

Measuring a bulk's geometry from the outside

Simon Caron-Huot

On work with Joydeep Chakravarty and Keivan Namjou
2501.13182, 2502.14963

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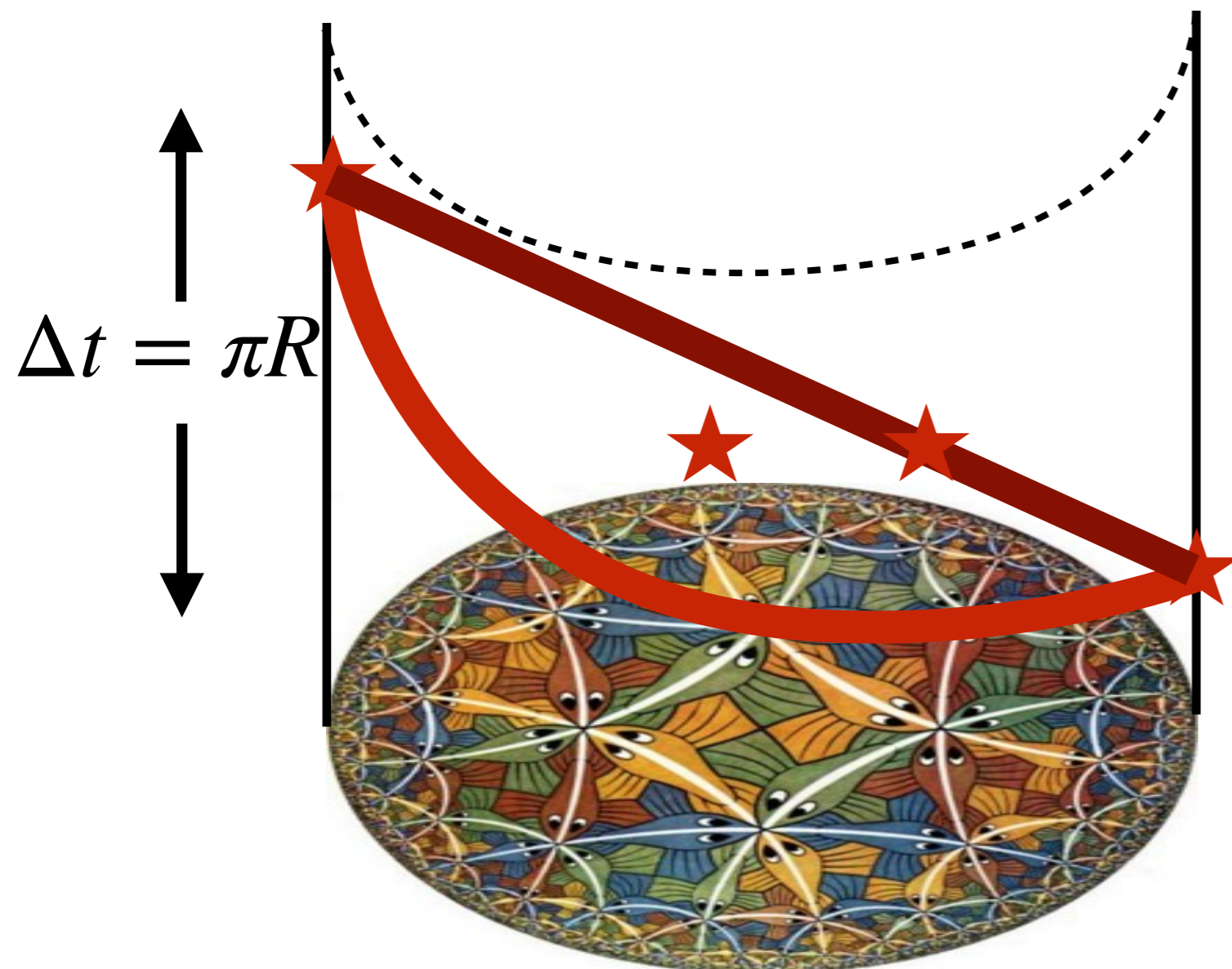
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McGill

Relativity in AdS/CFT

In empty AdS, a null geodesic cuts through the bulk in the same time it takes to go around!



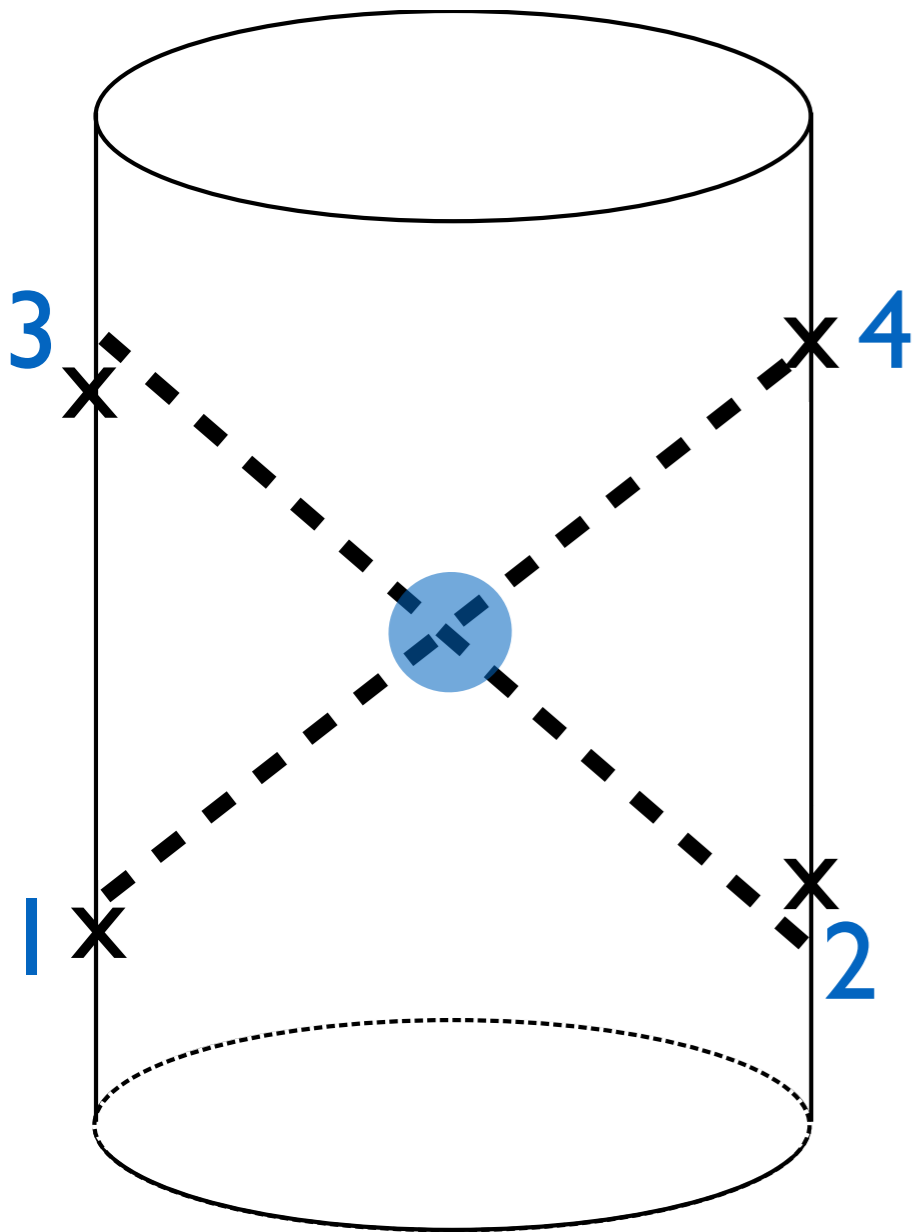
In CFT, an axiom:

$$[\mathcal{O}, \mathcal{O}'] = 0 \text{ spacelike}$$

In bulk: gravity can't allow faster-than-light signaling!

[Gao-Wald, ...]

CFT / S-matrix bootstrap



For spacelike $x_2 - x_3$,

$$\langle 0 | \mathcal{O}_4 [\mathcal{O}_2, \mathcal{O}_3] \mathcal{O}_1 | 0 \rangle = 0$$

yields bootstrap eq.:

$$\sum_{\Delta, \ell} \begin{array}{c} 3 \\ \diagup \\ \text{---} \Delta, \ell \text{---} \\ \diagdown \\ 1 \end{array} = \sum \begin{array}{c} \diagdown \\ \text{---} \\ \diagup \end{array}$$

For a holographic CFT*, imply constraints from S-matrix analyticity

[SCH, Mazac, Rastelli & Simmons-Duffin '21]

*a CFT where all contributions from spin $\ell > 2$ have large $\Delta > \Delta_{\text{higher-spin}}^{\text{gap}}$

Constraints on GR modifications

[Camanho, Edelstein, Maldacena & Zhiboedov '14]

[SCH, Li, Parra-Martinez & Simmons-Duffin '22]²

$$S = \int \frac{\sqrt{-g}}{G_N} (R + \lambda \text{Riem}^2 + \dots + (\partial\phi)^2 + g\phi \text{Riem}^2 + \dots)$$

– In d=5 flat space: $|\lambda_{\text{Riem}^2}| \leq \approx 11.5/M_{\text{higher-spin}}^2$

\leftrightarrow In $\text{AdS}_5/\text{CFT}_4$: $|a - c|/c \leq \approx 23/(\Delta_{\text{higher-spin}}^{\text{gap}})^2$

Mass of first fundamental* particle of spin ≥ 4

*decays to two-gravitons with strength $> \#M^2\sqrt{G_N}$

– In d=4: $\lambda_{\text{Riem}^3} \leq 25/M_{\text{higher-spin}}^8 \log(M/\mu_{\text{IR}})$

