Superposed quantum evolutions across regular and chaotic regimes

arXiv:2603.13209

CAP 2026

Amit Anand

Anne-Catherine de la Hamette, Robert Mann and Shohini Ghose

Institute for Quantum Computing
Physics and Astronomy Department
University of Waterloo

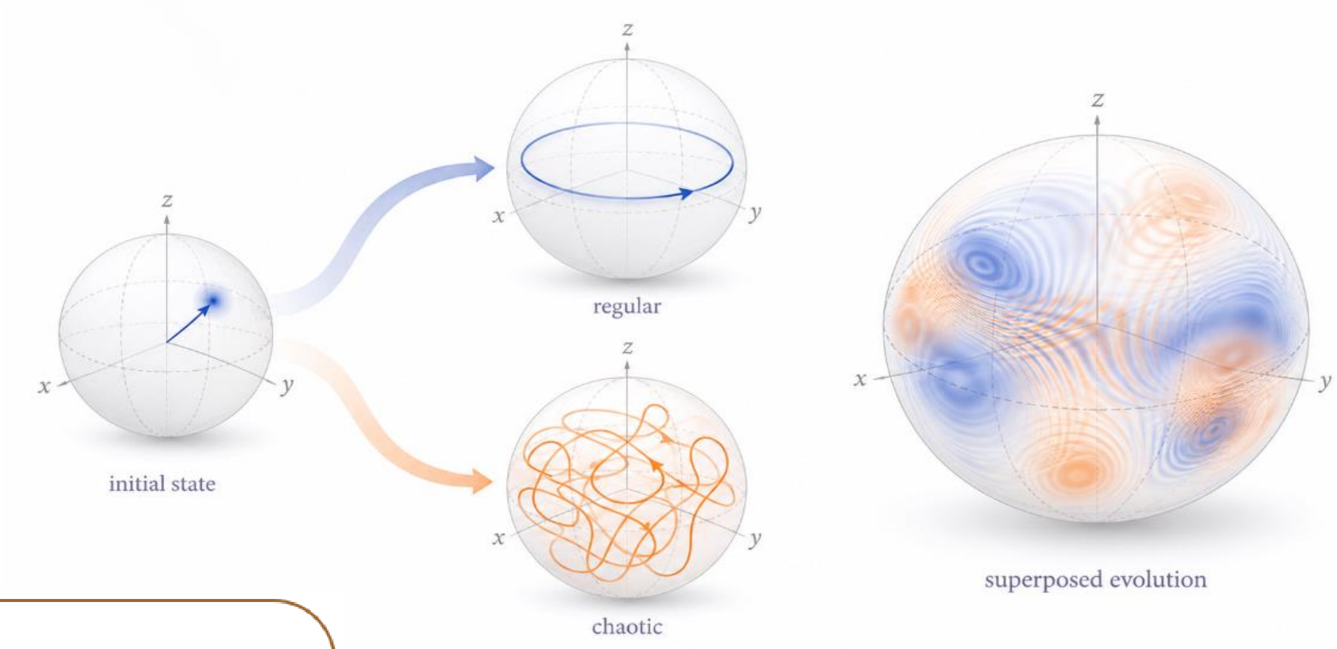
June 23, 2026

UNIVERSITY OF WATERLOO | FACULTY OF SCIENCE

IQC Institute for Quantum Computing

Question: What happens when a quantum state evolves through a coherent superposition of two different dynamics?

We study a new setting where the two branches correspond to **regular** and **chaotic** dynamics.



This allows us to ask:

- **How does classical chaos modify the quantum interference pattern between two evolutions?**
- Understanding this may provide insight into quantum-classical correspondence, scrambling, and quantum control.

Flow of the talk

Introduction of kicked top

Quantum and classical dynamics

Mach-Zehnder setup with kicked top evolutions

Post selection and classical mixture

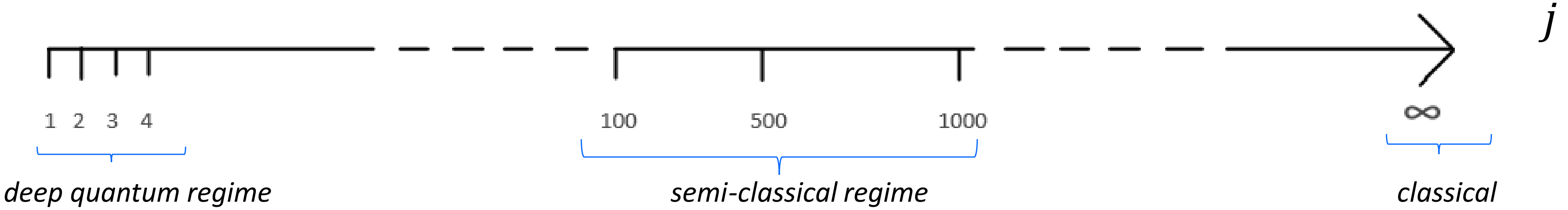
Entropy generation and observations

Quantum kicked top

- A **finite dimensional system** with fixed total spin (angular momentum) j.
- Periodically driven system

$$H = \underbrace{\hbar \frac{\alpha J_y}{\tau}}_{\text{precession about y-axis}} + \underbrace{\hbar \frac{\kappa J_z^2}{2j} \left(\sum_{n=-\infty}^{n=\infty} \delta(t - n\tau) \right)}_{\text{periodic impulsive non-linear twists}}$$

- Chaotic in classical limit $(\frac{\hbar}{j} \rightarrow 0)$
- Used to study quantum chaos.



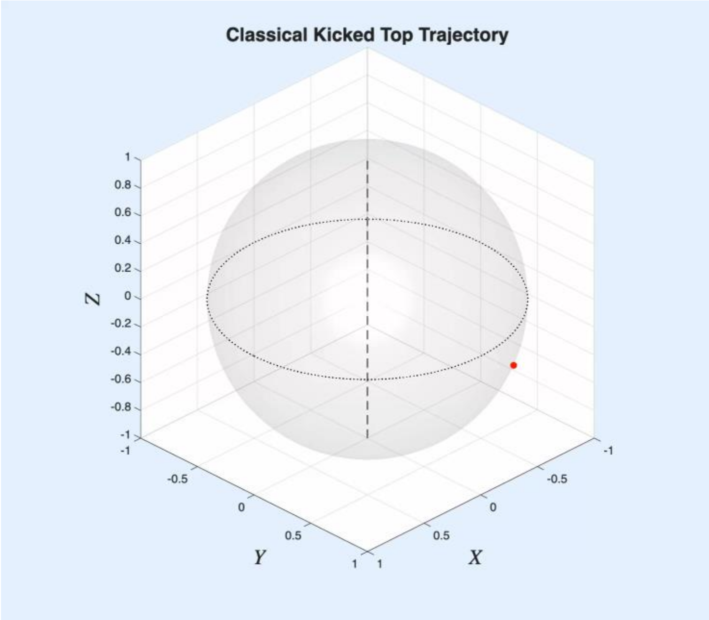
Haake, Z. Physik B - Condensed Matter 65, 381–395 (1987)

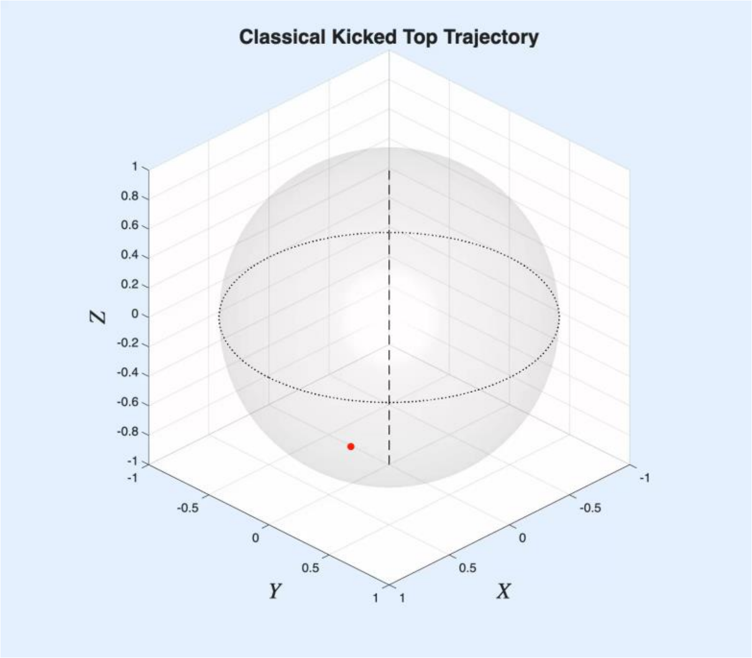
Classical dynamics ($j \to \infty$)

$$X_{n+1} = Z_n \cos(\kappa X_n) + Y_n \sin(\kappa X_n),$$
$$Y_{n+1} = Y_n \cos(\kappa X_n) - Z_n \sin(\kappa X_n),$$
$$Z_{n+1} = -X_n.$$

