# Bosonization of Noise Effects in Nonlocal Quantum Dynamics

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Quantum systems that interact nonlocally with an environment are paradigms for exploring collective phenomena. They naturally emerge in various physical contexts involving long-range, many-body interactions. We consider a general class of such open systems characterized by a coupling to the environment that is inversely proportional to the square root of the environment size. We show that the induced system dynamics has a universal bosonic nature: the same evolution arises from coupling the system to a collection of noninteracting bosonic modes, independently of the microscopic structure of the original environment. This emergent "bosonization" of the environment's influence results from the scaling of the coupling in the thermodynamic limit and is a manifestation of the quantum central limit theorem. While the effect has been observed in specific models before, we show that it is, in fact, a universal feature.

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Collective phenomena in quantum many-body systems arise when many microscopic degrees of freedom interact. The dynamics cannot be reduced to the properties of individual particles, but instead it reflects long-range correlations, entanglement, and quantum coherence spread over many constituents. A classic example is superradiance in quantum optics, where a group of atoms interacts with light of wavelength much larger then the atom separation, causing the group to collectively emit a light pulse of much higher intensity than would be expected from independently emitting atoms [1–8]. The loss of quantum coherence, called decoherence, between individual systems (qubits) through their interaction with a common noise source (environment, bath), is another collective phenomenon with wide-reaching consequences [9–12].

Collective phenomena induced by long-range interactions are often described with *mean-field theories* [13–21], which are also used to derive effective nonlinear evolution equations of complex quantum systems in the mathematical literature [22–28]. A system interacting with $M$ components (particles, modes, etc.) of a reservoir typically has an interaction energy of the order of $M$. In order to balance the interaction energy and the system energy as $M \to \infty$, mean-field models consider a scaling of the coupling strength $g \propto 1/M$. This scaling suppresses all but the lowest (zero) order correlation effects of the reservoir on the effective system dynamics—even though the latter depends on reservoir correlation functions of all orders. As a consequence, dynamical effects arising from fluctuations (bath correlations) cannot be captured by the mean-field approximation. For instance, mean-field theory predicts the relaxation to a stationary state in models of a quantum gas in an optical cavity; however, in reality nonstationarity is observed for long times, due to higher order correlation effects [29]. Systems coupled indirectly via a common reservoir become entangled [30]; however, entanglement in a multipartite system remains unaffected by a mean-field-type coupling to an environment [29,31]. In order to include effects caused by fluctuations, one can consider the scaling $g \propto 1/\sqrt{M}$, which unveils the result of first-order bath correlations on the system dynamics. This scaling is said to be of *fluctuation*, or *mesoscopic*, type. The terminology stems from the notion of *fluctuation observables*, which are extensive observables divided by the square root of the volume [16,17,32–37]. A rigorous treatment of the fluctuation regime is more demanding than the mean-field one, because correlation effects within the reservoir have to be taken into account. On a heuristic and numerical level, higher order cumulant expansions have also been considered for a class of central spin systems [38].

In those systems a single "central" spin interacts nonlocally ("one-to-all") with a large number of "bath" or environmental spins, making up the reservoir R [39–49]. The mean field and fluctuation regimes of a central spin $1/2$ coupled to bath spins $1/2$ is analyzed in [50]. It is shown that in the fluctuation regime ($g \propto 1/\sqrt{M}$) and $M \to \infty$, the effect of the reservoir can be encoded in a single bosonic mode communicating with the central spin,

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which results in an equivalent Jaynes-Cummings model. The setup in [50] is specific in several respects. Other than being two-level systems, S and R are coupled linearly via the collective bath ladder operators, conserving the total number of excitations. The reservoir dynamics is taken to be purely dissipative, without a Hamiltonian component, purely driven by a pump and decay term between the two levels, and the reservoir is taken to be in the resulting equilibrium.

