Not all four dimensional Black Holes can spin

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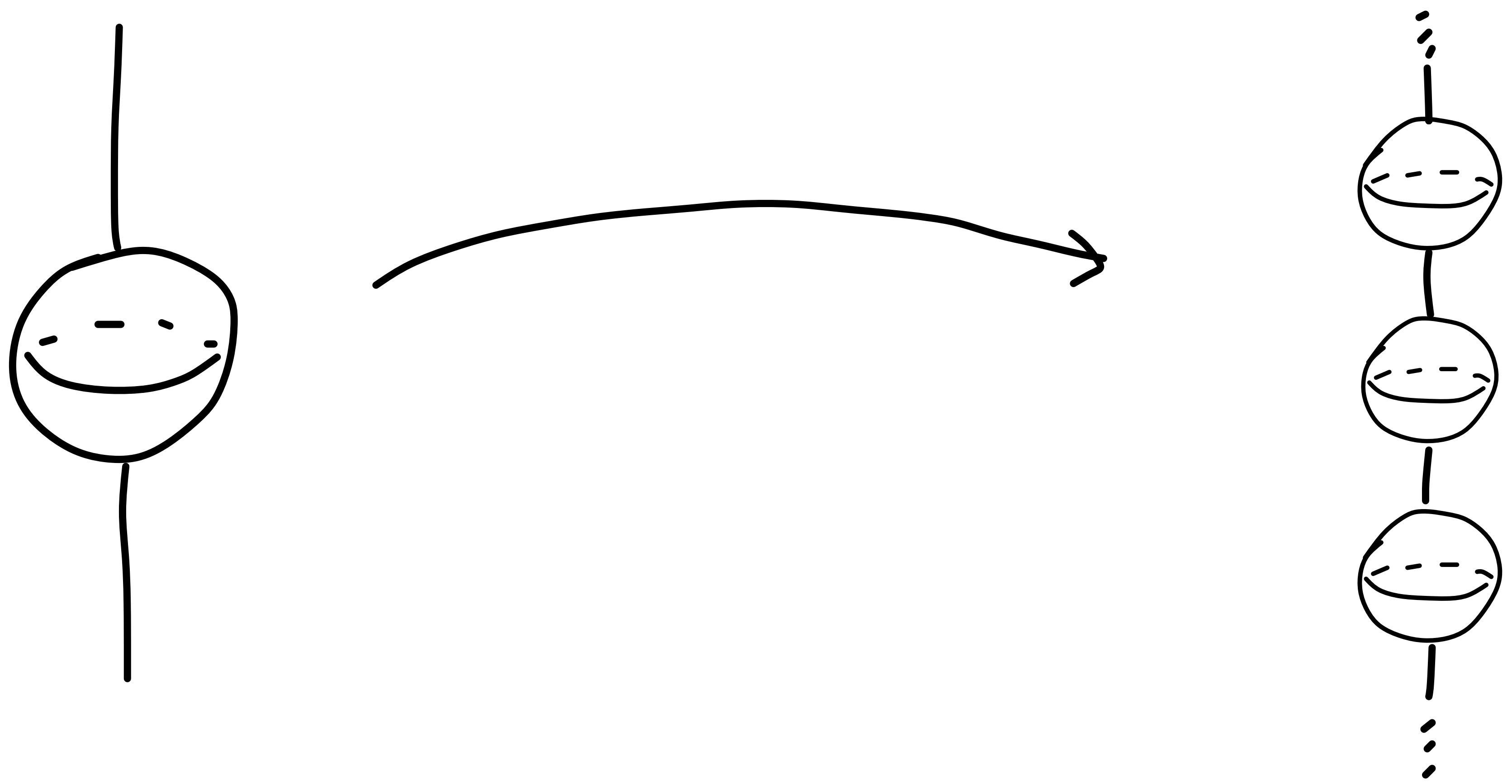
Introduction: can any black hole rotate?

