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Stochastic Dark Matter

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Plan

Motivation

- Spacetime discreteness could lead to covariant Brownian motion
- Particles would deviate from continuum geodesics

Formalism

- Stochastic calculus gives a unique stochastic geodesic equation
- Covariant diffusion equation in phase space

Application

- Diffusion of dark matter particles
- Implications for matter density and power spectrum
- Possible explanation for the S_8 tension

Stochasticity as a signature of a discrete background

2d Brownian motion in a fluid

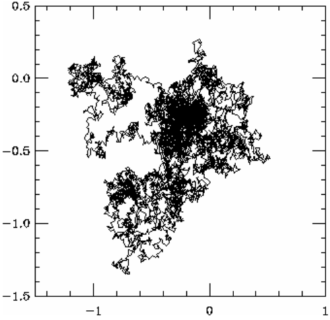


Image: (Garcia-Palacios, 2007)

1+1d Brownian motion on a causal set

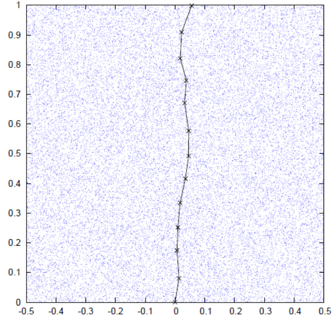


Image: (Philpott, 2010a)

Brownian motion in a fluid indicates the existence of molecules in a fluid.

Brownian motion in vacuum could indicate spacetime discreteness.

Brownian motion

Simplest Brownian motion:

$$\begin{cases} dx^i = dW^i \\ \langle dW^i dW^j \rangle = 2D\delta^{ij} dt \end{cases} \quad \leftrightarrow \frac{\partial f}{\partial t} = D\partial_x^2 f \tag{1}$$

In phase space (Klein-Kramers eq.):

$$\begin{cases} dx^i = v^i dt \\ dv^i = -\gamma v^i + dW^i \\ \langle dW^i dW^j \rangle = 2D\delta^{ij} dt \end{cases} \quad \leftrightarrow \frac{\partial f}{\partial t} + v^i \frac{\partial f}{\partial x^i} - (\gamma v^i) \frac{\partial f}{\partial v^i} = 3\gamma f + D\partial_v^2 f \tag{2}$$

Stochastic geodesics

Dudley (1966) proved that no Lorentz invariant stochastic process exists in Minkowski spacetime.

$$\begin{cases} dx^\mu = \frac{1}{m} p^\mu d\tau \\ dp^\mu + \frac{1}{m} p^\sigma p^\nu \Gamma^\mu_{\sigma\nu} d\tau = b^\mu d\tau + dW^\mu \end{cases} \tag{3}$$

with statistics

$$\left\langle dW^\mu \right\rangle = 0 \ , \qquad \left\langle dW^\mu dW^\nu \right\rangle = 2D^{\mu\nu} d\tau \ . \tag{4}$$

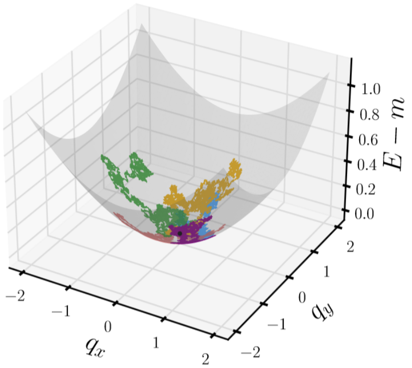
Using Itô's lemma, conservation of mass gives:

$$\begin{cases} p_\mu b^\mu + D^\mu{}_\mu = 0, \\ p_\mu D^{\mu\nu} = 0. \end{cases} \tag{5}$$