In this Letter we explain that arbitrary quantum systems S mesoscopically coupled to reservoirs R of arbitrary $M \to \infty$ constituents, evolve as if they were linearly coupled to a set F of oscillators in their ground state. The equivalent bosonic fluctuation reservoir F is constructed in a simple, explicit, and physically transparent manner. We show that for generic SR models where components of R have $Q \geq 2$ energy levels, $Q-1$ independent oscillators will arise in the equivalent fluctuation reservoir F. We show that entanglement is created within bipartite systems S by the indirect mesoscopically scaled contact with reservoirs. Our results hold for arbitrary initial states of R, which are stationary under the uncoupled, Hamiltonian reservoir dynamics. The fluctuation reservoir F depends explicitly on the initial state of R. We also identify the quantum central limit theorem as the universal mechanism underlying the equivalence R $\leftrightarrow$ F. Our key observation is that the SR coupling operators behave as noncommutative, normally distributed random variables ($M \to \infty$) [51–55]. Bosonic systems, such as collections of oscillators in quasifree (Gaussian) states, are likewise characterized by normally distributed correlation functions. By matching these correlations, we show that we can replace the original reservoir R by an equivalent set of harmonic oscillators F. Our results are mathematically exact.

*Model*—A quantum system S is coupled to a quantum reservoir R consisting of $M$ independent, identical components [see Fig. 1 (left part)]. The total complex is described by the Hilbert space

$$\mathcal{H}_{\mathrm{SR},M} = \mathcal{H}_{\mathrm{S}} \otimes \mathcal{H}_{\mathrm{R},M}, \qquad \mathcal{H}_{\mathrm{R},M} = \mathcal{H}_{\mathrm{R}} \otimes \cdots \otimes \mathcal{H}_{\mathrm{R}} \tag{1}$$

($M$-fold product). We assume that $\dim \mathcal{H}_{\mathrm{S}} < \infty$ and $\dim \mathcal{H}_{\mathrm{R}} < \infty$. The dynamics is generated by the interacting hermitian Hamiltonian on $\mathcal{H}_{\mathrm{SR},M}$,

$$H = H_{\mathrm{S}} + H_{\mathrm{R},M} + V_M, \tag{2}$$

where $H_{\mathrm{S}}$ is the (any) Hamiltonian of S and

$$H_{\mathrm{R},M} = \sum_{m=1}^{M} h_{\mathrm{R}}^{[m]} \tag{3}$$

is the sum of single-component Hamiltonians $h_{\mathrm{R}}^{[m]}$. Here, $h_{\mathrm{R}}$ is a fixed, arbitrary, Hermitian operator, and we use the notation

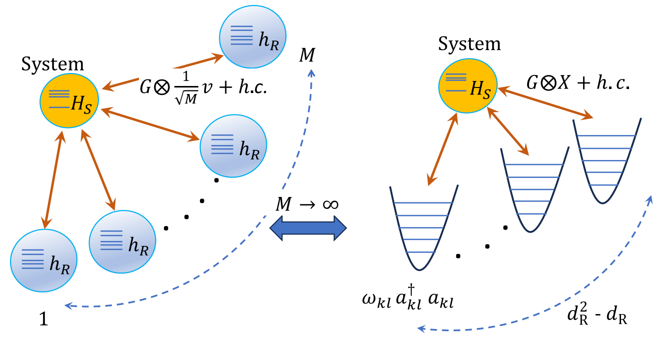


FIG. 1. Illustration of the main result. A system S with Hamiltonian $H_{\mathrm{S}}$ interacting with two different reservoirs. Left sketch: S interacts equally with the $M$ elements of a reservoir, each a $d_{\mathrm{R}}$-level system with Hamiltonian $h_{\mathrm{R}}$. The interaction operator is $\propto 1/\sqrt{M}$, called a mesoscopic scaling. Right sketch: S interacts with a reservoir of (maximally) $d_{\mathrm{R}}^2$ independent bosonic modes indexed by $(k, l)$, all in their ground state. Each oscillator corresponds to a transition $E_k \to E_l$ between energy levels of $h_{\mathrm{R}}$ allowed by the interaction SR operator $v$. The interaction operator $X$ of S with the bosonic modes is linear in the creation and annihilation operators. The left-right arrow $\leftrightarrow$ indicates our *main result*: the reduced dynamics of S obtained from the left model (as $M \to \infty$) and the right model is the same. The result holds for arbitrary S and regardless of the fine details of the components of R.

