Large Momentum Diffusion from the Dipole Force of Travelling Waves

2026 CAP Congress | Congrès de l'ACP 2026

June 21-26, 2026

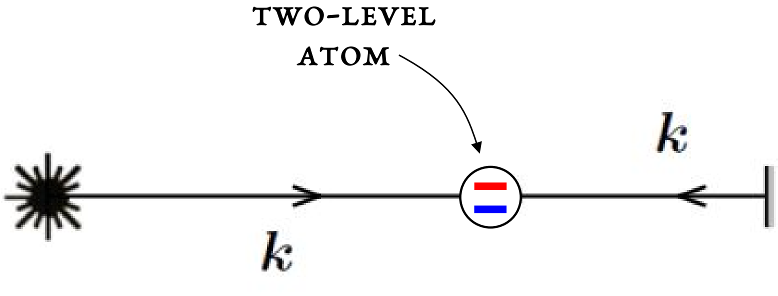
Louis Marmet

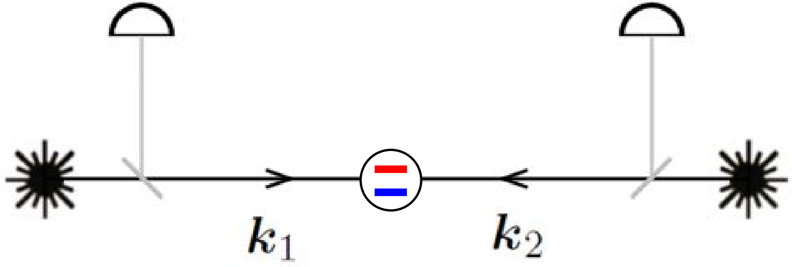
York University

Session (DAMOPC) R2-3 | Thu, June 25, 4:45 – 5:00 PM

U. Ottawa - Learning Crossroads (CRX) Building, 100 Louis-Pasteur Private, Ottawa, ON K1N 9N3

Large Momentum Diffusion from the Dipole Force of Travelling Waves



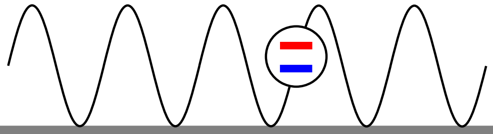


DETUNING $\Delta = \omega - \omega_0$

Standing wave

Dipole force is conservative

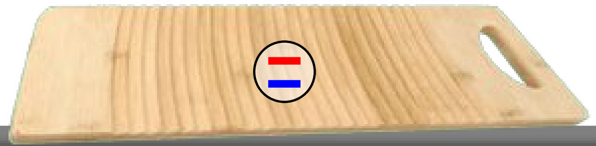
Quantum fluctuations produce **$1/\Delta^2$** heating



Travelling waves

Dipole force may become non-conservative

Photon transfers produces **$1/\Delta$** heating



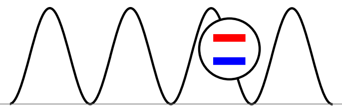
Standing Wave: A single Quantum State

Fields are subject to a boundary condition such as
a reflection on a mirror

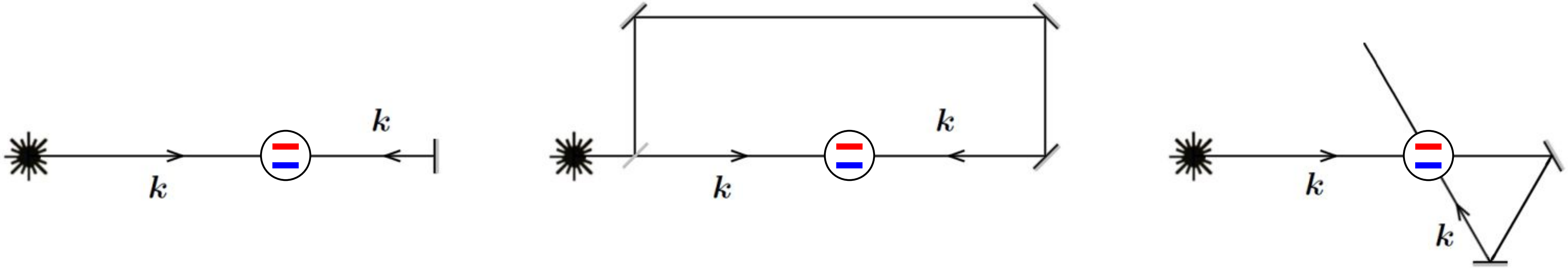
$$\hat{A}(x,t) = \sqrt{2}\,\alpha\hat{a}_c\cos(kx)\,\mathrm{e}^{-i\omega t}$$

