

# Anomaly mediation and Seiberg Witten theory

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University

# Big picture:

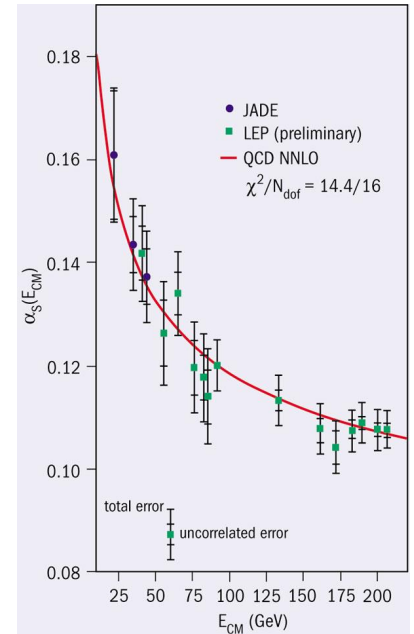
Understanding the IR structure  
of asymptotically free gauge  
theories

# Example: chiral QCD

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- 
- Running coupling: conformal anomaly
  - At  $\Lambda_{QCD}$ , perturbation theory breaks
  - Very different low-energy description



Renormalization group flow

UV

Renormalizable perturbative theory of quarks and gluons

$\Lambda_{QCD}$

IR

Pions - sigma model on  $\frac{SU(3)_L \times SU(3)_R}{SU(3)_V}$

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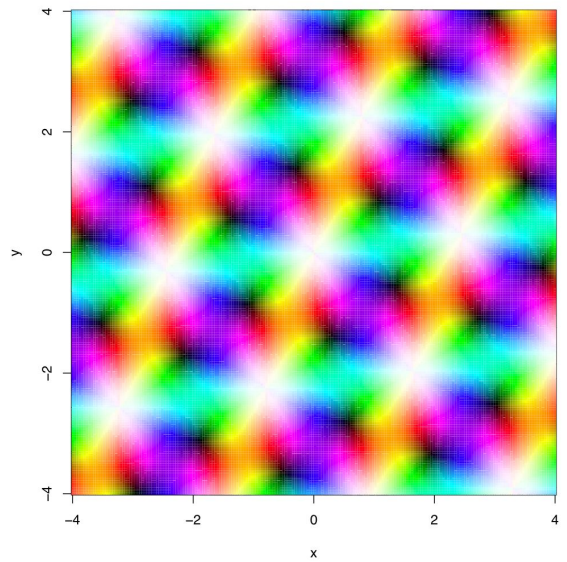
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Pions - sigma model on  $\frac{SU(3)_L \times SU(3)_R}{SU(3)_V}$

Most everyone believes this. Noone knows how to prove it.

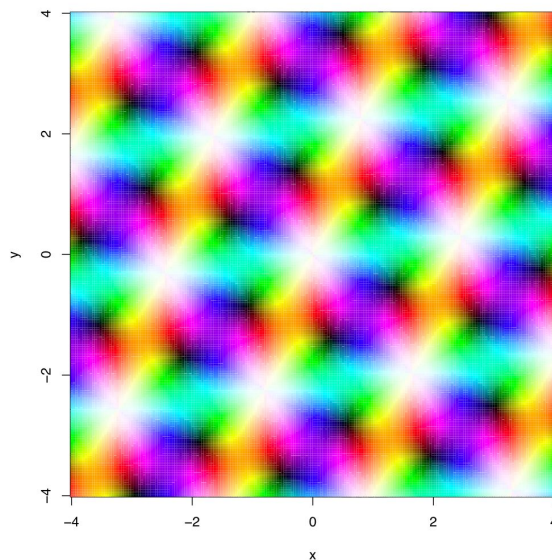
# Supersymmetry (SUSY)