$$X = J_x/j, \quad Y = J_y/j, \quad Z = J_z/j$$

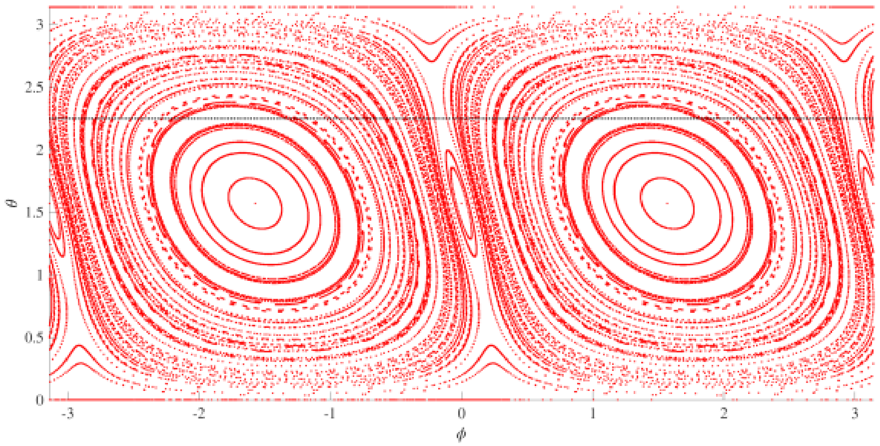
$$X^2 + Y^2 + Z^2 = 1$$



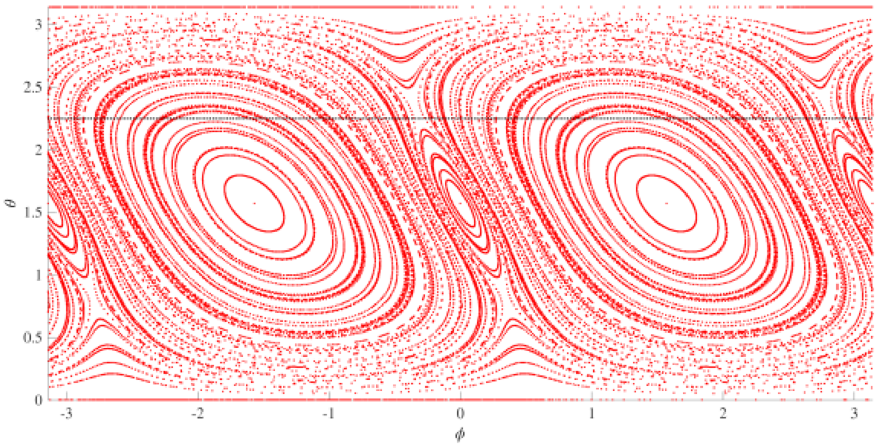


Haake, Z. Physik B - Condensed Matter 65, 381–395 (1987)

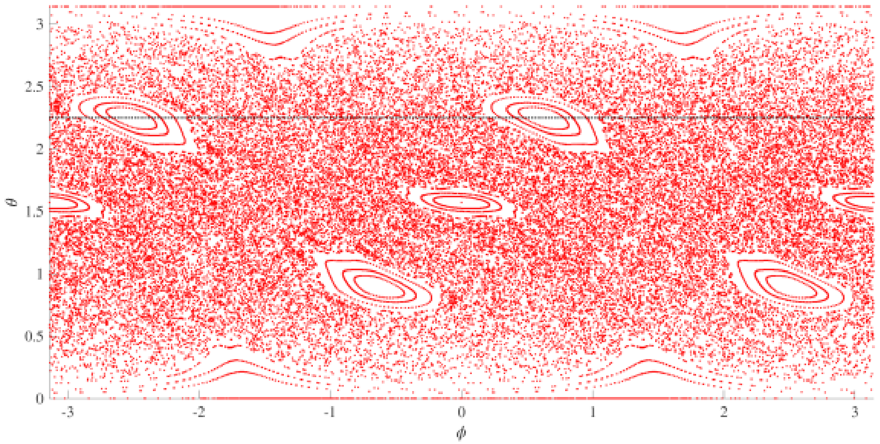
Stroboscopic phase space



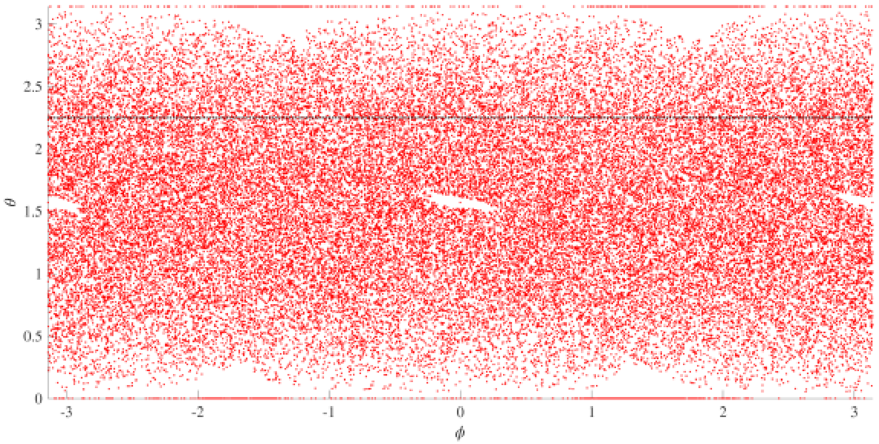
(a) $\kappa = 0.5$



(b) $\kappa = 1$



(c) $\kappa = 3$



(d) $\kappa = 6$

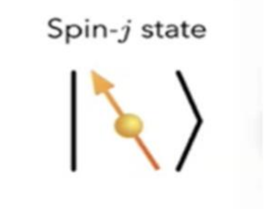
Quantum kicked top

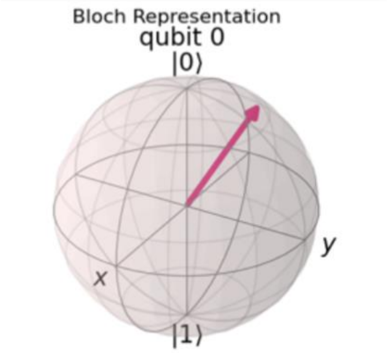
- The QKT system with spin j can be considered as an $N = 2j$ qubit state lying in the symmetric subspace.
- Qubit representation is useful to study entanglements in Kicked Top.

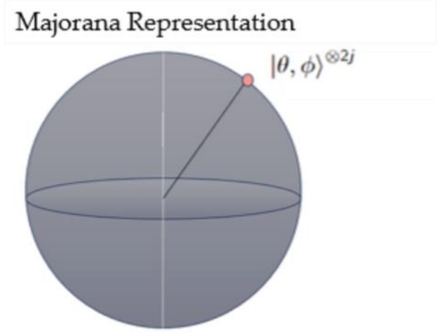
$$J_\alpha = \frac{1}{2}\sum_{i=1}^{2j} \sigma_{i\alpha}, \quad \alpha \in \{x, y, z\}$$

$$H = \hbar\frac{\alpha J_y}{\tau} + \hbar\frac{\kappa J_z^2}{2j}\left(\sum_{n=-\infty}^{n=\infty} \delta(t - n\tau)\right)$$

$$H = \hbar\frac{\kappa}{8j}\left(2j + \sum_{\substack{i,k=1 \\ i\neq k}}^{2j} \sigma_{iz}\otimes\sigma_{kz}\right)\sum_{n=-\infty}^{\infty}\delta(t - n\tau) + \hbar\frac{\alpha}{2}\sum_{i=1}^{2j}\sigma_{iy}.$$





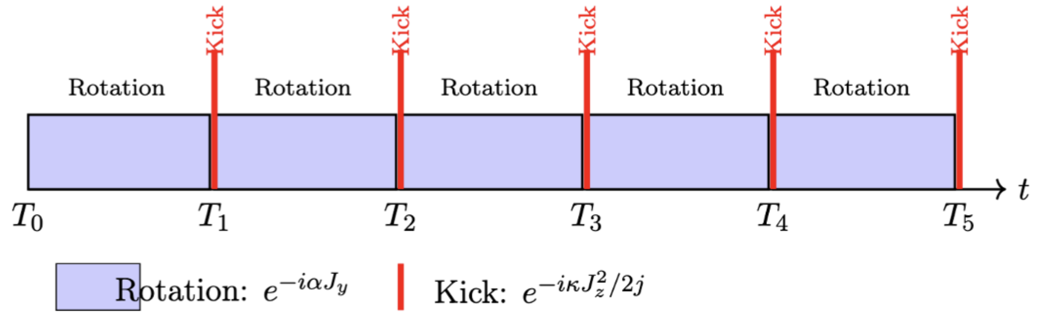


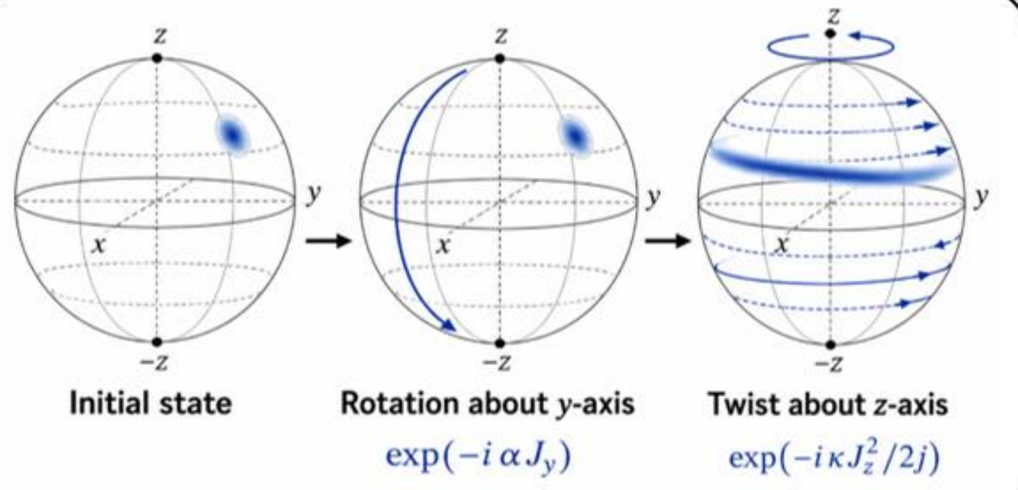
Majorana, E.: Nuovo Cimento 9, 43 (1932)