$g_{\phi\text{Riem}^2} \leq \#/M_{\text{higher-spin}}^4 \log(M/\mu_{\text{IR}})$

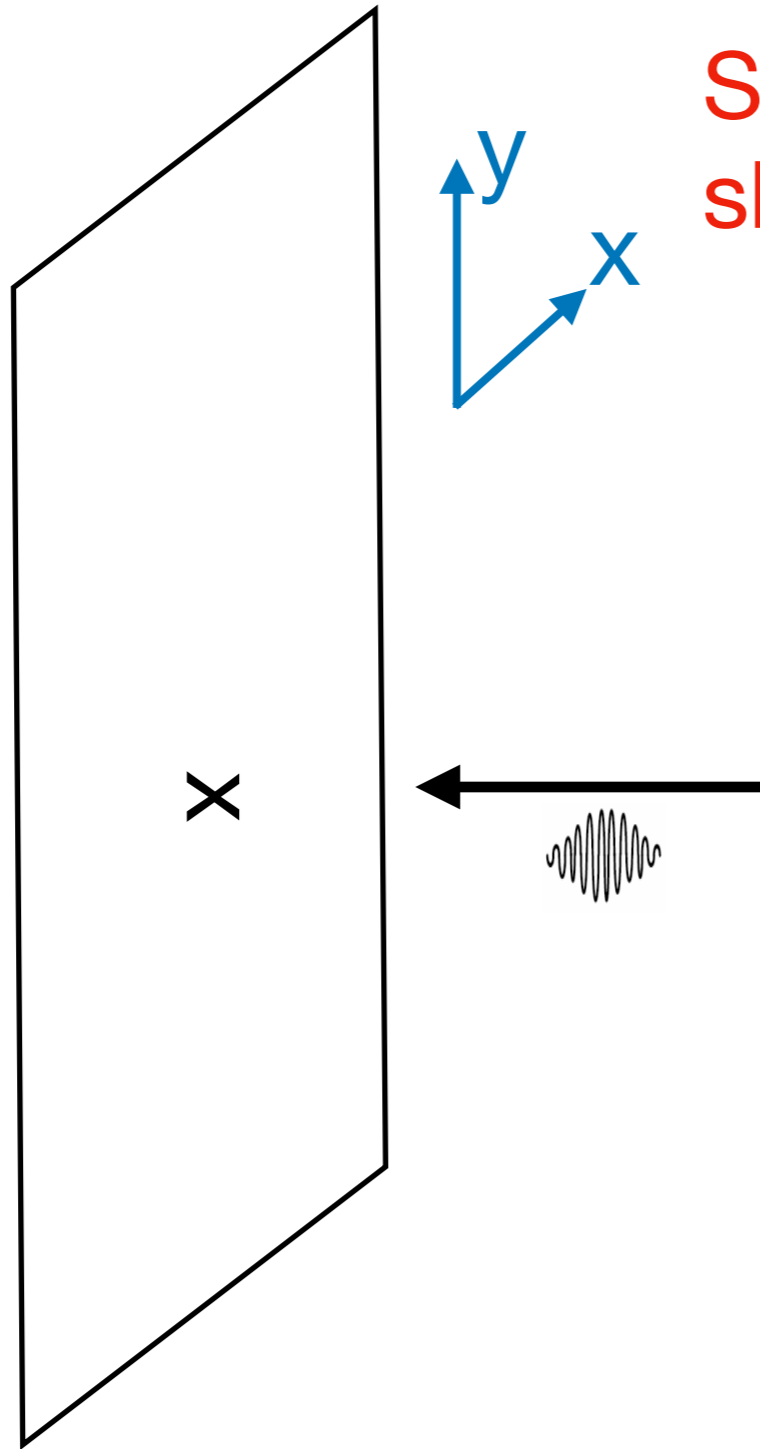
...

Why extend to other, non-AdS geometries?

- Do CFT axioms imply Einstein's equation beyond $2 \rightarrow 2$ scattering in vacuum?
- Constrain CFT correlators in non-conformal states ($T \neq 0$) or QFTs with RG flows
- For fun: how would one measure bulk geometry from experiments at boundary?

1. Context and motivation
 - bounds from thought experiments in empty AdS
2. Simple experiments on a holographic system that measure its geometry
3. WKB description and bulk amplitudes
4. Bulk geometry from “boundary hyperboloids”

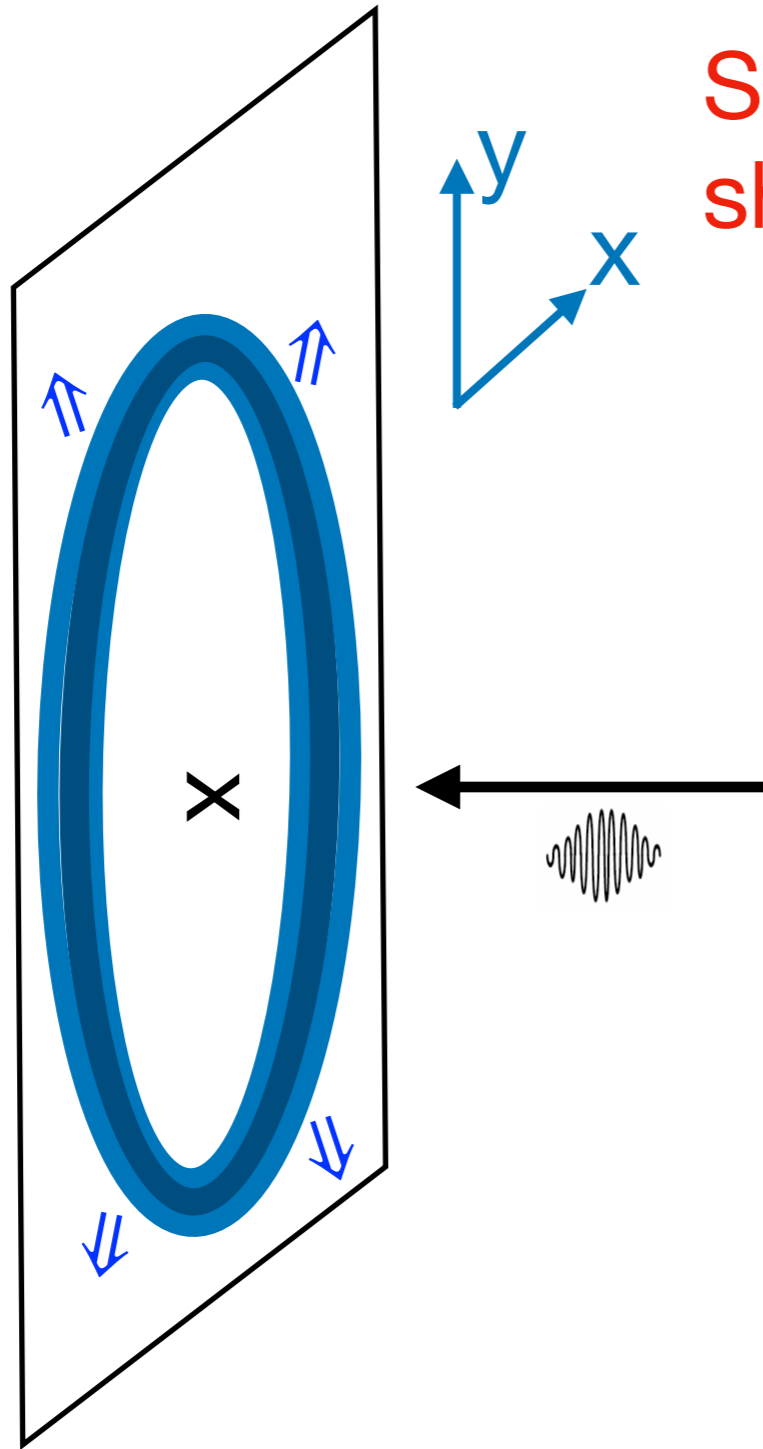
A holographic poster hangs on a wall...



Suppose we *had* a 2+1-dimensional sheet of "holographic material".

Hang it on a wall and throw electrons at it.

A holographic poster hangs on a wall...



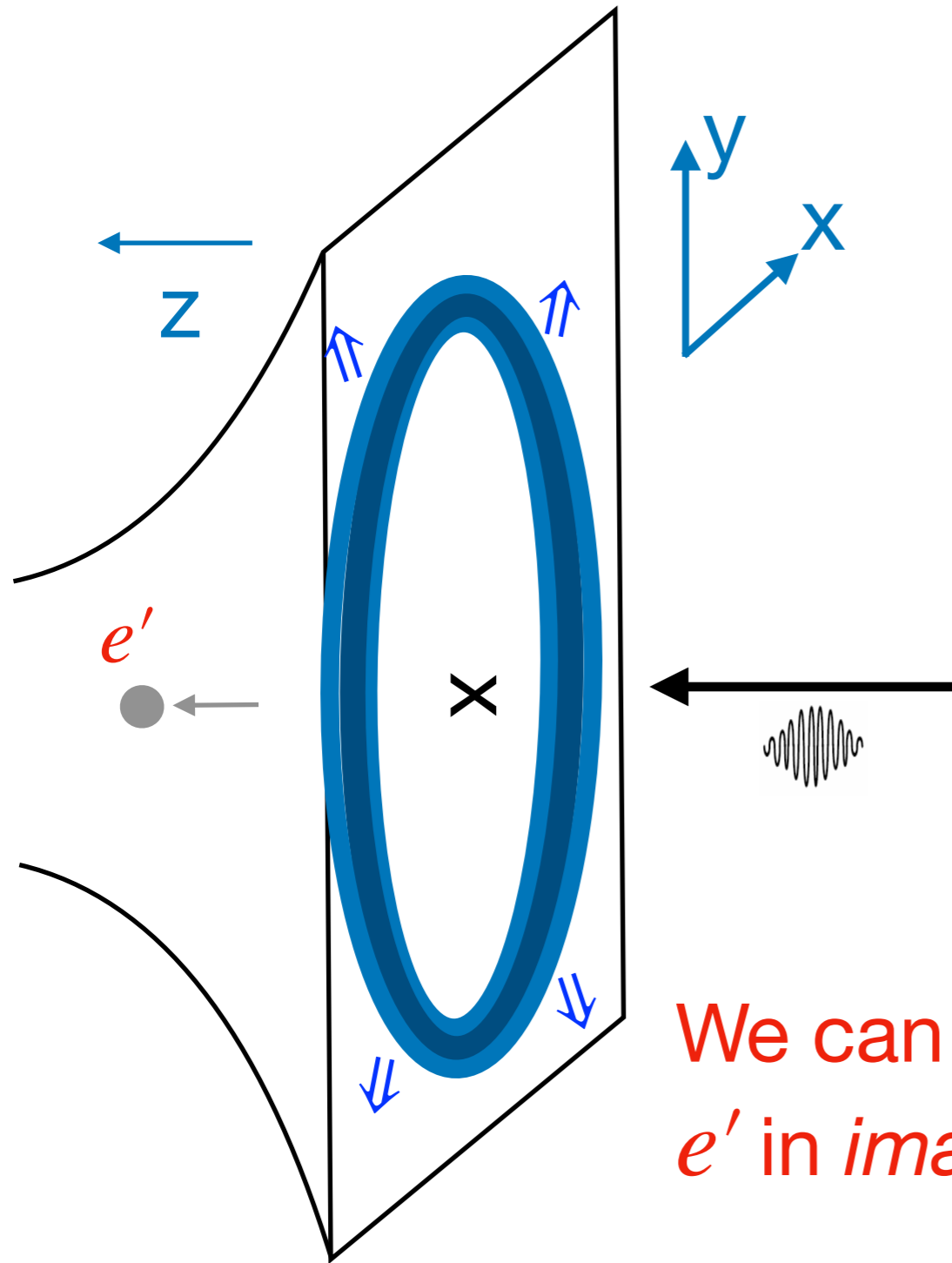
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Particle absorbed by holographic sample would produce a circular wave of energy spreading at c'

[Hofman-Maldacena '08]

A holographic poster hangs on a wall...