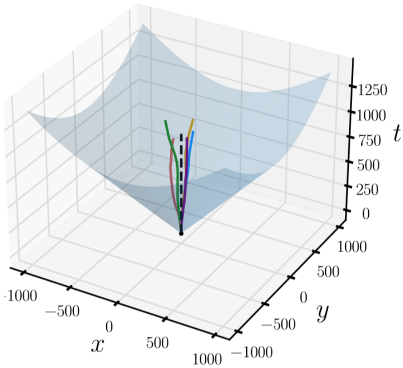
Minimally coupled case

Geometrically, b^μ and $D^{\mu\nu}$ should be constructed out of p^μ, $p^\mu p^\nu$, and $g^{\mu\nu}$.

$$b^\mu = \frac{3\kappa}{m^2} p^\mu \ ,$$
$$D^{\mu\nu} = \kappa\left(g^{\mu\nu} + \frac{1}{m^2} p^\mu p^\nu\right) \ . \quad (6)$$



(a)



(b)

Covariant Diffusion Equation

Corresponding Fokker-Planck equation:

$$L[f] \equiv p^{\mu} \frac{\partial}{\partial x^{\mu}} f + \Gamma^{\sigma}_{\mu\nu} p_{\sigma} p^{\mu} \frac{\partial}{\partial p_{\nu}} f = m\kappa \nabla^2_{\mathbb{H}_3} f. \tag{8}$$

This is a modified Boltzmann equation by a diffusion term in momentum space: A covariant diffusion equation

κ is the diffusion constant M^2/L.

In flat spacetime: Sorkin (1986), Dowker et al. (2004), Philpott et al. (2009).
In mathematical literature: Franchi and Le Jan (2007). Generalization of

Klein-Kramers; relativistic but frame-dependent: Debbasch et al. (1997), Dunkel and Hänggi (2005), Herrmann (2010), Cai et al. (2023).

Flat-space bounds: Kaloper and Mattingly (2006) .

Application in Cosmology

In principle, all massive particles could undergo this diffusion.

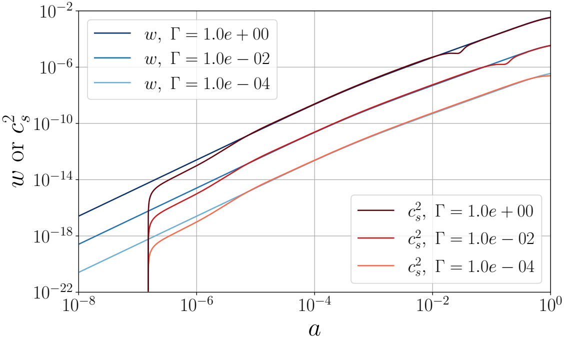
First step: Stochastic dark matter.

At the background, covariant diffusion corrects the continuity equation:

$$\dot{\rho} + 3H(\rho + P) = \Gamma(\rho - \frac{3}{2}P), \quad \Gamma = \frac{3\kappa}{m^2} \tag{9}$$

Number conservation $\dot{n} + 3Hn = 0$ still holds.

Background Solutions



In non-relativistic approximation

$$f_0 \propto \frac{1}{\sqrt{\frac{4}{3}\pi\Gamma T}^3} e^{-\frac{a^2 q^2}{\frac{4}{3}\Gamma T m^2}}; \quad dT = a^3 d\eta \tag{10}$$

Temperature is non-zero: $k_B \mathcal{T} \sim \Gamma m \frac{T}{a^2}$.

Diffusion is controlled by: Γ / H_0.

Perturbations

Solving Boltzmann Hierarchy: redundantly specify both κ, m. But only $\Gamma = 3\kappa/m^2$ matters for observables!

Generalized Boltzmann Hierarchy (GBH) (de Senna Nascimento, 2021): Real space only!

$$f_{\ell,n} \sim \int d^3q \; E\left(\frac{q}{E}\right)^{2n+\ell} P_\ell\left(\cos\theta\right)\delta f \tag{11}$$

$$\dot{f}_{\ell,n} = (\text{gravitational source}) + \sum_{\substack{\ell'=\ell-1,\ell,\ell+1 \\ n'=n-1,n,n+1}} c_{\ell',n'}(\Gamma) f_{\ell',n'}. \tag{12}$$

Should be truncated at some finite n_{max}, ℓ_{max}.