Quantum dynamics

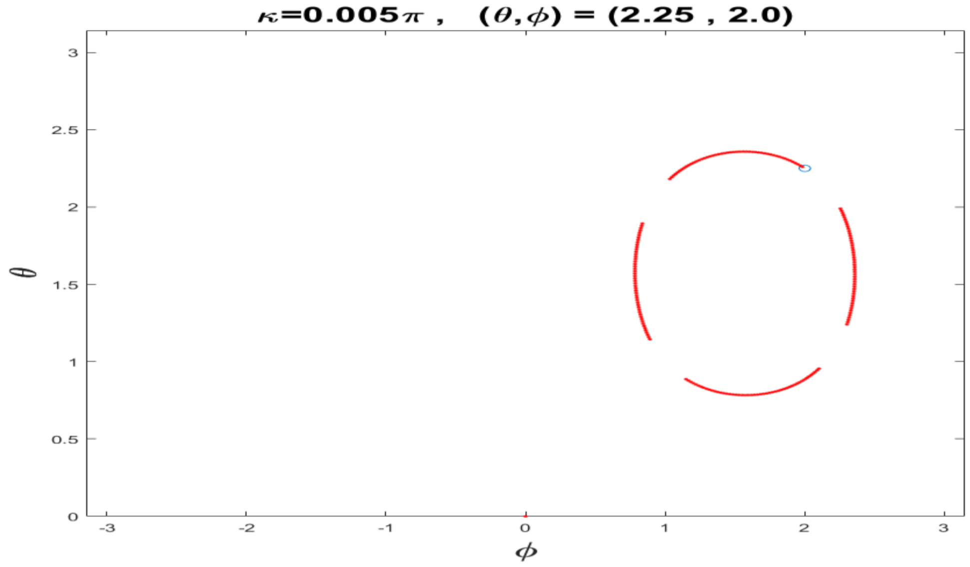
Floquet unitary operator for one kick -

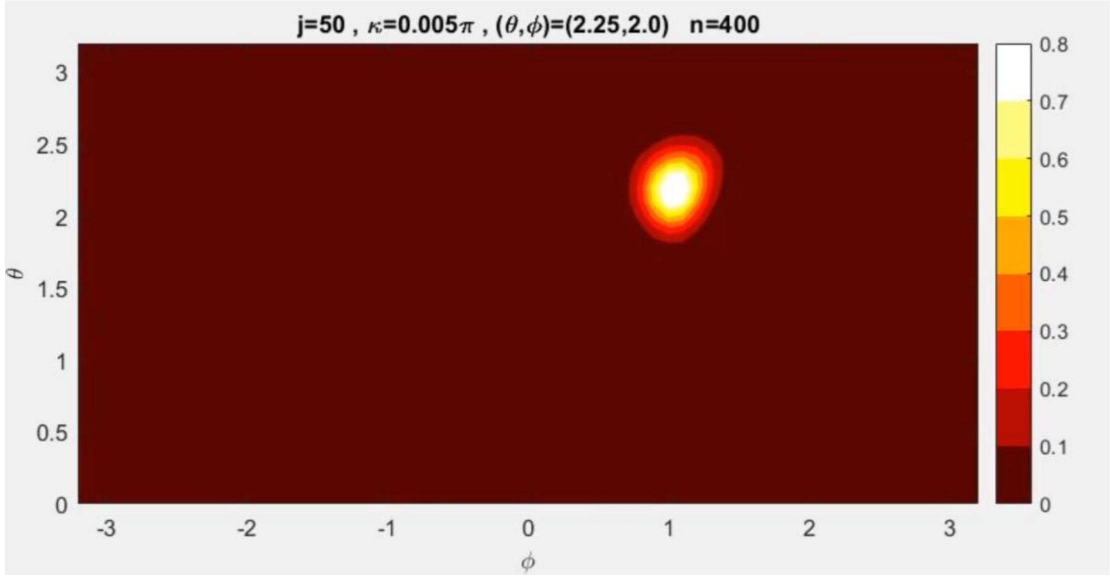
$$U = \exp\Big(-i\frac{\kappa}{2j}J_z^2\Big)\exp\Big(-i\alpha J_y\Big)$$