$$\hat{a}_c = (\hat{a}_1 + \hat{a}_2)/\sqrt{2}$$

conservative potential



$$N_{sw} = \hat{a}_c^\dagger \hat{a}_c = 1$$



B.W. Shore, P. Meystre, and S. Stenholm, "*Is a quantum standing wave composed of two traveling waves?*"
J. Opt. Soc. Am. B, vol. 8, no. 4, pp. 903–910, Apr. 1991, doi: 10.1364/JOSAB.8.000903

Travelling Waves: Two Interfering Quantum Waves

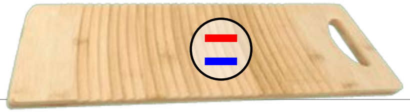
Natural choice when the field is the combination of two independent waves

$$\hat{A}_{rw}(x,t) = \alpha\left[\hat{a}_1 \exp(ik_1x) + \hat{a}_2 \exp(ik_2x)\right] \mathrm{e}^{-i\omega t}$$

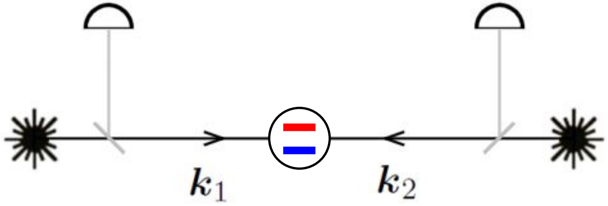
$$\hat{a}_1|n_1, n_2\rangle_R = \sqrt{n_1}|n_1 - 1, n_2\rangle_R$$

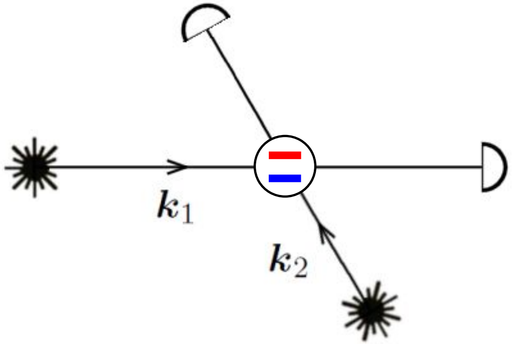
$$\hat{a}_2|n_1, n_2\rangle_R = \sqrt{n_2}|n_1, n_2 - 1\rangle_R$$

non-conservative potential



$$N_{rw} = \hat{a}_1^\dagger\hat{a}_1 + \hat{a}_2^\dagger\hat{a}_2 = n_1 + n_2$$





B.W. Shore, P. Meystre, and S. Stenholm, "*Is a quantum standing wave composed of two traveling waves?*" J. Opt. Soc. Am. B, vol. 8, no. 4, pp. 903–910, Apr. 1991, doi: 10.1364/JOSAB.8.000903

Interaction Hamiltonian

ONE-PHOTON RABI FREQUENCY $\Omega_0 = \mu\mathcal{E}/\hbar$

Standing wave

$$H^{sw} = \frac{\hat{p}^2}{2M} + \frac{1}{2}\hbar\omega_0\sigma_z + \hbar\omega\hat{a}^\dagger\hat{a} - \frac{\hbar\Omega_0}{4}(\sigma_+\hat{a} + \sigma_-\hat{a}^\dagger)[\mathrm{e}^{ik\hat{x}} + \mathrm{e}^{-ik\hat{x}}]$$