Analytic solution

Line-of-sight solution to δf with heat kernel:

$$\delta f(T,k,Q) = \int d^3Q' \left(e^{-\frac{1}{3}\Gamma k^2 h(T,T_i)}\right) \left(\frac{e^{-\frac{|Q-Q'|^2}{\frac{4}{3}\Gamma m^2 (T-T_i)}}}{\left(\frac{4}{3}\pi\Gamma m^2 (T-T_i)\right)^{\frac{3}{2}}}\right) \underbrace{e^{-ik\mu y(Q';T_f,T_i)+ik\mu y(Q;T_f;T)}}_{\text{free streaming}} \underbrace{\delta f(T_i,k,Q')}_{\text{SW-like Initial cond.}}$$

$$+ \int_{T_i}^{T} dT' \left(e^{-\frac{1}{3}\Gamma k^2 h(T,T')}\right) \int d^3Q' \left(\frac{e^{-\frac{|Q-Q'|^2}{\frac{4}{3}\Gamma m^2 (T-T')}}}{\left(\frac{4}{3}\pi\Gamma m^2 (T-T')\right)^{\frac{3}{2}}}\right) e^{-ik\mu y(Q';T_f,T')+ik\mu y(Q;T_f;T)} \underbrace{\frac{ik\mu m}{a'^2}\frac{\partial f_0}{\partial \tilde{q}}\phi_k(T')}_{\text{ISW-like source}}$$

heat kernel

Asymptotic solutions

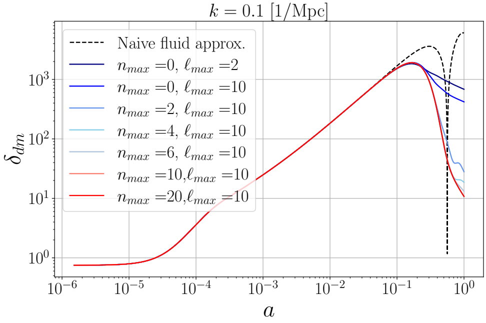
At the background level, only Γ/H_0 is important.

For perturbations: $\left(\frac{\lambda_D}{\lambda}\right)^2 \sim a\Gamma k^2 \eta^3$ matters!

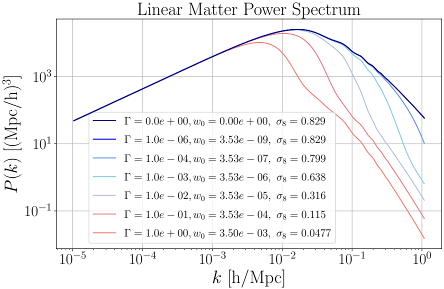
Using saddle point approximation, at scales below λ_D:

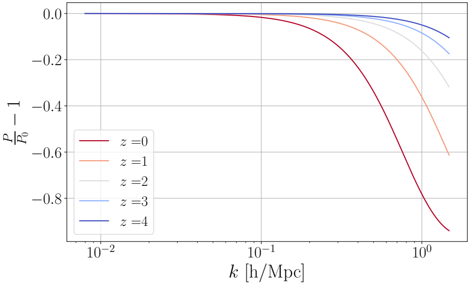
$$\delta_c \approx \frac{-21}{2a\Gamma\eta}\phi \tag{13}$$

For $\Gamma = 0.1\text{km/s/Mpc}$:



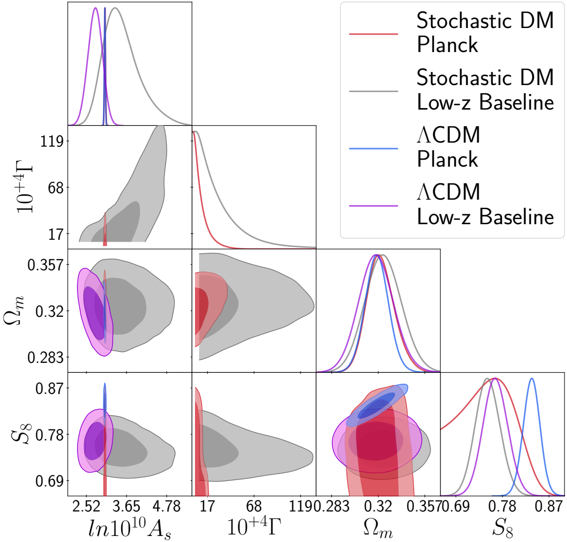
Matter Power spectrum



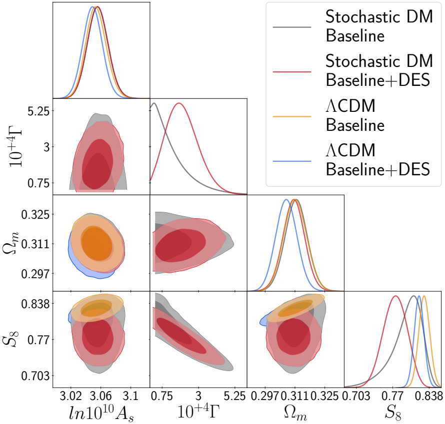


The matter power spectrum suppression could help with the S_8 tension. This is the discrepancy in the amount of matter clustering in ΛCDM as predicted from CMB compared to late-time weak-lensing measurements.

S_8 tension: ΛCDM vs Stochastic DM



MCMC: Planck + Late-time probes[1]



$$\text{Best fit: } \Gamma/H_0 \sim 10^{-6} \;\rightarrow\; \tau_f \sim \left(\Gamma^{-1} t_{pl}^4\right)^{1/5} \sim 10^{13} t_{pl}.$$

[1] KiDS1000 (Li et al., 2023), SDSS DR7 (Howlett et al., 2015), BOSS DR12 LOWZ and LRG samples (Alam et al., 2017), eBOSS DR16 LRG and Quasar (QSO) samples (Bautista et al., 2021; Hou et al., 2021), eBOSS DR16 Ly-α (Des Bourboux et al., 2020), and uncalibrated PantheonPlus SNIa catalog

Conclusions

- Covariant Brownian motion of DM particles has a universal description with a single free parameter.
- This results in spontaneous heating of dark matter particles.
- Matter power spectrum gets suppressed at small scales.
- The model is promising in resolving the S_8 tension.

Future directions

- N-body simulations? Implications for small-scale problems of CDM? Core-cosp problem?
- Stochastic behavior for fields? Wave-like dark matter?
- Stochastic Einstein equations? Think back: source of the non-conservation is discrete geometry.

$$G_{\mu\nu} + \xi_{\mu\nu} = T_{\mu\nu}, \quad T_{\mu\nu}(x) = \rho u_\mu u_\nu \tag{14}$$

Thank You!

Spacetime might be discrete at Planck scale. Where to look for discreteness?

Behavior of quantum fields on discrete/quantum spacetime? (Albertini et al., 2024; Parikh et al., 2020)

Simplify: Behavior of point particles on a classically discrete spacetime.

Geodesic $\leftrightarrow$ longest chain between two events (Myrheim, 1978; Brightwell and Gregory, 1991).

Too much global!

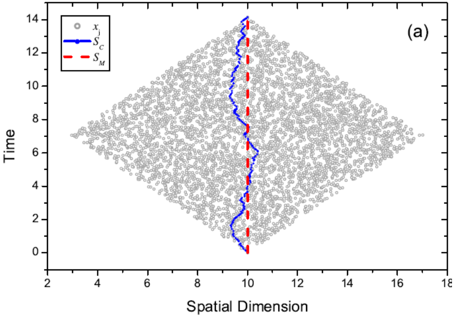
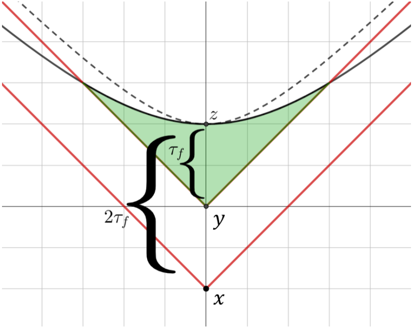
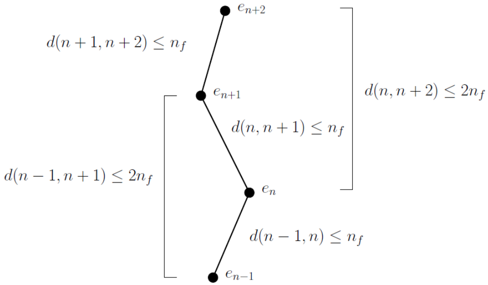


Image: (Ilie et al., 2006).

