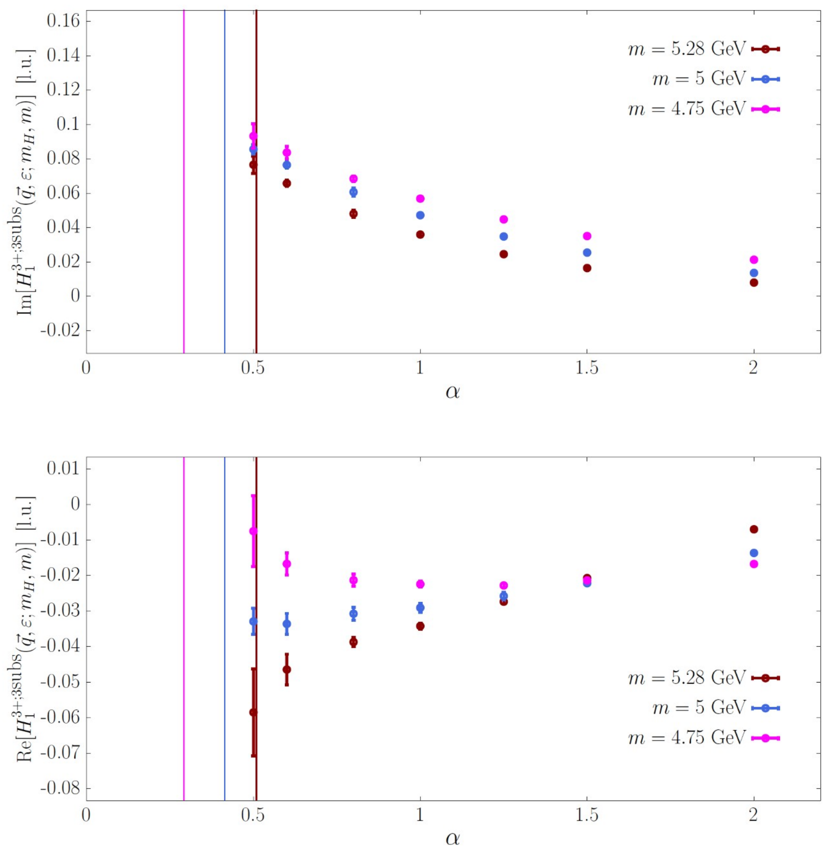

FIG. 18. α-dependence of our results for the contribution from $O_1^{(c)}$ contribution to the subtracted amplitude $B_s \to \eta_{ss'} \ell^+ \ell^-$ in the region of large m. The top (bottom) panel show our result for the imaginary (real) part of the smeared amplitude. The different colors correspond to different values of m in the high-m region. The vertical lines correspond for each m to the α value leading to $\epsilon(m) = \Delta(m)$, were $\Delta(m)$ is estimated according to Eq. (153).