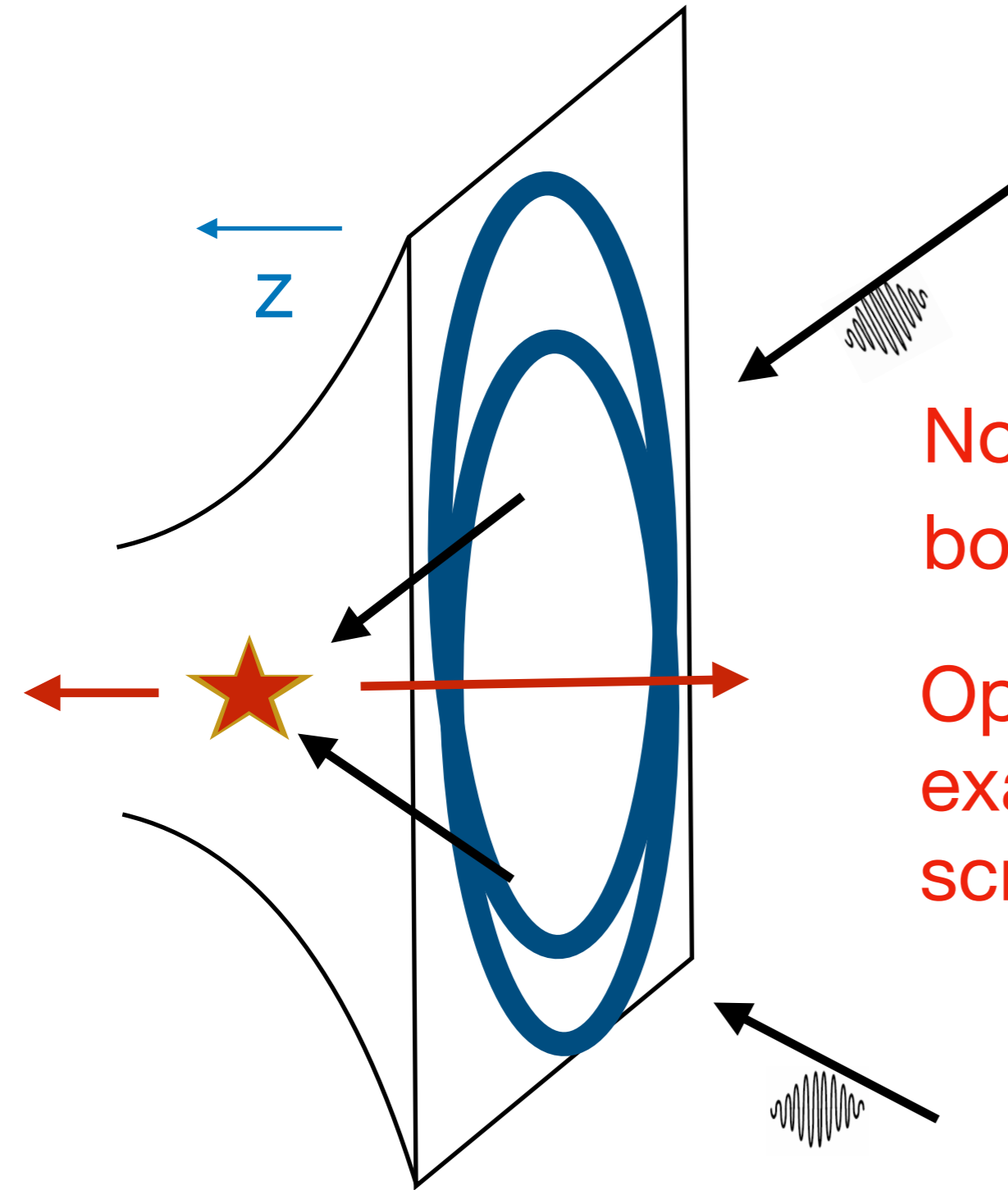
Particle absorbed by holographic sample would produce a circular wave of energy spreading at c'

[Hofman-Maldacena '08]

We can think of it as the image of a point e' in imagined negatively curved spacetime

depth z = size of boundary imprint

Actually probing the bulk dynamics

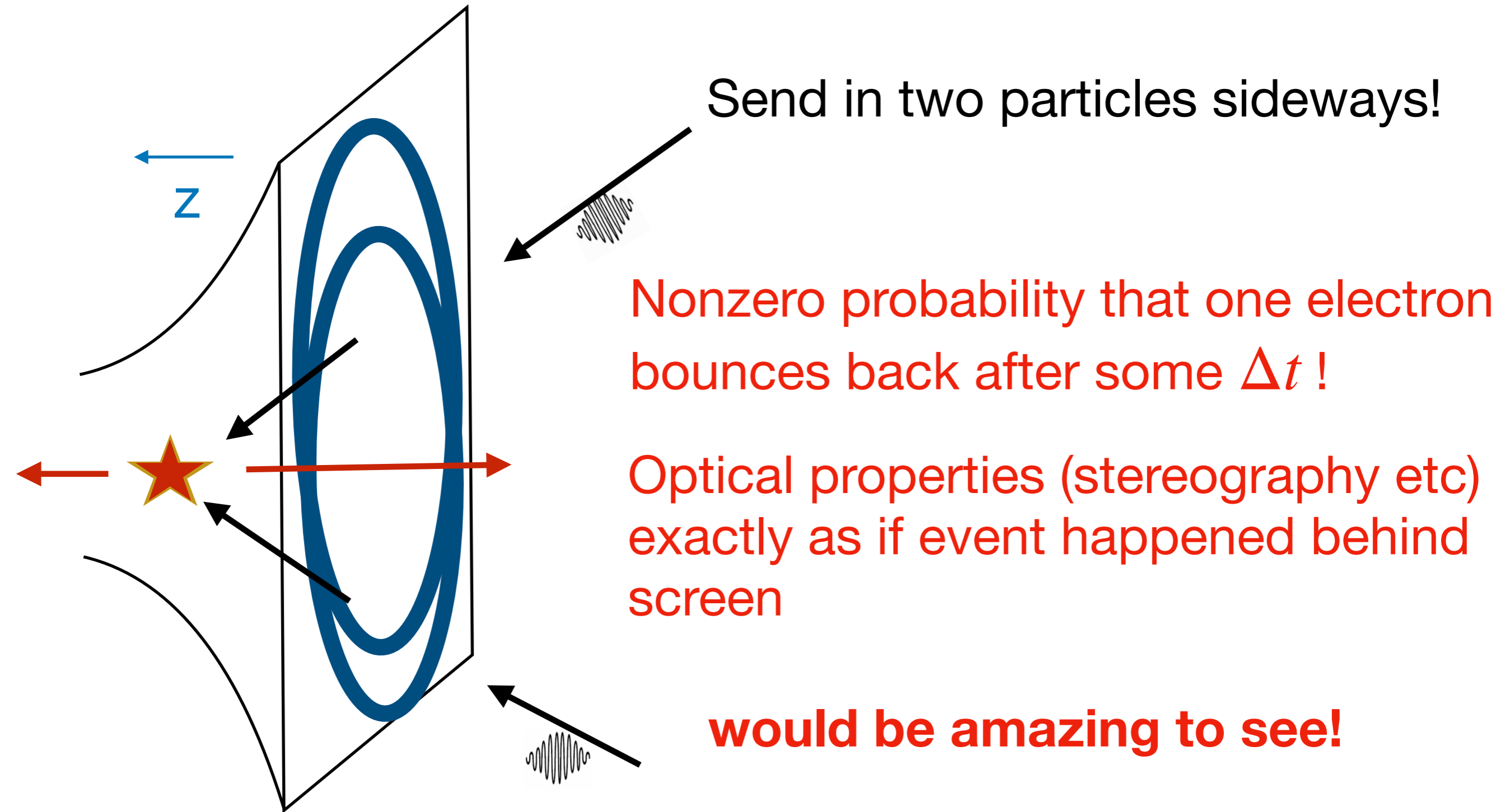


Send in two particles sideways!

Nonzero probability that one electron bounces back after some Δt !

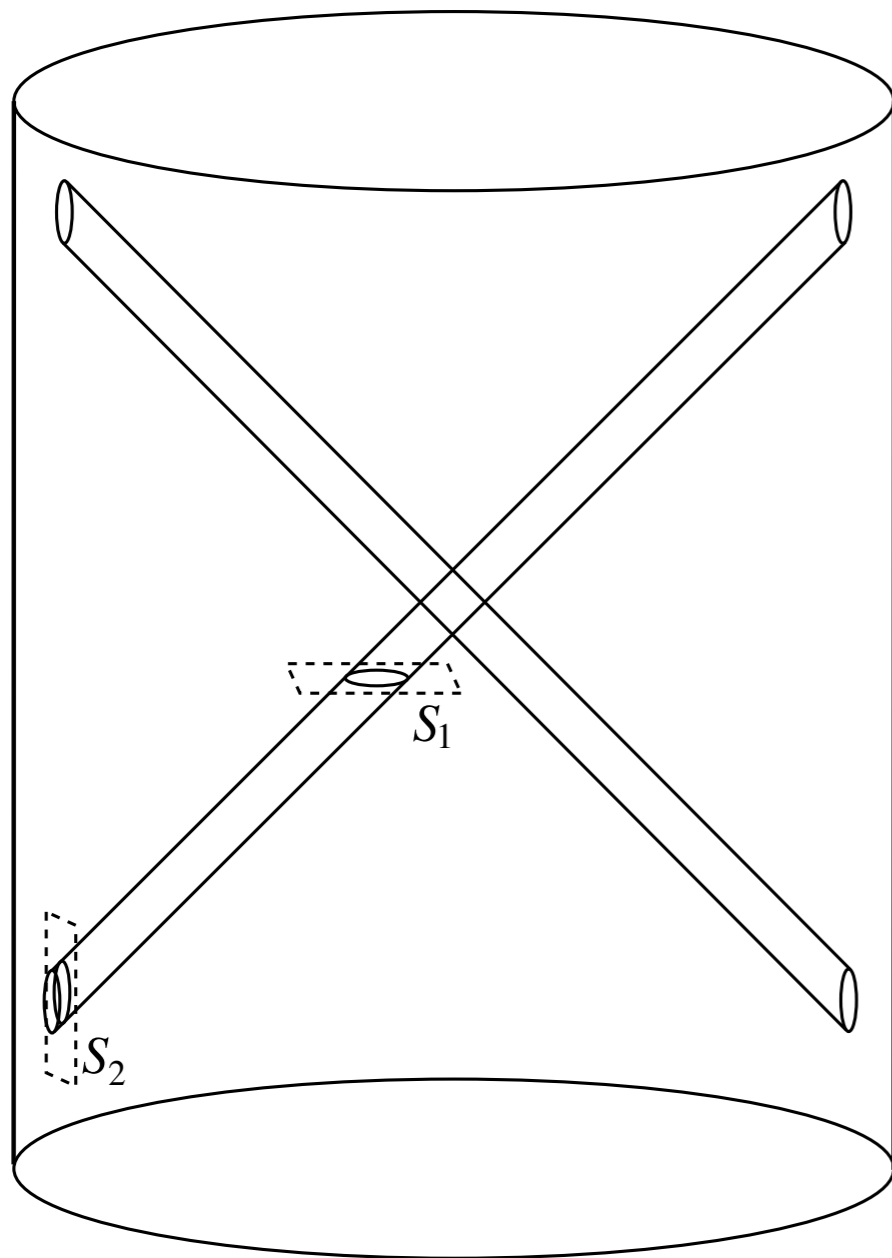
Optical properties (stereography etc) exactly as if event happened behind screen

