Contribution ID: **59** Type: **Poster Presentation**

Existence, Invariance, and Reduction in a Fast–Slow HPT-Axis-Model

Thursday, 26 February 2026 15:15 (9 minutes)

Singular perturbed dynamical systems provide a principled way to analyze mathematical models with widely separated times scales, where solutions typically evolve rapidly toward a slow manifold and then drift along it. The hypothalamus–pituitary–thyroid (HPT) axis is a central endocrine control loop that maintains thyroid hormone homeostasis via negative feedback: hypothalamic and pituitary signals regulate thyroid hormone production, while circulating thyroid hormones suppress upstream stimulation. Autoimmune thyroiditis can introduce slower changes in thyroid function and tissue state, motivating a fast–slow modeling perspective in which rapid hormonal regulation interacts with gradual diseasedriven dynamics. Building on earlier fast–slow studies of endocrine feedback, this poster completes the existence theory and provides a fully rigorous singular-perturbation argumentation for an ordinary differential equation model of a negative-feedback loop describing regulation of the hypothalamus–pituitary–thyroid (HPT)-axis under slow autoimmune disease progression. A positively invariant region capturing physiologically meaningful states is first established, and explicit sufficient conditions for local existence and uniqueness of solutions are derived. This results in a well-posed problem and provides the technical foundation required for a justified time-scale separation. Leveraging this groundwork, we then state verifiable criteria that guarantee normal hyperbolicity of the critical manifold and attraction by the fast subsystem, and we apply Tikhonov-Fenichel type arguments to establish convergence of trajectories of the full system to those of the reduced model in the singular limit. In this way, the poster extends earlier analyses by making the assumptions transparent, tightening the logical links between full and reduced dynamics, and delivering a complete and selfcontained justification of the reduction.

Affiliation

Author: HORVATH, Clara

Presenter: HORVATH, Clara