Alice and Rob Revisited

How Quantum Reference Frames can provide New Insights into Entanglement Degradation

June 23, 2026

CAP 2026 (U. Ottawa)

Everett A. Patterson

UNIVERSITY OF WATERLOO | FACULTY OF SCIENCE

Outline

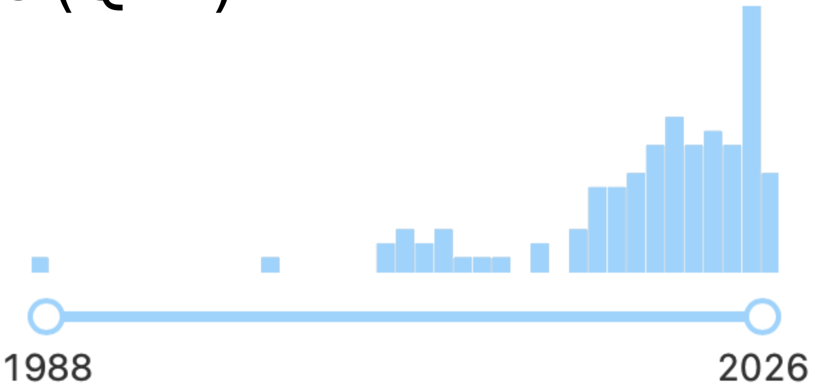
- (1) Introduce Quantum Reference Frames (QRFs)
  + Entanglement Degradation
- (2) Results in Relativistic Quantum Information (RQI)
  and in Quantum Information (QI)
- (3) Discussion

Why Quantum Reference Frames?

- All physics is described *relative* to some Reference Frame
  - Implicitly or Explicitly
- Assume the Universe is fundamentally quantum
- Self-consistent description of Quantum Theory?

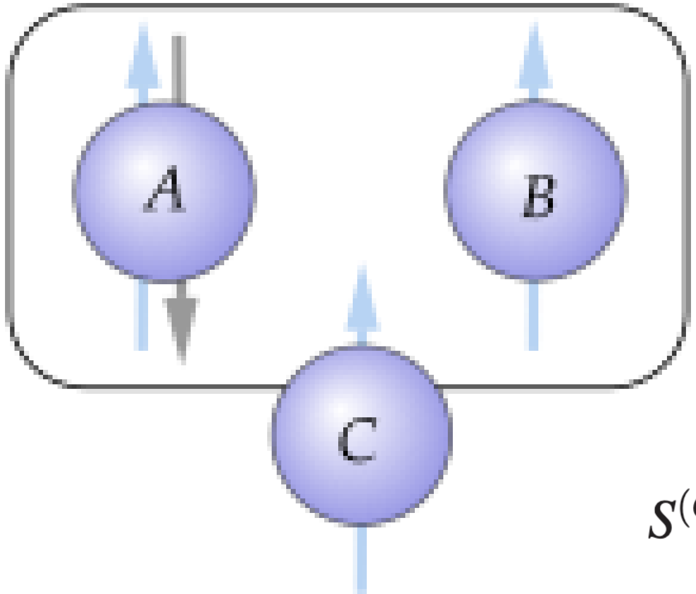
- Requires notion of Quantum Reference Frame (QRF)

- Renewed interest in last 10 years

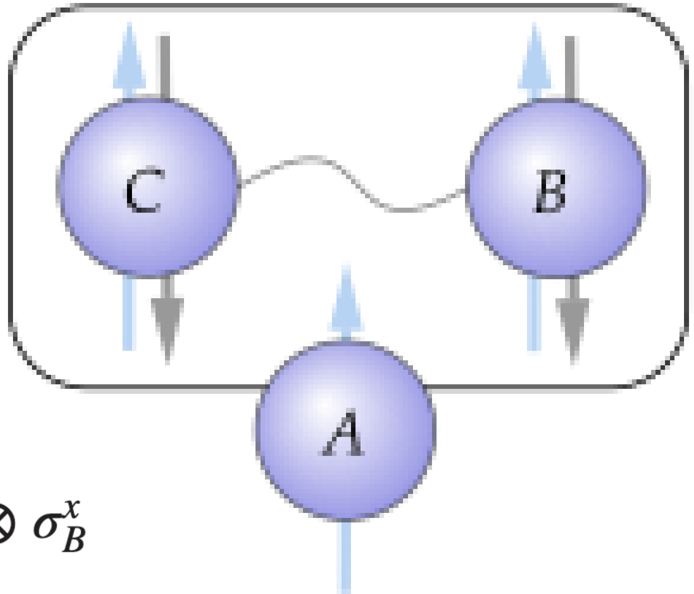


Example (QRF)

