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Bosonization of Noise Effects on Nonlocal Open Quantum Dynamics

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Bosonization of Noise Effects in Nonlocal Quantum Dynamics

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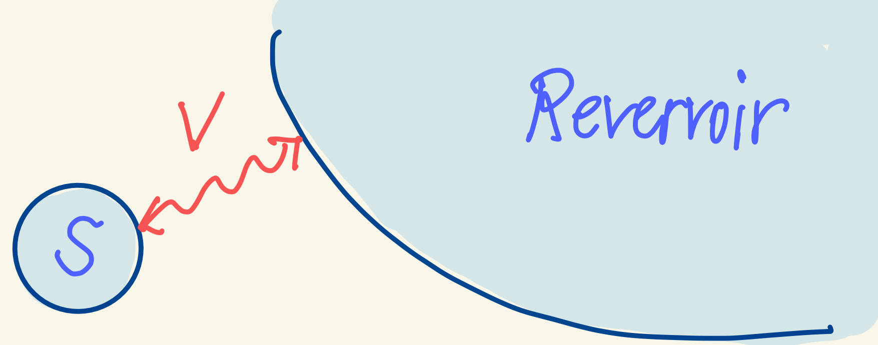


Open quantum system

Hamiltonian: $H = H_S + H_R + V$

Initial density matrix: $\rho_S \otimes \rho_R$

Reduced system state: $\rho_S(t) = \mathrm{Tr}_R\left[e^{-itH}(\rho_S \otimes \rho_R)\,e^{itH}\right]$

Role of reservoir correlations

Dyson expansion

multi-commutator

interaction picture

$$\rho_S(t) = \sum_{n\geq 0} (-i)^n \int_{0\leq t_n \leq \dots \leq t_1 \leq t} \mathrm{Tr}_R \left[V(t_1), \left[V(t_2), \dots \left[V(t_n), \rho_S \otimes \rho_R \right] \dots \right] \right]$$

Multi-time reservoir correlation functions

$$\mathrm{Tr}_R \left(\rho_R \, V(t_{i_1}) \dots V(t_{i_n}) \right) \equiv \langle V(t_{i_1}) \dots V(t_{i_n}) \rangle_R$$

Uncorrelated :

$$\langle V(t_{i_1}) \cdots V(t_{i_n}) \rangle_R = \langle V(t_{i_1}) \rangle_R \cdots \langle V(t_{i_n}) \rangle_R$$

Pairwise correlated :

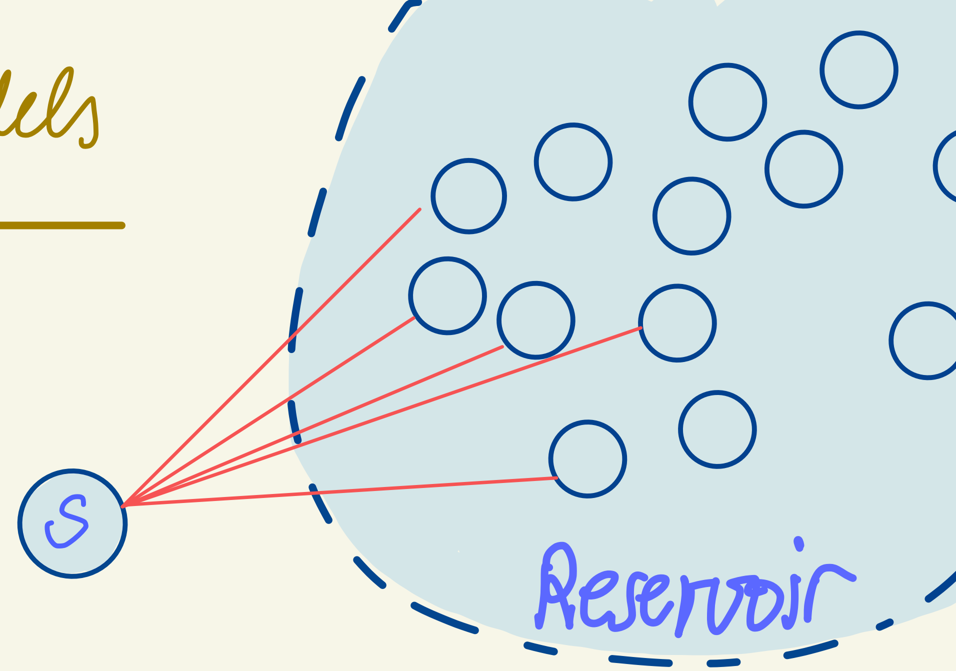
$$\langle V(t_{i_1}) \cdots V(t_{i_n}) \rangle_R = F\left(\langle V(t_i) V(t_j) \rangle_R \right)$$

Higher order correlations ...

