Evolution of twist 3 distributions in non-forward kinematics

Markus Diehl

Deutsches Elektronen-Synchrotron DESY

work done together with Simone Rodini
with earlier contributions by Lorenzo Zoppi and Peter Plößl

Volodyafest, Regensburg, 28 November 2025

HELMHOLTZ

Plan

- Higher twist: what and why?
- Evolution of twist 3 distributions: some old and some new results
- Evolution in non-forward kinematics: singular behaviour

along the way:

- What does all this have to do with the work of Volodya?

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- Higher twist: what and why?
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along the way:

- What does all this have to do with the work of Volodya?
  Short answer: some of Volodya's most cited work is on evolution and on higher twist:

Plan

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- Evolution in non-forward kinematics: singular behaviour

note:

- this talk is about higher twist for collinear distributions, i.e.
  - no explicit k_T dependence
  - matrix elements of operators with fields along a light-ray

disclaimer:

- no attempt is made to systematically cite all relevant literature

  citations will be rather biased towards recent work in our Forschergruppe and work by Volodya

What is higher twist?

- different definitions are used in different contexts
  roughly speaking, higher-twist distributions
  - appear in power suppressed observables
    twist 3: suppressed by $1/Q$
    twist 4: suppressed by $1/Q^2$
    relative to leading twist 2 terms
  - are associated with operators for > 2 partons
    and/or including "bad" components of fields
    what is "good" or "bad"?

twist 2 operators: 2 fields with good components
twist 3 operators: 3 fields with good components or
2 fields with one good and one bad component

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    what is "good" or "bad"?
    - gluons: use gauge $A^+ = 0$, good component: A^i with $i = 1, 2$,
      bad component A^-, likewise in D^i, D^-, F^{+i}, F^{+-}, ...
    - quarks: Dirac spinor projectors $P_+ = \frac{1}{2}\gamma^-\gamma^+$, $P_- = \frac{1}{2}\gamma^+\gamma^-$
      good components $\psi_+ = P_+\psi$, bad components $\psi_- = P_-\psi$
      in bilinears e.g. $\bar\psi\gamma^+\psi = \bar\psi_+\gamma^+\psi_+$ and $\bar\psi\gamma^i\psi = \bar\psi_-\gamma^i\psi_+ + \bar\psi_-\gamma^i\psi_+$

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Why higher twist?

often heard: higher twist

- is complicated
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but: higher twist

- is needed in some contexts
  - B meson decays work by Volodya and others
  - various spin and angular asymmetries
    in DIS (structure fct. g_2), SIDIS, Drell-Yan, single hadron production
    e.g. W Vogelsang et al.; A Ali, V Braun, G Hiller 1991 for g_2
  - high k_T (or small distance) limit of TMDs
    (transverse-momentum dependent distributions)
    e.g. work by A Vladimirov et al.
    famous example: Sivers distribution $\rightsquigarrow$ Qiu-Sterman function

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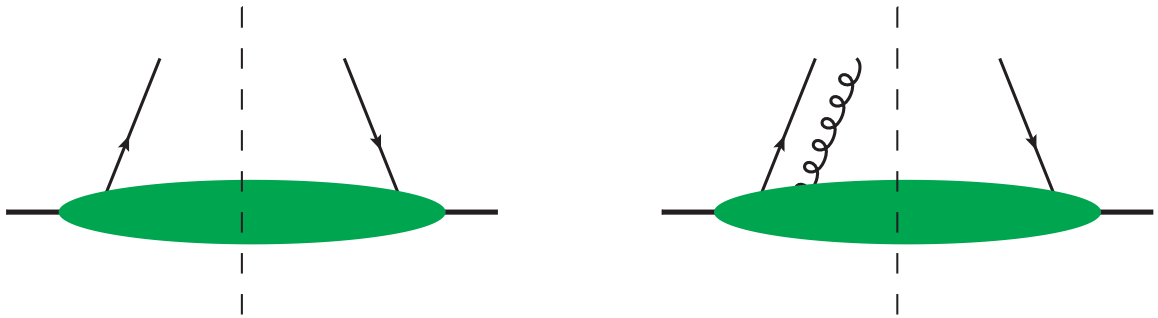
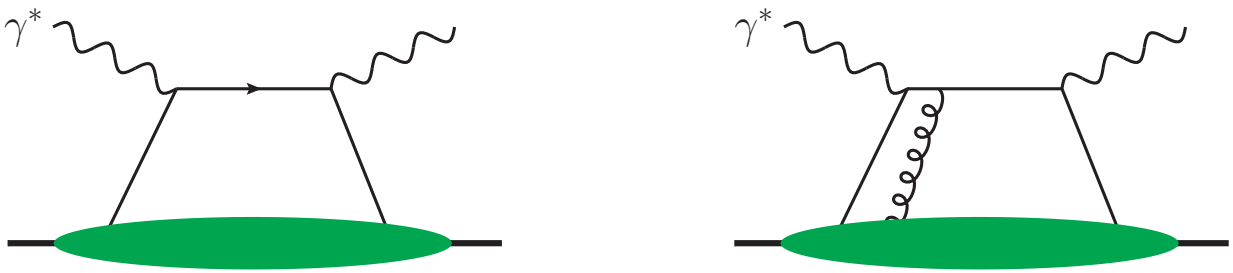
but: higher twist