$$X_{\mathrm{R}}^{[m]} = \mathbf{1} \otimes \cdots \otimes \mathbf{1} \otimes X_{\mathrm{R}} \otimes \mathbf{1} \otimes \cdots \otimes \mathbf{1}, \tag{4}$$

where the operator $X_{\mathrm{R}}$ sits on the $m$th factor in the $M$-fold product. The system-reservoir interaction operator in (2) is given by

$$V_M = \sum_{q=1}^{Q} G_q \otimes \frac{1}{\sqrt{M}} \sum_{m=1}^{M} v_q^{[m]} + \mathrm{H.c.}, \tag{5}$$

for some $Q \geq 1$, where $G_q$ and $v_q$ are (not necessarily Hermitian) operators on $\mathcal{H}_{\mathrm{S}}$ and $\mathcal{H}_{\mathrm{R}}$, respectively.

We consider initial system-reservoir states (density matrices) of the form

$$\rho_{\mathrm{SR},M} = \rho_{\mathrm{S}} \otimes \rho_{\mathrm{R},M}, \qquad \rho_{\mathrm{R},M} = \rho_{\mathrm{R}} \otimes \cdots \otimes \rho_{\mathrm{R}}, \tag{6}$$

with an arbitrary system state $\rho_{\mathrm{S}}$ and where each component of R is in the same state $\rho_{\mathrm{R}}$, which is assumed to be stationary (for example a thermal equilibrium state),

$$e^{-ith_{\mathrm{R}}} \rho_{\mathrm{R}} e^{ith_{\mathrm{R}}} = \rho_{\mathrm{R}}. \tag{7}$$

The interaction operators $v_q$ are normalized (shifted, centralized) to have vanishing expectation,

$$\mathrm{tr}_{\mathrm{R}}(\rho_{\mathrm{R}} v_q) = 0. \tag{8}$$

The object of interest is the reduced system density matrix in the thermodynamic limit,

$$\rho_{\rm S}(t) = \lim_{M\to\infty} {\rm tr}_{{\rm R},M}\left(e^{-itH_{{\rm SR},M}}\rho_{{\rm SR},M}e^{itH_{{\rm SR},M}}\right), \tag{9}$$

where ${\rm tr}_{{\rm R},M}$ stands for the partial trace over the reservoir degrees of freedom.

*Consequences of symmetry and scaling*—We now discuss the mechanism and ideas that lead to our main result (19). For ease of presentation we do this here for an interaction (5) with $Q=1$ and $G=G^\dagger$, $v=v^\dagger$. A mathematical proof of the general case is given in [56]. The model has two important features: (i) *Symmetry*. The full Hamiltonian (2) and the reservoir initial state (6) are symmetric under the exchange of reservoir components. (ii) $1/\sqrt{M}$ *scaling*. The interaction (5) is scaled with this prefactor (termed *mesoscopic*).

The contribution of the reservoir to the dynamics is encoded in the multitime correlation functions

$$C_M(t_1,\ldots,t_n) \coloneqq {\rm tr}_{{\rm R},M}\left(\rho_{{\rm R},M}\frac{1}{\sqrt{M}}S(t_1)\cdots\frac{1}{\sqrt{M}}S(t_n)\right), \tag{10}$$

where $S(t)=\sum_{m=1}^M v^{[m]}(t)$ [see Eq. (5)], with $v(t)=e^{ith_{\rm R}}ve^{-ith_{\rm R}}$. The product $S(t_1)\cdots S(t_n)$ in (10) is a multiple sum over indices $m_1,\ldots,m_n$. Each index $m$ indicates the reservoir element on which the operator $v^{[m]}$ acts [see Eq. (4)]. At first sight one might think that (10) would diverge as $M\to\infty$, because the multiple sum has $M^n$ terms and the scaling only provides a factor $M^{-n/2}$. However, as ${\rm tr}[\rho_{\rm R}v(t)]=0$ [see Eqs. (7) and (8)] and by the symmetry and product form of $\rho_{{\rm R},M}$, (6), terms in the multiple sum vanish whenever at least one $m$ differs from the others. So indices need to be paired up or appear in larger clusters for a nonzero contribution. A combinatorial argument shows that for $M\to\infty$ all terms with larger clusters disappear as their number grows less than $O(M^{n/2})$. The scaling $M^{-n/2}$, with $M\to\infty$, selects precisely the terms where the indices $m_1,\ldots,m_n$ are paired, with each pair corresponding to the product of two operators $v(t)v(t')$ acting on the same reservoir "site" or component. This implies that the multicorrelation functions $C_M(t_1,\ldots,t_n)$ are expressed for $M\to\infty$, via the two-point function