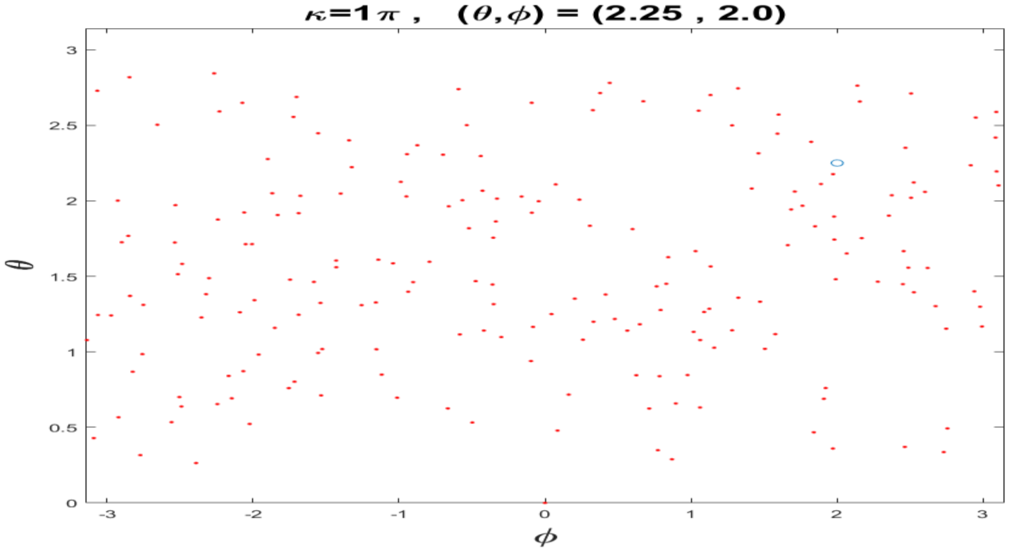


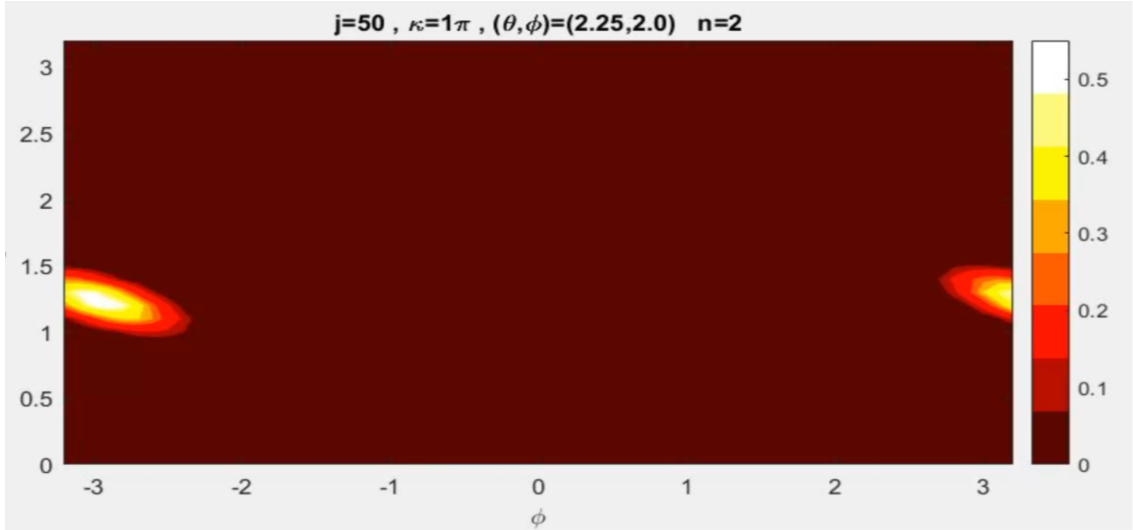


Comparison with classical dynamics





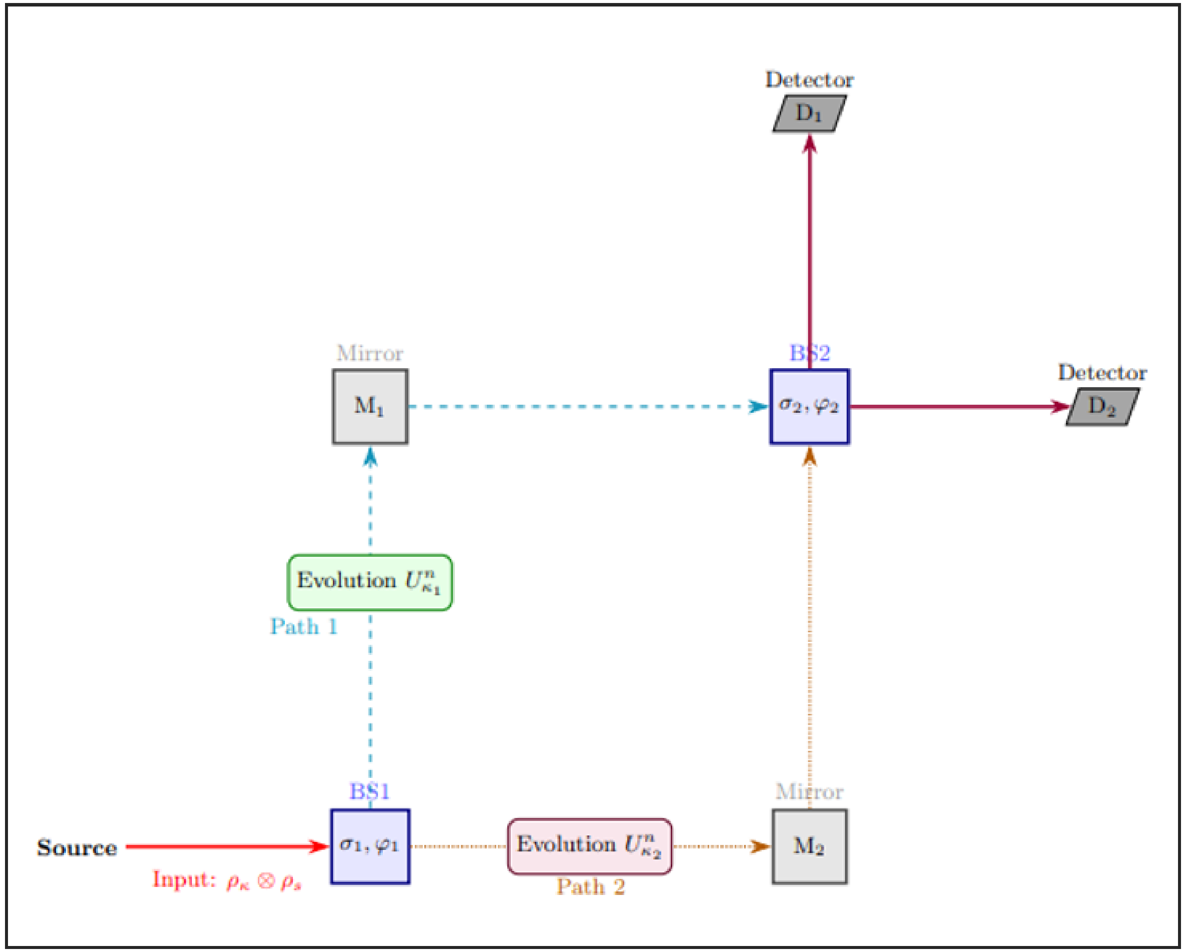




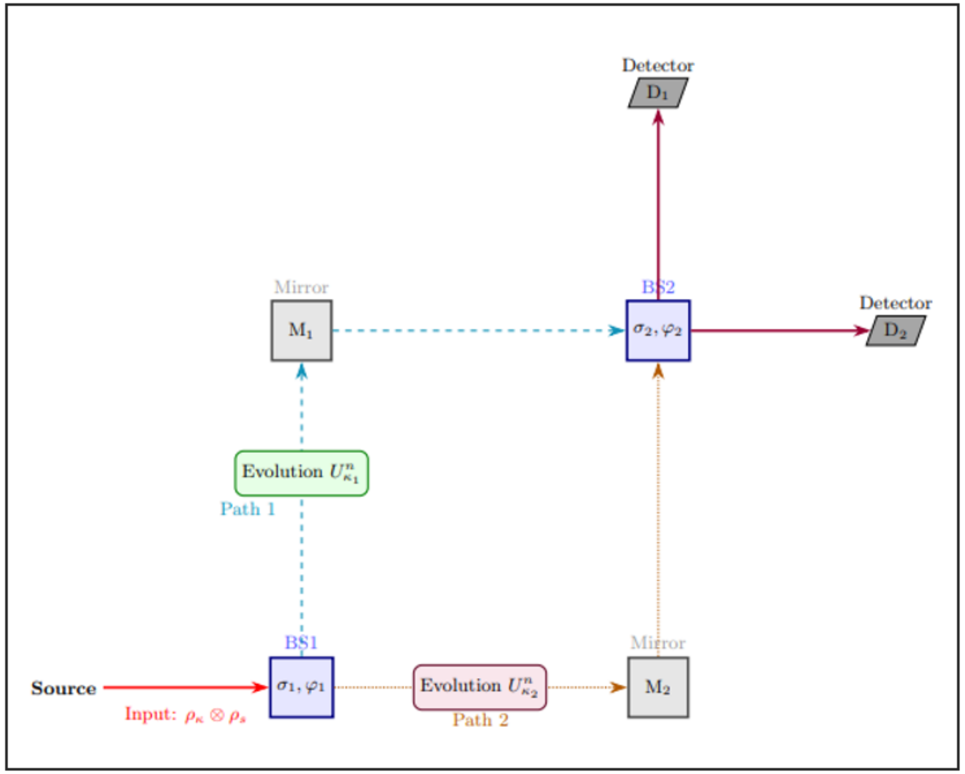
1 corresponds to SCS.

Question: what happens when we superposed two chaotic evolution?

Quantum kicked top in the Mach-Zender interferometer



Quantum kicked top in the Mach-Zender interferometer



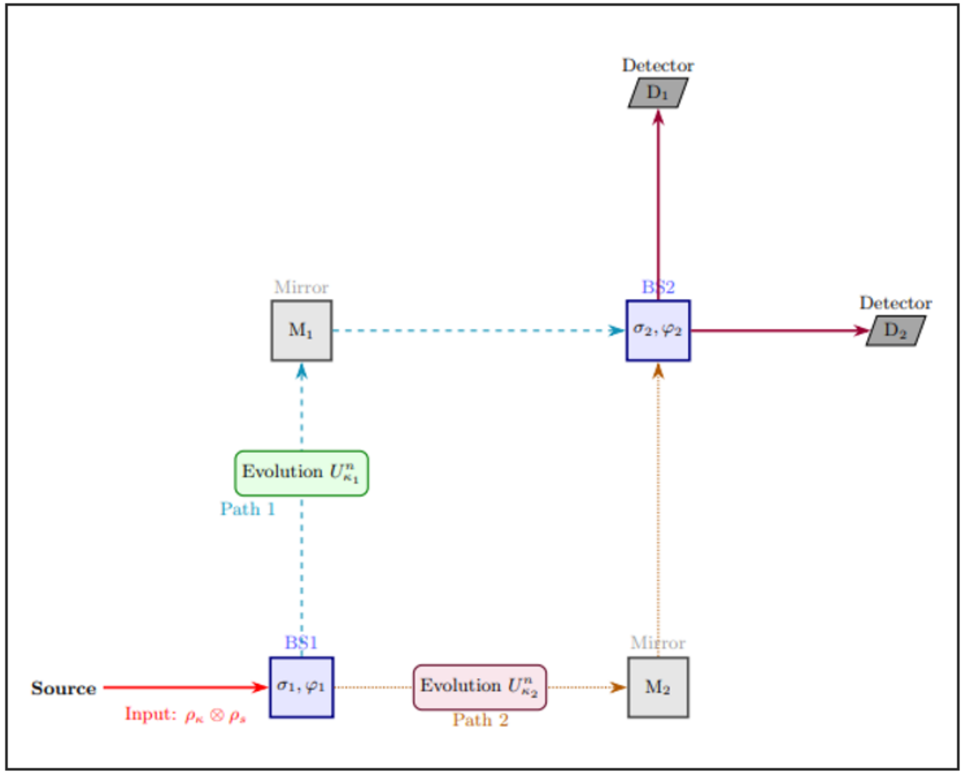
Initial state:

$$\rho_{\kappa+s}(0) = \rho_\kappa(0) \otimes \rho_s(0)$$

$$\hat{M}_{BS_1} = \begin{pmatrix} \cos(\sigma_1/2) & e^{-i\varphi_1}\sin(\sigma_1/2) \\ e^{i\varphi_1}\sin(\sigma_1/2) & -\cos(\sigma_1/2) \end{pmatrix}$$

$$\begin{aligned} &\rho'_{\kappa+s}(0) \\ &= \hat{M}_{BS_1}\,\rho_{\kappa+s}(0)\,\hat{M}^\dagger_{BS_1} \\ &= \Big(\cos^2\!\left(\tfrac{\sigma_1}{2}\right)|1\rangle\langle 1| + e^{-i\varphi_1}\cos\!\left(\tfrac{\sigma_1}{2}\right)\sin\!\left(\tfrac{\sigma_1}{2}\right)|1\rangle\langle 2| \\ &\quad + e^{i\varphi_1}\cos\!\left(\tfrac{\sigma_1}{2}\right)\sin\!\left(\tfrac{\sigma_1}{2}\right)|2\rangle\langle 1| + \sin^2\!\left(\tfrac{\sigma_1}{2}\right)|2\rangle\langle 2|\Big) \otimes \rho_s \end{aligned}$$