- reveals aspects of QCD dynamics beyond ordinary (twist 2) distributions
  - physics interpretation of certain twist 3 matrix elements:
    - average k_T^2 of quarks in a transversely polarised proton
    - average force acting on a quark ejected from a transv. pol. proton

    M Burkardt 2003, 2008
  - quantum interference between one and two partons in proton:
    e.g. twist 2 PDF $\sim \Psi_q^* \Psi_q$ vs. twist 3 $\sim \Psi_q^* \Psi_{qg}$



Non-forward kinematics: twist 3 GPDs



- non-forward twist 3 distributions appear in DVCS
  contribute to specific angular modulations
  e.g. J Schönleber, R Szafron 2024; earlier: D Müller et al., M Polyakov et al.
  $1/Q$ power corrections to DVCS $\rightsquigarrow$ work by Volodya et al.
- certain twist 3 GPDs are related to orbital angular momentum
  sum rules by Ji and Jaffe/Manohar, spin-orbit correlations
  Y Hatta et al; M Engelhardt, S Liuti et al; C Lorcé, B Pasquini et al

Twist 3: two- vs. three-parton distributions

- twist 3 distributions with two parton fields
  like $g_T(x)$ (forward) or $E_{2T}(x,\xi,t)$ (GPD)
  are "simpler" than those with three parton fields
  e.g. for constructing a model ansatz, fitting, numerical calculations, ...
- at LO in α_s, the twist 3 part of DVCS can be expressed in terms of two-parton GPDs only

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- but: at twist 3
  - the scale evolution of two-parton dists. depends on three-parton dists.
  - by contrast, three-parton dists. evolve autonomously
- cannot reduce three-parton distributions to two-parton ones

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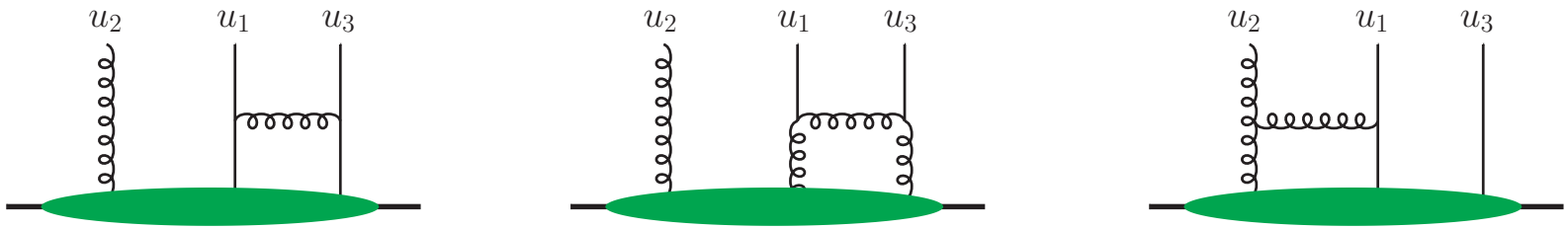
  $\rightsquigarrow$ we should be able to evolve three-parton distributions of twist 3
  both in forward and in non-forward kinematics
- two-parton dists. (of twist 3) can be expressed in terms of
  twist 2 distributions (sometimes called Wandzura-Wilczek part)
  and three-parton distributions (sometimes called "genuine twist 3" part)
  can be established at operator level, I Balitsky and V Braun 1988