$$c(t,t') \coloneqq {\rm tr}_{\rm R}(\rho_{\rm R}v(t)v(t')) \tag{11}$$

by Wick's theorem (see Ref. [56] for a simple proof), just as in quantum field theory. Letting $\mathcal{P}_n$ denote the set of all pairings $\pi$, we have

$$\lim_{M\to\infty} C_M(t_1,\ldots,t_n) = \begin{cases} \sum_{\pi\in\mathcal{P}_n}\prod_{j=1}^{n/2} c(t_{\pi(2j-1)},t_{\pi(2j)}) & n \text{ even} \\ 0 & n \text{ odd}. \end{cases} \tag{12}$$

The formula (12) expresses the quantum central limit theorem. We prove it in Proposition 2 in [56].

*Equivalent bosonic (fluctuation) modes* F—We explain now how to build a collection of harmonic oscillators that reproduces the two-point function (11). In the common eigenbasis $\{\chi_j\}$ of $\rho_{\rm R}$ and $h_{\rm R}$ we have

$$\rho_{\rm R} = \sum_{j=1}^{d_{\rm R}} p_j|\chi_j\rangle\langle\chi_j|, \qquad h_{\rm R} = \sum_{j=1}^{d_{\rm R}} E_j|\chi_j\rangle\langle\chi_j|, \tag{13}$$

where the $0\le p_j\le 1$ are the populations of $\rho_{\rm R}$ and the $E_j\in\mathbb{R}$ are the energies of $h_{\rm R}$ (possibly with $E_j=E_{j'}$ for different indices). Denoting the matrix elements of $v$ as $v_{kl}=\langle\chi_k|v|\chi_l\rangle$ the two-point function (11) reads

$$c(t,t') = \sum_{k,l=1}^{d_{\rm R}} p_k|v_{kl}|^2 e^{-i(E_l-E_k)(t-t')}. \tag{14}$$

We associate one harmonic oscillator $a_{kl}^\dagger, a_{kl}$ to each pair $(k,l)\in\mathcal{M}' \coloneqq \{(k,l)\,{\rm s.t.}\,p_k\neq 0, v_{kl}\neq 0\}$, that is to each allowed ($v_{kl}\neq 0$) transition $E_k\to E_l$ provided $E_k$ populated in $\rho_{\rm R}$. We take the frequency of the oscillators to be just the Bohr transition frequency, $\omega_{kl}=E_l-E_k$. This means physically that when the oscillators are coupled by an energy exchange interaction with S, they can induce the same transitions in S as the original reservoir. The oscillator Hamiltonian is $H_{\rm F}=\sum_{(k,l)\in\mathcal{M}'}\omega_{kl}a_{kl}^\dagger a_{kl}$. We introduce the field operator $\phi(t)=\sum_{(k,l)\in\mathcal{M}'}\sqrt{p_k}v_{kl}e^{i\omega_{kl}t}a_{kl}^\dagger + {\rm H.c.}$ It is easily seen that its two-point function in the oscillator ground state $\rho_{\rm F}=|0\rangle\langle 0|\otimes\cdots\otimes|0\rangle\langle 0|$ is precisely (14), ${\rm tr}(\rho_{\rm F}\phi(t)\phi(t'))=c(t,t')$. This implies that the new bath F consisting of all the oscillators, coupled to S linearly via the interaction operator $G\otimes\phi(0)$, induces the same dynamics (9) in S as the original reservoir. One shows this by realizing that the Dyson series expansions of the dynamics of S resulting from the SF interaction and from the SR interaction are exactly the same. Indeed, these expansions express the system dynamics in terms of multicorrelation functions of the corresponding reservoirs (R or F), and for both reservoirs, those multicorrelation functions are expressed in terms of the two-point functions via Wick's theorem [as in (12)]. By design the two-point functions of R and F are the same, hence so are the multicorrelation functions and thus so are the Dyson series expansions. We present the technical details of the arguments above in Secs. S and T of [56]. Above is the outline of the result and the main ideas for $Q=1$, $G^\dagger=G$, and $v^\dagger=v$ in (5). Now we present the general case.

