Complex quasi-momentum and edge-bulk hybridization in generalized SSH models

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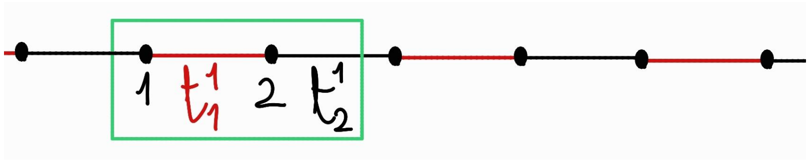
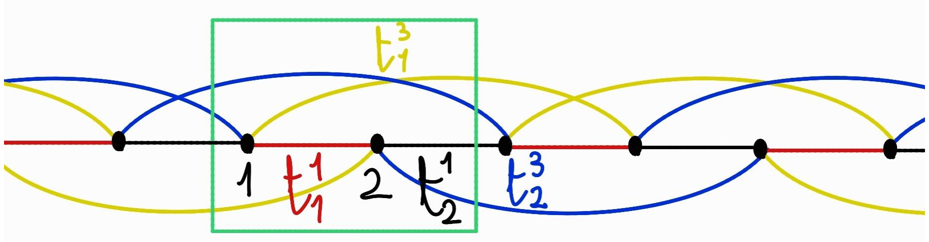
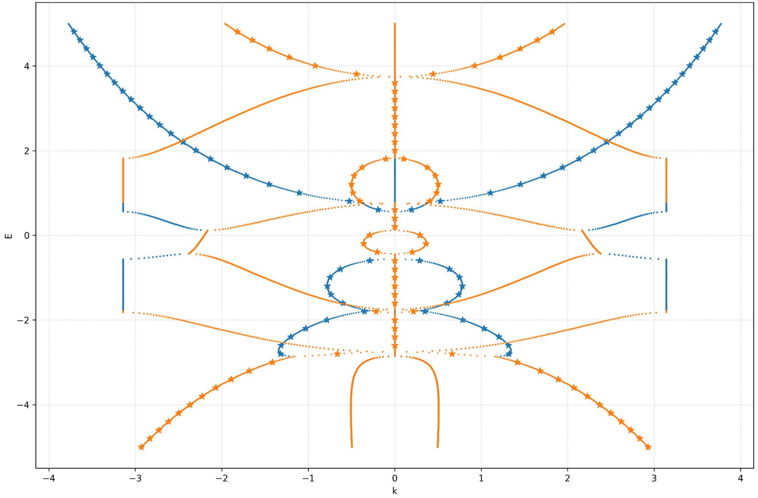
Motivation



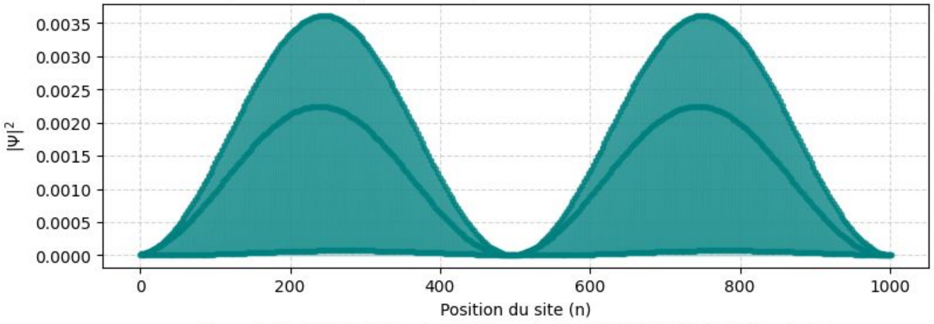
Standard SSH



Example of generalised SSH



Band spectrum



Hybridize state

Bloch Hamiltonian

$$\hat{H} = \sum_{j=-\infty}^{+\infty} \sum_{s=1}^{+\infty} \sum_{n=1}^{N} t_n^s |j; n+s >< j; n| + h.c.$$

Tight binding hamiltonian

$$H_{n;n'} = \sum_{r=0}^{\lceil s_{max}/N \rceil} e^{-i(r+\Theta(n-n'))k} t_n^{n'-n+N(\Theta(n-n')+r)} + h.c.$$

Bloch hamiltonian element

$$H = \begin{bmatrix} 0 & t_1^1 + t_2^1 e^{ik} \\ t_1^1 + t_2^1 e^{-ik} & 0 \end{bmatrix}$$