Three-parton distributions: scale evolution



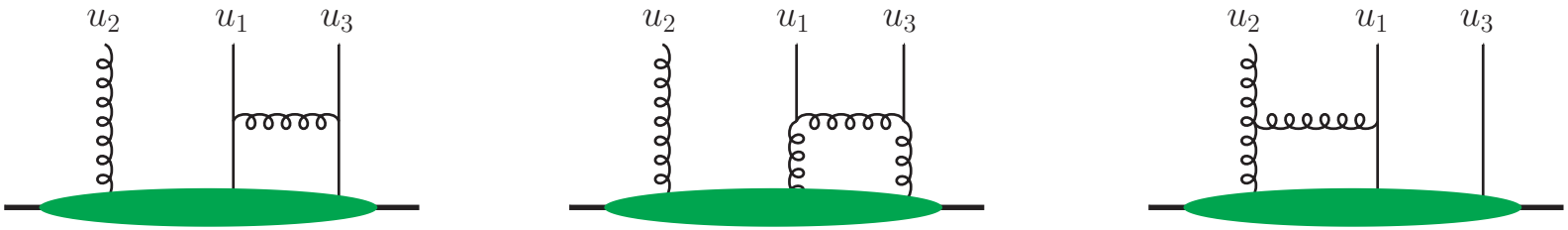
- at LO in α_s: ladder graphs connecting any 2 out of the 3 partons
  kernels unknown at NLO, not discussed further here
- each ladder can have nonzero momentum transfer in the t channel
  $\rightarrow$ almost the same kernels as in evolution of twist 2 GPDs
  - colour octet in the t channel for twist 3, colour singlet for twist 2
  - no $qg \to qg$ ladders at twist 2
- kernels long known: Bukhvostov, Frolov, Lipatov, Kuraev (BFLK) in 1985
  recomputed independently by Volodya et al (BMRP)
  V Braun, A Manashov, J Rohrwild, 2009
  V Braun, A Manashov, B Pirnay, 2009
- the same kernels (in more colour channels) describe the evolution of n-parton distributions of twist n

Three-parton distributions: scale evolution in practice

for numerical investigations

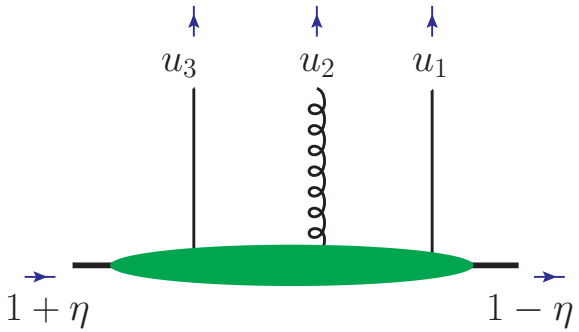
- code by B Pirnay, 2013
  for transverse proton polarisation; only $n_f = 2$ flavours
- forward kinematics: S Rodini, L Rossi, A Vladimirov 2024
  for all polarisation channels and general n_f
  independent codes in FORTRAN and in C
- ongoing project (MD and S Rodini):
  code for forward and non-forward kinematics
  not yet finished, but on the way to a code we came across several interesting points $\rightsquigarrow$ rest of this talk

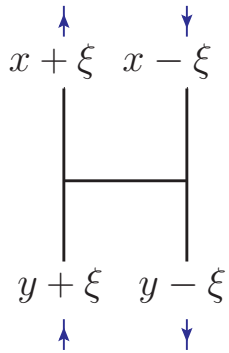
all these codes work in space of momentum fractions



Momentum fractions

use different "coordinate systems" for momentum fractions





arrows indicate momentum flow

- "global coordinates":
  - mom. fractions u_i w.r.t. to average hadron momentum $\frac{1}{2}(p+p')$
  - skewness variable η; analogue of ξ in twist 2 GPDs
  - mom. conservation $\Rightarrow u_1+u_2+u_3=2\eta$
- "ladder coordinates":
  - for each evolution ladder that leaves u_i untouched use mom. fractions $x_i \pm \xi_i$ on upper and $y_i \pm \xi_i$ on lower side of ladder
  - normalised such that $x_i, y_i, \xi_i \in [-1,1]$

Momentum fractions

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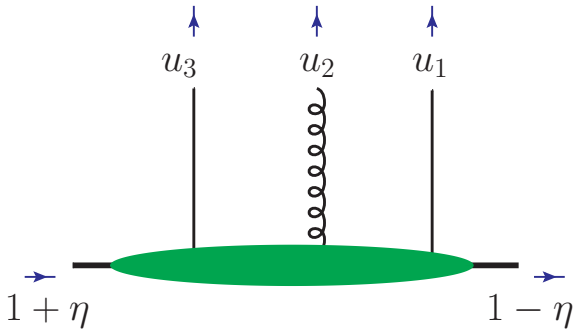