ATOMIC MOMENTUM ATOMIC STATE PHOTON NUMBER INTERACTION

Travelling waves

$$H^{rw} = \frac{1}{2M}\left[\hat{p} - \hbar k_1\hat{a}_1^\dagger\hat{a}_1 - \hbar k_2\hat{a}_2^\dagger\hat{a}_2\right]^2 + \frac{1}{2}\hbar\omega_0\sigma_z + \hbar\omega(\hat{a}_1^\dagger\hat{a}_1 + \hat{a}_2^\dagger\hat{a}_2) - \frac{\hbar\Omega_0}{2\sqrt{2}}\left[\sigma_+(\hat{a}_1 + \hat{a}_2) + \sigma_-(\hat{a}_1^\dagger + \hat{a}_2^\dagger)\right]$$

Dipole Force: Adiabatic Elimination of the Atomic Operator

DETUNING $\Delta = \omega - \omega_0$

MOMENTUM TRANSFER $q = k_1 - k_2$

Standing wave

$$H^{sw} = \frac{\hat{p}^2}{2M} + \frac{\hbar\Omega_0^2}{8\Delta}\hat{a}^\dagger\hat{a}\left[e^{iq\hat{x}} + e^{-iq\hat{x}}\right]$$

ATOMIC MOMENTUM

INTERACTION

DIPOLE FORCE *

$$\vec{F} = -\vec{\nabla}U \propto \vec{k}\frac{\Omega_0^2}{\Delta}$$

Travelling waves

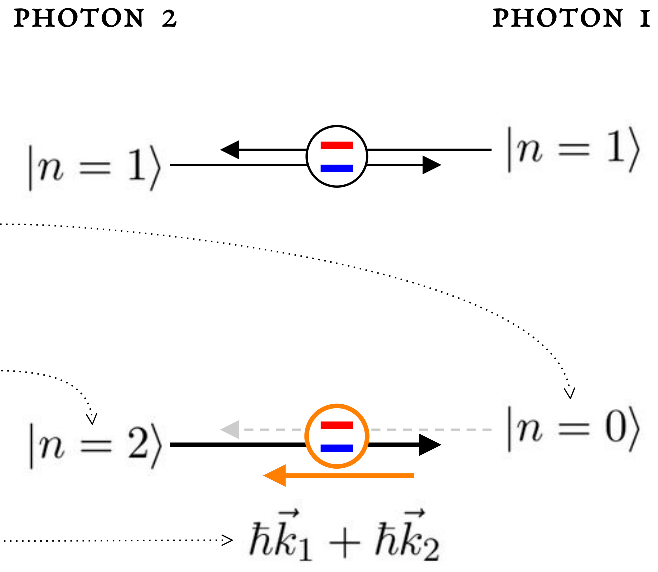
$$H^{rw} = \frac{1}{2M}\left[\hat{p} - \hbar k_1\hat{a}_1^\dagger\hat{a}_1 - \hbar k_2\hat{a}_2^\dagger\hat{a}_2\right]^2 + \frac{\hbar\Omega_0^2}{4\Delta}\left[\hat{a}_1^\dagger\hat{a}_2 + \hat{a}_2^\dagger\hat{a}_1\right]$$

STIMULATED EMISSION

* Neglecting spontaneous emission

Dipole Force: Stimulated Emission and Momentum Recoil

$$H^{rw} = \frac{1}{2M}\left[\hat{p} - \hbar k_1 \hat{a}_1^\dagger \hat{a}_1 - \hbar k_2 \hat{a}_2^\dagger \hat{a}_2\right]^2 + \frac{\hbar \Omega_0^2}{4\Delta}\left[\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1\right]$$



Momentum Diffusion: Quantum Fluctuations of the Force

MOMENTUM DIFFUSION

$$D_p = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left[\langle \Delta\boldsymbol{p}\cdot\Delta\boldsymbol{p}\rangle - \langle\Delta\boldsymbol{p}\rangle\cdot\langle\Delta\boldsymbol{p}\rangle\right]$$

EQUATION OF MOTION FOR THE DENSITY MATRIX

$$\Delta\tilde{\rho}(t_0) = \frac{1}{i\hbar}\int_{t_0}^{t_0+\Delta t}\mathrm{d}t\left[\tilde{H}^{sw}(t),\tilde{\rho}(t)\right]$$

Photon transfers 1→2 and 2→1 are correlated. Solve Bloch equations $\overline{w}$ correlations.

