Resonant boundary effects of graviton-photon conversion

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June 23rd, 2026

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Summary of the presentation

- Summary of the Gertsenshtein effect (graviton-photon coupling)
- How it is amplified by a resonant boundary effect

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The coupling between gravity and electromagnetism is described by the Einstein field equation and Maxwell's equations in curved spacetime:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa^2 \left(F_{\mu\lambda} F_{\nu}{}^{\lambda} - \frac{1}{4} g_{\mu\nu} F_{\lambda\rho} F^{\lambda\rho} \right), \quad \nabla_{\nu} F_{\mu}{}^{\nu} = 0, \qquad (1)$$

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Unless one of those two photons is from a background field instead of a wave...

Summary of the Gertsenshtein effect (graviton-photon coupling)

The dynamics is controlled by a dimensionless parameter

$$\lambda := \frac{\kappa}{\sqrt{2}\,k}\sqrt{(B_y + E_z)^2 + (B_z - E_y)^2}\,, \tag{2}$$

where E_y, E_z, B_y and B_z are components of the background electromagnetic field (this assumes that the waves propagate along the positive x axis) and k is the wave's wavenumber. It is almost always true that $\lambda \ll 1$, so we expand around λ.

Summary of the Gertsenshtein effect (graviton-photon coupling)

The background electromagnetic field splits the dispersion relation (which would be just $\omega - k = 0$ in a vacuum) into two (birefringence):

$$\omega - k = \mp\lambda k + k\,O(\lambda^2). \tag{3}$$

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This is a **weak** effect; it is suppressed by λ.

How it is amplified by a resonant boundary effect

Consider a vacuum region at $x < 0$ and a constant background electromagnetic field at $x > 0$. Hence, the background electromagnetic tensor is

$$\overline{F}_{\mu\nu} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{bmatrix} \Theta(x). \tag{4}$$

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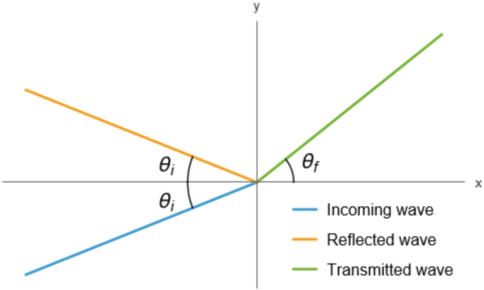
We send a pure gravitational wave from $x \to -\infty$ and making an angle θ_i with the positive direction of the x axis. Part of it is reflected back to $x \to -\infty$ and makes an angle θ_i in with the x axis in the other direction. Another part of it is transmitted towards $x \to +\infty$ and makes an angle θ_f. Without loss of generality, we take the $x - y$ plane to be the plane in which the waves propagate.



Conservation of energy and momentum tangential to the boundary imposes (Snell's law):

$$\begin{aligned} \sin\theta_f &= (1 \mp \lambda + O(\lambda^2)) \sin\theta_i \,, \\ \cos\theta_f &= (1 \mp \lambda + O(\lambda^2)) \sqrt{\cos^2\theta_i \pm 2\lambda + O(\lambda^2)} \,. \end{aligned} \tag{5}$$

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We can see that beyond the critical angle $\theta_c := \arccos\sqrt{2\lambda}$, half of the eigenstates (the ones with the lower signs) become evanescent on the other side of the boundary: $\sin\theta_f > 1$ and $\cos^2\theta_f < 0$. Therefore, **these eigenstates are totally reflected**.

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We now want to calculate the various reflection coefficients, which describe what fraction of the incoming gravitational wave's energy is reflected; and among how much is reflected back as a gravitational wave and how much is converted into electromagnetic radiation. There are also transmission coefficients, but these are not interesting for this specific setup (in which the background electromagnetic in $x > 0$ goes forever).

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We consider the specific case in which the boundary physics (currents, energy-momentum tensor) does not interfere at all with the waves (i.e. total transparency; **this is not always true**). This is equivalent to having the waves and their first derivatives be continuous across the boundary:

$$\Delta h_{ab} = 0, \quad \Delta a_a = 0, \quad \Delta(\partial_x h_{ab}) = 0, \quad \Delta(\partial_x a_a) = 0, \tag{6}$$

where Latin indices (a, b, c, etc.) represent boundary coordinates.