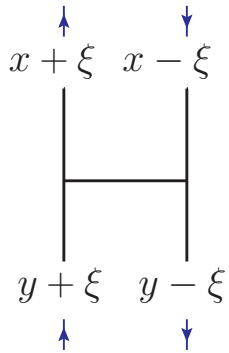

arrows indicate momentum flow

- LO evolution equation schematically reads

$$\frac{d}{d\ln\mu^2}\,\phi(u_1,u_2,u_3;\mu) = \frac{\alpha_s(\mu)}{4\pi}\sum_{i=1,2,3}\int_{-1}^{1} dy_i\; K_i(x_i,y_i,\xi_i)\,\phi(y_i,\xi_i;\mu)$$

labels and sums for parton types and polarisations to be added

using x_i, y_i, ξ_i on r.h.s. avoids explicit δ functions in integral
suitable for discretisation and numerical integration

Operators for three-parton distributions

- associated light-ray operators schematically read:

$$\bar{q}gq : F^{+i,a}\,\bar{\psi}\,\Gamma\,t^a\,\psi \quad \text{with} \quad \Gamma = \gamma^+, \gamma^+\gamma_5, \gamma^+\gamma^j$$

$$ggg : (f^{abc} \text{ or } d^{abc}) \times F^{+i,a}\,F^{+j,b}\,F^{+k,c}$$

i and j are transverse Lorentz indices

note on bilinears: $\bar{\psi}\,\Gamma\psi \sim q(x)$ vs. $F^{+i}F^{+j} \sim xg(x)$ in twist-two PDFs

Support properties

- support: $-1 \le u_i - \eta \le 1$ and $-1 \le u_j + u_k - 2\eta \le 1$ for all i and $j \neq k$
- symmetric representation in barycentric coordinates (Volodya et al)

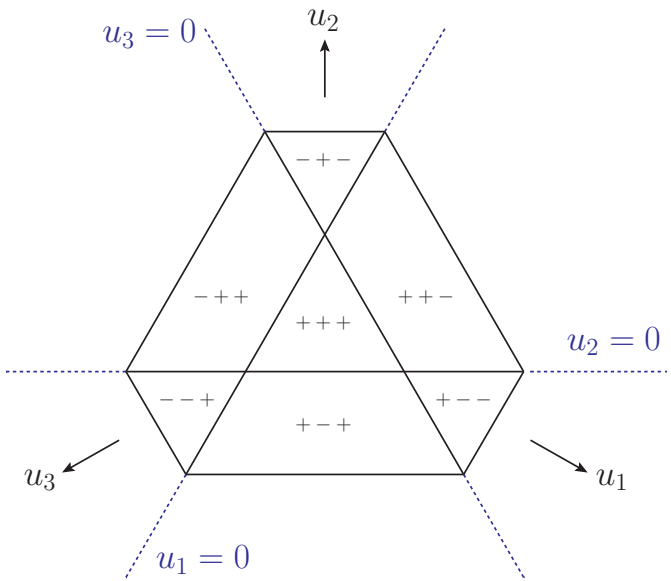

figure for $\eta > 0$

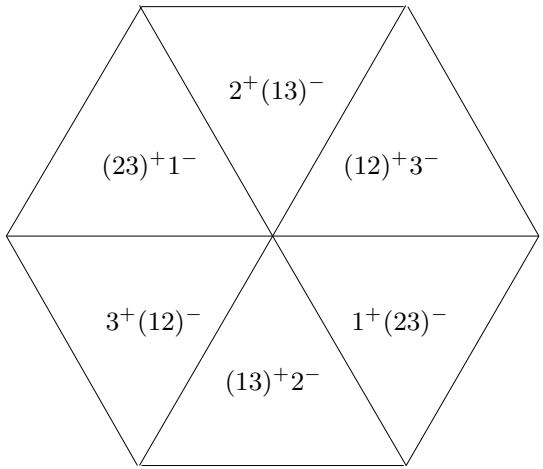

figure for $\eta = 0$ from
V Braun et al, arXiv:1103.1269

- signs refer to momentum fractions u_1, u_2, u_3
- height of hexagon $= 2$
  height of central triangle $= 2|\eta|$ (disappears for $\eta = 0$)