Cepollaro *et al.* 2025 PRL



→



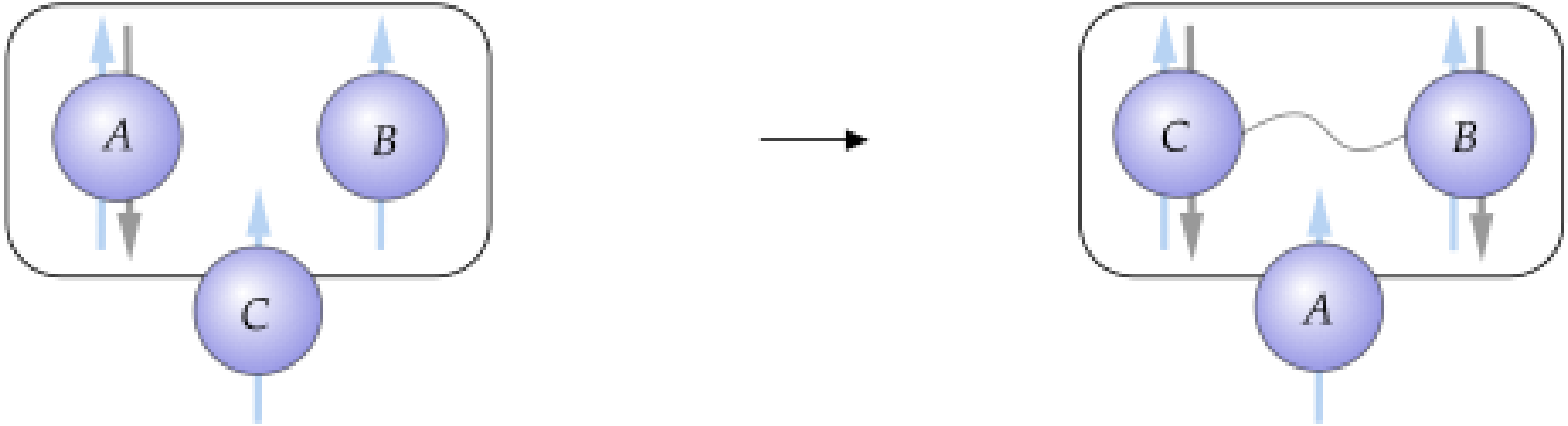
$$S^{(C)\to(A)} = |0\rangle_{CA}\langle 0| \otimes \mathbb{1}_B + |1\rangle_{CA}\langle 1| \otimes \sigma^x_B$$

$$|\psi\rangle^{(C)}_{AB} = \frac{|0\rangle_A + |1\rangle_A}{\sqrt{2}}\,|0\rangle_B$$

$$|\psi\rangle^{(A)}_{BC} = \frac{|0\rangle_B|0\rangle_C + |1\rangle_B|1\rangle_C}{\sqrt{2}}$$

Entanglement & Coherence (QRF)

Cepollaro *et al.* 2025 PRL



$$|\psi\rangle_{AB}^{(C)} = \frac{|0\rangle_A + |1\rangle_A}{\sqrt{2}} |0\rangle_B$$

$$|\psi\rangle_{BC}^{(A)} = \frac{|0\rangle_B |0\rangle_C + |1\rangle_B |1\rangle_C}{\sqrt{2}}$$