Standing wave
(counterpropagating beams)

$$D_p = (\hbar k)^2\frac{\Gamma}{8}\frac{\Omega^2}{\Delta^2}$$

DETUNING $\Delta = \omega - \omega_0$

DECAY RATE Γ

J.P. Gordon and A. Ashkin, "*Motion of atoms in a radiation trap*," Phys. Rev. **A21**, no. 5, p. 1606, May 1980. doi: 10.1103/PhysRevA.21.1606

Momentum Diffusion: Travelling Waves, 1st Order

EQUATION OF MOTION FOR THE DENSITY MATRIX

$$\Delta\tilde{\rho}(t_0) = \frac{1}{i\hbar}\int_{t_0}^{t_0+\Delta t} \mathrm{d}t \left[\tilde{H}^{sw}(t), \tilde{\rho}(t)\right]$$

Photon transfers 1→2 and 2→1 are correlated.

$$D_p = (\hbar k)^2 \frac{\Gamma}{8}\frac{\Omega^2}{\Delta^2}$$

DETUNING $\Delta = \omega - \omega_0$

DECAY RATE Γ

Momentum Diffusion: Travelling Waves, 2nd Order

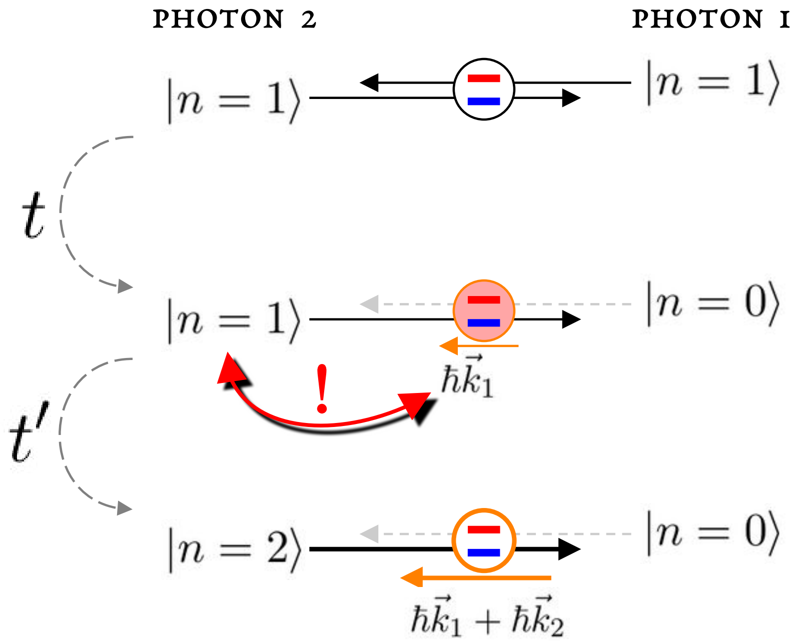
EQUATION OF MOTION FOR THE DENSITY MATRIX + 2ND ORDER

$$\Delta\tilde{\rho}(t_0) = \frac{1}{i\hbar}\int_{t_0}^{t_0+\Delta t} \mathrm{d}t \left[\tilde{H}^{rw}(t), \tilde{\rho}(t_0)\right] - \frac{1}{\hbar^2}\int_{t_0}^{t_0+\Delta t} \mathrm{d}t' \int_{t_0}^{t'} \mathrm{d}t \left[\tilde{H}^{rw}(t'), \left[\tilde{H}^{rw}(t), \tilde{\rho}(t_0)\right]\right]$$

Photon transfers 1→2 and 2→1 are independent.

$$D_p = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left[\langle \Delta \boldsymbol{p} \cdot \Delta \boldsymbol{p}\rangle - \langle \Delta \boldsymbol{p}\rangle \cdot \langle \Delta \boldsymbol{p}\rangle\right]$$

$$\frac{\Omega^2}{\Delta} \qquad \Delta \boldsymbol{p}^2 = (2\hbar\vec{k})^2$$



C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, “*Atom - Photon Interactions: Basic Process and Applications*.” Weinheim: Wiley-VCH, 2004. doi: 10.1002/9783527617197