Quantum kicked top in the Mach-Zender interferometer

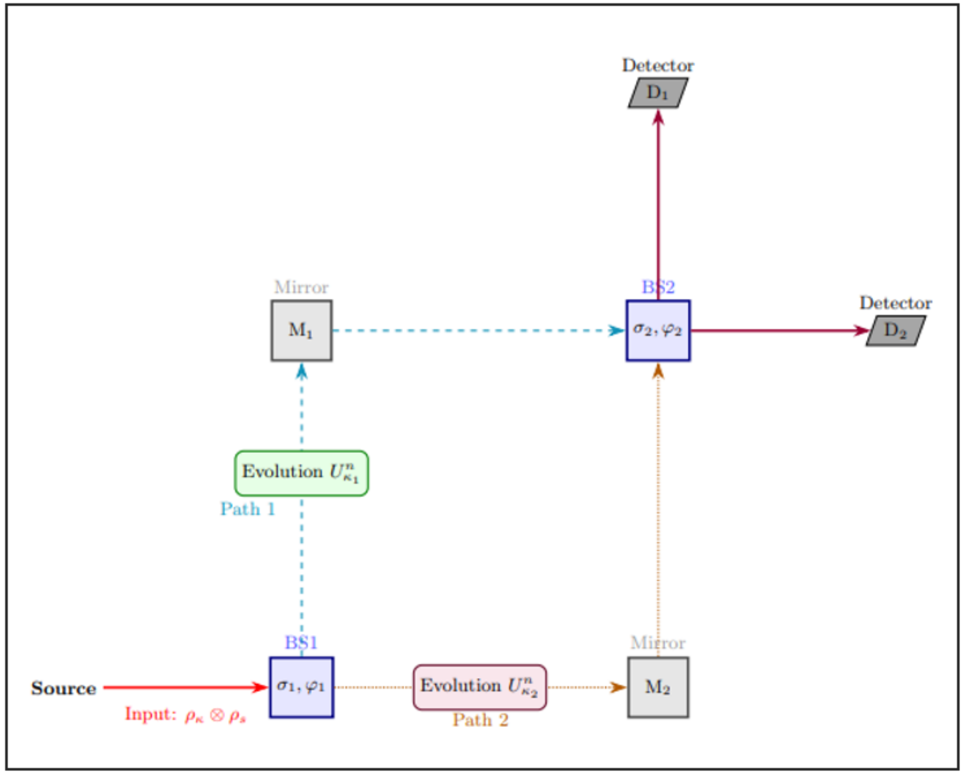


$$\begin{aligned}&\rho'_{\kappa+s}(0)\\ &= \hat{M}_{BS_1}\,\rho_{\kappa+s}(0)\,\hat{M}^\dagger_{BS_1}\\ &= \Big(\cos^2\!\left(\tfrac{\sigma_1}{2}\right)|1\rangle\langle 1| + e^{-i\varphi_1}\cos\!\left(\tfrac{\sigma_1}{2}\right)\sin\!\left(\tfrac{\sigma_1}{2}\right)|1\rangle\langle 2|\\ &\quad + e^{i\varphi_1}\cos\!\left(\tfrac{\sigma_1}{2}\right)\sin\!\left(\tfrac{\sigma_1}{2}\right)|2\rangle\langle 1| + \sin^2\!\left(\tfrac{\sigma_1}{2}\right)|2\rangle\langle 2|\Big)\otimes\rho_s\end{aligned}$$

$$U = |1\rangle\langle 1|\otimes e^{-i\frac{\kappa_1}{2j}\hat{J}_z^2}e^{-i\alpha\hat{J}_y} + |2\rangle\langle 2|\otimes e^{-i\frac{\kappa_2}{2j}\hat{J}_z^2}e^{-i\alpha\hat{J}_y}$$

$$\rho'_{\kappa+s}(n) = \begin{pmatrix}\cos^2\!\left(\tfrac{\sigma_1}{2}\right)\rho_s^{11}(n) & \cos\!\left(\tfrac{\sigma_1}{2}\right)\sin\!\left(\tfrac{\sigma_1}{2}\right)e^{-i\varphi_1}\rho_s^{12}(n)\\ \cos\!\left(\tfrac{\sigma_1}{2}\right)\sin\!\left(\tfrac{\sigma_1}{2}\right)e^{i\varphi_1}\rho_s^{21}(n) & \sin^2\!\left(\tfrac{\sigma_1}{2}\right)\rho_s^{22}(n)\end{pmatrix}$$

Quantum kicked top in the Mach-Zender interferometer



$$\rho'_{\kappa+s}(n) = \begin{pmatrix} \cos^2\left(\frac{\sigma_1}{2}\right)\rho_s^{11}(n) & \cos\left(\frac{\sigma_1}{2}\right)\sin\left(\frac{\sigma_1}{2}\right)e^{-i\varphi_1}\,\rho_s^{12}(n) \\ \cos\left(\frac{\sigma_1}{2}\right)\sin\left(\frac{\sigma_1}{2}\right)e^{i\varphi_1}\,\rho_s^{21}(n) & \sin^2\left(\frac{\sigma_1}{2}\right)\rho_s^{22}(n) \end{pmatrix}$$

$$\hat{M}_{BS_2} = \begin{pmatrix} \cos(\sigma_2/2) & e^{-i\varphi_2}\sin(\sigma_2/2) \\ e^{i\varphi_2}\sin(\sigma_2/2) & -\cos(\sigma_2/2) \end{pmatrix}$$

$$\begin{aligned} &\tilde{\rho}_s^{\mathrm{ps}}(n;\sigma_1,\varphi_1;\sigma_2,\varphi_2) \\ &= \cos^2\left(\frac{\sigma_2}{2}\right)\cos^2\left(\frac{\sigma_1}{2}\right)\rho_s^{11}(n) + \sin^2\left(\frac{\sigma_2}{2}\right)\sin^2\left(\frac{\sigma_1}{2}\right)\rho_s^{22}(n) \\ &+ \frac{1}{4}\sin(\sigma_2)\sin(\sigma_1)\left(e^{i(\varphi_2-\varphi_1)}\rho_s^{12}(n) + e^{-i(\varphi_2-\varphi_1)}\rho_s^{21}(n)\right) \end{aligned}$$

$$\rho_s^{\mathrm{ps}}(n;\sigma_1,\varphi_1;\sigma_2,\varphi_2) = \frac{\tilde{\rho}_s^{\mathrm{ps}}(n;\sigma_1,\varphi_1;\sigma_2,\varphi_2)}{\mathrm{Tr}[\tilde{\rho}_s^{\mathrm{ps}}(n;\sigma_1,\varphi_1;\sigma_2,\varphi_2)]}$$

Entropy difference

Superposition with post-selection:

$$\tilde{\rho}_s^{\mathrm{ps}}(n;\sigma_1,\varphi_1;\sigma_2,\varphi_2)$$
$$= \cos^2(\tfrac{\sigma_2}{2})\cos^2(\tfrac{\sigma_1}{2})\rho_s^{11}(n) + \sin^2(\tfrac{\sigma_2}{2})\sin^2(\tfrac{\sigma_1}{2})\,\rho_s^{22}(n)$$
$$+\frac{1}{4}\sin(\sigma_2)\sin(\sigma_1)\Big(e^{i(\varphi_2-\varphi_1)}\rho_s^{12}(n) + e^{-i(\varphi_2-\varphi_1)}\rho_s^{21}(n)\Big)$$

Von Neumann entropy:

$$S(\rho) = -\,\mathrm{Tr}(\rho\log\rho)$$

Single-qubit entropy difference:

$$\Delta S(n) = S\big(\mathrm{Tr}_{N-1}(\rho_s^{\mathrm{cl}})\big) - S\big(\mathrm{Tr}_{N-1}(\rho_s^{\mathrm{ps}})\big)$$

Classical mixture:

$$\rho_s^{\mathrm{cl}}(n;\sigma_1,\varphi_1) = \cos^2(\sigma_1/2)\,\rho_s^{11} + \sin^2(\sigma_1/2)\,\rho_s^{22}$$

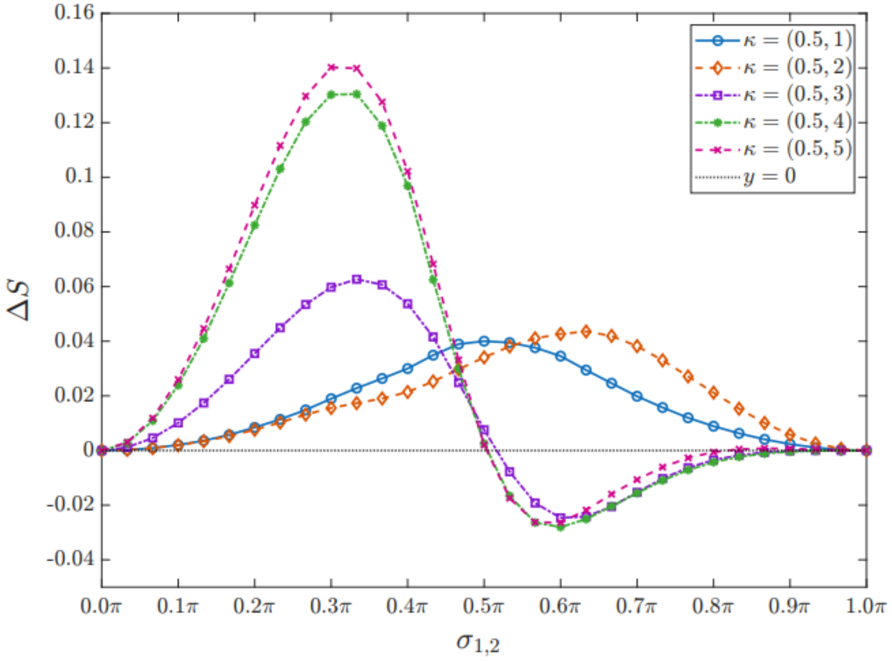
$\Delta S > 0:$ classical mixture has more entropy,

