The Problem Of Time In Quantum Gravity: A Black Hole Case

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UNIVERSITY OF ALBERTA

The Goal

Study

- the dynamics of and fate of singularity
- in the interior of a quantum black hole
- using the relational approach (problem of time)

Outline

1. Problem of Time and Quantum Relational Approach
2. Classical Schwarzschild Interior
3. Quantum Schwarzschild Interior
4. Singularity-Related Gauge-Invariant Quantum Observables

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- $t =$parameter of the gauge group of evolution $e^{-it\hat{H}}$.

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  Hilbert space a tensor product space

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- Evolution of some $\hat{f}\left(q_S, p_S\right)$ (sometimes $\hat{f}\left(q_S, p_S, q_C, p_C\right)$) w.r.t. $\hat{T}\left(q_C, p_C\right)$

Ideal Quantum Clock

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4. $|t\rangle$ do not overlap $\langle t \mid t' \rangle = \delta\left(t - t'\right)$
5. Non-ideal clock $\hat{T}$: constructed out of POVMs; 2, 3, 4 not true

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for which we employ refined algebraic quantization, in which

$$\Psi_{\phi}^{\text{phys}} = \frac{1}{2\pi}\int_{\mathbb{R}} ds\,\hat{U}^{\dagger}\left(s\right)\phi \quad \forall\phi\in\Phi=\text{set of nice decaying functions }\subset\mathscr{H}_{\text{kin}}$$

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where $\hat{U}(s)=e^{-is\hat{\mathcal{H}}}$

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2. **Classical Schwarzschild Interior**
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Classical Interior Metric

Metric of the Schwarzschild interior

$$ds^2 = -N^2(\lambda)\frac{a(\lambda)}{b(\lambda)}d\lambda^2 + \frac{b(\lambda)}{a(\lambda)}dx^2 + a(\lambda)^2 d\Omega^2$$

Inside the BH

$\lambda =$timelike, $\quad x =$spacelike $\quad N =$lapse function

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Corresponding GR action

$$S = \alpha \int d\lambda \left(N - \frac{\dot{a}\dot{b}}{N} \right), \qquad \alpha = \frac{v}{4G}, \text{ and } v := \int_{x_{\text{min}}}^{x_{\text{max}}} dx$$

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The classical singularity at $\lambda = 0$,
for black hole solution: $\lambda \in (-\sqrt{2GM}, 0]$
for white hole solution: $\lambda \in (0, \sqrt{2GM}]$

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Momenta are

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To get a separable Hamiltonian

$$N \to \bar{N} = -\frac{N}{\alpha} p_a \Longrightarrow H = \bar{N}\mathcal{H} = \bar{N}\left(\frac{\alpha^2}{p_a} + p_b\right)$$

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Clock: b

System: (a, p_a)

Outline

Interior Quantization: $\mathscr{H}_{\text{kin}}$

$\mathscr{H}_{\text{kin}}$: square-integrable functions of p_a and p_b

$$\begin{aligned}\mathscr{H}_{\text{kin}} = \mathscr{H}_a \otimes \mathscr{H}_b =& L^2\left(\mathbb{R}^2, dp_a dp_b\right)\\ =& \left\{\psi : \mathbb{R}^2 \to \mathbb{C} \middle| \int_{\mathbb{R}^2} dp_a dp_b \, \psi^*\left(p_a, p_b\right) \psi\left(p_a, p_b\right) < \infty\right\}\end{aligned}$$

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Algebra

$$\left[\hat{a}, \hat{p}_a\right] = i\hbar\hat{\mathbb{I}}, \qquad \left[\hat{b}, \hat{p}_b\right] = i\hbar\hat{\mathbb{I}},$$

and operators

$$\begin{aligned}\hat{a}\psi =& i\hbar\frac{\partial\psi}{\partial p_a}, & \hat{p}_a\psi =& p_a\psi,\\ \hat{b}\psi =& i\hbar\frac{\partial\psi}{\partial p_b}, & \hat{p}_b\psi =& p_b\psi.\end{aligned}$$

Interior Quantization: Clock States

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and the clock operator is

$$\hat{T} = \hat{b} + \hbar\, J\left(\hat{H}_C\right)$$

Interior Quantization: Inverse Operator

Given

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We look for nice ϕ that: decay to 0 faster than any p_a^m for $m \in \mathbb{Z}$

- At $p_a \to \pm\infty$,
- At $p_a \to 0$

Interior Quantization: Seed Space Φ

Our nice functions (dense in $\mathscr{H}_{\text{kin}}$) are

$$\phi \in \Phi = D_a \otimes D_b \subset \mathscr{H}_{\text{kin}}$$

where the set of Thiemann-Neuser functions

$$D_a = \text{span}\left\{ p_a^n e^{-\frac{p_a^2}{2\sigma^2}} e^{-\frac{\tilde{\sigma}^2}{2p_a^2}} \middle| n \in \mathbb{Z},\ \sigma, \tilde{\sigma} \in \mathbb{R}\backslash\{0\} \right\},$$