$\Leftrightarrow$ Property of reservoir state ρ_R

Mean field & mesoscopic models

R: $M \to \infty$ constituents



Structured

Symmetry:

$$\rho_{R,M} = \rho_R \otimes \cdots \otimes \rho_R$$

$$V_M = G \otimes \alpha \sum_{m=1}^{M} v^{[m]},$$

$$v^{[m]} = 1 \otimes \cdots \underbrace{v}_{m} \cdots \otimes 1$$

system op.

scaling factor

Reservoir correlation function

$$\sum_{m_1,\dots,m_n=1}^{M} \mathrm{Tr}_{R,M}\left(\rho_R \otimes \cdots \otimes \rho_R \; v^{[m_1]}(t_1) \cdots v^{[m_n]}(t_n) \right)$$

For most terms ($O(M^n)$) the m_j are distinct

$\Rightarrow \mathrm{Tr}_{R,M}(\cdots) = \langle v(t_1) \rangle \cdots \langle v(t_n) \rangle$ each

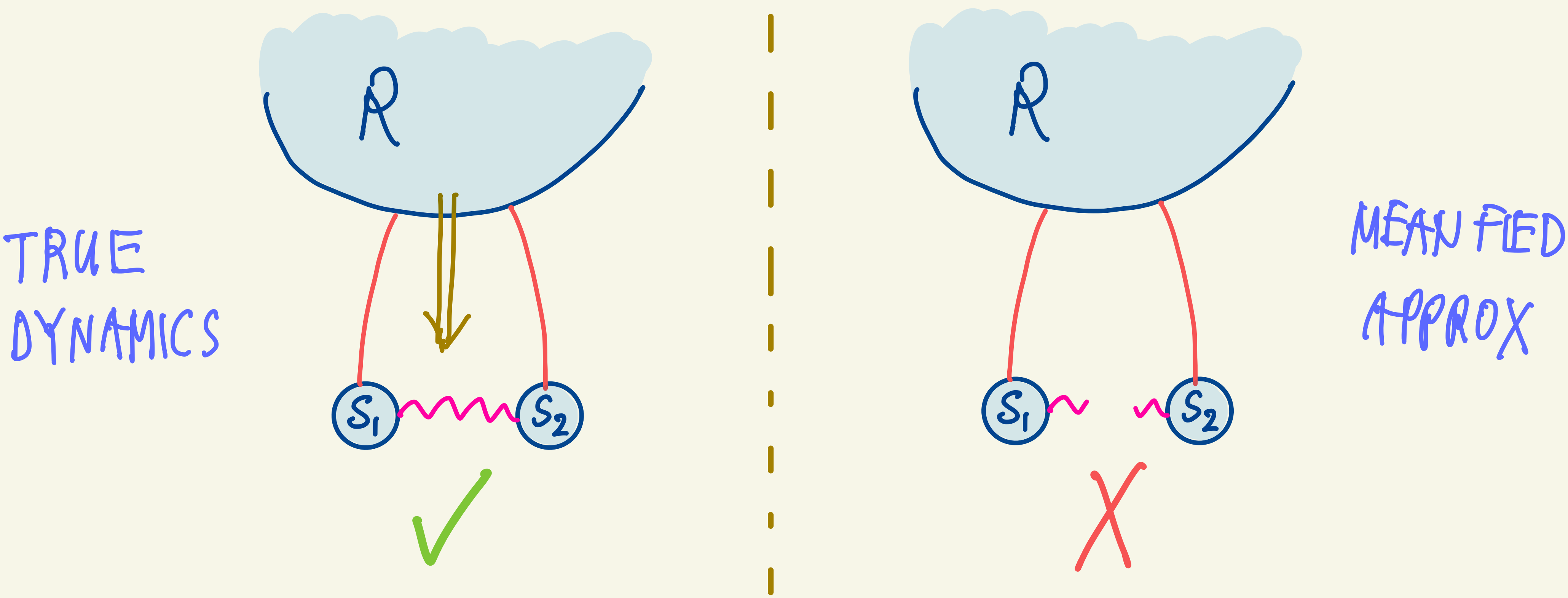
Scaling $v \mapsto \frac{1}{M} v$ selects those terms as $M \to \infty$

Mean field scaling

$$H = H_S + H_R + G \otimes \frac{1}{M} \sum_{m=1}^{M} v^{[m]}$$

$M \to \infty$ removes effects due to bath correlations.