*Main result*—Consider the models (1)–(9). We introduce the "fluctuation reservoir" F, which is made of independent harmonic oscillators $a_{kl}^\dagger, a_{kl}$ in their ground state

$$\rho_{\rm F} = |0\rangle\langle 0|\otimes\cdots\otimes|0\rangle\langle 0|. \tag{15}$$

The oscillators $a_{kl}^\dagger$, $a_{kl}$ are indexed by $(k,l)$ such that $E_k$ is an energy level occupied in $\rho_R$ ($p_k \neq 0$), and at least one of the interaction operators $v_q$ in (5) allows the transition $E_k \to E_l$ or $E_l \to E_k$. In other words, the oscillators are indexed by $(k,l)$ in the set of modes,

$$\mathcal{M} := \{(k,l)\,\text{s.t.}\, p_k \neq 0 \quad \text{and} \\ [v_q]_{kl} \text{ or } [v_q]_{lk} \neq 0 \text{ for some } q = 1,\ldots,Q\}. \quad (16)$$

The oscillator $(k,l)$ has the frequency $\omega_{kl} = E_l - E_k$, and we define the Hamiltonian

$$H_F = \sum_{(k,l)\in\mathcal{M}} \omega_{kl} a_{kl}^\dagger a_{kl}.$$

We couple the system S to F via the interacting Hamiltonian

$$H_{SF} = H_S + H_F + \sum_{q=1}^{Q} G_q \otimes X_q + G_q^\dagger \otimes X_q^\dagger, \quad (17)$$

with $X_q$ defined as

$$X_q = \sum_{(k,l)\in\mathcal{M}} \sqrt{p_k}\big([v_q]_{lk} a_{kl}^\dagger + [v_q]_{kl} a_{kl}\big). \quad (18)$$

The $[v_q]_{kl} = \langle\chi_k|v_q|\chi_l\rangle$ are the matrix elements of the interaction operator $v_q$ [58]. Our main result is:

*Theorem*—The system dynamics $\rho_S(t)$ defined in (9) is equivalently given, for all $t\in\mathbb{R}$, by

$$\rho_S(t) = \mathrm{tr}_F\big(e^{-itH_{SF}}(\rho_S \otimes \rho_F)e^{itH_{SF}}\big), \quad (19)$$

where $H_{SF}$ is the interacting SF Hamiltonian (17) and $\rho_F$ is the ground state (15).

*Discussion and examples*—The theorem shows that one can use a bath of oscillators in their ground state to model the dynamics of any system S within the class of SR models (1)–(9) presented above. The form of the equivalent fluctuation reservoir F is not unique. F should be Gaussian (quasifree) and have the same two-point function as R. Intuitively, the reservoir F should provide the system S with the same set of transition energies $E_l - E_k$ and the same transition probabilities as R does, in order to drive the same system dynamics. A realization of F is the ground state oscillator bath (15)–(18), where the mode $a_{kl}^\dagger$ enables the transition $E_k \to E_l$. We call this the standard representation of F, as it is suitable for all models. In special situations (for parameter relations compatible with thermal transition probabilities), one may realize F as a collection of thermal modes at appropriate inverse temperatures $\beta_{kl}$, each availing S with transition energies $E_k - E_l$ and $E_l - E_k$ simultaneously. We explain this with more precision in Sec. T of [56]. The nonuniqueness of F is not surprising, as in general, different coupled system-reservoir dynamics can generate the same reduced system dynamics. This is akin to different multivariate probability distributions having a fixed marginal.