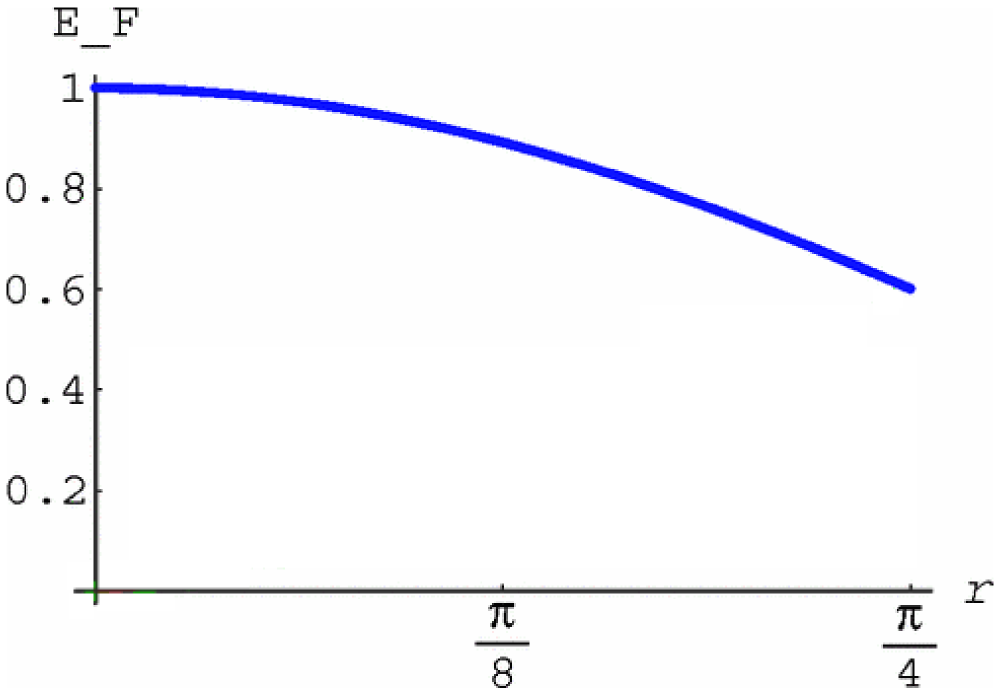
$$\mathcal{C}_e^{(C)} + \mathcal{E}_e^{(C)} = \mathcal{C}_e^{(A)} + \mathcal{E}_e^{(A)}$$

20 years ago!

Entanglement Degradation

Fuentes, Mann 2005 PRL
Alsing, Fuentes, Mann, Tessier 2006 PRA

- “Entanglement is observer-dependent quantity in non-inertial frames”



~ acceleration

- “Consequence of the Unruh effect”

Entanglement Degradation

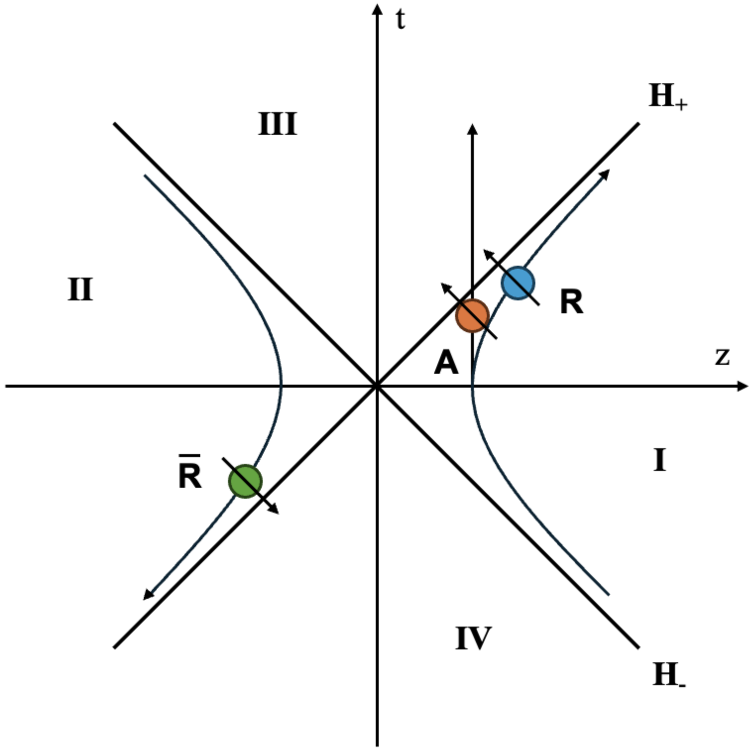
Fuentes, Mann 2005 PRL
Alsing, Fuentes, Mann, Tessier 2006 PRA

- Fermionic field in flat spacetime
- Alice and Rob share a pair of entangled field modes

$$|\phi\rangle_{AR} = \frac{1}{\sqrt{2}}(|0\rangle_A |0\rangle_R + |1\rangle_A |1\rangle_R)$$