Example: entanglement suppression



Reservoir correlation function, again:

$$\sum_{m_1,\dots,m_n=1}^{M} \mathrm{Tr}_{R,M}\left(\rho_R \otimes \cdots \otimes \rho_R \; v^{[m_1]}(t_1) \cdots v^{[m_n]}(t_n) \right)$$

$\langle v(t) \rangle = 0 \Rightarrow$ previously dominant contribution $= 0$

2nd order correlations: | pairwise equal indices m :
$O(M^{n/2})$ terms in multiple sum

Scaling $v \mapsto \frac{1}{\sqrt{M}} v$ selects those terms as $M \to \infty$

Fluctuation scaling

$$H = H_S + H_R + G \otimes \frac{1}{\sqrt{M}} \sum_{m=1}^{M} v^{[m]}$$

$M \to \infty$ describes effects of second order bath correlations

$$\mathrm{Tr}_{R,M}\left(\rho_R \otimes \cdots \otimes \rho_R \left(\frac{1}{\sqrt{M}} \sum_{m=1}^{M} v(t_1)^{[m]}\right) \cdots \left(\frac{1}{\sqrt{M}} \sum_{m=1}^{M} v(t_n)^{[m]}\right)\right) \to ?$$

Quantum Central Limit Theorem

Proposition 2 *Let w_j, $j = 1, \ldots, n$, be operators on $\mathcal{H}_{\mathrm{R}}$ such that $\omega_{\mathrm{R}}(w_j) \equiv \mathrm{tr}_{\mathrm{R}}(\rho_{\mathrm{R}} w_j) = 0$ and set*

$$W_j = \frac{1}{\sqrt{M}} \sum_{m=1}^{M} w_j^{[m]}.$$

Then we have

$$\lim_{M \to \infty} \mathrm{tr}_{\mathrm{R},M}\Big(\rho_{\mathrm{R},M} W_1 \cdots W_n\Big) = \begin{cases} \displaystyle\sum_{\pi \in \mathcal{P}_n} \prod_{j=1}^{n/2} \omega_{\mathrm{R}}\Big(w_{\pi(2j-1)} w_{\pi(2j)}\Big) & \text{for } n \text{ even} \\ 0 & \text{for } n \text{ odd} \end{cases}$$

Wick's theorem

Symmetry (initial state & S-R interaction)