Actually probing the bulk dynamics



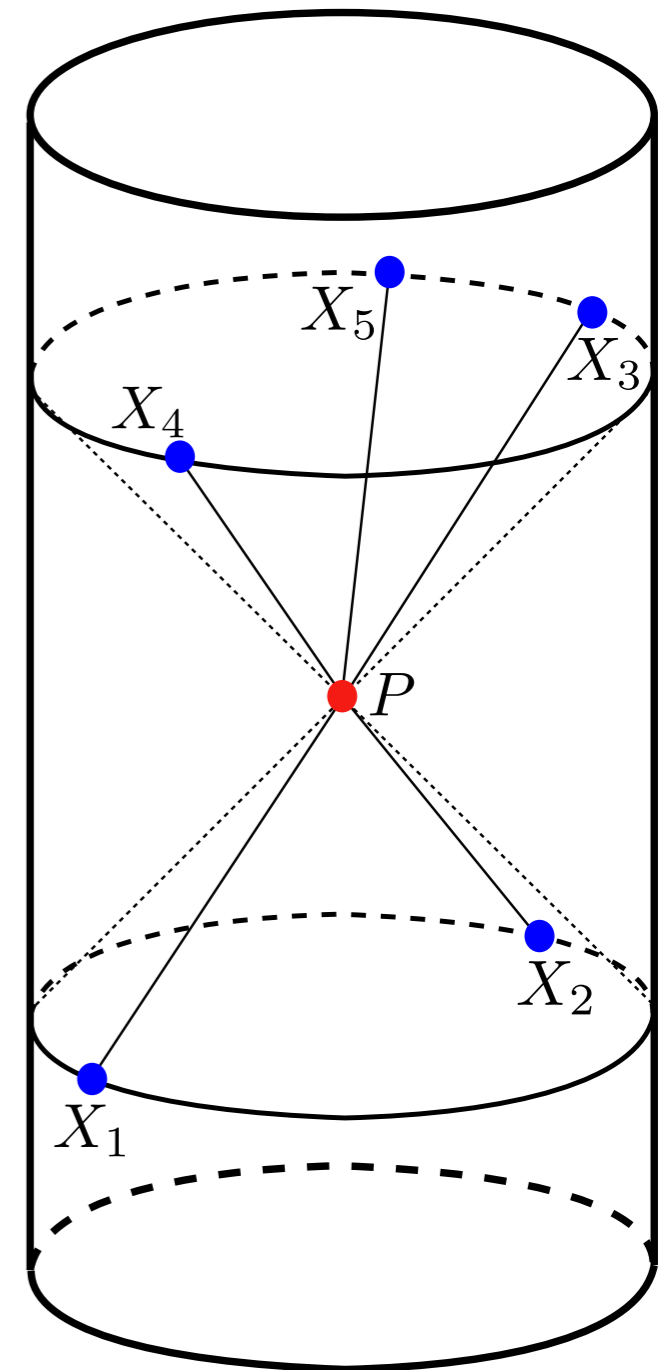
Today's goal: *measure* (not “reconstruct”) bulk metric from such experiments

Wavepacket scattering



[Polchinski '99]

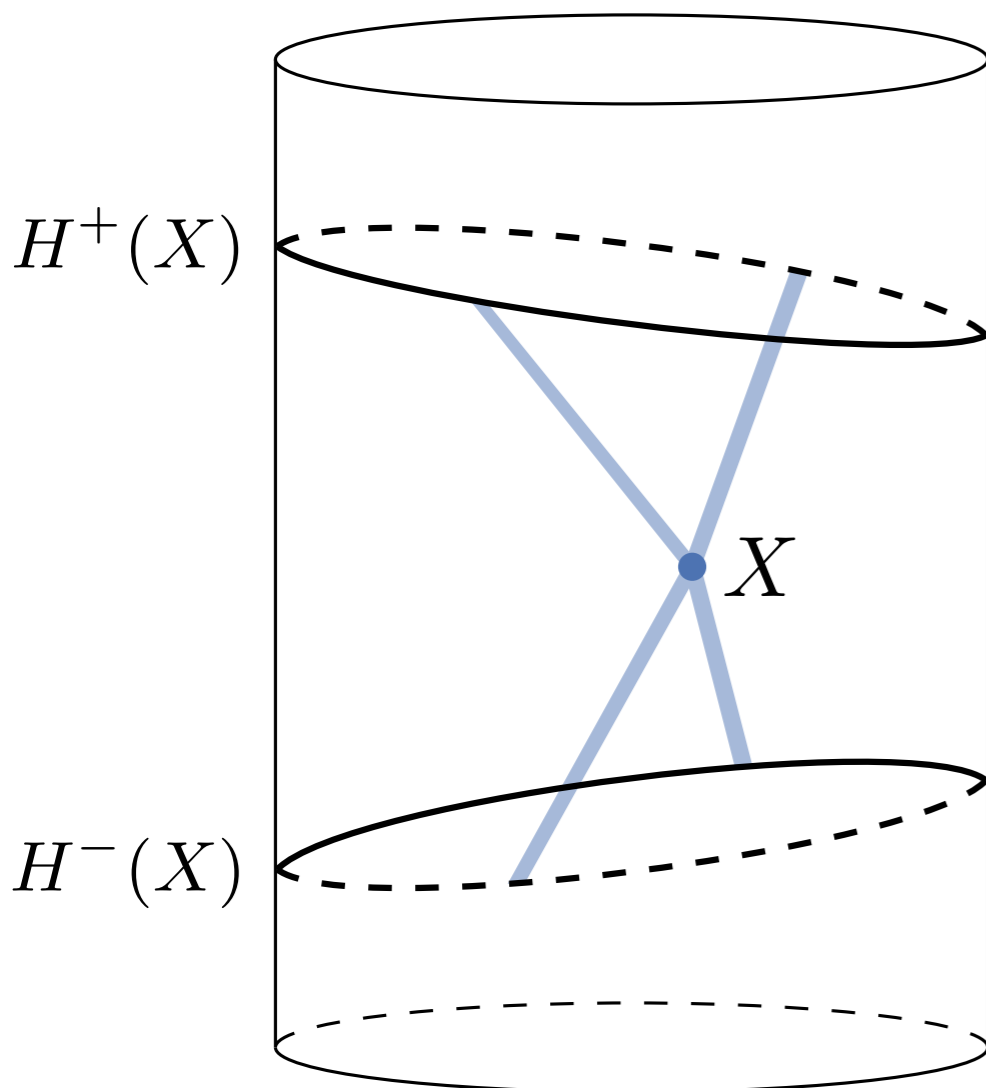
VS.



[Maldacena, Simmons-Duffin & Zhiboedov '15]

- Each boundary wavepacket picks a bulk lightray
(contrast with boundary $\mathcal{O}(x) \leftrightarrow$ bulk lightcone)

Geometry of wavepacket scattering



Wavepacket correlators peak when all boundary points approach the lightcone of a common bulk point X

$H^\pm(X) = (\text{lightcone from } X) \cap (\text{boundary})$

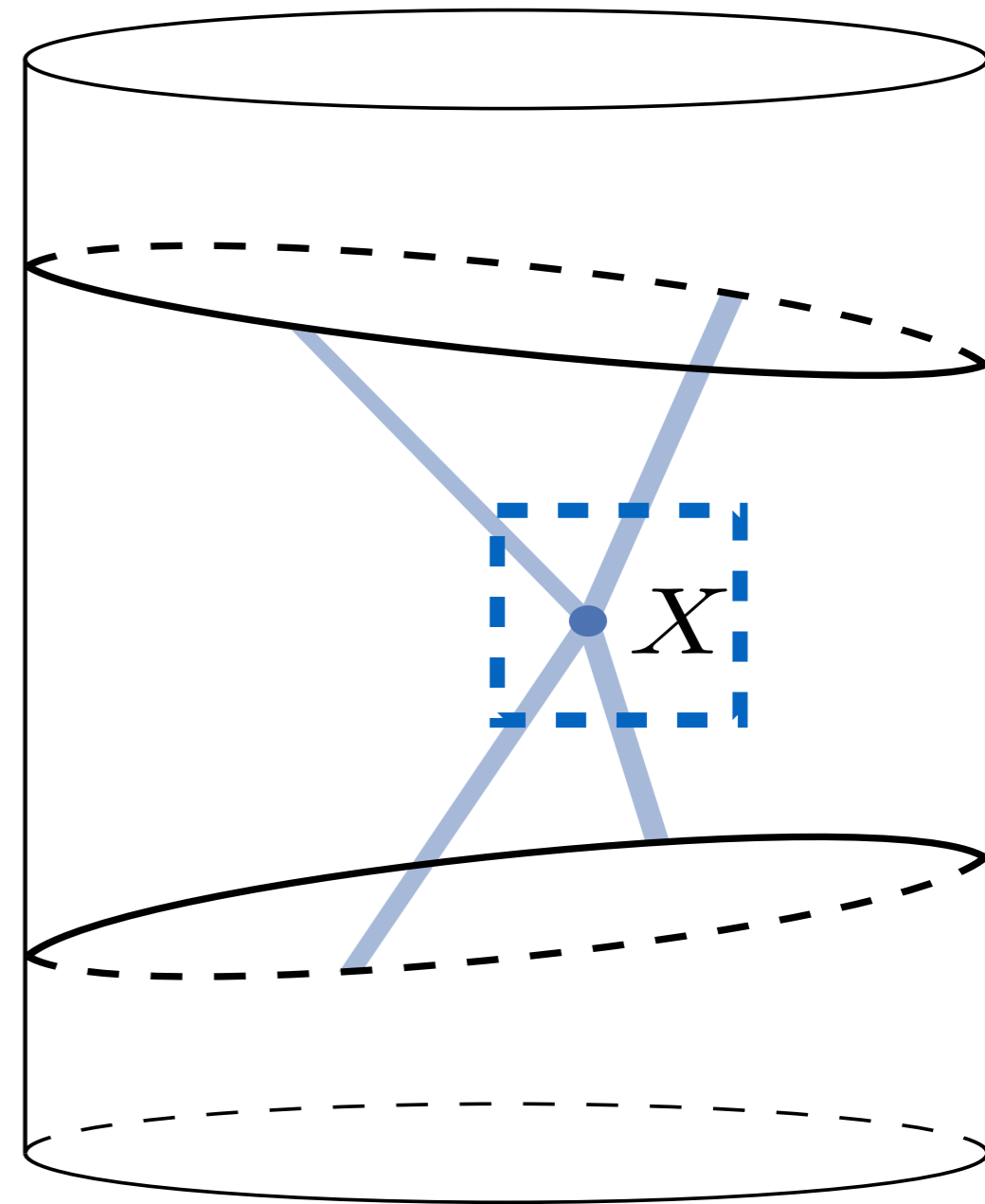
"boundary hyperboloids"

Shape of hyperboloids determines bulk conformal metric

[Englehardt & Horowitz '16-...]