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decay faster than any power of p_a^n for $n \in \mathbb{Z}$ at both $p_a \to \pm\infty$ and $p_a \to 0$ and D_b is the set of Schwartz functions

$$D_b = \left\{ \chi_k^{(m)}(p_b) \sim p_b^m e^{-\frac{p_b^2}{2k^2}}, \quad m \in \mathbb{N},\ k \in \mathbb{R}\backslash\{0\} \right\}$$

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Thus

$$\phi\left(p_a, p_b\right) = N_\psi \sum_{m,n} c_{nm} \phi_{\sigma,\tilde{\sigma}}^{(n)}\left(p_a\right) \chi_k^{(m)}\left(p_b\right)$$

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Now can safely represent

$$\widehat{\frac{1}{p_a}}\phi = \frac{1}{p_a}\phi, \qquad \psi \in D_a$$

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Also

$$\begin{aligned} A_{\text{min}} =& 4\pi\left(\Delta a\right)^2 \\ =& 4\pi\ell_p^2\left(\frac{p_p}{\sigma}\right)^2, \qquad \text{for large } n \end{aligned}$$

Kretschmann Scalar

In the same way, for expectation value of the gauge-invariant version of $\hat{K} = R_{\mu\nu\rho\sigma}\widehat{R^{\mu\nu\rho\sigma}}$

$$\left|\left\langle \hat{F}_{K,T}\left(\tau\right)\right\rangle_{\text{phys}}\right| < \infty$$

Thus $\left\langle \hat{F}_{K,T}\left(\tau\right)\right\rangle_{\text{phys}}$ remains finite over the whole interior

Expansion Scalar

For congruence of radial null infalling geodesics

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If τ_0 corresponds to the "location" of classical singularity

$$\begin{aligned}\left|\mathfrak{Re}\left\langle \hat{F}_{\vartheta,T}\left(\tau\right)\right\rangle_{\text{phys}}\right| &<\infty\\ \mathfrak{Re}\left\langle \hat{F}_{\vartheta,T}\left(\tau_0\right)\right\rangle_{\text{phys}} &=0\\ \frac{d}{d\tau}\mathfrak{Re}\left\langle \hat{F}_{\vartheta,T}\left(\tau_0\right)\right\rangle_{\text{phys}} &>0\\ \mathfrak{Re}\left\langle \hat{F}_{\vartheta,T}\left(\tau-\tau_0\right)\right\rangle_{\text{phys}} &= -\,\mathfrak{Re}\left\langle \hat{F}_{\vartheta,T}\left(-\left(\tau-\tau_0\right)\right)\right\rangle_{\text{phys}}\end{aligned}$$

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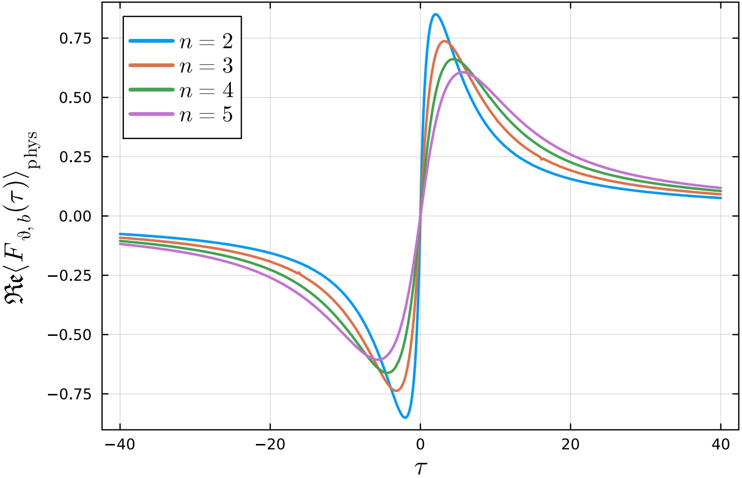
\mathfrak{Re}\left\langle \hat{F}_{\vartheta,T}\left(\tau_0\right)\right\rangle_{\text{phys}} &= 0 \\
\frac{d}{d\tau}\mathfrak{Re}\left\langle \hat{F}_{\vartheta,T}\left(\tau_0\right)\right\rangle_{\text{phys}} &> 0 \\
\mathfrak{Re}\left\langle \hat{F}_{\vartheta,T}\left(\tau-\tau_0\right)\right\rangle_{\text{phys}} &= -\,\mathfrak{Re}\left\langle \hat{F}_{\vartheta,T}\left(-\left(\tau-\tau_0\right)\right)\right\rangle_{\text{phys}}
\end{aligned}$$

This is a BH→WH bounce!

Expansion Scalar

In fact numerical calculations confirms: classical singularity replaced by a bounce to a WH



Summary

- We studied the interior of the Schwarzschild BH:
    - Relational approach
    - Standard Schrodinger representation
    - Thiemann-Neuser states to deal with inverse operators
    - Refined algebraic quantization
- Computed 3 singularity-related gauge-invariant observables
    - Area of 2-spheres: strictly positive, proportional to Δa
    - Kretschmann: regular everywhere
    - Expansion scalar: regular everywhere, changing sign at classical singularity
- Singularity is replaced by a BH→WH bounce
- It seems that the above ingredients resolve the singularity without the need to resort to other types of quantization
- But further works to challenge this is needed/welcome