Momentum Diffusion: Travelling Waves

Travelling waves
(counterpropagating beams)

$$D_p^{tw} \sim (\hbar k)^2 \frac{\Omega^2}{8\Delta}$$

DETUNING $\Delta = \omega - \omega_0$

Spontaneous scattering
(radiation pressure)

$$D_p^{rp} = (\hbar k)^2 \frac{\Gamma}{4} \frac{\Omega^2}{\Delta^2}$$

DECAY RATE Γ

Standing wave
(counterpropagating beams)

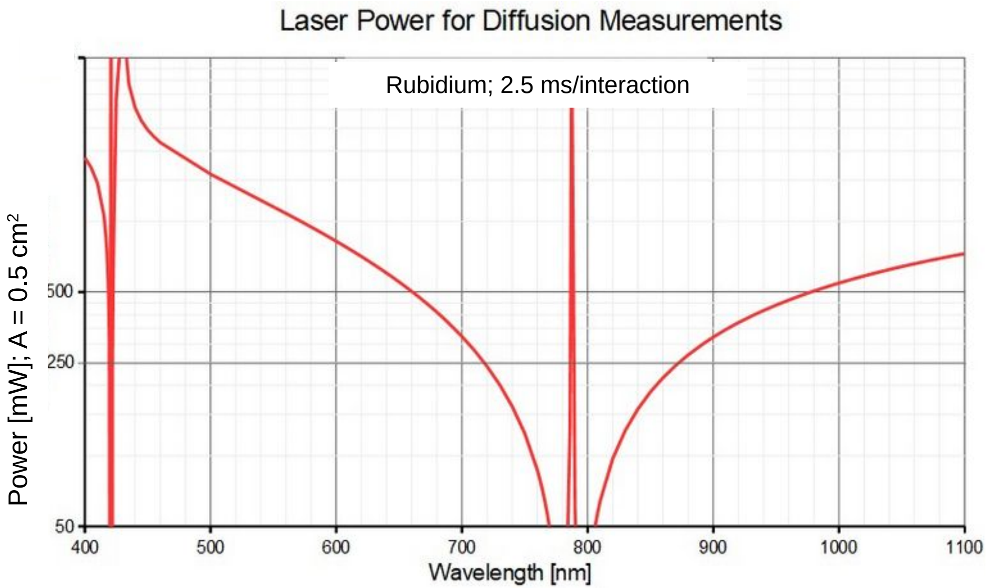
$$D_p = (\hbar k)^2 \frac{\Gamma}{8} \frac{\Omega^2}{\Delta^2}$$

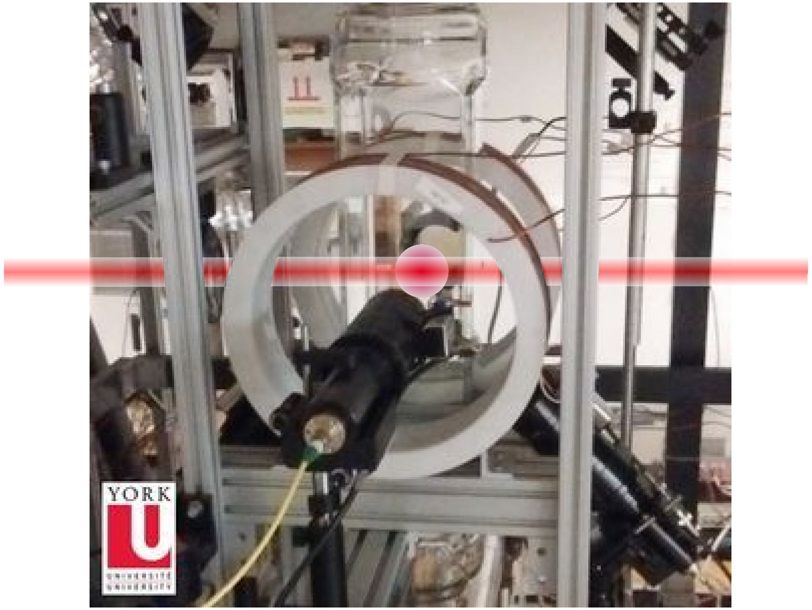
J. Dalibard and C. Cohen-Tannoudji, "*Dressed-atom approach to atomic motion in laser light: the dipole force revisited*," J. Opt. Soc. Am. B, vol. 2, no. 11, pp. 1707–1720, Nov. 1985, doi: 10.1364/JOSAB.2.001707.