Standard SSH

$$H = \begin{bmatrix} 0 & t_1^1 & t_1^2 + t_3^2 e^{ik} & t_4^1 e^{ik} \\ t_1^1 & 0 & t_2^1 & t_2^2 + t_4^2 e^{ik} \\ t_1^2 + t_3^2 e^{-ik} & t_2^1 & 0 & t_3^1 \\ t_4^1 e^{-ik} & t_2^2 + t_4^2 e^{-ik} & t_3^1 & 0 \end{bmatrix}$$

4 SSH-4-4(2,2)

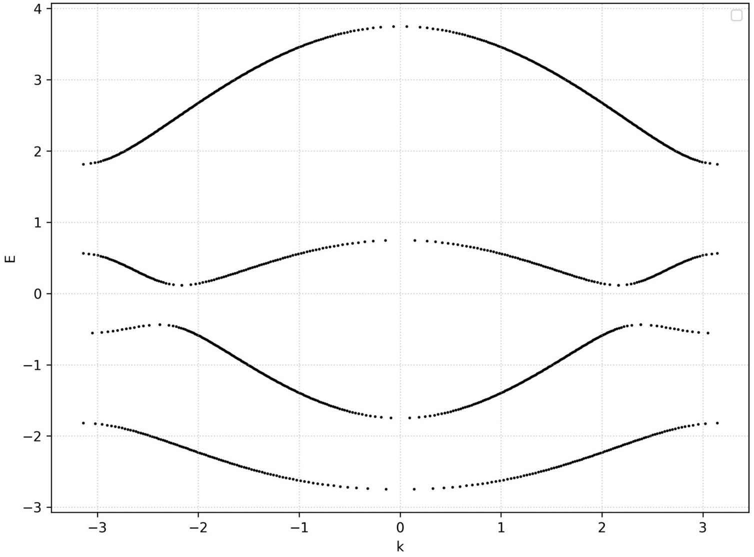
Quasi-momentum multiplicity

$$\sum_{n=0}^{N}\sum_{s=0}^{n_m} a_{n,s}E^n(\cos k)^s = 0$$

Dispersion relation

$$a(E)\cos^2(k) + b(E)\cos(k) + c(E) = 0$$

Example for $n_m = 2$



Band structure exemple

$$k_1 = \arccos\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)$$

$$k_2 = -\arccos\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)$$

$$k_3 = \arccos\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$$

$$k_4 = -\arccos\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$$

Four solutions for each energy

Complex quasi-momentum

$$\cos k \in [-1;1] \rightarrow k \in [-\pi;\pi]$$

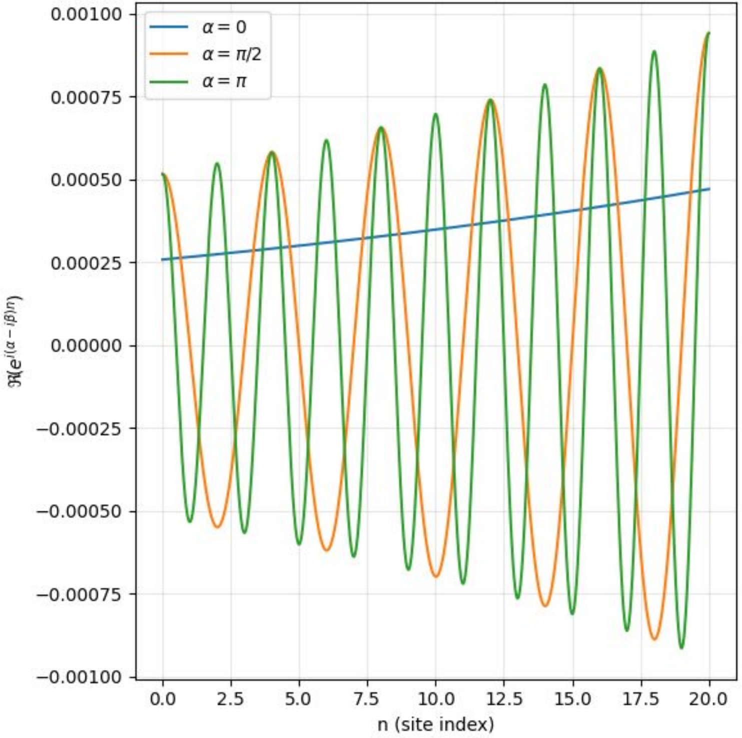
Standard bulk states

$$\cos k > 1 \rightarrow k = i\kappa \quad \kappa \in [0;+\infty]$$
$$\cos k < -1 \rightarrow k = \pi + i\kappa \quad \kappa \in [0;+\infty]$$