Intrinsic Model of Particle Propagation

Follow the “longest chain” only at a non-local scale $\tau_f \sim n_f t_p$. (Philpott et al., 2009).





Maximize $d(e_{n+1}, e_{n+2})$. Demand separation of scales:

$$t_p \ll \tau_f. \tag{15}$$

Diffusion equations

First attempt: Does this explain the distribution of cosmic rays? (Dowker et al., 2004)

$$m \frac{\partial}{\partial \tau} \rho + p^{\mu} \frac{\partial}{\partial x^{\mu}} \rho = m \kappa \nabla^2_{\mathbb{H}_3} \rho \tag{16}$$

Bound from cold Hydrogen gas in laboratory: $\kappa \lesssim 10^{-44} GeV^3$.
This is the unique Lorentz invariant diffusion equation! (Philpott et al., 2009)
For $\mathbb{M}_2$, it was shown (Philpott, 2010b)

$$\kappa \sim m^2 \frac{t_p^4}{\tau_f^5}. \tag{17}$$

The equation is independent of causal sets. Can model any Lorentz-invariant diffusive QG effect!
Aim: curved spacetime.

Stochastic Processes, Physics of Fokker-Planck eq

Claim: The covariant equation that describes diffusion of free massive particles in a curved spacetime is unique *assuming* minimal coupling.
Let ρ be the distribution in phase space $\mathcal{P}$, with coordinates Z^A.

Fokker-Planck:

$$\partial_\tau \rho = \underbrace{-\partial_A\left(v^A \rho\right)}_{\text{drift/transport}} + \underbrace{\partial_A \partial_B \left(K^{AB}\rho\right)}_{\text{diffusion}}. \tag{18}$$

Transformation properties: (Sorkin, 1986; Graham, 1977)

$$K'^{MN} = \frac{\partial Z'^M}{\partial Z^A}\frac{\partial Z'^N}{\partial Z^B} K^{AB} \tag{19}$$

A true vector: $u^A = v^A - \frac{1}{\sqrt{|\hat{g}|}}\partial_B(K^{AB}\sqrt{|\hat{g}|})$. What can u^A, K^{AB} be?

General Relativistic Phase Space

Phase space: $\Gamma_m^+ = \{(x,p) \in T^*\mathcal{M} | g^{\mu\nu}(x)p_\mu p_\nu = -m^2, p^0 > 0\}$. Includes time coordinate! (Acuna-Cardenas et al., 2022).
Project on a tetrad ${e_a}^\mu(x)$: $q_a = {e_a}^\mu p_\mu$. So $Z^A = (x^\mu, q_i),\ i = 1,2,3$.
What can u, K be to be covariant under these transformations?

$$v^A = \lim_{\delta\tau\to 0} \frac{1}{\delta\tau}\left\langle \delta Z^A \right\rangle \quad \to \quad v^\mu = p^\mu/m$$
$$K^{AB} = \lim_{\delta\tau\to 0} \frac{1}{2\delta\tau}\left\langle \delta Z^A \delta Z^B \right\rangle \quad \to \quad K^{\mu B} = 0. \tag{20}$$

Therefore, K^{ij} has to be a tensor on the mass shell $\eta^{ab}q_a q_b = -m^2$.
This is a maximally symmetric hyperbolic space.

$$K = \begin{pmatrix} K^{\mu\nu} = 0 & K^{\mu i} = 0 \\ 0 & K^{ij} = \kappa(\delta^{ij} + \frac{1}{m^2}q^i q^j) = \kappa\sigma^{ij} \end{pmatrix}. \tag{21}$$

$u = L$ to recover geodesic motion!