Setup

- Start with state $|\Psi\rangle$ dual to a classical asymptotical AdS geometry: the “background” around which we expand
- Add particle excitations that follow localized trajectories to ‘X’
- Neglect long-range interactions while particles well-separated*
- Neglect spacetime curvature in neighbourhood of X



Boundary wavepackets

$$O_{x,p,\sigma} = \int d^d \delta x \psi_{p,\sigma}(\delta x) O(x + \delta x)$$

Gaussians with given center, momentum and width

$$\psi_{p,\sigma}(\delta x) = \exp \left(-ip_\mu \delta x^\mu - \frac{1}{2} \sigma_{\mu\nu}^{-1} \delta x^\mu \delta x^\nu \right)$$

Use $O_{p,\sigma}^\dagger(x) |\Psi\rangle$ to create bulk excitation starting from boundary point x

- Bulk wavefunction:

$$\phi_{x,p,\sigma}(X) = \langle \Psi | \phi(X) O_{p,\sigma}^\dagger(x) | \Psi \rangle$$



- Assume WKB in bulk (geometric optics)
- Use exact solution near AdS boundary (where WKB fails)
 (apart from that factor, similar formulas for other ‘boundaries’:
 infinity in Minkowski, tube around a local observer, etc)

Bulk WKB wavefunction

- Neglecting bulk interactions on the way to X , need to solve wave equation in bulk, e.g.

$$\partial_M g^{MN} \sqrt{-g} \partial_N \phi_{x,p,\sigma}(X) = 0$$

- Compute using single-particle path integral:

$$\phi_{x,p,\sigma}(X_f) \propto \int_{x,p,\sigma'}^{X_f} DX e^{iS}$$

Worldline action

$$S[X, \eta] = \frac{1}{2} \int d\sigma \left(\frac{g_{MN}}{\eta} \frac{dX^M}{d\sigma} \frac{dX^N}{d\sigma} - \eta m^2 \right)$$

- Equivalent to $-m\tau$ but smooth massless limit
- $S(X_i; X_f)$ = action of on-shell path from X_i to X_f
- Bulk momentum: $\delta S(X_i; X_f) = P_f \cdot \delta X_f - P_i \cdot \delta X_i$

$$P_M = g_{MN}(X) \frac{dX^N}{d\lambda} \quad (= \text{Noether charge if symmetries, but symmetries not needed})$$

$(d\lambda = \eta d\sigma)$

- Satisfies $g^{MN} P_M P_N + m^2 = 0$

Hamilton-Jacobi function

- Canonical transformation: fix boundary momentum p and bulk position X

$$S(p; X) = [S(x; X) + p_\mu x^\mu]_{x^\mu = x^\mu(p; X)}$$

- Procedure: find geodesic emanating from X that hits boundary with momentum p^μ [parallel to bdy]
- A nontrivial function in massless case
[unlike $S(X_i; X_f) = 0$ which is only defined for null-separated points]
- Definition uniformly applies to massive case
[complex boundary endpoint x^μ : quantum tunneling]

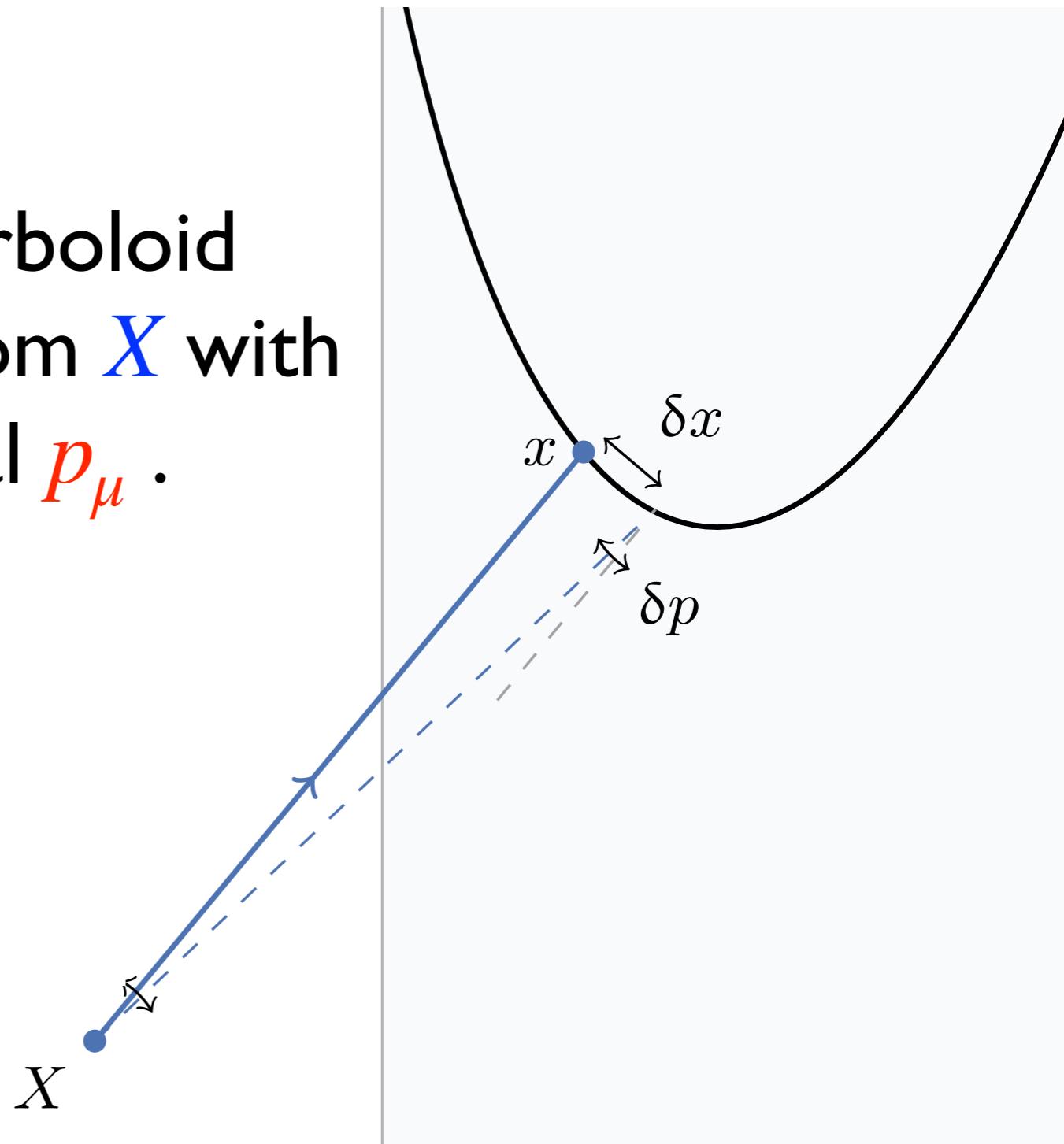
$S(p; X)$ encodes all optical properties of bulk seen from boundary.

$$x^\mu(p; X) = \frac{\partial S(p; X)}{\partial p_\mu};$$

parametrizes boundary hyperboloid (intersection of light cone from X with boundary) in terms of normal p_μ .

$$P_M(p; X) = \frac{\partial S(p; X)}{\partial X^M}$$

corresponding bulk momentum near X .



Solution for bulk wavefunction

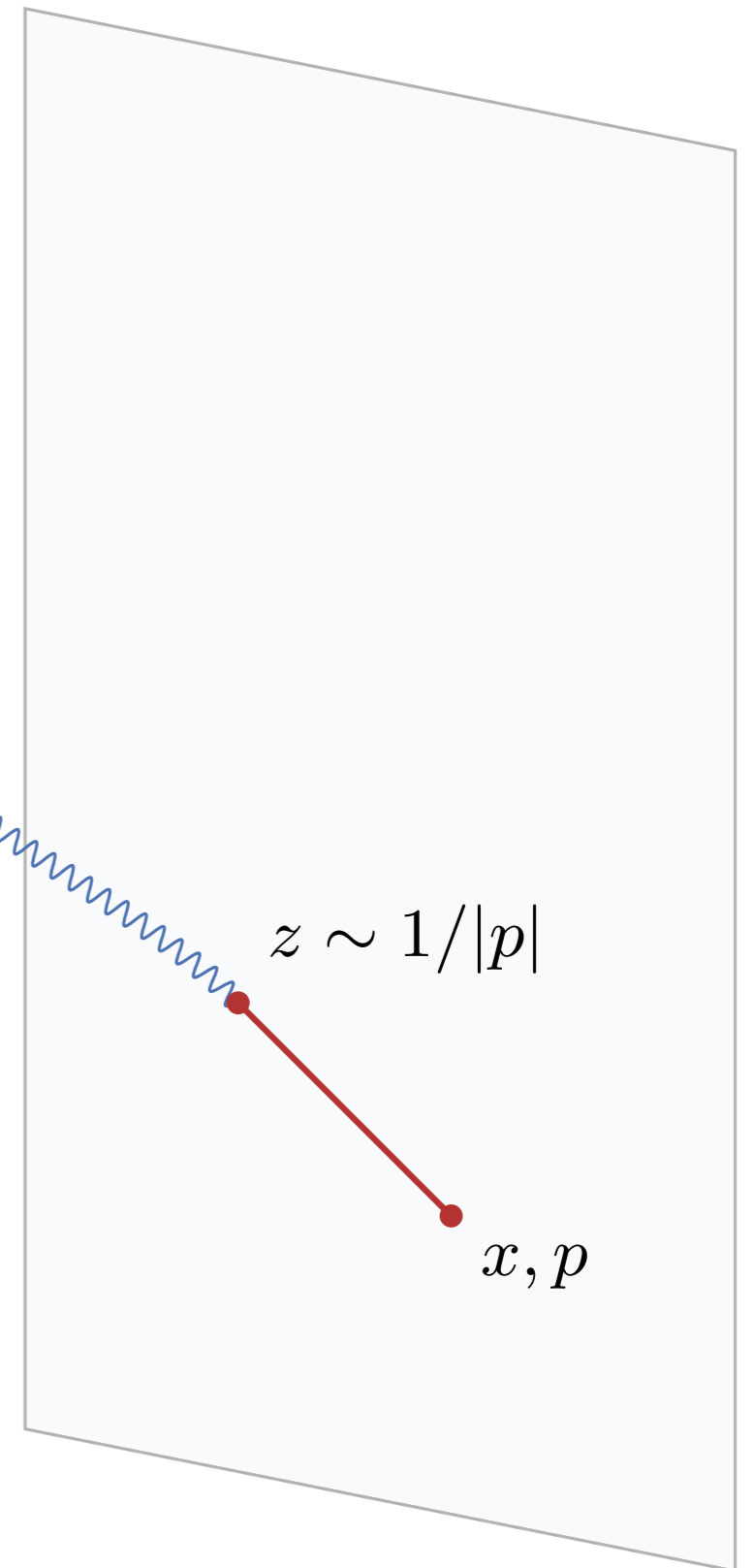
$$\langle \Psi | \phi(X) O_p^\dagger(x) | \Psi \rangle$$

$$= C_{\Delta,p}^\pm \sqrt{\mathcal{D}(p; X)} \exp(iS(p; X))$$

Normalization
from AdS
boundary

van-Vleck-
Morette
determinant
 $\det \frac{\partial^2 S}{\partial p_\mu \partial X^N}$

