Dynamical effects on gravity and the possibility of gravitational detection of earthquakes

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- Gravitational waves were detected for the first time in 2016 which offered a new validation of general relativity but also a new tool to probe the cosmos.
- But what could those detectors tell us about processes happening much closer to us?
- The gravitational effect of masses moving close to the detectors is often seen as a source of Newtonian noise but, in the case of earthquakes, we might be able to detect a signal.

- Since gravity is a very weak force and decays as $\frac{1}{r^2}$, our experiment has to either be very close or it has to involve extremely massive objects.
- In the case of earthquakes, there is more than 10 billion tonnes of rock moving at speeds of meters per second over a distance measured in meters.

Since those gravitational effect are extremely small, we can treat them as linear perturbations on a flat spacetime.

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{1}$$

Choosing the harmonic gauge $(\partial^\mu(\bar{h}_{\mu\nu}) = \partial^\mu(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h^\lambda_\lambda) = 0)$, the einstein equation simplifies to :

$$\Box \bar{h}_{\mu\nu} = -16\pi G T_{\mu\nu} \tag{2}$$

For a point mass, the stress-energy tensor takes the simple form :

$$T_{00} = \gamma M c^2 \qquad T_{0i} = T_{i0} = \gamma M v_i c \qquad T_{ij} = \gamma M v_i v_j \tag{3}$$

Similarly to the Liénard-Wiechert potential, we solve the differential equation using the retarded Green function.

$$\bar{h}_{\mu\nu}(x) = 16\pi G \int D_r(x - x')T_{\mu\nu}(x')d^4x' \tag{4}$$

Here the stress energy tensor depends on the retarded time (t_r) and the position at time t_r so the perturbations will also be dependent on them. We find :

$$\bar{h}_{\mu\nu}(x) = \left.\frac{4GM\gamma\beta_\mu\beta_\nu}{R - \boldsymbol{\beta}\cdot\boldsymbol{R}}\right|_{t_r,\, x(t_r)} \tag{5}$$

where $\beta = \frac{v}{c}$ and $\boldsymbol{R}$ is the distance between the observer and the source.

The equation of motion of a mass due to the perturbation of the metric will just be the geodesic.

$$\frac{dp_i}{dt} = -E\Gamma^i_{00} = -m\left[-\frac{1}{2}\partial_i\bar{h}_{00} + \partial_0\bar{h}_{0i} - \frac{1}{4}\partial_i\bar{h}^\lambda_\lambda\right]\Bigg|_{t_r,\,x(t_r)} \tag{6}$$

It is important to note that the derivatives in the equation of motion are with respect to the coordinates at which the observer made the measurements. Defining $\kappa = 1 - \frac{\boldsymbol{\beta}\cdot\boldsymbol{R}}{R}$ and calculating all the derivatives we find the following equation :

$$\begin{aligned}\frac{d\vec{p}}{dt} = -GmM\Bigg[&\frac{4R\gamma\dot{\vec{\beta}}}{R^2\kappa^2} + \frac{4R\gamma^3(\vec{\beta}\cdot\dot{\vec{\beta}})\vec{\beta}}{R^2\kappa^2} + \frac{\gamma^3(\vec{\beta}\cdot\dot{\vec{\beta}})\vec{R}}{R^2\kappa^2} + \frac{2\gamma(\vec{\beta}\cdot\dot{\vec{\beta}})\vec{R}}{R^2\kappa^2}\\ &-\frac{\gamma\vec{\beta}}{R^2\kappa^2} + \frac{4\gamma(\vec{R}\cdot\dot{\vec{\beta}})\vec{\beta}}{R^2\kappa^3} + \frac{\gamma\vec{n}}{R^2\kappa^3} + \frac{\gamma(\vec{n}\cdot\dot{\vec{\beta}})\vec{R}}{R^2\kappa^3}\\ &+\frac{\gamma\beta^2(\vec{n}\cdot\dot{\vec{\beta}})\vec{R}}{R^2\kappa^3} + \frac{4\gamma(\vec{n}\cdot\vec{\beta})\vec{\beta}}{R^2\kappa^3}\Bigg]\end{aligned} \tag{7}$$