Vector u

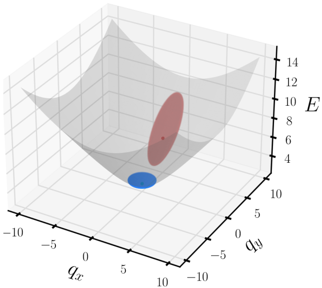
Admissible coordinate transformations?

$$(x^\mu, q_a) \to \Big(x'^\alpha(x),\ q'_r(x,q) = e'_r(x).e^a(x)\ q_a\Big). \tag{22}$$

The vector u transforms as

$$u'_r = \underbrace{\frac{\partial q'_r}{\partial x^\mu}}_{=\partial_\mu\left(e'_r.e^a\right)q_a} u^\mu + \frac{\partial q'_r}{\partial q_a} u_a \tag{23}$$

$$u = \frac{1}{m} L, \quad L = p^\mu \frac{\partial}{\partial x^\mu} + \Gamma^\sigma_{\mu\nu} p_\sigma p^\mu \frac{\partial}{\partial p_\nu}$$



A non-minimal Brownian motion:

$$D^{\mu\nu} = \kappa'\left(g^{\mu\alpha} + \frac{1}{m^2}p^\mu p^\alpha\right)\left(g^{\nu\beta} + \frac{1}{m^2}p^\nu p^\beta\right) R_{;\alpha} R_{;\beta}\ , \tag{24}$$

$$b^\mu = \frac{1}{m^2} p^\mu D^\nu{}_\nu = \frac{\kappa'}{m^2}\left(R_{;\alpha}R^{;\alpha} + \frac{1}{m^2}\left(p^\alpha R_{;\alpha}\right)^2\right) p^\mu\ . \tag{25}$$

Energy-Momentum Non-Conservation

How to conserve energy-momentum?

$$\nabla_\mu T^\mu_{\ \nu} = m\kappa \int \frac{d^3q}{E}\, p_\nu \nabla^2_{\mathbb{H}_3} f. \tag{26}$$

Look for a phenomenological dark energy model, with minimal deviation from ΛCDM. Main physics: diffusion of matter.
Unimodular non-conservation? $\nabla_\nu \Lambda = -8\pi G \nabla_\mu T^\mu_{\ \nu}$ doesn't work.
Ansatz from imperfect fluids (Sawicki et al., 2013; Zimdahl et al., 2019), with $w_x = -1$, $c_x^2 = 1$.

$$T^x_{\mu\nu} = p_x g_{\mu\nu} + (p_x + \rho_x) u_\mu u_\nu + \mathcal{Q}_\mu u_\nu + \mathcal{Q}_\nu u_\mu, \quad u^\mu \mathcal{Q}_\mu = 0. \tag{27}$$

The dynamics of dark energy comes solely from the balance equation: $\nabla^\mu T^{dm}_{\mu\nu} = -\nabla^\mu T^x_{\mu\nu}$. For example, the continuity eq reads

$$\bar{\rho}_x' = -a\Gamma\bar{\rho}_{dm}. \tag{28}$$

Background Solutions

In non-relativistic approximation

$$f_0 \propto \frac{1}{\sqrt{\frac{4}{3}\pi\Gamma T}^{3}} e^{-\frac{a^2 q^2}{\frac{4}{3}\Gamma T m^2}}; \qquad (29)$$

where $dT = a^3 d\eta$

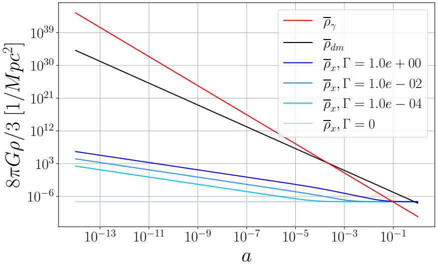
Temperature is non-zero:

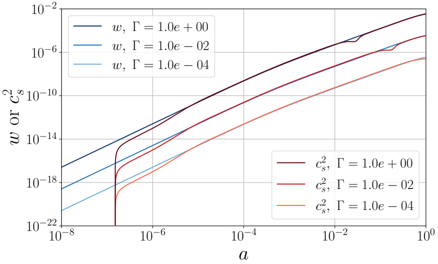
$$k_B \mathcal{T} = \frac{2\pi}{3}\Gamma m \frac{T}{a^2}. \qquad (30)$$

Can we select $w = \frac{P}{\rho}$, $c_s^2 = \frac{\delta P}{\delta \rho}$ to be zero?

No!

$mn = \rho - \frac{3}{2}P + (\text{higher moments})$.





Asymptotic solutions

During matter domination:

$$e^{-\frac{1}{3}\Gamma k^2 h(T,T_i)} \approx e^{-\frac{1}{315}(a\Gamma k^2\eta^3 - a'\Gamma k^2\eta'^3)} \tag{31}$$

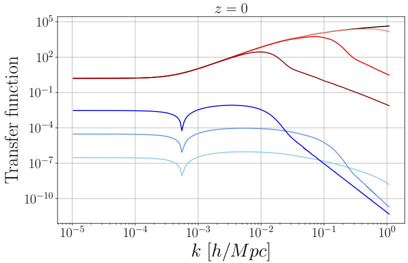
At background level, only Γ/H_0 is important.
At perturbations: $\left(\frac{\lambda_D}{\lambda}\right)^2 \sim a\Gamma k^2\eta^3$ matters!

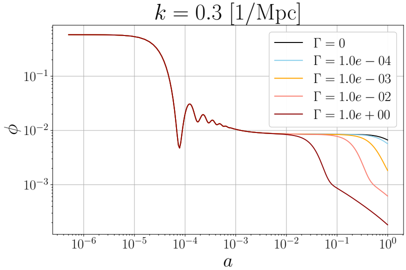
$e^{-\frac{1}{3}\Gamma k^2 h(T,T_i)} \ll 1 \Rightarrow$ Initial condition is exponentially suppressed
$e^{-\frac{1}{3}\Gamma k^2 h(T,T')} \ll 1$ unless $T' \simeq T$ $\Rightarrow$ only late ISW is relevant.

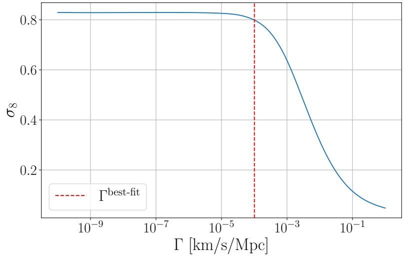
Using saddle point approximation

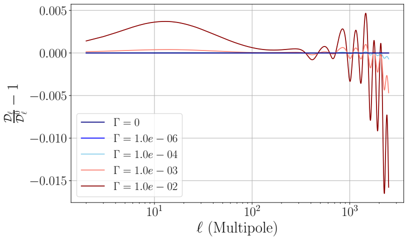
$$\delta_c \approx \frac{-21}{2a\Gamma\eta}\phi, \quad \frac{\delta p}{\rho} \approx -\phi \tag{32}$$

Perturbation plots

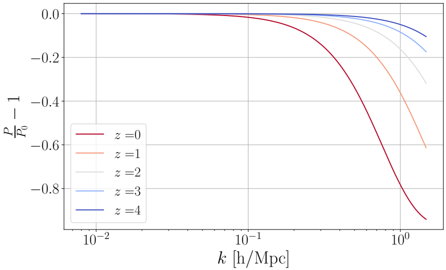




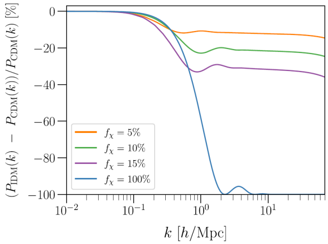




Comparison with IDM and DDM

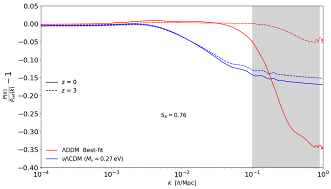


IDM (He et al., 2023)