Hamilton-
Jacobi
function



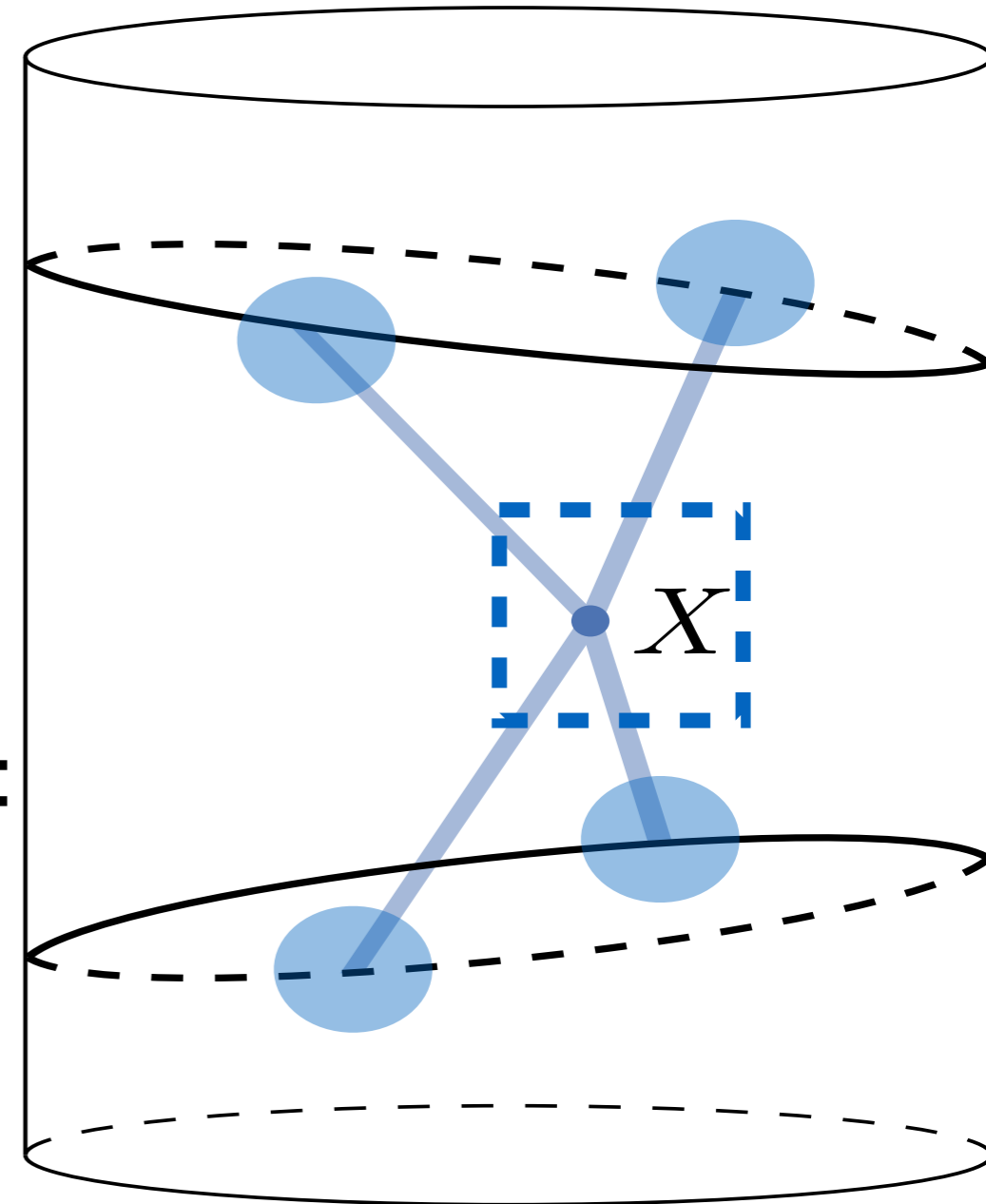
Main result: n-point correlators

CFT wavepacket correlator near H^\pm :

$$\langle \Psi | \prod_{j=m+1}^{m+n} O_{x_j^+, p_j, \sigma_j} \prod_{j=1}^m O_{x_j^-, p_j, \sigma_j}^\dagger | \Psi \rangle$$

equal to **bulk scattering amplitude**
integrated against **calculable Gaussian**:

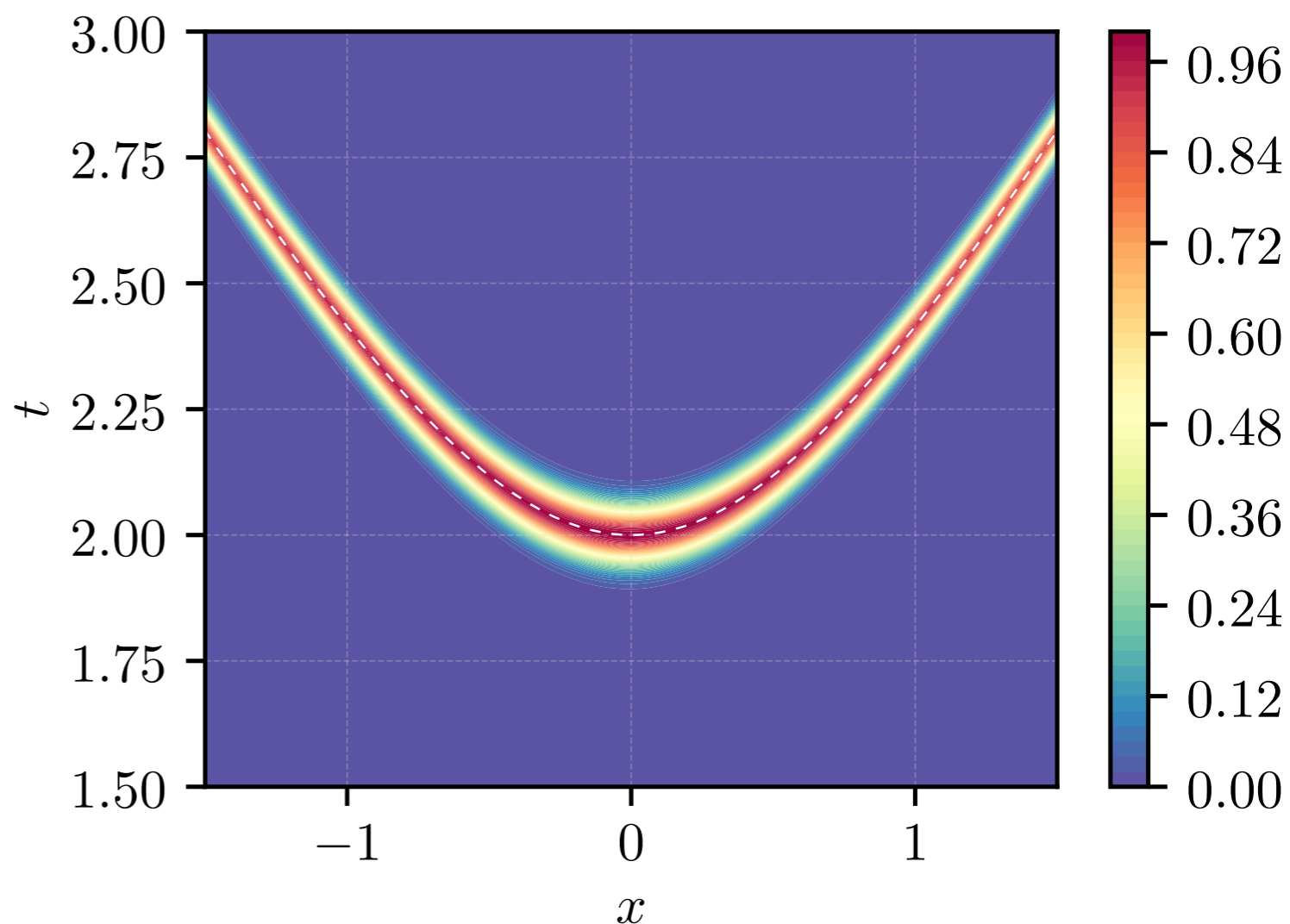
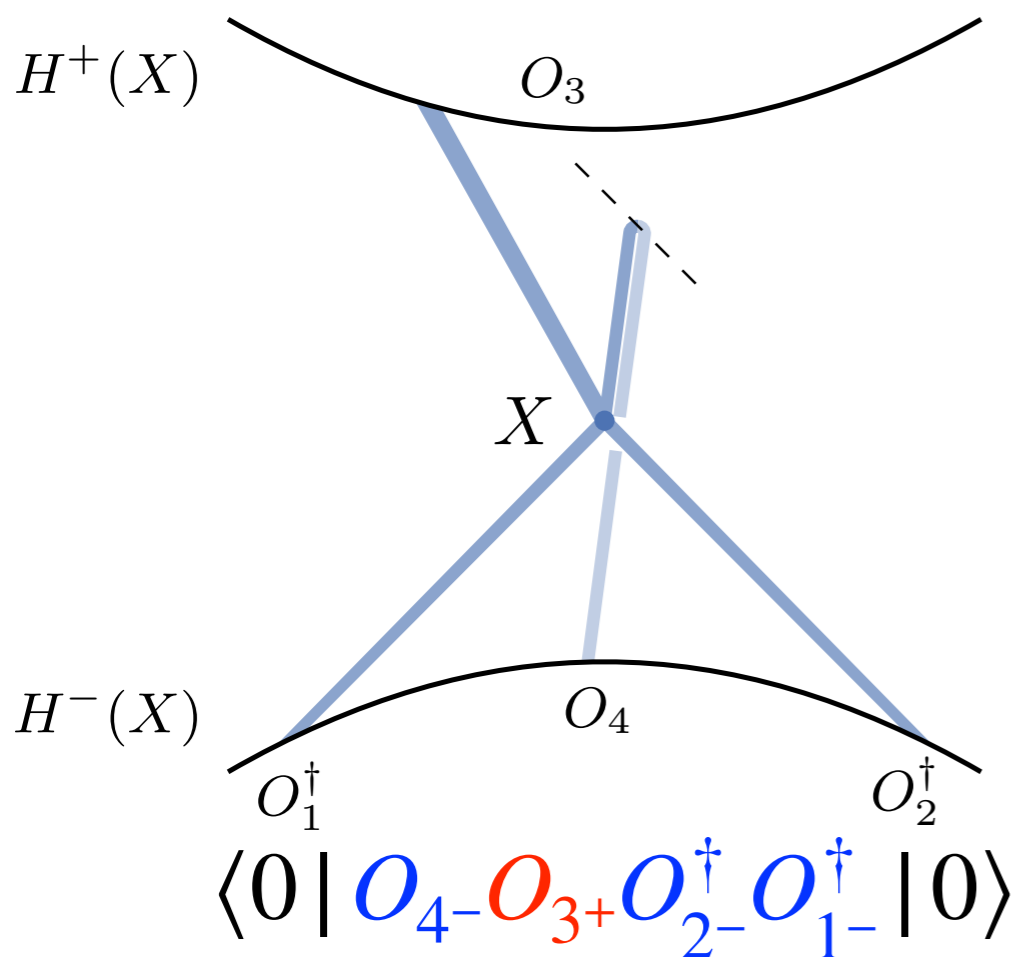
$$\approx \int \left[\prod dP_j \psi_j^{\text{bulk}} \right] \sqrt{-g(X)} \\ \times \delta^{d+1} \left(\sum P_j \right) i\mathcal{M}^{\text{bulk}}(\{P_j\})$$



[Chakravarty, SCh, Namjou '25]

- Similar formula for inclusive (OTOC) correlators/amplitudes

Measuring hyperboloids in empty AdS



Sudden, near-simultaneous signal at $t=2.0$ is smoking gun of local bulk dynamics. [does not happen in eg. 3D Ising CFT]

This particular “event” had bulk energy $\sqrt{s}R_{\text{AdS}} \approx 170$.