Standard edge states

$$\cos k \in \mathbb{C} \rightarrow k = \alpha + i\beta \quad \alpha \in [-\pi;\pi]; \beta \in \mathbb{R}$$

(Free) Complex solution



Frequence transition

Complex state as a consequence of multiplicity

$$H = \begin{bmatrix} 0 & t_1^1 & t_1^2 + t_3^2 e^{ik} & t_4^1 e^{ik} \\ t_1^1 & 0 & t_2^1 & t_2^2 + t_4^2 e^{ik} \\ t_1^2 + t_3^2 e^{-ik} & t_2^1 & 0 & t_3^1 \\ t_4^1 e^{-ik} & t_2^2 + t_4^2 e^{-ik} & t_3^1 & 0 \end{bmatrix}$$

Hamiltonian

$$C = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Cost Matrix

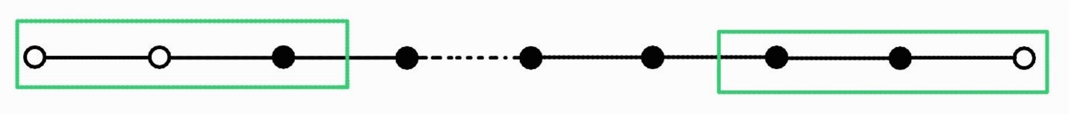
$$\sum_{n=0}^{N} \sum_{s=0}^{n_m} a_{n,s} E^n (\cos k)^s = 0$$

Find n_m

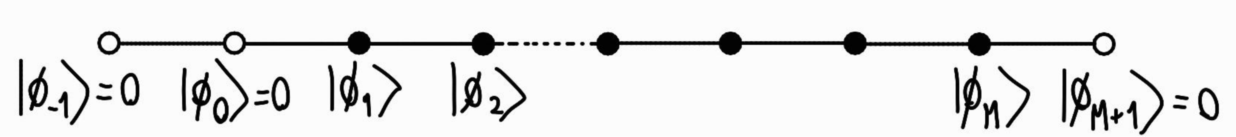
$$k = \arccos\left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right) \quad b^2 < 4ac$$