If we only consider the terms of β up to the second order and terms of $\dot{\beta}$ up to the first order, we arrive at the following :

$$\frac{dp_i}{dt} \approx GMm\Bigg[-\frac{n_i}{R^2}+\frac{4\dot{\beta}_i}{R}+\frac{8(\vec{n}\cdot\vec{\beta})\dot{\beta}_i}{R}+\frac{4(\vec{n}\cdot\dot{\vec{\beta}})\beta_i}{R}+\frac{6(\vec{n}\cdot\vec{\beta})\beta_i}{R^2} -\frac{3(\vec{n}\cdot\vec{\beta})n_i}{R^2}-\frac{\beta^2 n_i}{2R^2}-\frac{(\vec{n}\cdot\dot{\vec{\beta}})n_i}{R}-\frac{3(\vec{n}\cdot\vec{\beta})(\vec{n}\cdot\dot{\vec{\beta}})n_i}{R}+\frac{\beta_i}{R^2} -\frac{3(\vec{\beta}\cdot\dot{\vec{\beta}})n_i}{R}-\frac{4(\vec{n}\cdot\vec{\beta})(\vec{\beta}\cdot\dot{\vec{\beta}})n_i}{R}\Bigg] \tag{8}$$

The interesting result is that the correction terms that we have found depends also on the speed and the acceleration of the mass. We also see that at very low speed and acceleration, this expression reduces to Newton's expression for the gravitational force.

- When looking at earthquakes that have a high seismic moment (>7), we see that the movements starts at an epicentre before propagating along the fault at speeds close to the shearing speed.
- Previously in our calculations, we were only considering a moving point mass but now we have many smaller masses moving at different moments over a time frame of about one minute.
- But in general relativity, energy also affects spacetime and earthquakes release a great amount of potential energy which we have to take into account.

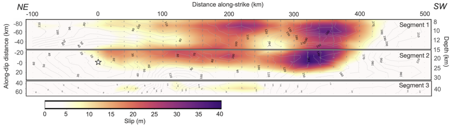


Figure: figure of the displacements of several points along the fault surface

- We now need a way to include the stored energy that is released. Furthermore, the data for the slip needs to be calculated for regions farther away from the fault surface.
- To do so, we treat a seismic event as a dislocation in a lattice[1].

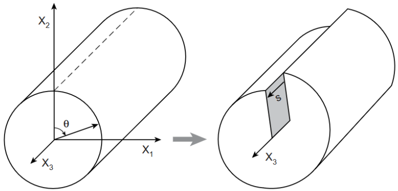


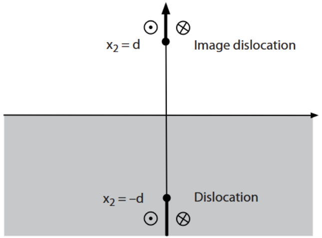
Figure: Figure of a dislocation along the x_3 axis.

[1] Paul Segall. *Earthquake and Volcano Deformation*. Publication Title: Earthquake and Volcano Deformation. Paul Segall. ISBN: 9781400833856. Princeton University Press ADS. Jan. 1, 2010

When starting from one side of the fault, if we were to do a complete turn we would have moved in the x_3 direction by the full amount of the displacement which leads to the following equation :

$$u_3 = \pm \frac{s\theta}{2\pi}. \\ \pm \frac{s}{2\pi} \arctan \frac{x_1}{x_2} \quad (9)$$

Since the slip stops once it reaches the surface, we need to add a dislocation in the opposite direction at the same distance but above the surface.