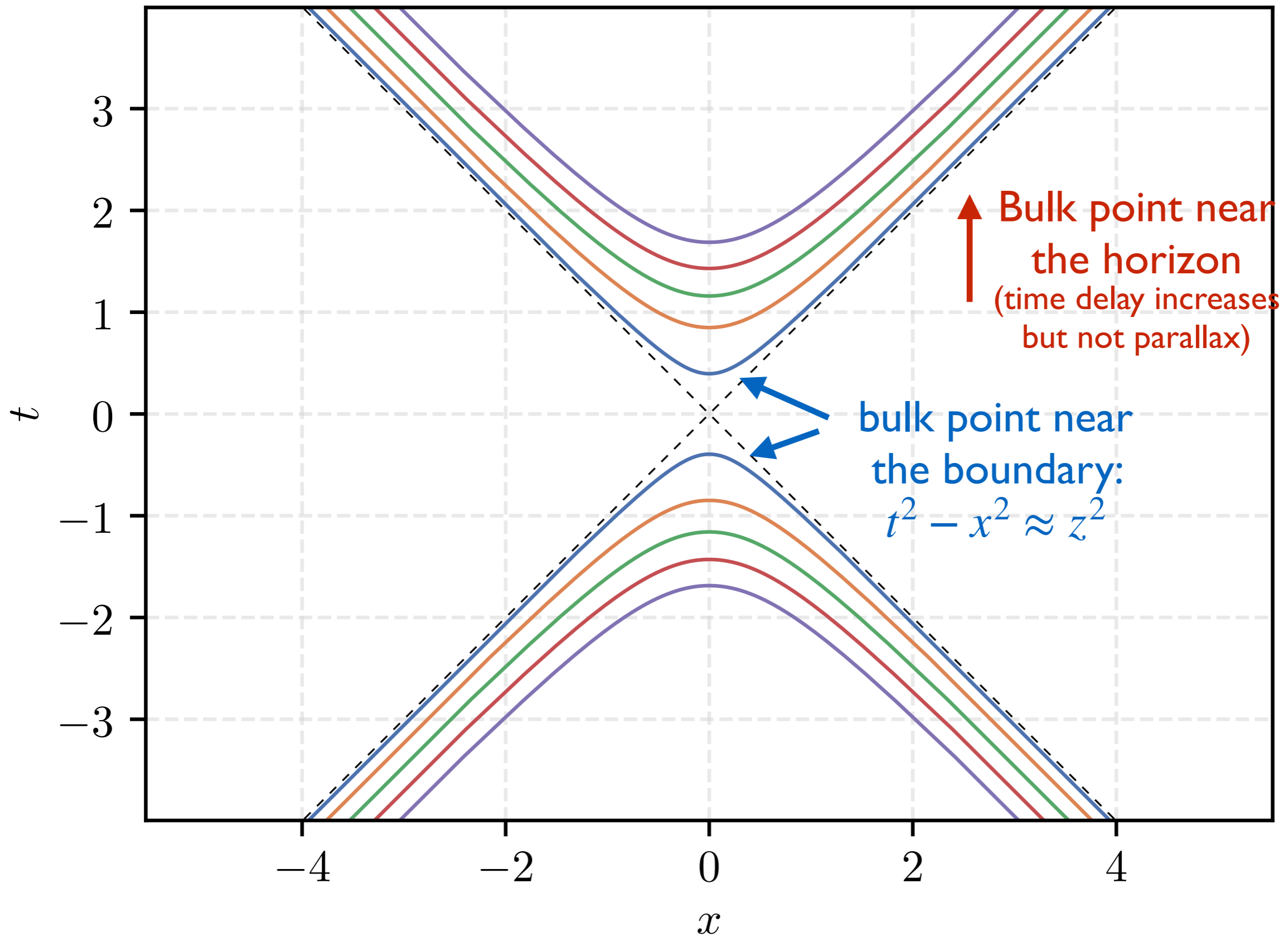
Requiring this to be $<$ string mass ($2\lambda^{\frac{1}{4}}$ in $\text{AdS}_5 \times S^5$) $\rightarrow \lambda \gtrsim 10^8$ (!?)

Bulk coordinates

- Radial coordinate can be defined from boundary, much like we observe distances to objects
- Ex: $r = \frac{1}{2}$ back-and-forth time
[=time between past&future branches at specific shooting point]
- Ex: $r = \text{parallax} \frac{\Delta x}{\Delta \theta}$
[=curvature of hyperboloid]
- Many possible choices of boundary-anchored coordinates

Planar AdS₅ black hole hyperboloids

($z_h = 1$)



Measuring the bulk metric

Any bulk displacement $\delta X = (\delta x, \delta t, \delta z)$

= small change in hyperboloid

δX is null *iff* two hyperboloids are tangent at a point

[Engelhardt&Horowitz]

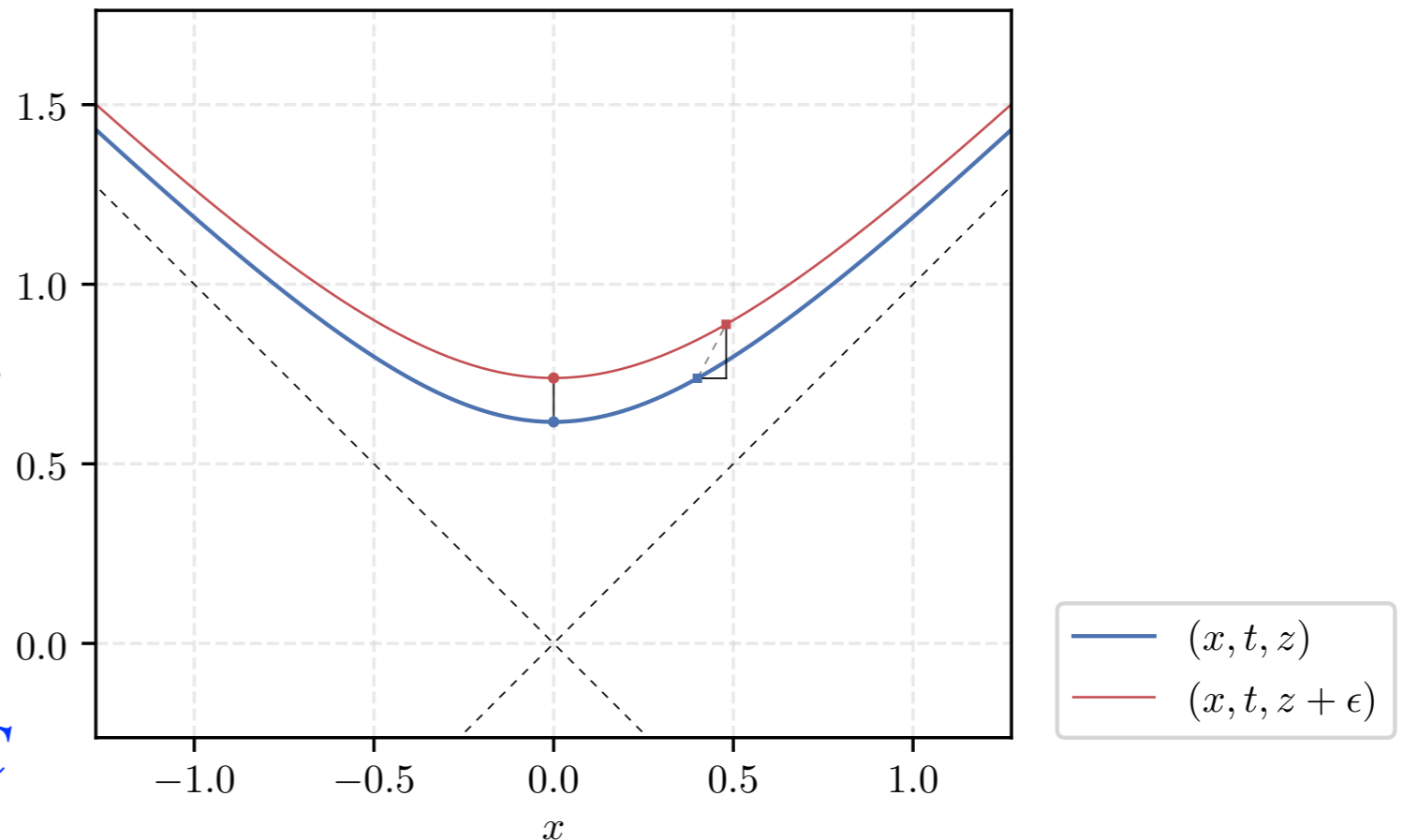
Ex: (w/translation symmetry):

assume near X :

$$ds^2 = -A dt^2 + B dz^2 + C dx^2$$

From $x = 0$: $\frac{A}{B} = \frac{(\delta z)^2}{(\delta t)^2}$

Any other point then fixes A/C



[Of course $A/B = 1$ if z =time-of-flight: “tortoise coordinates”.