Evolution kernels: validation and specific form

- we extracted the kernels from the BFLK paper and checked their consistency with
  - the GPD kernels in A Belitsky, A Radyushkin 2005
    and in V Bertone et al 2022/23
  - the twist-three kernels of BMRP
- form of the flavour singlet kernels for $gg \leftarrow \bar{q}q$, $\bar{q}q \leftarrow gg$ and $gg \leftarrow gg$ is not unique:
  - in given channel (colour, spin, C parity), distributions are even or odd in "ladder coordinate" for gg or $\bar{q}q$ pair
  - in appropriate kernels can thus add antisymmetric or symmetric terms in y_i
    leaving $\int dy_i \; K_i(x_i, y_i, \xi_i)\, \phi(y_i, \xi_i)$ unchanged
  - use this to simplify the kernels
    and to eliminate spurious divergences at $y_i = \pm\, \xi_i$

Evolution kernels: validation and specific form

- form of the flavour singlet kernels for $gg \leftarrow \bar{q}q$, $\bar{q}q \leftarrow gg$ and $gg \leftarrow gg$ is not unique:
  - . . . eliminate spurious divergences at $y = \pm\,\xi$
  - example: for $|x| \leq \xi$ and $\xi > 0$, kernel $K_{\bar{q}q \leftarrow gg}$ has term $\propto$

$$\frac{1}{(y-\xi)^2}\,\frac{1}{y+\xi} = \frac{1}{2\xi^2}\,\frac{y}{y^2-\xi^2} - \frac{1}{2\xi^2}\,\frac{y-2\xi}{(y-\xi)^2} \qquad \text{for } y \leq x \leq \xi$$

$$\frac{1}{(y+\xi)^2}\,\frac{1}{y-\xi} = \frac{1}{2\xi^2}\,\frac{y}{y^2-\xi^2} - \frac{1}{2\xi^2}\,\frac{y+2\xi}{(y+\xi)^2} \qquad \text{for } y \geq x \geq -\xi$$

  First terms on r.h.s. are singular at $y = -\xi$ or $y = \xi$ inside integration region
  - can drop if distribution is even in y
  - cancel against another term if distribution is odd in y
  - absence of spurious singularities benefits numerical implementation

Behaviour of distributions at $u_k = 0$

$$\frac{d}{d\ln\mu^2}\,\phi(u_1,u_2,u_3) = \frac{\alpha_s}{4\pi}\sum_i \int_{-1}^{1} dy_i\; K_i(x_i,y_i,\xi_i)\,\phi(y_i,\xi_i)$$

- $u_k = 0$ corresponds to $x_i = \pm\,\xi_i$
- setting $x_i = \pm\,\xi_i$ does not commute with $\int dy_i$ for all kernels
- for $|x| \le \xi$ and $\xi > 0$,
  convolutions with $K_{\bar{q}q\leftarrow gg}$, $K_{gg\leftarrow gg}$ and $K_{qg\leftarrow qg}$ contain term $\propto$

$$\int_{-1}^{x} dy\,\frac{x-\xi}{(y-\xi)^2}\,\phi(y,\xi) = \int_{-1}^{x} dy\,\frac{x-\xi}{(y-\xi)^2}\,\big[\phi(y,\xi)-\phi(x,\xi)\big] - \frac{1+x}{1+\xi}\,\phi(x,\xi)$$

  Finite contribution for $x \to \xi$, given by 2nd line

- aside: for $qg \leftarrow qg$, limit $x \to \xi$ corresponds to gluon momentum $\to 0$
  this limit is taken in the Qiu-Sterman fct.
  setting u_2 in the kernel gives wrong evolution eq.

  V Braun, A Manashov, B Pirnay 2009

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- writing all convolutions as

$$\int dy\; K(x,y,\xi)\,\phi(y,\xi) = \int dy\; K(x,y,\xi)\,\big[\phi(y,\xi)-\phi(x,\xi)\big] + \mathcal{C}(x,\xi)\,\phi(x,\xi)$$

  makes leading behaviour for $x \to \xi$ explicit:

  - quark mom. $\to 0$: convolution is continuous, but 1st derivative in x divergent: $\mathcal{C}(x,\xi) \sim -|x-\xi|\,\ln|x-\xi|$
  - gluon mom. $\to 0$: convolution and 1st derivative in x are continuous, but 2nd derivative in x divergent: $\mathcal{C}(x,\xi) \sim -(x-\xi)^2\,\ln|x-\xi|$