- Rob undergoes uniform acceleration
  -> Rindler modes

$$|\phi_r\rangle_{AR\overline{R}} = \frac{1}{\sqrt{2}}(\cos r|000\rangle + \sin r|011\rangle + |110\rangle),$$



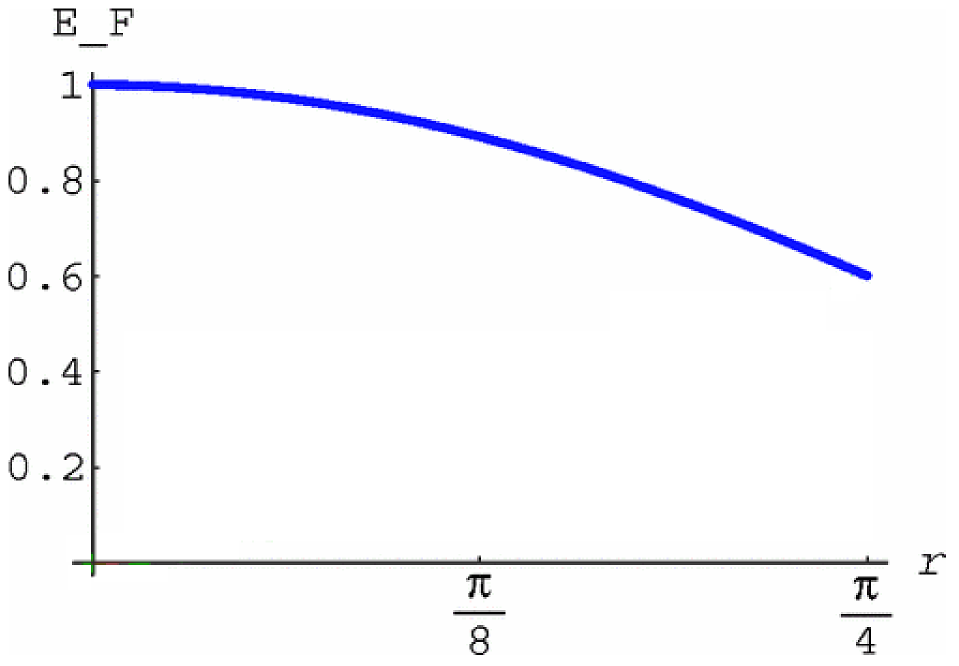
Entanglement Degradation

Fuentes, Mann 2005 PRL
Alsing, Fuentes, Mann, Tessier 2006 PRA

- Joint state $$|\phi_r\rangle_{AR\overline{R}} = \frac{1}{\sqrt{2}}(\cos r|000\rangle + \sin r|011\rangle + |110\rangle),$$

- Want entanglement between Alice and Rob
- Trace over anti-Rob

- Compute entanglement of reduced density matrix



Entanglement Deg. w QRFs

- Same set up as before

$$|\phi_r\rangle_{AR\overline{R}} = \frac{1}{\sqrt{2}}(\cos r|00\underline{0}\rangle + \sin r|01\underline{1}\rangle + |11\underline{0}\rangle),$$

$$\frac{1}{\sqrt{2}}(\cos r\,|000\rangle + \sin r\,|\underline{100}\rangle + |110\rangle)$$

- But '**Perspective Assignment**' rather than trace:

$$|\phi_r\rangle_{AR}^{(\overline{R})} = \frac{1}{\sqrt{2}}(\cos r|\underline{00}\rangle + \sin r|\underline{10}\rangle + |\underline{11}\rangle)$$