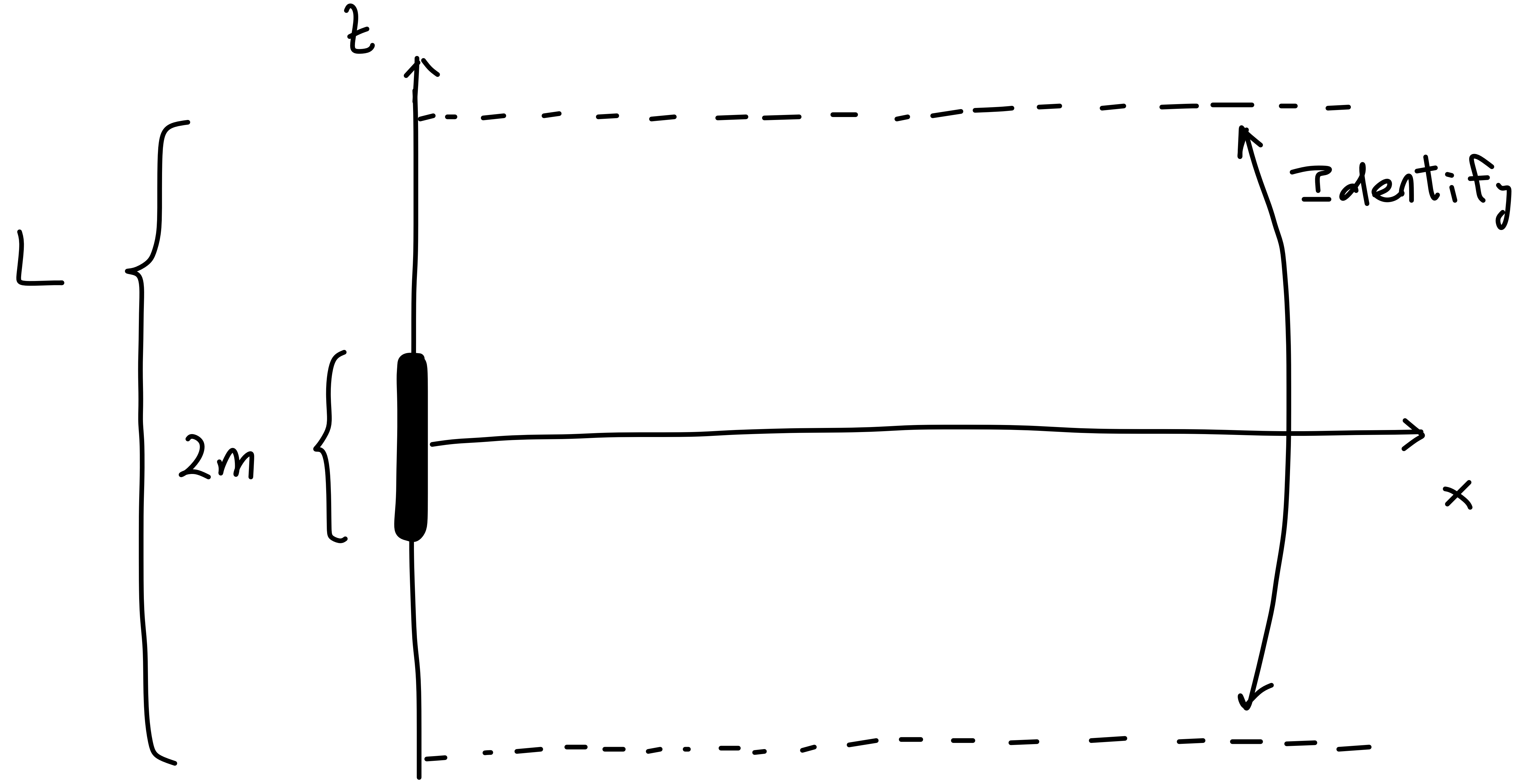
- Kerr solution (1963) as a rotating Schwarzschild solution (1915).
- Usually, the class of asymptotically flat spacetimes admit stationary deformations.
- Myers (1987) and Korotkin/Nicolai (1994) found *periodic* Schwarzschild solution:



- The asymptotic behaviour is Kasner (not flat !) —> can they admit rotation?