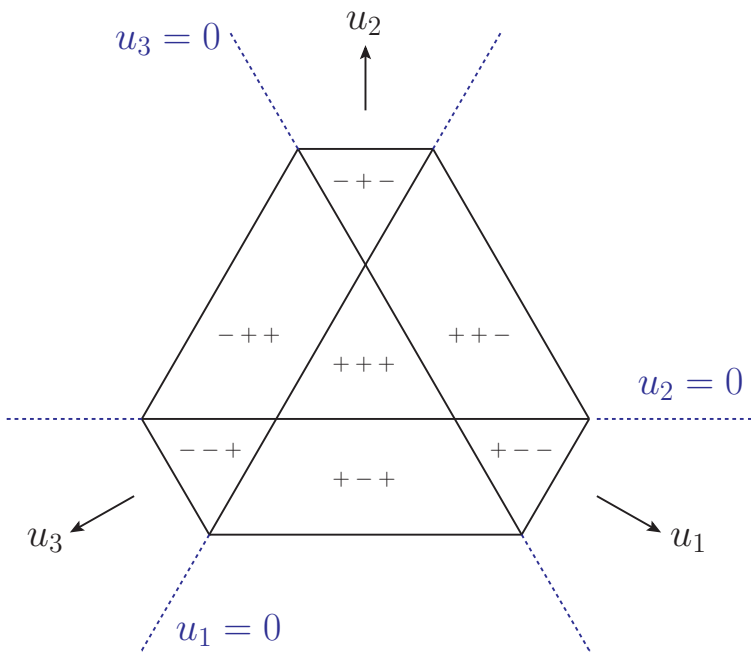
  likewise for $x \to -\xi$

- relevant for three-parton distributions and for twist 2 GPDs
- in line with results for GPDs in V Braun et al, hep-ph/0204191 and M Kirch, A Manashov, A Schäfer hep-ph/0509333

Behaviour of distributions at $u_k = 0$

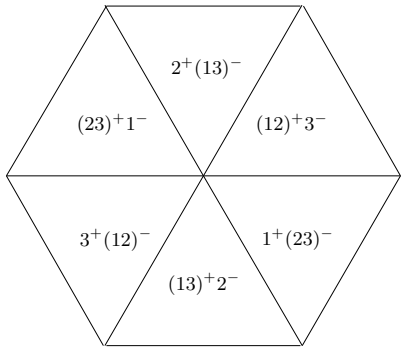
upshot:

- $d \log \mu^2 / d\phi$ is non-analytic at points $x_i = \pm \xi_i$ $(i = 1, 2, 3)$
  i.e. across the lines where $u_k = 0$ $(k = 1, 2, 3)$



Behaviour of distributions for $u_j = u_k = 0$

DANGEROUS LIMIT



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  this point not relevant for scattering processes or for evolution

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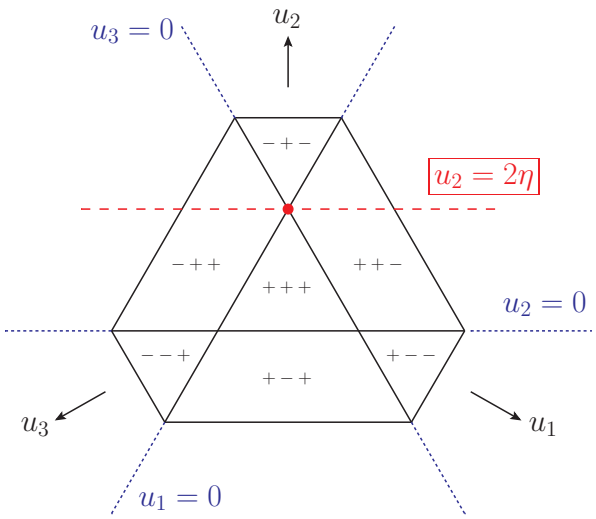

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  these points occur in evolution integrals for other points

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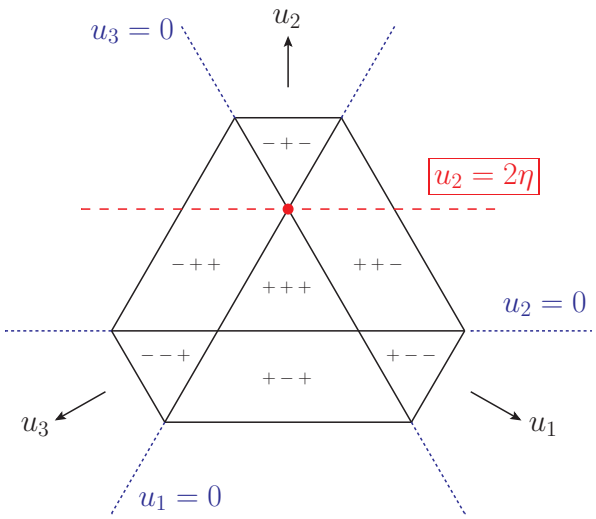