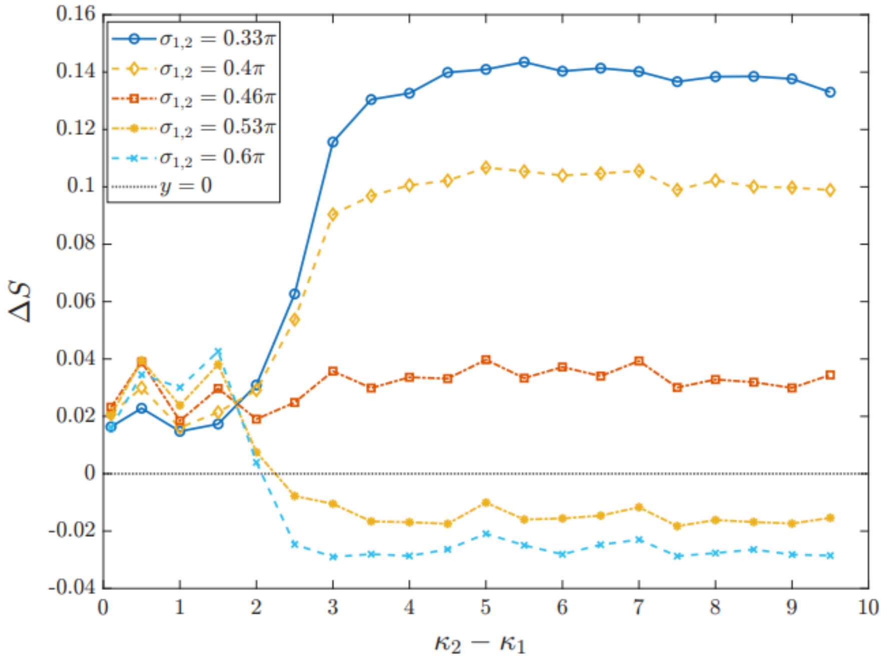
$\Delta S < 0:$ post-selected coherent superposition has more entropy

Case 1: $\sigma_1 = \sigma_2$ & $\varphi_1 = \varphi_2 = 0$

Single-qubit entropy difference:

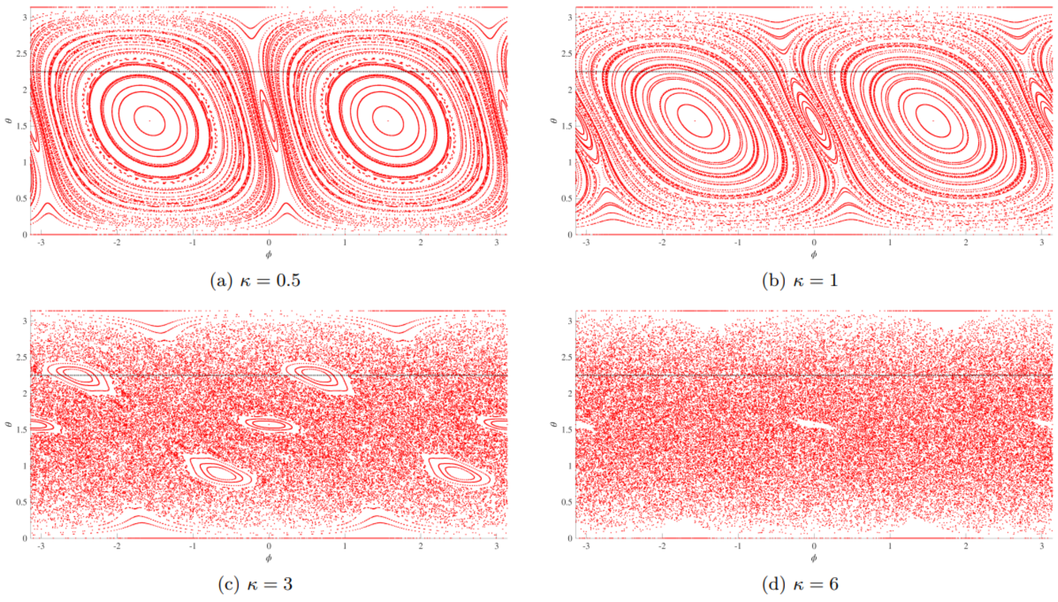
$$\Delta S(n) = S\big(\mathrm{Tr}_{N-1}(\rho_s^{\mathrm{cl}})\big) - S\big(\mathrm{Tr}_{N-1}(\rho_s^{\mathrm{ps}})\big)$$



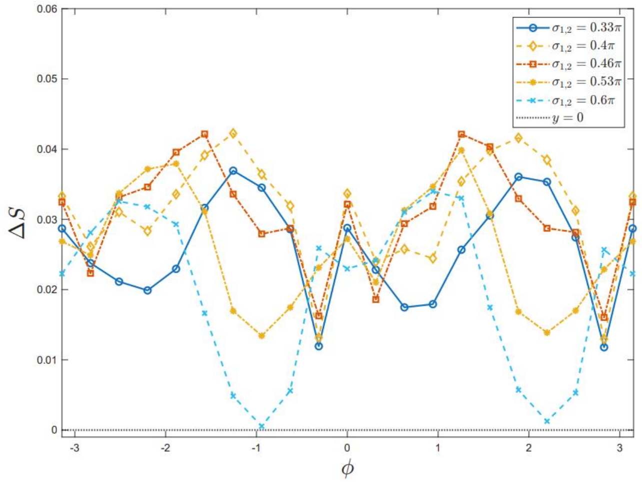


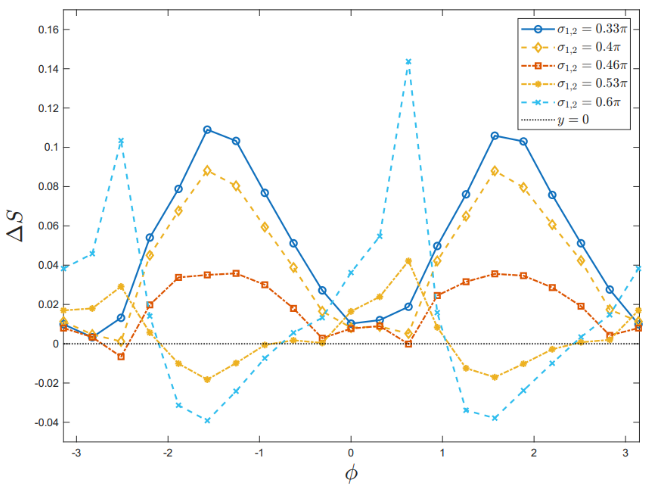
Regular + chaotic superposition can generate more entropy than classical mixing, when the chaotic branch has sufficient weight.

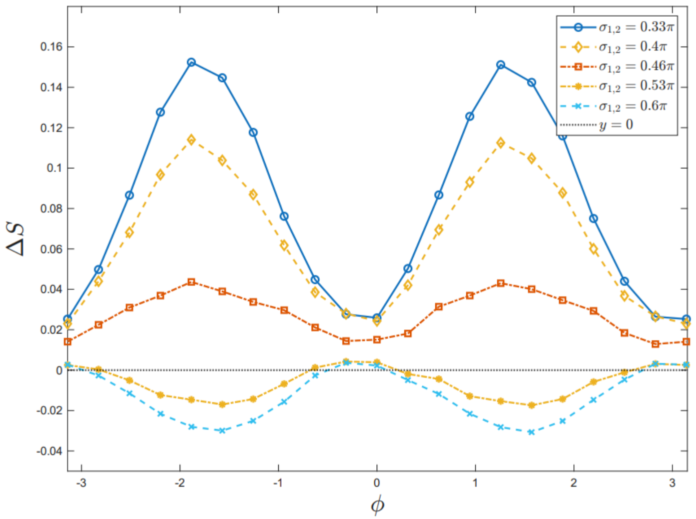
Case 1: $\sigma_1 = \sigma_2$ & $\varphi_1 = \varphi_2 = 0$



(a) $\kappa = 0.5$ (b) $\kappa = 1$ (c) $\kappa = 3$ (d) $\kappa = 6$







Regular + chaotic superposition can generate more entropy than classical mixing, when the chaotic branch has sufficient weight.

Summary

Key idea	**Message**
Coherent dynamics beyond classical mixing	A post-selected interferometer produces a coherent combination of two kicked-top evolutions. The result is controlled by interference terms, not only by probabilistic weights.
Regular vs chaotic branches	Entropy enhancement appears when the two branches belong to different dynamical regimes and post-selection favors the chaotic branch.
Phase-space sensitivity	The generated entropy depends on the underlying classical phase-space structure, such as regular islands, mixed regions, and chaotic seas.
Impact	This provides a new way to probe how chaos modifies quantum interference, with possible relevance for quantum-classical correspondence, scrambling, and coherent control.