Complex k solution

Finite systems

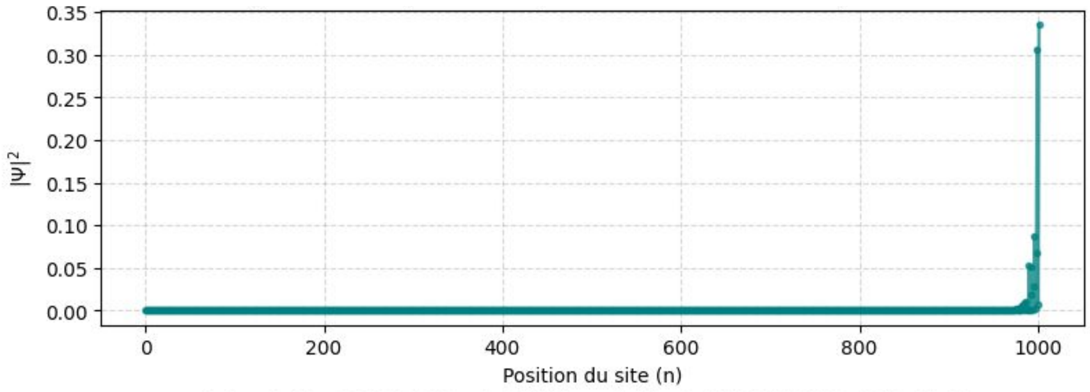


How the chain end



Boundary condition

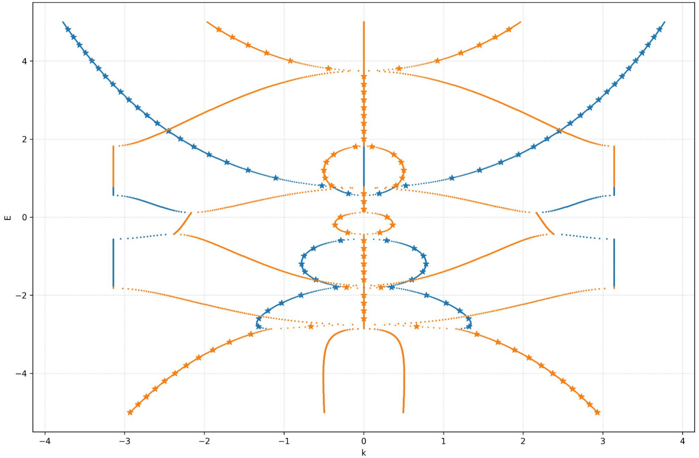
Standard edge states



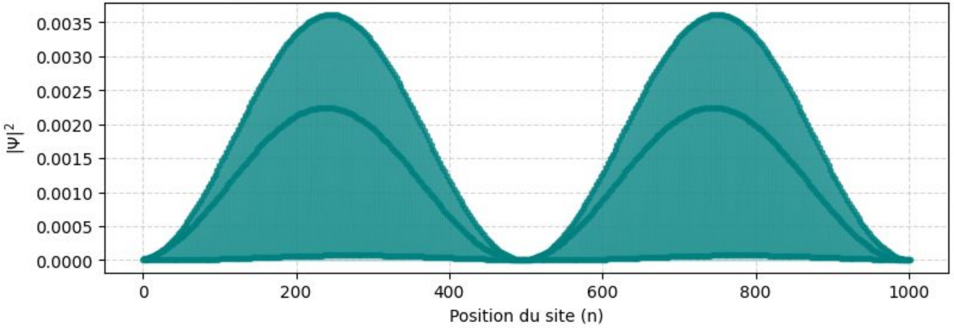
Edge-to-bulk hybridization

$$|\Psi\rangle = \sum_{n=-n_m}^{n_m} a_n |\psi_n\rangle$$

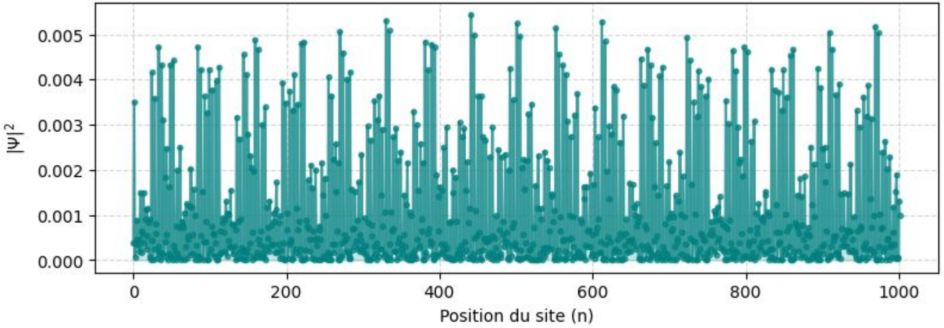
Linear combination



Overlapping type of states



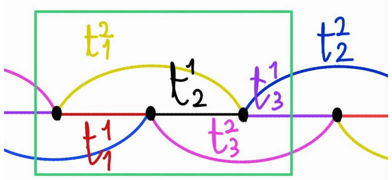
Coherent interference

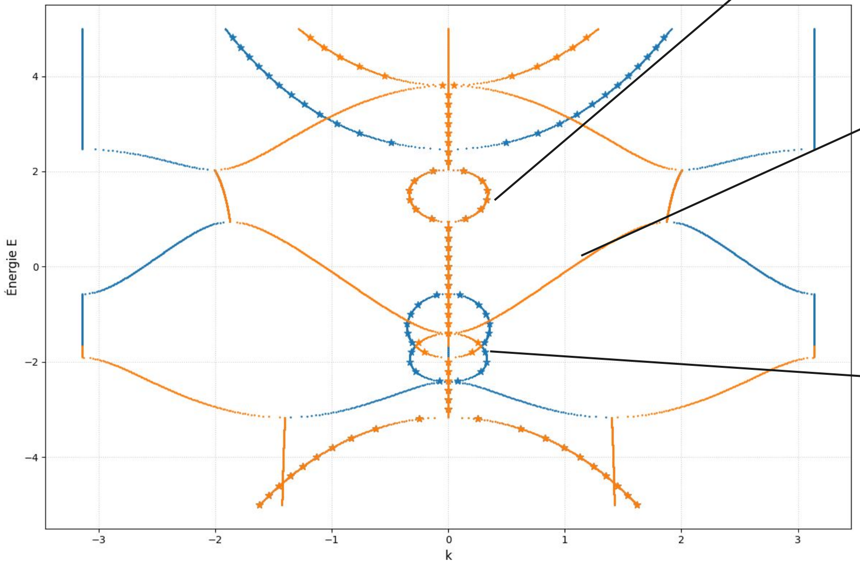


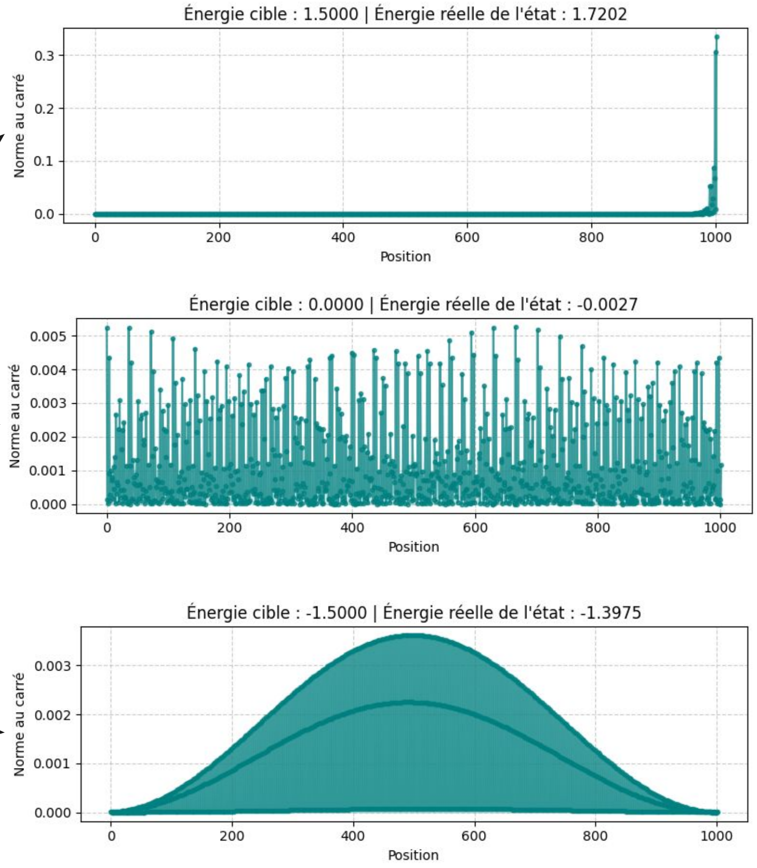