Setup

- Vacuum, static, axisymmetric
- Topology $\mathbb{R}_t \times S^1_\phi \times S^1_z \times \mathbb{R}_x$
- One spherical horizon per period
- Solution space given by (L, m) (non-physical)
- Physical parameters: (D, M, A)
- Stationary deformation: angular momentum J as new parameter



$$g_{Sch} = -\,e^{\sigma_{Sch}} + e^{-\sigma_{Sch}}\left(e^{2k}(dz^2 + dx^2) + \rho^2 d\phi^2\right)$$

$$\Delta\sigma_{Sch} + \left\langle \nabla\sigma_{Sch}, \nabla \ln \rho \right\rangle = 0$$

$$\sigma = \sigma_{Sch} + \sum_{n\geq 1} \sigma_{Sch}(z - nL, x) + \sigma_{Sch}(z + nL, x) - \frac{4m}{nL}$$

Stationary equations

General stationary and axisymmetric metric:

$$g = -Vdt^2 + 2Wdtd\phi + \eta d\phi^2 + \frac{1}{\eta}\underbrace{e^{2\gamma}(dx^2 + dz^2)}_{q}$$

$V = (\rho^2 - W^2)/\eta, \quad W = \eta\Omega, \quad d\Omega = -\rho(\star d\omega)/\eta, \quad f = \ln\rho$

Reduced Einstein Equations:

$$Ric[g] = 0 \quad \Rightarrow \quad \begin{cases} \Delta\eta + \langle \nabla f, \nabla\eta \rangle = \dfrac{|\nabla\eta|^2 - |\nabla\omega|^2}{\eta} \\ \Delta\omega + \langle \nabla f, \nabla\omega \rangle = 2\dfrac{\langle \nabla\omega, \nabla\eta \rangle}{\eta} \\ Ric[q] - \nabla\nabla f = \dfrac{\nabla\eta\nabla\eta + \nabla\omega\nabla\omega}{\eta^2} + \nabla f\nabla f \end{cases}$$

Main Theorem: non-existence result

Theorem. ([JP, Reiris, **CQG 42 (2025) 8**, arXiv:2407.16960])

MKN black holes with $L/m < 4$ cannot be put into stationary rotation, along a deformation family with L/m varying continuously and with q metrically complete at infinity.

Proof. See [arXiv:2407.16960] for details. Estimates for geometric quantities.

Theorem. *MKN black holes with $D/\sqrt{A} < 1/12$ cannot be put into stationary rotation*

Proof. Estimates of D in terms of L, m.

Heuristics: Smarr formula

- Far away from axis —> massive cylinder rotating with $J > 0$.
- Lewis/van Stockum models for asymptotic: one *assumes* ρ is coordinate

$$V \propto \rho^{1-\alpha}, \quad W \propto \rho^{1-\alpha}, \quad \eta \propto \rho \sinh(\alpha \ln(\rho)), \quad e^{2\gamma - \ln \eta} \propto \rho^{(\alpha^2-1)/2}, \text{ with } 0 < \alpha < 1$$

- Komar mass relative to ∂_t

$$m + 2\Omega J = \frac{\kappa A}{4\pi} + 2\Omega J \underset{\rho_0 \to 0}{=} \frac{1}{4} \int_{\rho=\rho_0} \nabla^{[\mu} \partial_t^{\nu]} d\Sigma_{\mu\nu} \underset{\rho_0 \to \infty}{=} (1-\alpha)\frac{L}{4}$$

$$\boxed{\Omega J > 0 \quad \Rightarrow \quad m < \frac{L}{4}}$$

Further into periodic solutions…

- Multiple horizons solutions ([arXiv:2602.22501])
- Numeric analysis for existence of solutions:
  - Heat flow for harmonic map ([arXiv:2210.1289]), O. Ortiz, JP, M. Reiris.
  - Inverse scattering method ([arXiv:2505.05698]), D.Korotkin, JP
- Conjectures and open problems:
  - What about extremal periodic black holes?
  - Formal proof of existence when $m < L/4$
  - Wick rotation —> interest in geometry of 4-manifolds (instantons)

Thank you !