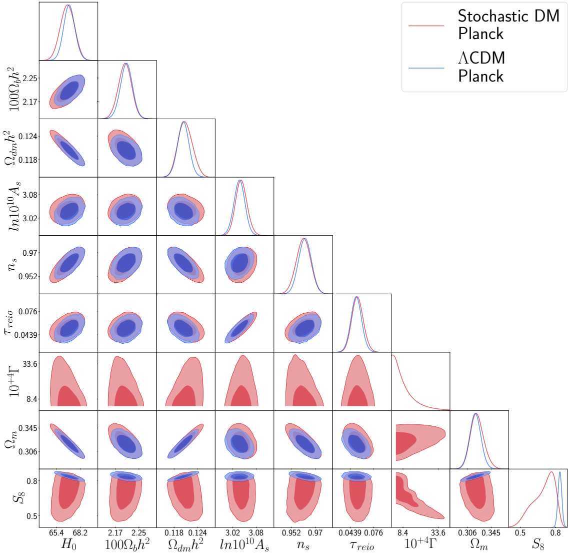


Lyα allows at most 20% suppression of matter power spectrum at $z \sim 3$.

DDM (Abellan et al., 2022)

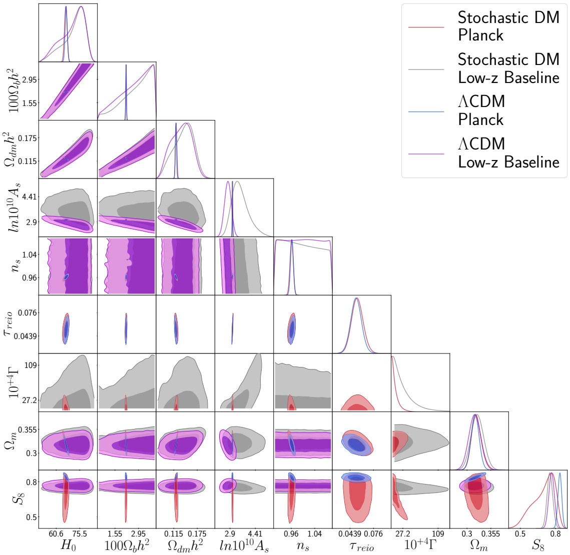


Planck

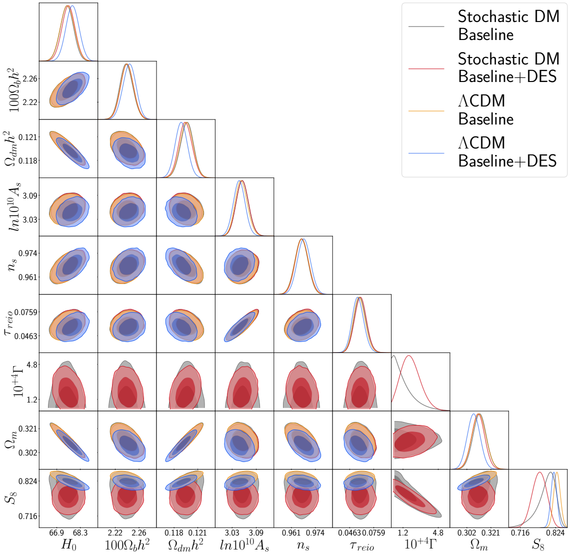


Stochastic DM
Planck
ΛCDM
Planck
H_0
$100\Omega_b h^2$
$\Omega_{dm}h^2$
$ln10^{10}A_s$
n_s
τ_{reio}
$10^{+4}\Gamma$
Ω_m
S_8
65.4
68.2
2.17
2.25
0.118
0.124
3.02
3.08
0.952
0.97
0.0439
0.076
8.4
33.6
0.306
0.345
0.5
0.8

Planck vs Low-z



Baseline+DES



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