Incoherent interference

Numerical examples

$3 \quad SSH3-3(3, SC_1^2+2)$







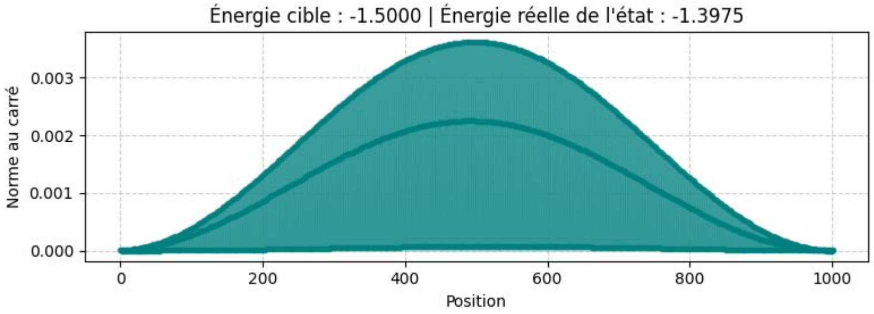
Conclusion

$$\sum_{n=0}^{N}\sum_{s=0}^{n_m} a_{n,s}E^n(\cos k)^s = 0$$

Characteristic polynomial

$$\cos k \in \mathbb{C} \rightarrow k = \alpha + i\beta \quad \alpha \in [-\pi;\pi];\beta \in \mathbb{R}$$

Emerging solution



States hybridization

Thank you for listening.

Link

Link to the online présentation

https://docs.google.com/presentation/d/1PeKo5IOmD8AJbwsNEKTEt-JNofG_ABnjXUK4R-DMopA/edit?usp=sharing