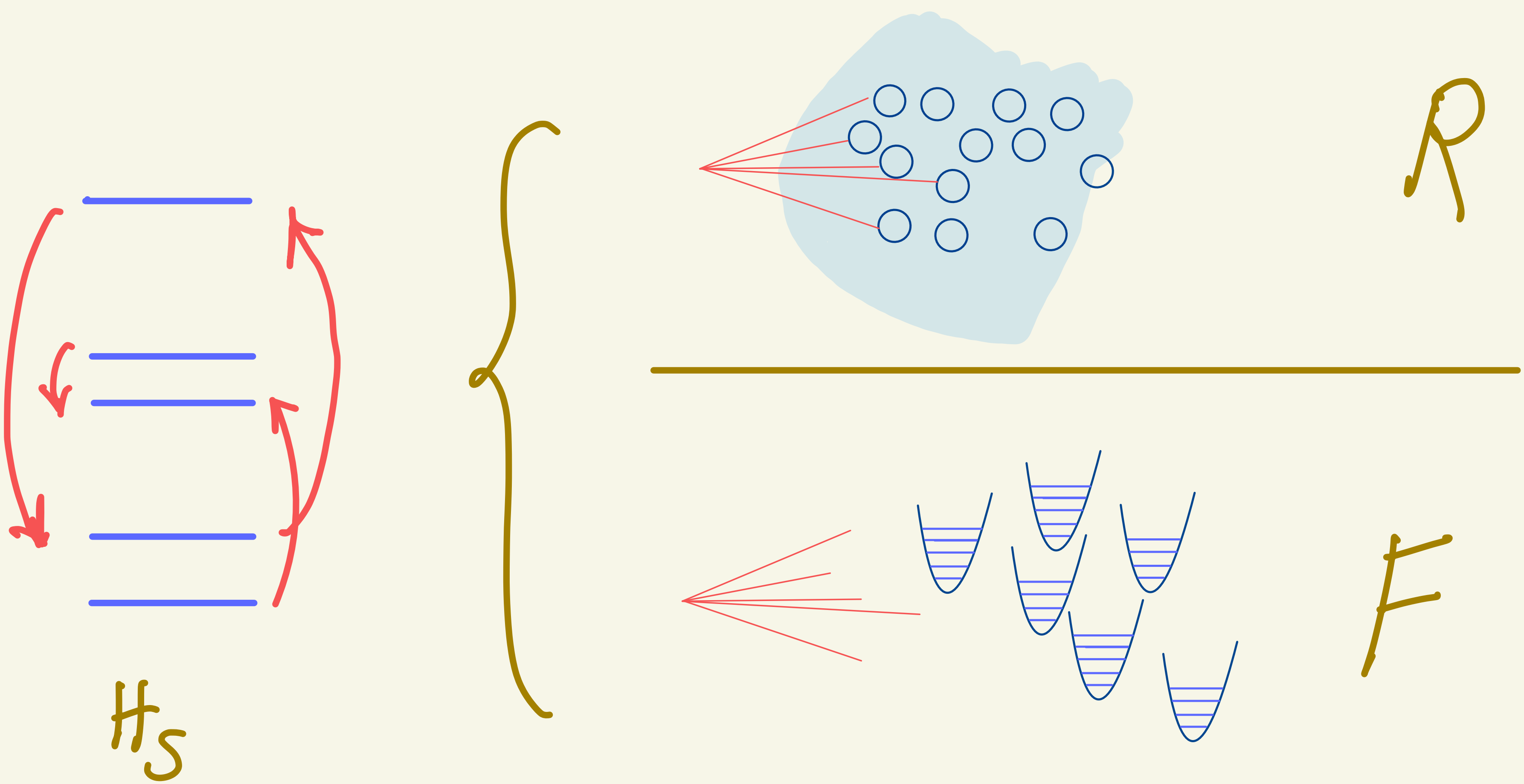
+ fluctuation scaling ($\frac{1}{\sqrt{N}}$)

= Bosonic, Gaussian structure of reservoir correlations

Can replace R by generic
Gaussian (Harmonic Oscillator!)
Reservoir: $R \rightsquigarrow F$

Equivalent "Fluctuation" Reservoir F

Collection of harmonic oscillators coupled to S which generate SAME transitions in S as R does



Design of fluctuation reservoir F

Original reservoir R	Equivalent F reservoir
$\rho_R = \sum_j p_j \, |x_j\rangle\langle x_j|$ (stat.)	$\forall \, (k, \ell) \in \mathcal{M}$
$h_R = \sum_j E_j \, |x_j\rangle\langle x_j|$	$\longrightarrow \; a_{k\ell}, \; a^\dagger_{k\ell}$
$\mathcal{M} = \{ (k, \ell) : \; p_k \neq 0 \;\; \& \;\; \langle x_\ell , v \, x_k \rangle \neq 0 \}$	$\omega_{k\ell} = E_\ell - E_k$
Allowed transitions $E_k \to E_\ell$	$H_F = \sum_{(k,\ell) \in \mathcal{M}} \omega_{k\ell} \, a^\dagger_k \, a_\ell$

	$S \leftrightarrow R$	$S \leftrightarrow F$
Interaction	$G \otimes \frac{1}{\sqrt{M}} \sum_{m=1}^{M} v^{[m]}$	$G \otimes \underbrace{\sum_{(k,\ell)\in\mathcal{M}} \sqrt{p_k}\,\langle \chi_k, v\chi_\ell\rangle\, a^{\dagger}_{k\ell} + \text{H.C.}}_{\phi \text{ field operator}}$
Initial state	$\rho_S \otimes (\rho_R \otimes \cdots \otimes \rho_R)$	osci_vacuum $\rho_S \otimes \vert 0\rangle\langle 0\vert \otimes \cdots \otimes \vert 0\rangle\langle 0\vert$

Original system dynamics

$$\rho_{\mathrm{S}}(t) = \lim_{M\to\infty} \mathrm{tr}_{\mathrm{R},M}\left(e^{-itH_{\mathrm{SR},M}}\rho_{\mathrm{SR},M}e^{itH_{\mathrm{SR},M}}\right)$$