- Extension of poincare



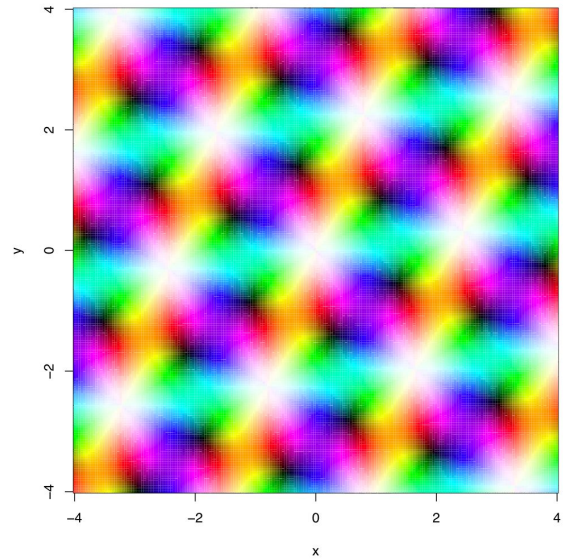
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- Poinc. invariance is powerful. SUSY is even more powerful.
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# Supersymmetry

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- Poincaré invariance is powerful. SUSY is even more powerful.
- Allows derivation of exact, nonperturbative results.
  
- We'll see an example as we proceed.



SUSY = 0th order approximation. Can we get to 1st order?

Possible answer: Anomaly mediated supersymmetry breaking (AMSB)

# Anomaly mediated SUSY breaking (AMSB)

- Rotate conformal anomaly into superspace: superconformal anomaly
- This is AMSB

- Only 1 parameter,  $m_{3/2}$

- + Lagrangian + beta functions ->

Classical conformal violation

$$V_{tree} = m_{3/2} \left( \partial_i W h^{i\bar{j}} \bar{\partial}_{\bar{j}} K - 3W \right) + \text{h.c.} + |m_{3/2}|^2 \left( \partial_i K h^{i\bar{j}} \bar{\partial}_{\bar{j}} K - K \right)$$

$$m_\lambda = \frac{-\beta(g)}{g} m_{3/2}$$

$$m_i^2 = \frac{-1}{4} \dot{\gamma}_i |m_{3/2}|^2$$

$$A_{ijk} = \frac{-1}{2} (\gamma_i + \gamma_j + \gamma_k) m_{3/2}$$

Loop level conformal violation

# AMSB for non-SUSY gauge theory

- [arXiv:2104.01179](https://arxiv.org/abs/2104.01179)
- Hitoshi Murayama - Introduces AMSB to SQCD
- Many similar studies since.

UV

Large  $m_{3/2}$ , apply AMSB to  
perturbative UV theory ->  
squarks+gaugino decouple

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**\$1,000,000 question:  
Far IR at large SUSY-breaking**

# Phase transitions as $m_{3/2} \rightarrow \infty$

- arXiv: 211.11214
- Bai & Stolarski,  
Georgi-Glashow like SU(5)  
theory
- Phase transition guaranteed  
(t'Hooft anomaly matching)

# Our project

- Seiberg Witten theory + AMSB
- arXiv:xxxx.xxxxx

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- Examine AMSB in a more controlled setting. What can we learn about the flow from small to large SUSY-breaking?