Open question: Does the effect sharpen in the limit of large-j / semi-classical limit?

IQC Institute for Quantum Computing

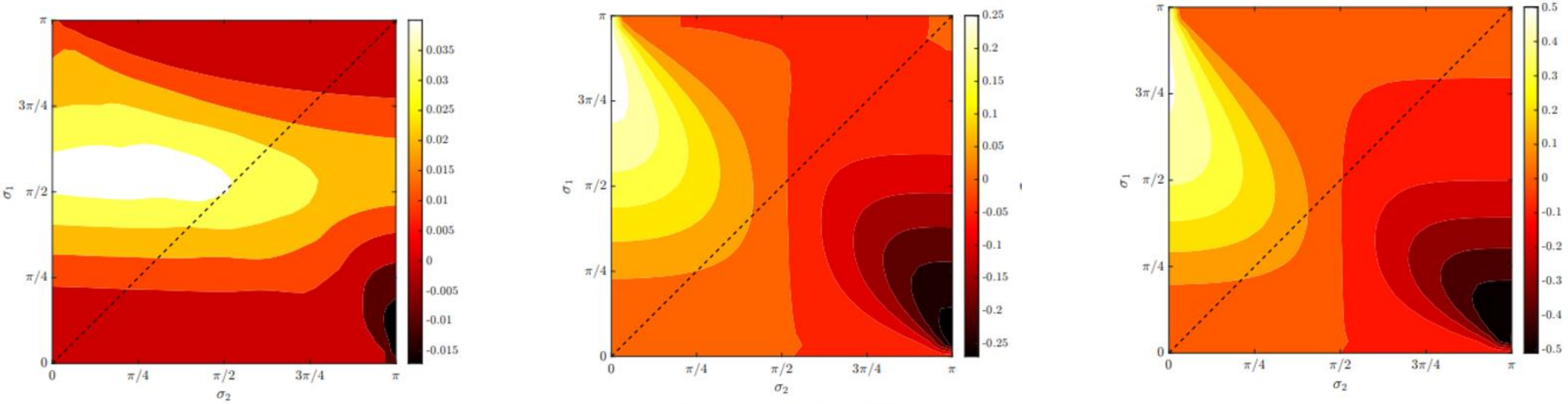
UNIVERSITY OF WATERLOO

FACULTY OF SCIENCE

Case 1: $\sigma_1 \neq \sigma_2$ & $\varphi_1 = \varphi_2 = 0$

Single-qubit entropy difference:

$$\Delta S(n) = S\big(\mathrm{Tr}_{N-1}(\rho_s^{\mathrm{cl}})\big) - S\big(\mathrm{Tr}_{N-1}(\rho_s^{\mathrm{ps}})\big)$$



(a) $(\kappa_1, \kappa_2) = (0.5, 1)$ (b) $(\kappa_1, \kappa_2) = (0.5, 3)$ (c) $(\kappa_1, \kappa_2) = (0.5, 6)$

Regular + chaotic superposition can generate more entropy than classical mixing, when the chaotic branch has sufficient weight.

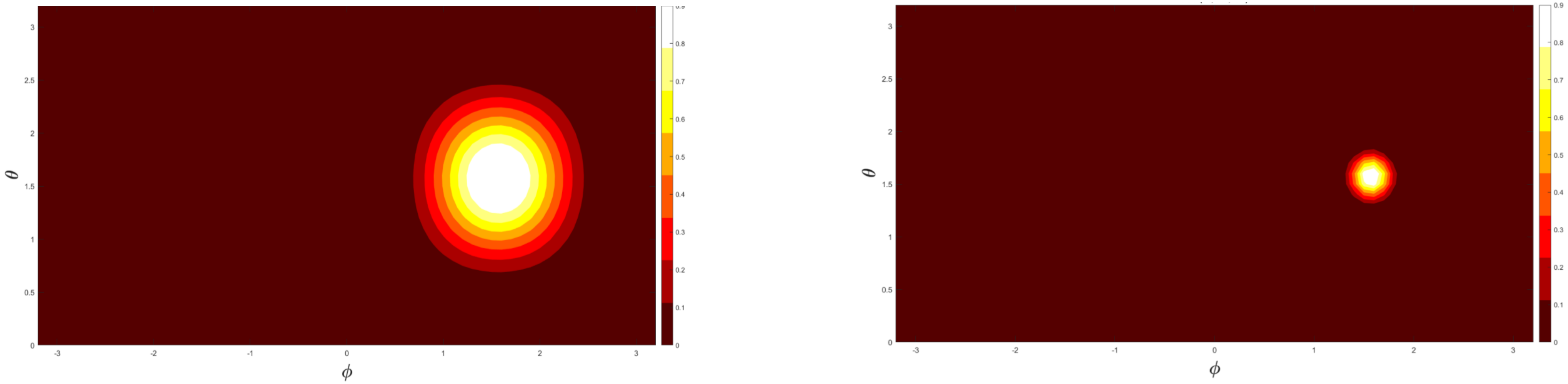
Spin coherent State

- Initial state

$$|j;\theta,\phi\rangle = R(\theta,\phi)|j,j\rangle \equiv \exp\left[i\theta(J_x \sin\phi - J_y \cos\phi)\right]|j,j\rangle.$$

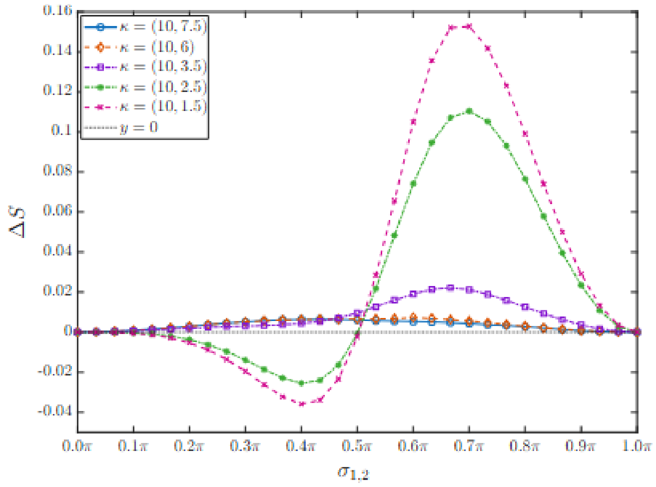
- Coherent states are minimum-uncertainty states

$$\Delta J_i \Delta J_k = \frac{\hbar}{2}|\Delta J_l|$$

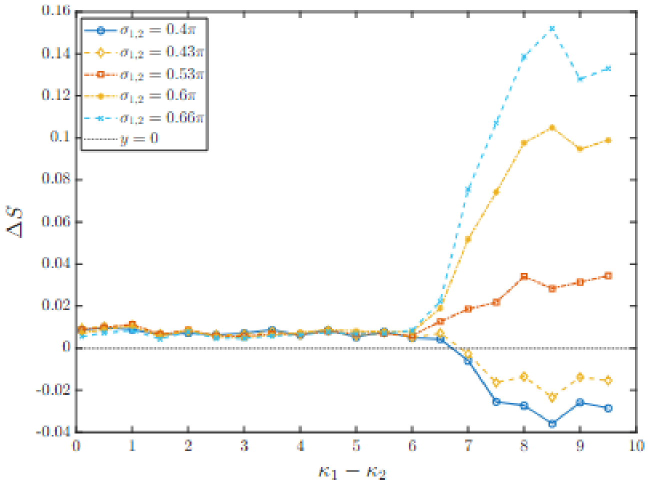


Husimi probability distribution for $j = 4$ and $j = 50$ and max value 1 corresponds to SCS.

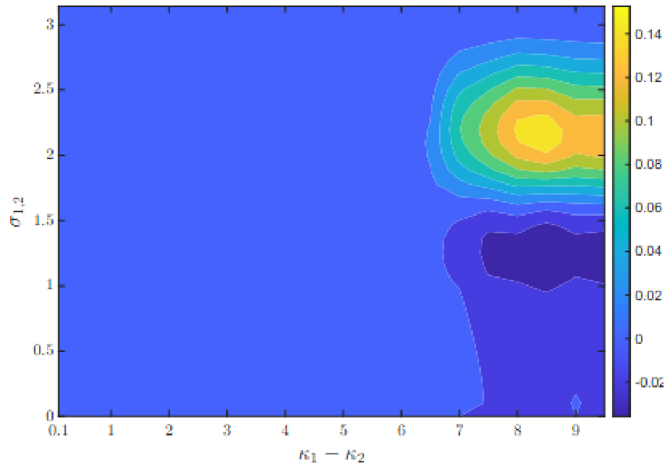
Classical mixture vs superposition



(a)