Theorem $\forall t \in \mathbb{R}$:

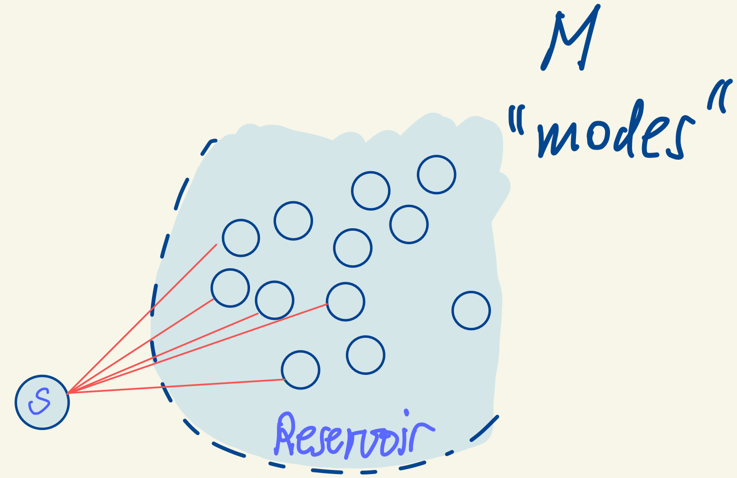
$$\rho_S(t) = \mathrm{Tr}_F\left(e^{-itH_{SF}} \, \rho_S \otimes \big(|0\rangle\langle 0| \otimes \cdots \otimes |0\rangle\langle 0|\big) \, e^{itH_{SF}} \right)$$

$$H_{SF} = H_S + H_F + G \otimes \phi$$

Open system dynamics is equivalently given by coupling to new 'universal' Bosonic, Gaussian reservoir.

Eg: Jaynes-Cummings model

Summary

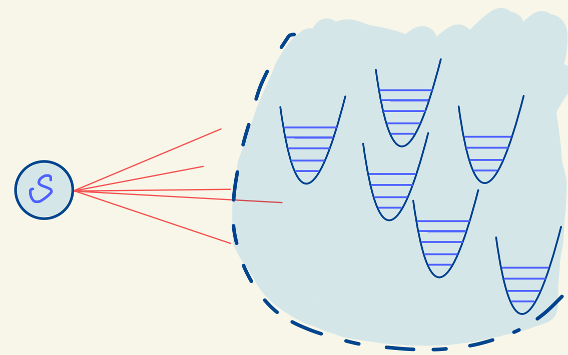


- System coupled to bath

  symmetry & mesoscopic

  (fluctuation) scaling $\frac{1}{\sqrt{M}}$

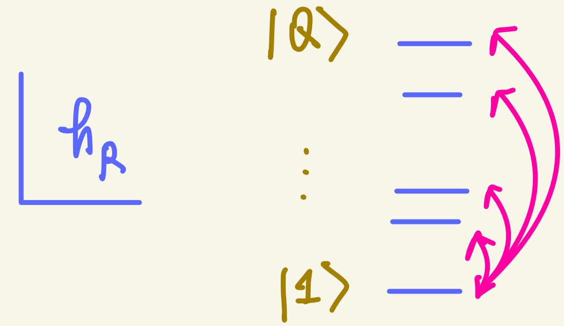
  $\equiv$ system coupled to bath of harmonic oscillators



- Captures dynamic effects caused by lowest bath correlations

- Bosonic, Gaussian reservoirs are generic & can be used to simulate open syst. dyn.

Explicit example



$|1\rangle \leftrightarrow |q\rangle$

S : arbitrary

R : Q-level system
$$h_R = \sum_{q=1}^{Q} E_q \, |q\rangle\langle q| \,, \quad \rho_R = |1\rangle\langle 1|$$

Interaction :
$$\sum_{q=2}^{Q} G_q \otimes \frac{1}{\sqrt{M}} \sum_{m=1}^{M} \big(|q\rangle\langle 1|\big)^{[m]} + H.C.$$

Equivalent F reservoir : Modes $a_q^\dagger, a_q$, $q = 2, \dots, Q$

$$H_{SF} = H_S + \sum_{q=2}^{Q} (E_q - E_1)\, a_q^\dagger a_q + \sum_{q=2}^{Q} G_q \otimes a_q^\dagger + H.C.$$