# Seiberg Witten theory

# Seiberg Witten theory - arXiv:9407087

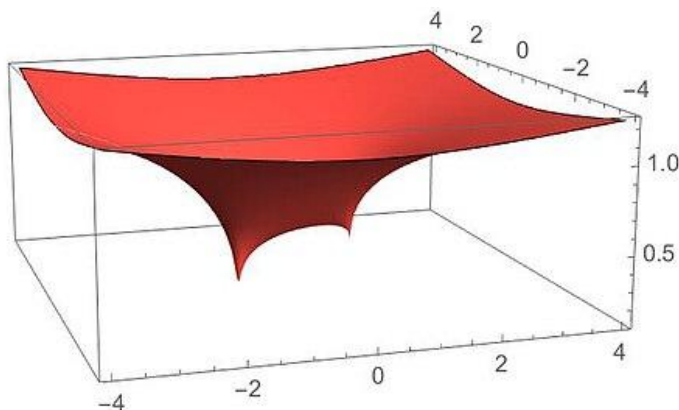
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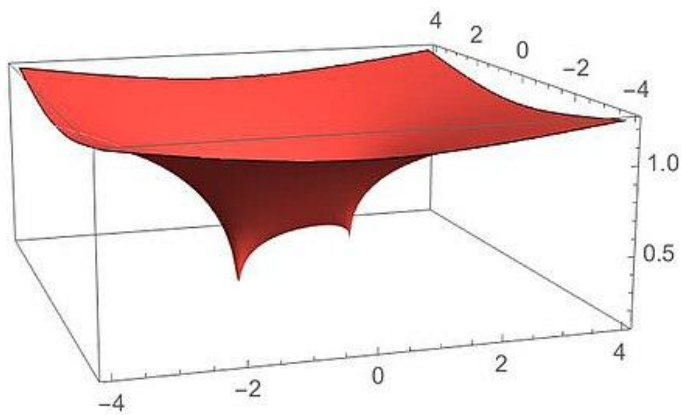
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- Scalar potential  $\rightarrow V \propto [\phi^\dagger, \phi]$  VEV breaks  $SU(2) \rightarrow U(1)$
- A moduli space of vacua (like goldstones, but space not symmetric)



# Seiberg Witten theory

- Exact electromagnetic duality

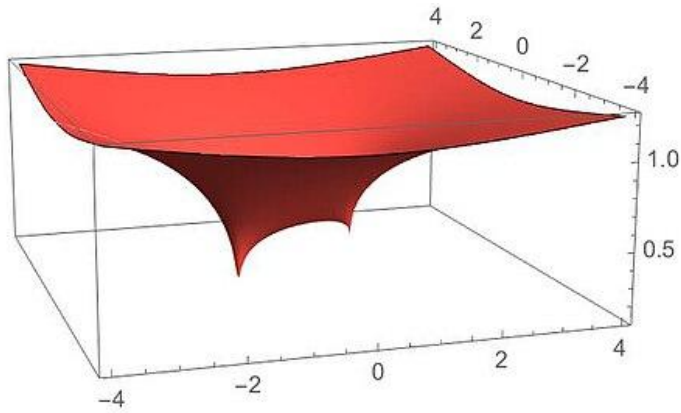
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# Seiberg Witten theory

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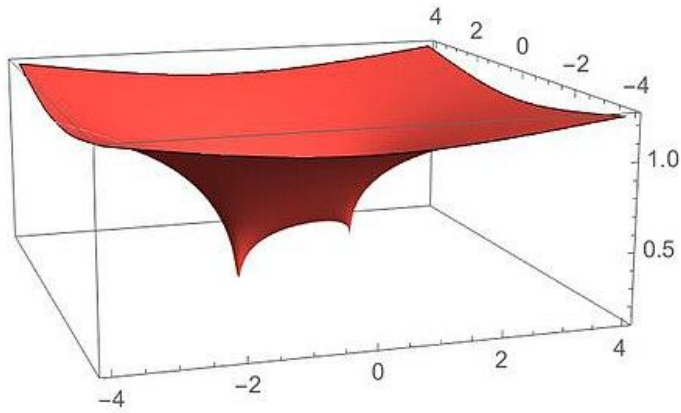
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# Seiberg Witten theory

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- Can exactly calculate metric
- Singularities = newly massless particles

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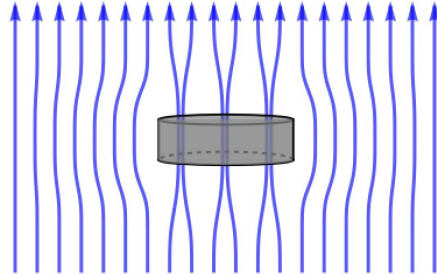


# Seiberg Witten deformation

- N=1-preserving mass term  $\Delta\mathcal{L} = \int d^2\theta \mu \text{Tr}(\Phi^2)$
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- Monopoles condense -> dual Meissner effect, confinement!!