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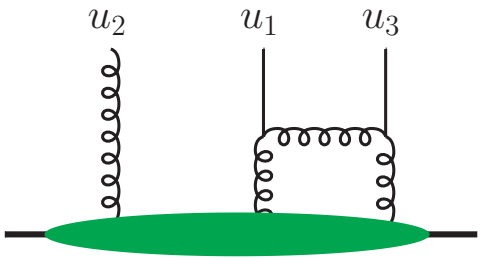
- evolution integrals have power-law singularity for $\phi_{\bar{q}gq}$ at zero quark momenta $u_1 = u_3 = 0$
- originates from $\bar{q}q \leftarrow gg$:



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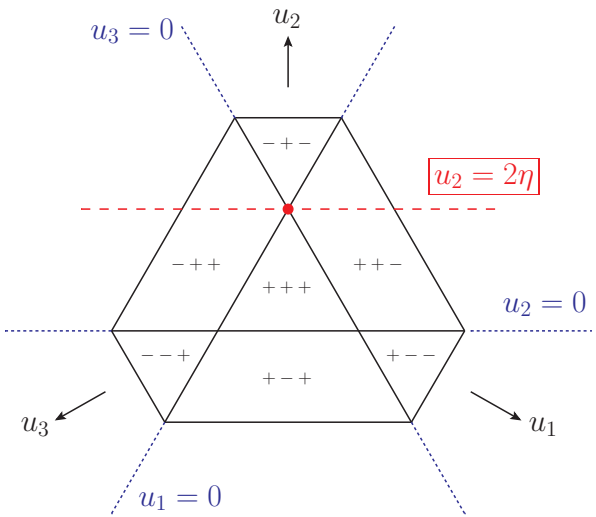

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- evolution integrals have power-law singularity for $\phi_{\bar{q}gq}$ at zero quark momenta $u_1 = u_3 = 0$
- simple to see for $\xi_2 = 0$ $(\Leftrightarrow u_2 = 2\eta)$: DGLAP evolution

$$\frac{d\phi_{\bar{q}gq}(x_2, \xi_2 = 0)}{d \ln \mu^2} \ni \int_{x_2}^1 \frac{dz}{z} P_{qg}\left(\frac{x_2}{z}\right) \frac{\phi_{ggg}(z, \xi_2 = 0)}{z} \qquad \text{use } P_{qg}(z) \sim 1/z$$

$$\sim \frac{1}{x_2} \int_{x_2}^1 \frac{dz}{z} \phi_{ggg}(z, \xi_2 = 0)$$

Behaviour of distributions for $u_j = u_k = 0$

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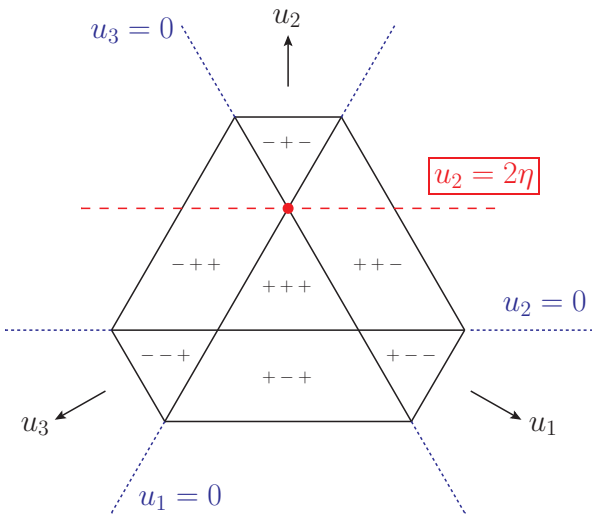

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- evolution integrals have power-law singularity for $\phi_{\bar{q}gq}$ at zero quark momenta $u_1 = u_3 = 0$
- using different ansätze for ϕ_{ggg} get from $\bar{q}q \leftarrow gg$ kernel:

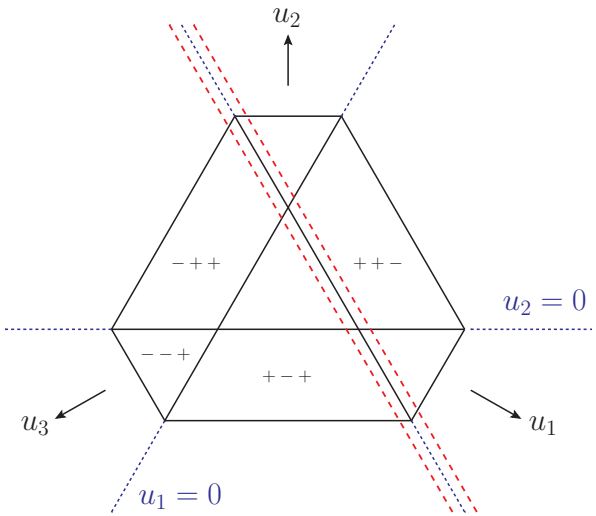
$$\frac{d\phi_{\bar{q}gq}(u_1, u_2, u_3)}{d \ln \mu^2} \ni \frac{1}{\delta} \ln^k \delta \qquad\qquad k = 0, 1, 2$$

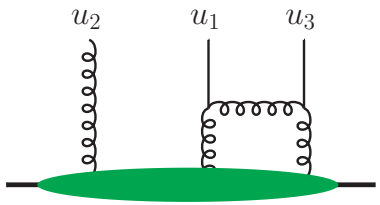
  for small $\delta =$ distance of point (u_1, u_2, u_3) from $(0, 2\eta, 0)$

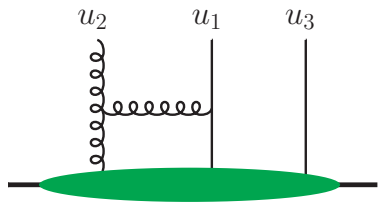
- if ϕ_{qgg} had the same singular behaviour, evolution integrals for $qg \leftarrow qg$ ladders along the lines $u_1 = 0$ and $u_3 = 0$ would be ill-defined

Behaviour of distributions for $u_j = u_k = 0$

DANGEROUS LIMIT







- using different ansätze for ϕ_{ggg} get from $\bar{q}q \leftarrow gg$ kernel:

$$\frac{d\phi_{\bar{q}gq}(u_1, u_2, u_3)}{d \ln \mu^2} \ni \frac{1}{\delta} \ln^k \delta \qquad k = 0, 1, 2$$

- do the terms from $qg \leftarrow qg$ ladders prevent the formation of such a singularity in $\phi_{\bar{q}gq}$?
- assume the above singular form of $\phi_{\bar{q}gq}$ and evaluate evolution integrals from $K_{qg \leftarrow qg}$ for u_1 or u_3 small but finite

$$\frac{d\phi_{\bar{q}gq}(u_1, u_2, u_3)}{d \ln \mu^2} \ni c \, \ln(\delta) \, \phi_{\bar{q}gq} + \text{less singular terms} \qquad \text{with } c = 1, 3$$

is slightly more singular than $\phi_{\bar{q}gq}$ and has opposite sign

$\rightsquigarrow$ should indeed prevent formulation of singularity at $\delta = 0$

Summary

- twist 3 terms are relevant in various contexts of inclusive and exclusive processes
- they contain interesting physics information
- for scale evolution must consider three-parton distributions
  LO kernels are known, closely related to those for twist 2 GPDs
- in kernels can eliminate all spurious singularities at $y = \pm\xi$
  remaining terms include $1/(y-\xi)$ and $1/(y-\xi)^2$ for $y \le x \le \xi$
  as well as $1/(y+\xi)$ and $1/(y+\xi)^2$ for $y \ge x \ge -\xi$
- writing convolution integrals as "plus distributions" allows us to
  - take limit $x \to \pm\xi$ under the integrals
  - read off behaviour of convolution for $x \to \pm\xi$
  find that $d\phi/d\ln\mu^2$ is non-analytic across the lines $u_k = 0$
  plausible to expect same behaviour for ϕ itself
- in non-forward kinematics, $\bar{q}gq$ distributions become enhanced at the point $u_1 = u_3 = 0$, but should stay finite there

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- they contain interesting physics information
- for scale evolution must consider three-parton distributions
  LO kernels are known, closely related to those for twist 2 GPDs

Volodya, I wish you all the best for the next phase of your career.
Enjoy physics and life beyond physics
and don't forget how beautiful higher twist can be.