Entanglement Deg. w QRFs

Global state

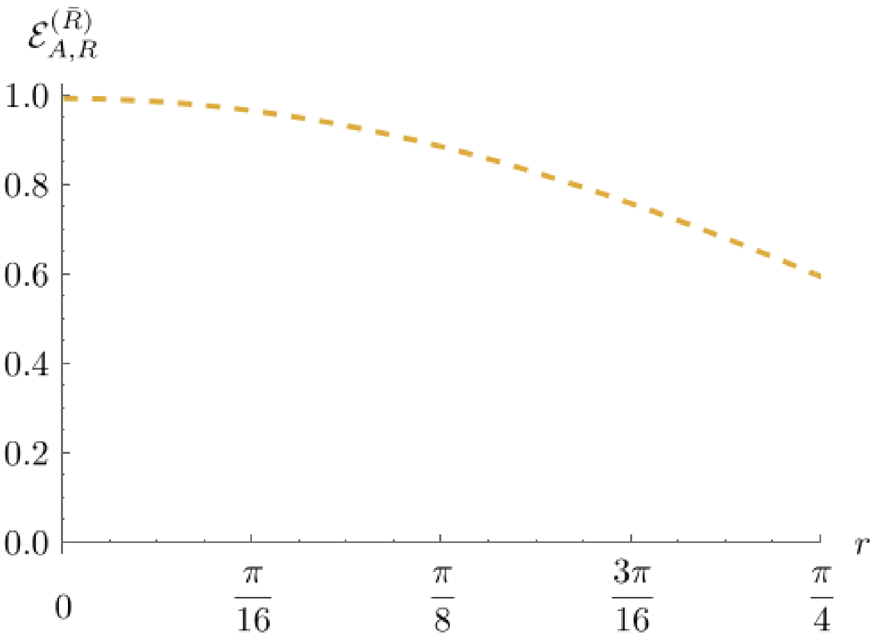
$$|\phi_r\rangle_{AR\overline{R}} = \frac{1}{\sqrt{2}}(\cos r|000\rangle + \sin r|011\rangle + |110\rangle),$$

Perspectival State

$$|\phi_r\rangle_{AR}^{(\overline{R})} = \frac{1}{\sqrt{2}}(\cos r|00\rangle + \sin r|10\rangle + |11\rangle)$$

Compute Entanglement Entropy:

- Entanglement Degrades!