The number of oscillators in F is the cardinality of $\mathcal{M}$, (16). It satisfies $|\mathcal{M}| \leq d_R \times \mathrm{rank}\rho_R$, where $d_R = \dim\mathcal{H}_R$ and the rank of $\rho_R$ is the number of nonzero populations $p_k$. If all transitions are allowed by the $v_q$ then $|\mathcal{M}| = d_R \times \mathrm{rank}\rho_R$. For "off-diagonal" interactions, $[v_q]_{kk} = 0$ for all $q$ and $k$ (and all other transitions allowed), F has $|\mathcal{M}| = (d_R - 1)\times \mathrm{rank}\rho_R$ oscillators. A pure state $\rho_R = |\psi_R\rangle\langle\psi_R|$ has rank one, and the reservoir F has at most $d_R$ oscillators. For a reservoir R made of spins, $d_R = 2$, we need two oscillators in F. If the interaction is off-diagonal, $[v_q]_{kk} = 0$, then F consists of a single oscillator. The corresponding SF model is then of the Jaynes-Cummings type as in [50]. On the other hand, thermal states $\rho_R \propto e^{-\beta h_R}$ ($\beta < \infty$) have full rank $d_R$ leading to a maximal number $d_R^2$ of oscillators (or $d_R^2 - d_R$ for off-diagonal $v_q$).

Qubit (spin) system: Consider both S and each component of R to be spins $1/2$ and let $H_S$ and $\rho_S$ be any spin $1/2$ Hamiltonian and density matrix. Take $h_R = \omega_R|\uparrow\rangle\langle\uparrow|$ with $\omega_R > 0$ and $\rho_R = w|\downarrow\rangle\langle\downarrow| + (1-w)|\uparrow\rangle\langle\uparrow|$ with $0 \leq w \leq 1$. For the interaction (5) take $Q = 1$ and $G = |\downarrow\rangle\langle\uparrow|$, $v = |\uparrow\rangle\langle\downarrow|$ so that excitations are exchanged between S and R. The reservoir populations and energy levels (13) in this example are, $p_1 = 1-w$, $p_2 = w$, $E_1 = 0$, $E_2 = \omega_R$. If $w \neq 0$, 1 (mixed state) then the set of modes (16) is $\mathcal{M} = \{(1,2),(2,1)\}$ and F consists of two oscillators $a^\dagger, a$ and $b^\dagger, b$ at frequencies $\omega_a = \omega_R$ and $\omega_b = -\omega_R$. The interaction operator (18) is $X = w^{1/2}a^\dagger + (1-w)^{1/2}b$. In the case $w = 0$, 1 (pure state) $\mathcal{M}$ reduces to a single point and F consists of only one oscillator. We introduce the new, effective modes $A = (1-2w)^{-1/2}[w^{1/2}a^\dagger + (1-w)^{1/2}b]$ and $B = (1-2w)^{-1/2}[(1-w)^{1/2}a + w^{1/2}b^\dagger]$ (for $0 < w < 1/2$) such that $[A,A^\dagger] = [B,B^\dagger] = \mathbf{1}$, $[A,B] = [A,B^\dagger] = 0$. In terms of these independent oscillators, the SF Hamiltonian becomes

$$H_{SF} = H_S - \omega_R(A^\dagger A - B^\dagger B) + G\otimes A + G^\dagger \otimes A^\dagger.$$

Therefore, S is coupled to a single effective mode $A$ [59].

Emergence of multiple fluctuation modes: Let S be arbitrary with any Hamiltonian $H_S$ and any initial state $\rho_S$. Take reservoir components to be $Q$-level systems ($Q \geq 2$), $h_R = \sum_{q=1}^{Q} E_q|q\rangle\langle q|$, and $\rho_R = |1\rangle\langle 1|$. Take an SR coupling that induces transitions between $|1\rangle$ and $|q\rangle$, $q \geq 2$, namely (5) with arbitrary $G_q$ and $v_q = |q\rangle\langle 1|$. The mode set (16) is $\mathcal{M} = \{(1,q)\colon q = 2,\ldots,Q\}$, and we denote by $a_q \equiv a_{1,q}$ the corresponding bosonic operators of F. The SF Hamiltonian (17) couples S to $Q-1$ independent modes of