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- UV (large  $m_{3/2}$ ) reproduces Seiberg Witten deformation in perturbation theory

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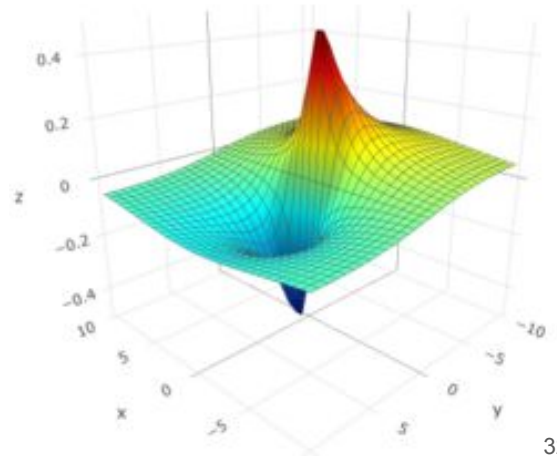
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- IR is similar, but not identical! SUSY is broken, condensate doesn't match.
- Let's look at UV more carefully

# AMSB + Seiberg Witten

$$\beta = Ag^3 + B \exp\left(-\frac{8\pi^2}{g^2}\right) + \dots$$

1 - loop  
(N=2 only runs  
at 1-loop)

Instantons



# AMSB + Seiberg Witten

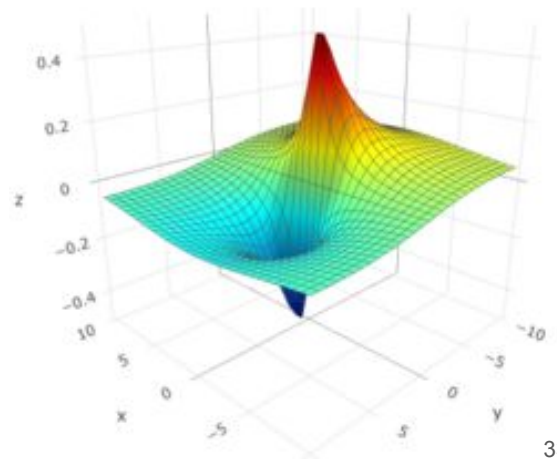
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Preserves N=1

Instantons

Breaks N=1



UV

Large  $m_{3/2}$  perturbative  $\rightarrow$  AMSB =  
Seiberg Witten

+Instantons  $\neq$  Seiberg Witten

$\Lambda$

Small  $m_{3/2}$   $\rightarrow$  VEV rolls to singularities.  
Monopoles condense, but theory  $\neq$  Seiberg  
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Flow of  $m_{3/2}$

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- But monopole condensate and spurious charge assignment do.

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- Transition can only take place when  $m_{3/2} \sim \Lambda$
  
- Regime involves interactions of mutually nonlocal degrees of freedom. No weakly coupled Lagrangian description.

# A cautionary tale - hidden phase transitions?

- In this theory, there was no phase transition per se.
- But a VEV was altered, as were its spurious transformation properties.
- Easy to imagine this type of behaviour causing phase transitions

## A cautionary tale - fictitious phase transitions?

- A global symmetry (N=1 supersymmetry) is broken at all  $m_{3/2}$
- Invisible in UV perturbation theory, much clearer in IR.
- Could there be situations where nonperturbative breaking of a symmetry in UV e.g. gives loophole to t'Hooft anomaly matching?

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- Valuable hints at how this all might work.
- Story for another time -> (R, super, conformal)
- Possibility of extending exact SUSY results to AMSB (nonperturbatively?)

# Summary

- AMSB + SUSY = way to study non-SUSY
- Extrapolating from small to large SUSY breaking -> dangerous.
- We applied AMSB to Seiberg Witten
- Large AMSB = Seiberg Witten deformation + instanton corrections
- Small AMSB  $\neq$  Seiberg Witten deformation (but qualitatively similar)
- No phase transition, but change in VEV and moduli space geometry invisible in perturbation theory
- Vague hints of potentially fruitful future lines of inquiry.

# UV IR spurion mismatch

Theory space