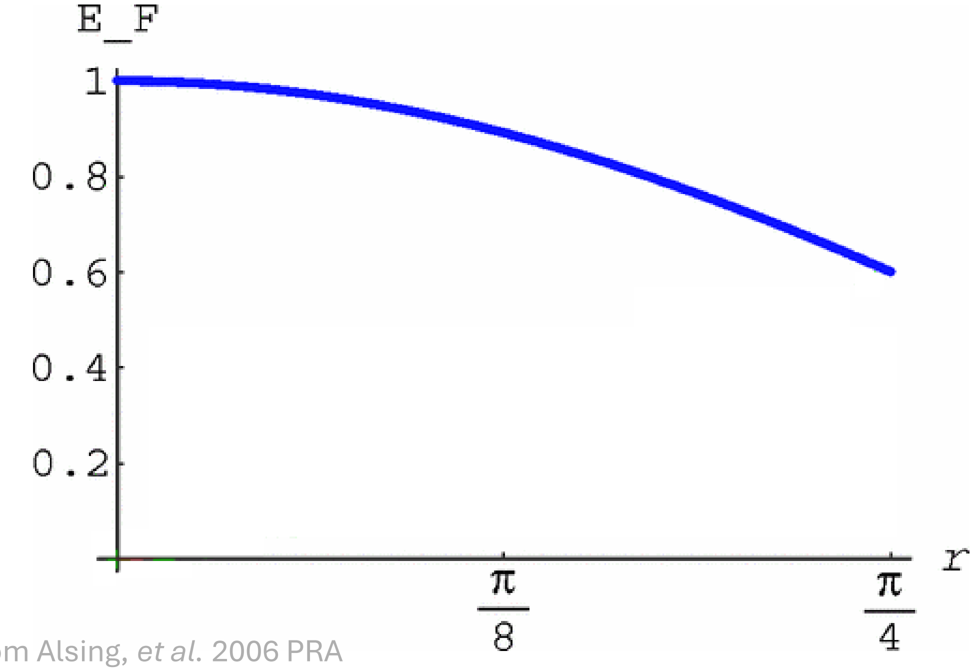
Entanglement Deg. w QRFs

Global state

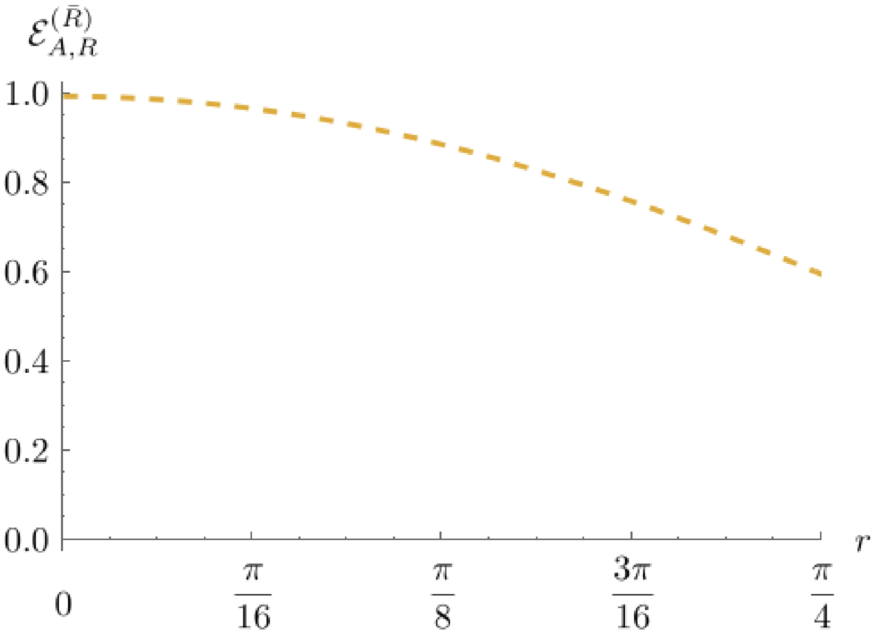
$$|\phi_r\rangle_{AR\overline{R}} = \frac{1}{\sqrt{2}}(\cos r|000\rangle + \sin r|011\rangle + |110\rangle),$$

Perspectival State

$$|\phi_r\rangle_{AR}^{(\overline{R})} = \frac{1}{\sqrt{2}}(\cos r|00\rangle + \sin r|10\rangle + |11\rangle)$$