F with frequencies $\omega_q = E_q - E_1$,

$$H_{\rm SF} = H_{\rm S} + \sum_{q=2}^{Q} \omega_q a_q^\dagger a_q + \sum_{q=2}^{Q} G_q \otimes a_q^\dagger + \text{H.c.}$$

Creation of intrasystem entanglement: The interaction of two systems $S_1$ and $S_2$ to a common environment R generally produces entanglement between $S_1$ and $S_2$, even if they are not coupled directly [30]. This effect is not captured in mean-field models. It is shown in [31] that the effective propagator of multipartite systems $S = S_1 + \cdots + S_N$ coupled in the mean-field way to a common reservoir, is the product of local unitaries (with time-dependent generators). The intrasystem entanglement is thus constant in time, and initially disentangled states of S stay so when interacting with R. The following simple example shows that the creation of intrasystem entanglement is restored by the interaction with a common bath in the fluctuation scaling. Take an arbitrary bipartite $S = S_1 + S_2$ with any uncoupled $H_{\rm S} = H_{\rm S_1} + H_{\rm S_2}$ and a reservoir with two-level components, $h_{\rm R} = E_1|1\rangle\langle 1| + E_2|2\rangle\langle 2|$, say in the ground state $\rho_{\rm R} = |1\rangle\langle 1|$. Take the coupling (5) with $Q = 1$, $G = \Gamma_1 + \Gamma_2$ (any coupling operators of $S_1$ and $S_2$) and $v = |1\rangle\langle 2|$. The fluctuation reservoir F has a single mode $a$, and the SF Hamiltonian (17) is ($\omega_{\rm F} = E_2 - E_1$),

$$H_{\rm SF} = H_{\rm S_1} + H_{\rm S_2} + \omega_{\rm F} a^\dagger a + (\Gamma_1 + \Gamma_2) \otimes (a^\dagger + a). \quad (20)$$

This Hamiltonian generates entanglement between $S_1$ and $S_2$ [60]

Nondemolition models: Consider $Q = 1$ and commuting $H_{\rm S}$ and $G$ [Eqs. (2) and (5)], $H_{\rm S} = \sum_{j=1}^N e_j|\psi_j\rangle\langle\psi_j|$, $G = \sum_{j=1}^N g_j|\psi_j\rangle\langle\psi_j|$, where $\{\psi_j\}_{j=1}^N$ is an orthonormal basis of $\mathcal{H}_{\rm S} = \mathbb{C}^N$ and $e_j \in \mathbb{R}$, $g_j \in \mathbb{C}$ ($G$ is not necessarily Hermitian). Let $h_{\rm R}$, $\rho_{\rm R}$, and $v$ be arbitrary, fixed. As $H_{\rm S}$ commutes with $H_{\rm SF}$ the populations of S (diagonal density matrix elements) in the dynamics $\rho_{\rm S}(t)$, (19), are independent of time. The coherences (off-diagonals) evolve independently, $\langle\psi_m|\rho_{\rm S}(t)|\psi_n\rangle = e^{-it(e_m - e_n)} D_{m,n}(t)\langle\psi_m|\rho_{\rm S}(0)|\psi_n\rangle$, where the decoherence function is

$$D_{m,n}(t) = \prod_{(k,l):k\neq l} D_{m,n}(k,l,t). \quad (21)$$

Consider the case $g_n \in \mathbb{R}$ and $v = v^\dagger$. Then the modulus of (21) is (see End Matter)

$$|D_{m,n}(t)| = \exp\left[-8(g_m - g_n)^2 \sum_{(k,l):k\neq l} p_k |v_{kl}|^2 \frac{\sin^2(t\omega_{kl}/2)}{\omega_{kl}^2}\right].$$