This adds a new term in our expression :

$$u_3(x_1, x_2) = -\frac{s}{2\pi}\left[\tan^{-1}\left(\frac{x_1}{x_2 + d_1}\right) - \tan^{-1}\left(\frac{x_1}{x_2 - d_1}\right)\right] \quad (10)$$

Since we break down the movement along the fault into regions of constant slip, we will need to add a second dislocation and its mirror dislocation. We place one dislocation at the beginning of the interval and one at the end. Using equation 10, we find the following expression for the slip at a distance x_1 from the fault :

$$\begin{aligned} u_3 = -\frac{s}{2\pi}\Bigg[&\tan^{-1}\left(\frac{x_1}{x_2 + d_1}\right) - \tan^{-1}\left(\frac{x_1}{x_2 - d_1}\right) \\ &- \tan^{-1}\left(\frac{x_1}{x_2 + d_2}\right) + \tan^{-1}\left(\frac{x_1}{x_2 - d_2}\right)\Bigg] \end{aligned} \quad (11)$$

Assuming that Hooke's law is a good enough approximation for the rocks making up the lithosphere, we can relate the slip directly to the strain from which we can determine the stored potential energy causing the slip.

$$\epsilon_{13} = \frac{1}{2}\frac{\partial u_3}{\partial x_1} \qquad \epsilon_{23} = \frac{1}{2}\frac{\partial u_3}{\partial x_2} \tag{12}$$

Using equation 11, the expression for the strain becomes :

$$\begin{aligned}\epsilon_{13} = -\frac{s}{4\pi}\Bigg[&\frac{x_2+d_1}{x_1^2+(x_2+d_1)^2} - \frac{x_2-d_1}{x_1^2+(x_2-d_1)^2}\\ &-\frac{x_2+d_2}{x_1^2+(x_2+d_2)^2} + \frac{x_2-d_2}{x_1^2+(x_2-d_2)^2}\Bigg]\end{aligned} \tag{13}$$

$$\begin{aligned}\epsilon_{23} = \frac{s\,x_1}{4\pi}\Bigg[&\frac{1}{x_1^2+(x_2+d_1)^2} - \frac{1}{x_1^2+(x_2-d_1)^2}\\ &-\frac{1}{x_1^2+(x_2+d_2)^2} + \frac{1}{x_1^2+(x_2-d_2)^2}\Bigg]\end{aligned} \tag{14}$$

The strain can be used to find the stored energy and the shear stresses.

$$T_{00} = \gamma\rho c^2\delta(x' - x) + \mu(\epsilon_{13}^2 + \epsilon_{23}^2) \qquad T_{i0} = T_{0i} = \gamma\rho c\beta_i\delta(x' - x)$$
$$T_{ij} = (\sigma_{ij} + \gamma\rho\beta_i\beta_j)\,\delta(x' - x) = (2\mu\epsilon_{ij} + \gamma\rho\beta_i\beta_j)\,\delta(x' - x) \tag{15}$$

Using the retarded Green function to once again solve equation 2, we find the following perturbations :

$$\bar{h}_{00} = \frac{4GM\gamma}{R - \boldsymbol{\beta}\cdot\boldsymbol{R}} + \frac{4GV\mu(\epsilon_{13}^2 + \epsilon_{23}^2)}{R - \boldsymbol{\beta}\cdot\boldsymbol{R}} \tag{16}$$

$$\bar{h}_{0i} = \bar{h}_{i0} = \frac{4GM\gamma\beta_i}{R - \boldsymbol{\beta}\cdot\boldsymbol{R}} \tag{17}$$

$$\bar{h}_{ij} = \frac{4GM\gamma\beta_i}{R - \boldsymbol{\beta}\cdot\boldsymbol{R}} + \frac{8G\mu\epsilon_{ij}}{R - \boldsymbol{\beta}\cdot\boldsymbol{R}} \tag{18}$$

Since we are interested in detecting the gravitational effects caused by an earthquake and because it is very close, we can use the instantaneous coordinates for the purpose of our calculations.

Rough approximations using the 2025 Kamchatka Peninsula earthquake gives us a change in length of 10^{-15}m which should be detectable by LIGO which can detect changes as small as 10^{-18}m.

Thank you for listening!