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This is enough information to calculate the reflection coefficients.

When $\theta_i < \theta_c$, the reflection coefficients are

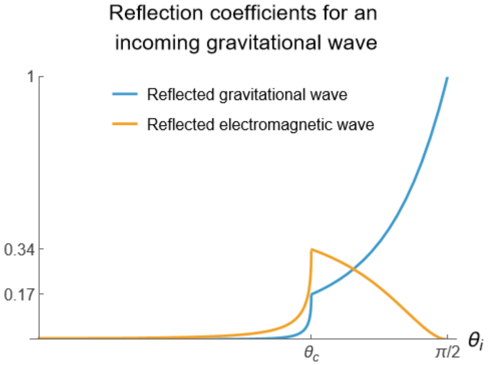
$$R^{(g)} = \frac{\left(1 - \sqrt{1 - \frac{4\lambda^2}{\cos^4\theta_i}}\right)^2}{\left(1 + \sqrt{1 + \frac{2\lambda}{\cos^2\theta_i}}\right)^2 \left(1 + \sqrt{1 - \frac{2\lambda}{\cos^2\theta_i}}\right)^2},$$
$$R^{(\gamma)} = \frac{\left(\sqrt{1 + \frac{2\lambda}{\cos^2\theta_i}} - \sqrt{1 - \frac{2\lambda}{\cos^2\theta_i}}\right)^2}{\left(1 + \sqrt{1 + \frac{2\lambda}{\cos^2\theta_i}}\right)^2 \left(1 + \sqrt{1 - \frac{2\lambda}{\cos^2\theta_i}}\right)^2}. \tag{7}$$

When $\theta_i > \theta_c$, the reflection coefficients are now

$$R^{(g)} = \frac{2\lambda}{\cos^2\theta_i \left(1 + \sqrt{1 + \frac{2\lambda}{\cos^2\theta_i}}\right)^2}, \quad R^{(\gamma)} = \frac{2}{\left(1 + \sqrt{1 + \frac{2\lambda}{\cos^2\theta_i}}\right)^2}. \tag{8}$$

How it is amplified by a resonant boundary effect

Graphically, this looks as follows.



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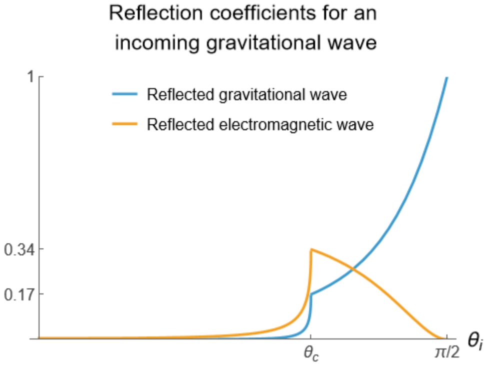

At the critical angle θ_c, $R^{(g)} = \dfrac{1}{3+2\sqrt{2}} \approx 17\%$ and $R^{(\gamma)} = \dfrac{2}{3+2\sqrt{2}} \approx 34\%$.

Approximately 34 % **of the gravitational wave's energy is reflected and converted into electromagnetic waves**.

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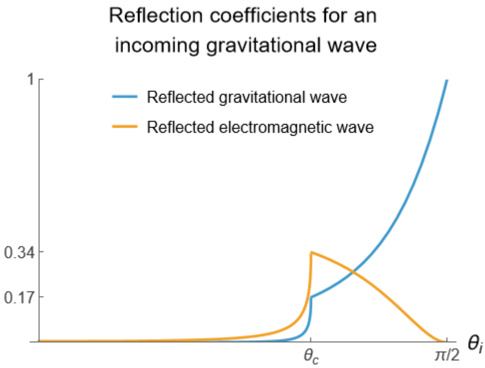

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Approximately 34 % **of the gravitational wave's energy is reflected and converted into electromagnetic waves**. However, the catch is that since $\lambda \ll 1$, θ_c is **really** close to 90 °. So we only have a significant conversion around a very narrow range of angles around the critical value. That makes it a **resonant boundary effect**.

Merci pour votre attention. Je vais répondre à vos questions.

Thank you for your attention. I will answer your questions.