Near horizon, algorithm will produce $A/C \rightarrow 0$ from $\delta x/\delta t \rightarrow 0$]

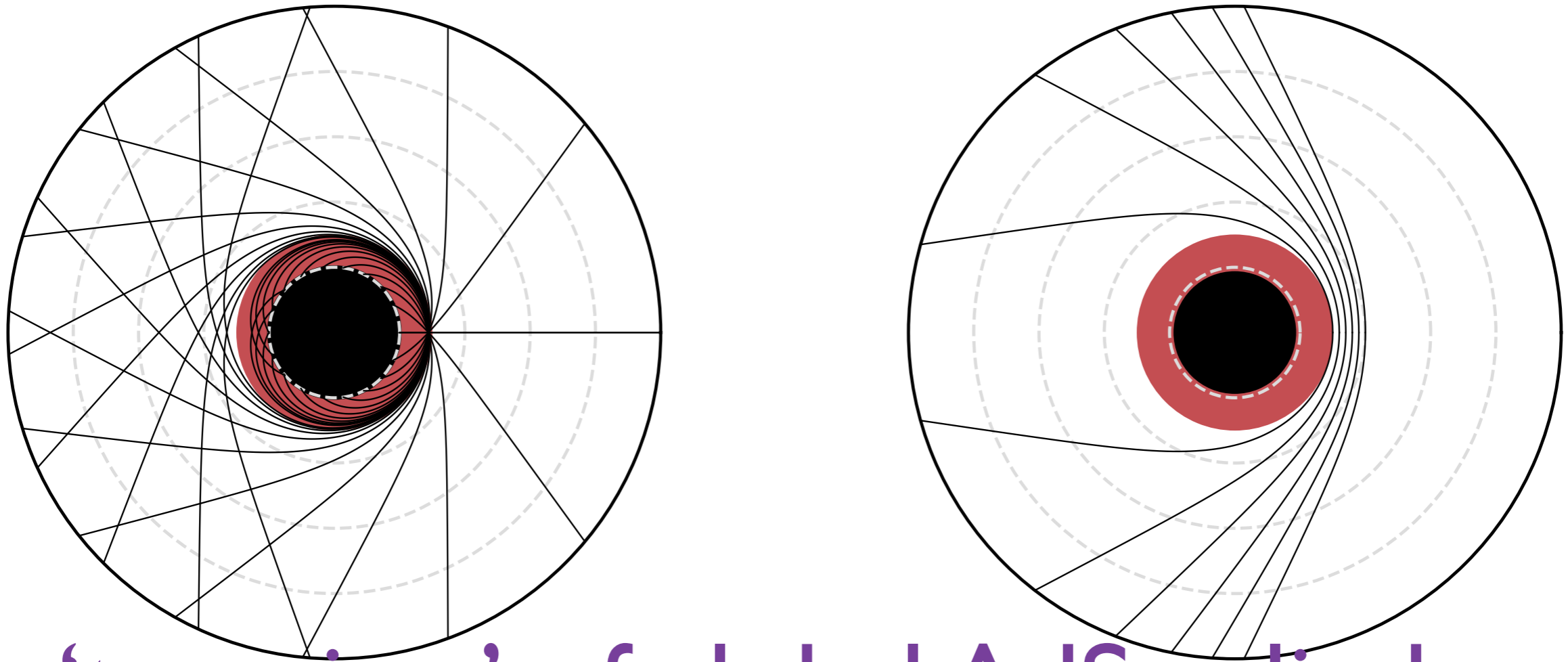
Measuring the bulk metric (2)

- Conformal metric at X determined by $(D + 2)(D - 1)/2$ measurements at discrete points on nearby hyperboloids!

(scale factor can be read off from correlator normalization)

- Over-constrained: succeeds only if metric actually exists!
- Difference between ‘measurement’ and ‘reconstruction’
[only use EOMs for probes, not background]
- Thm: For a family of hyperboloids $x^\mu(p; X)$, if any displacement δX that has a tangent point solves some quadratic constraint ($g_{MN}(X)\delta X^M\delta X^N = 0$ for some g_{MN}), then the hyperboloids represent the null geodesics of that metric

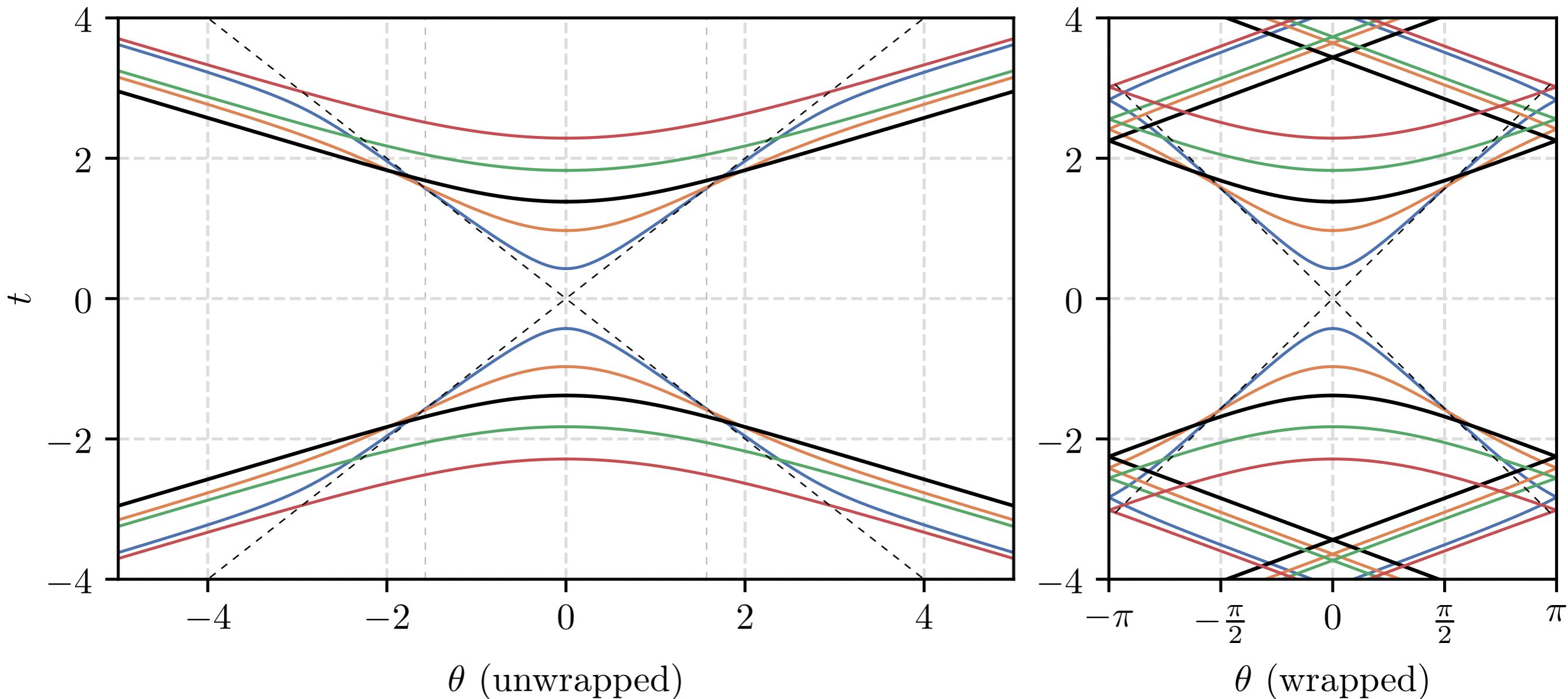
Fun with small black holes



'top view' of global AdS cylinder

Figure 4: Null geodesics in the AdS_5 -Schwarzschild black hole with $r_h = 0.2$. The black region represents the black hole interior and the red region indicates the interior of the photon sphere at r_{ps} . The left figure shows the geodesics originating from a point at the photon sphere. The figure on the right shows several geodesics that have a turning point due to the presence of the black hole.

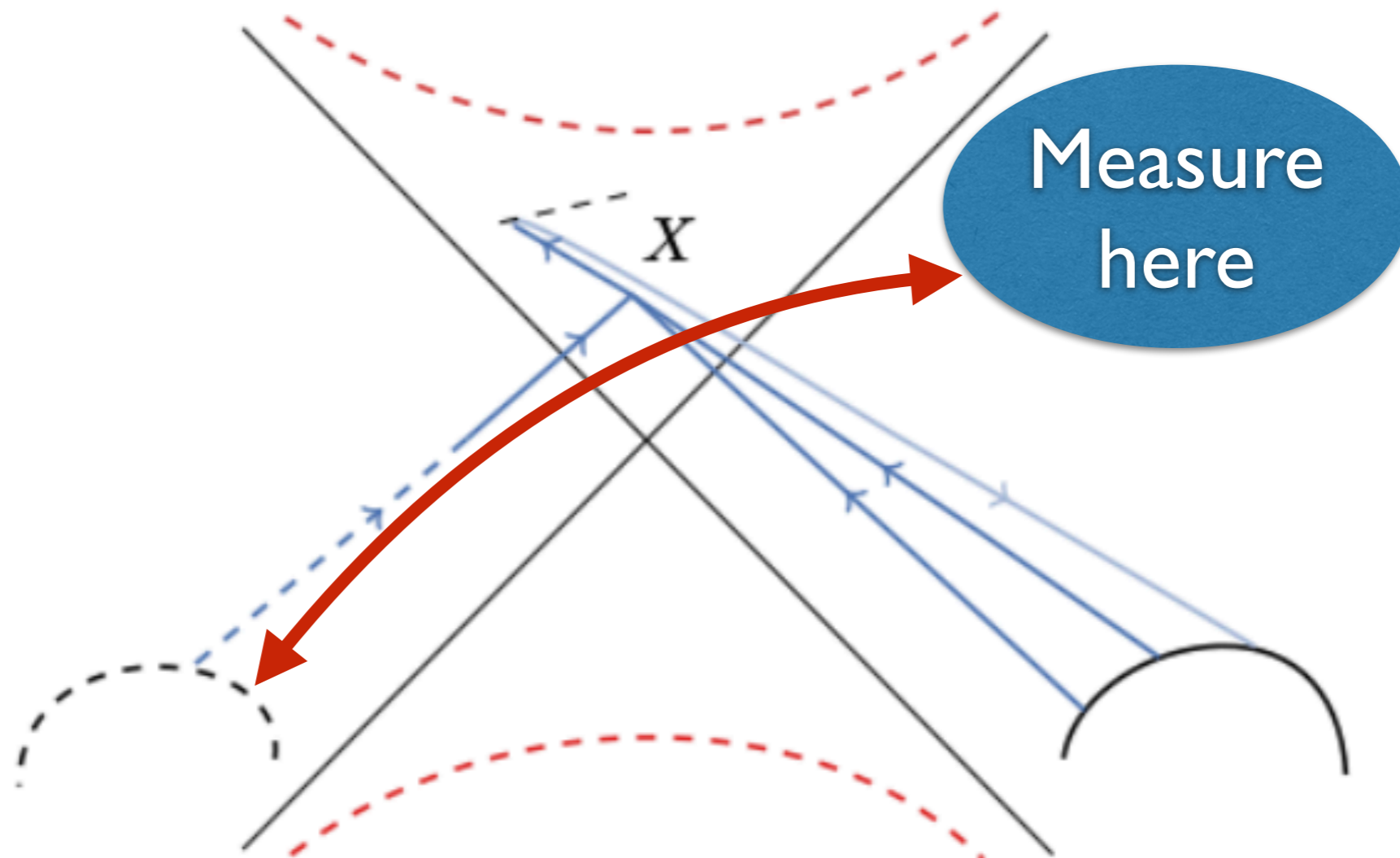
Fun with small black holes (2)



Asymptotic slope $dt/dx = \text{redshift factor at photon sphere}$.
 [No causality violation because θ is periodic: not prompt.]

Fun behind the horizon

[2512.10912 Chakravarty]



- No relation to in-falling observer, just a ‘continued metric’
- $\sim e^{-\beta E/2}$ contributions to 4pt exterior correlator
- An exterior question that “the interior metric is the answer to”

Summary & directions

- Studied boundary correlators integrated against wavepackets: an exercise in geometric optics
 - Features along lightcones (‘hyperboloids’) reveal bulk geometry
 - Factorization formula: exact solution near AdS boundary + WKB on geodesics + flat space amplitude near bulk point
-
- Why same bulk amplitude near any X (“equivalence principle”)?
 - Superpose Kaluza-Klein modes to localize in extra dimensions?
 - Derive bulk Raychaudhuri equations/focussing theorems from properties of boundary correlators?
 - Effect of long-range interactions (\rightarrow multiplicative ala SCET)?