Momentum Diffusion: Earth’s and Stellar Atmospheres

Measurement of the Large Momentum Diffusion for Travelling Waves

Heating of laser-cooled atoms in an atomic-interferometer



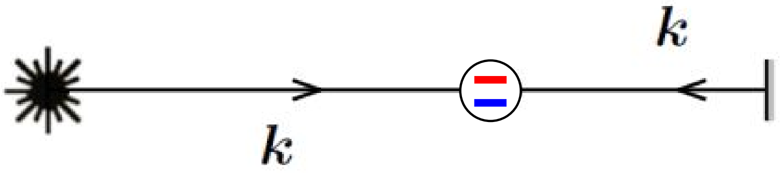


Large Momentum Diffusion from the Dipole Force of Travelling Waves

Standing wave

Dipole force is conservative

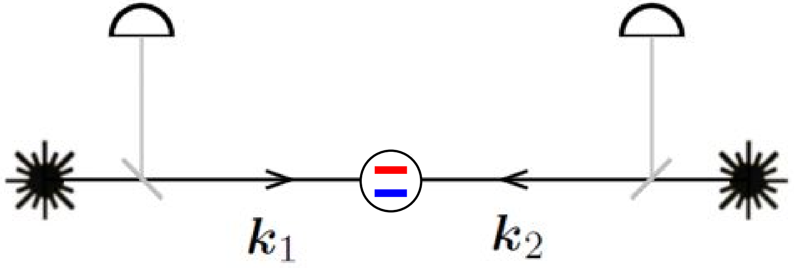
Quantum fluctuations produce **$1/\Delta^2$** diffusion



Travelling waves

Dipole force may become non-conservative

Photon transfers produces **$1/\Delta$** diffusion



Large Momentum Diffusion from the Dipole Force of Travelling Waves

Abstract

In the presence of a resonant light field, atoms of a gas spontaneously scatter photons in random directions, leading to a momentum diffusion Δ that causes the temperature to increase at a rate proportional to the spontaneous emission rate. When light is detuned from the atomic resonance by a frequency Δ, the scattering rate decreases rapidly resulting in $D_A \propto 1/\Delta^2$. In a standing wave configuration, atoms redistribute photons through stimulated emission and experience a dipole force proportional to $1/\Delta$. However, this redistribution process does not lead to momentum diffusion, as photons from a standing wave state $[\,|+\rangle + |-\rangle\,]/\sqrt{2}$ can only be transferred between counterpropagating components of the same quantum state. Since the average momentum of the state is zero, there is no momentum diffusion, indicating that the force is conservative. This property allows for experimental trapping of cold atoms with a very low heating rate.

This work addresses the case where the dipole force is generated by independent travelling waves in the weak field limit, described by a statistical mixture of photons within a reservoir. In this context, photon redistribution between travelling waves causes a momentum kick in a random direction, resulting in a dipole force that is no longer conservative. The irreversible process yields a diffusion coefficient proportional to the stimulated emission rate $D_B \propto 1/\Delta$, which is in general much larger than in the case of a standing wave. This can significantly increase the temperature of atoms interacting with broadband radiation.

We examine the implications of this large heating rate on the temperatures of Earth's and stellar atmospheres, as well as for general heating processes in astrophysics. Additionally, I propose an experiment using an atom interferometer that could detect momentum diffusion of atoms resulting from the travelling waves of independent counter-propagating beams generated by broadband laser sources.

Keywords: "dipole force" "travelling waves" "atomic heating"