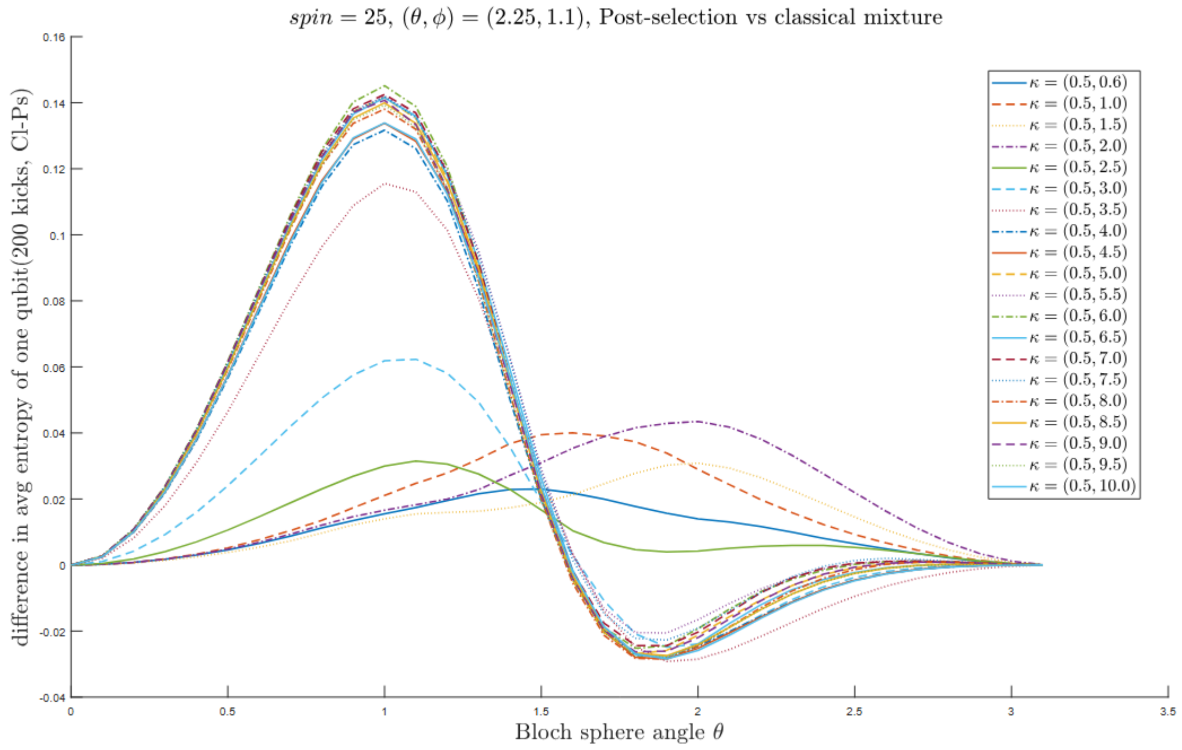


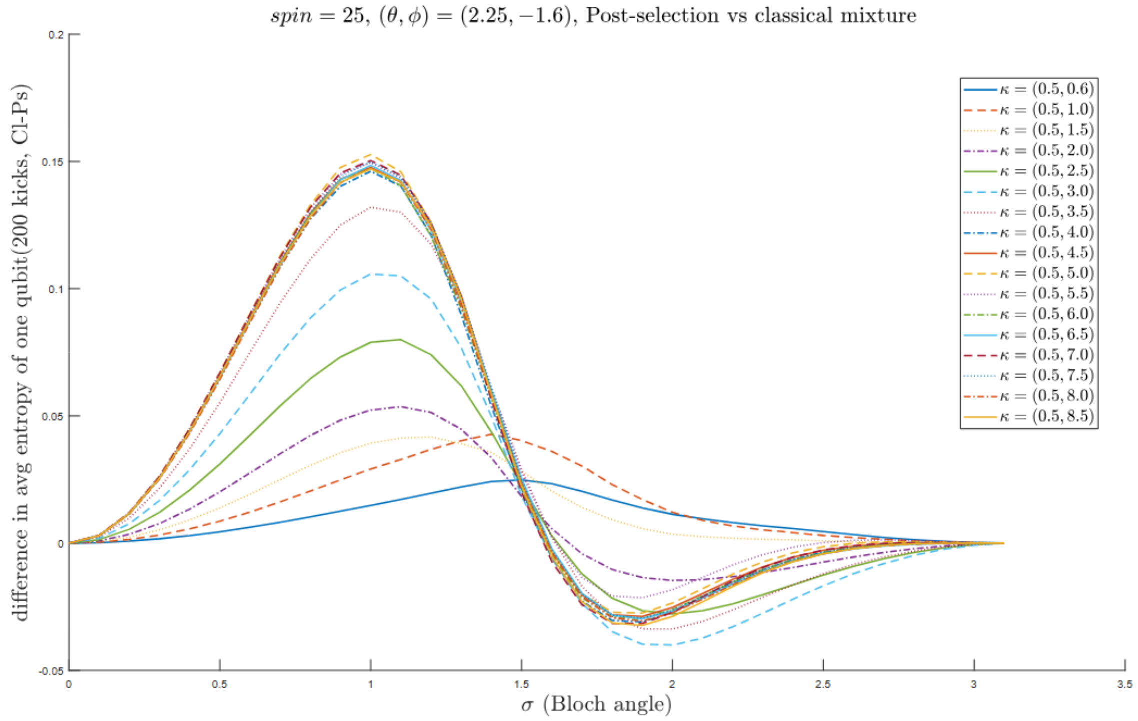
(b)



(c)

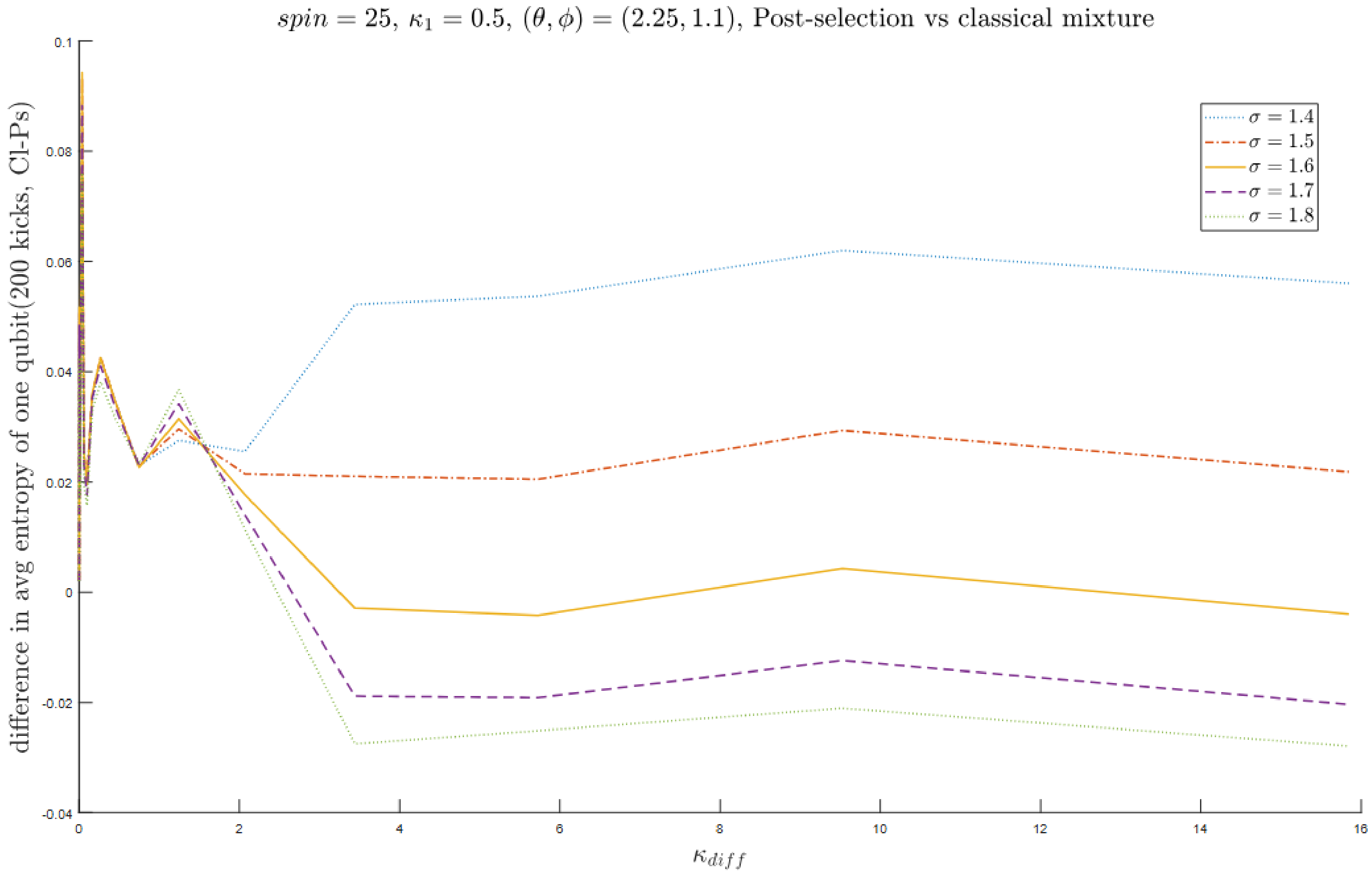
Classical mixture vs superposition





UNIVERSITY OF WATERLOO | FACULTY OF SCIENCE

Classical mixture vs superposition



UNIVERSITY OF WATERLOO | FACULTY OF SCIENCE

Classical mixture vs superposition