The system undergoes a quasiperiodic evolution in time, in contrast to what happens for open systems coupled to reservoirs having a continuum of modes, such as the spin boson model. Those systems show decoherence, that is $D_{m,n}(t) \to 0$ as $t \to \infty$. Nevertheless, for times $t \ll \min_{k\neq l}(1/|\omega_{kl}|)$ we have the onset of decoherence,

$$|D_{m,n}(t)| \sim \exp\left[-2(g_m - g_n)^2 t^2 \sum_{k,l=1}^{d_{\rm R}} |v_{kl}|^2 p_k\right] = e^{-t^2(2d_{\rm R}-1)u^2}, \quad (22)$$

where in the equality we took a homogeneous coupling, $|v_{kl}| = \begin{cases} u & k \neq l \\ 0 & k = l \end{cases}$. The decoherence rate (22) is largely model independent. It does not depend on the initial state $\rho_{\rm R}$ nor on the eigenvalues $E_j$ of $h_{\rm R}$. Equation (22) stays valid for longer times if the energy gap narrows, which is typically the case for increasing $d_{\rm R}$.

*Summary*—We study quantum systems S coupled to reservoirs R of size $M$ through nonlocal interactions, scaled mesoscopically, proportional to $1/\sqrt{M}$, in the limit $M \to \infty$. We show that the reduced density matrix of the system S evolves exactly as if it were interacting with a different reservoir F, consisting of independent bosonic modes initially in their ground states, and coupled through a simple, explicit, linear interaction. Our findings reveal that free bosonic reservoirs universally emerge from mesoscopically scaled, nonlocal couplings. We plan to use this framework to analyze the interplay of mean-field and fluctuation effects in systems far from equilibrium (transport) and describe thermodynamic effects at arbitrary coupling strengths.

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[58] For $Q = 1$ and $G = G^\dagger$, $v = v^\dagger$ the Hamiltonian (17) produces the situation described in the previous paragraph. Note also that $[v_q]_{lk} = \overline{([v_q]^\dagger)_{kl}}$ (the bar denotes the complex conjugate).

[59] We thank a referee for pointing this out to us.

[60] If both $S_1$ and $S_2$ are qubits, then the Hamiltonian (20) is the same as the one used in [30]. Using the standard positive partial transpose criterion one can readily check that entanglement between the qubits is created by the indirect contact with the single mode.

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## End Matter

The product structure in (21) comes from the fact that $\rho_{\mathrm{F}}$ is factorized and that all modes $(k, l)$ are independent. Each mode has its individual decoherence function, $D_{m,n}(k, l, t) = \langle 0|Y_{kl}(t, n)Y_{kl}(-t, m)|0\rangle$, where $Y_{kl}(t, n) = e^{it(\omega_{kl}a^\dagger a + \sqrt{p_k}\phi(\bar{g}_n\overline{v_{kl}} + g_n v_{lk})}$ and where $|0\rangle$ is the vacuum of the single mode with creation and annihilation operator $a^\dagger, a$ and $\phi(x) = xa^\dagger + \bar{x}a$ (for $x \in \mathbb{C}$). A standard polaron transformation type calculation gives (see for instance Lemma 1 of [61]) $D_{m,n}(k, l, t) = e^{iA}\langle 0|W \times [(e^{i\omega_{kl}t} - 1/i\omega_{kl})\sqrt{2p_k}\{(\bar{g}_n - \bar{g}_m)\overline{v_{kl}} + (g_n - g_m)v_{lk}\}]|0\rangle$ for some phase $A$ (which has an explicit albeit a bit cumbersome expression, depending on $m$, $n$, $k$, $l$, $t$) and where $W(x) = e^{i\phi(x)}$ is the Weyl operator. The average of $W(x)$ in the state $|0\rangle$ is the Gaussian $\langle 0|W(x)|0\rangle = e^{-\frac{1}{4}|x|^2}$, so we obtain $|D_{m,n}(k, l, t)| = \exp\{-2p_k[\sin^2(t\omega_{kl}/2)/\omega_{kl}^2]|(\bar{g}_n - \bar{g}_m)\overline{v_{kl}} + (g_n - g_m)v_{lk}|^2\}$. For the case $g_n \in \mathbb{R}$ and $v = v^\dagger$ we have $|(\bar{g}_n - \bar{g}_m)\overline{v_{kl}} + (g_n - g_m)v_{lk}|^2 = 4(g_m - g_n)^2|v_{kl}|^2$ and the expression in the main text follows.