Plot from Alsing, *et al.* 2006 PRA



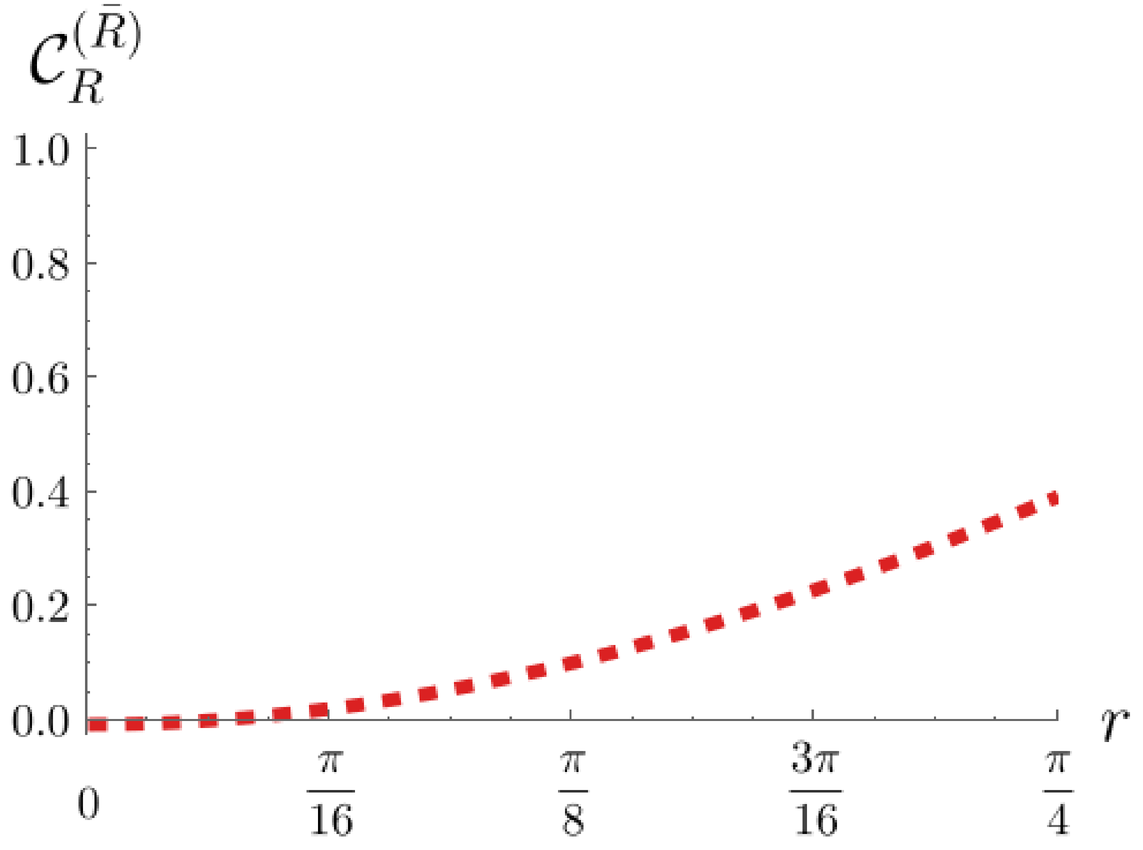
Coherence w QRFs

- What about coherence?

Coherence Generation(!) w QRFs

- What about coherence?

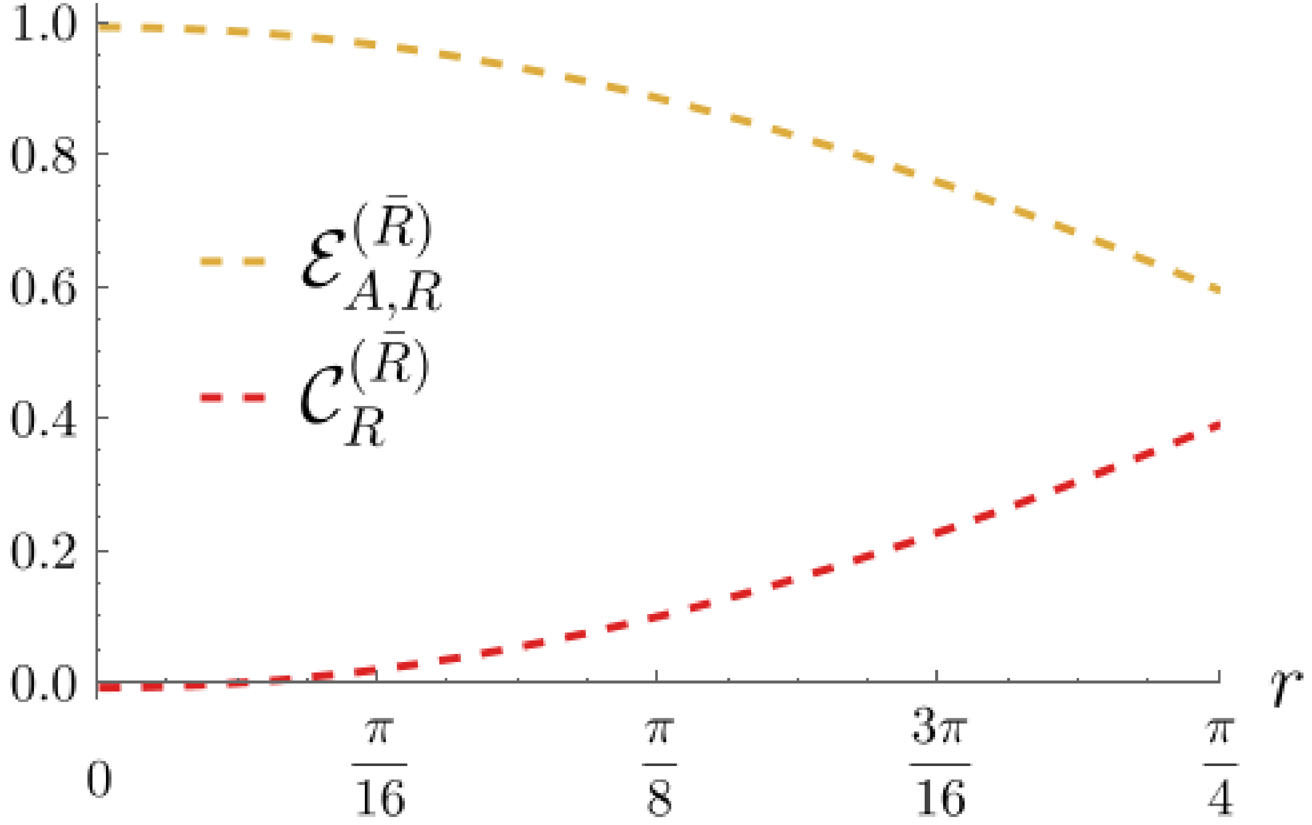
- Coherence increases with acceleration!



E+C for RQI

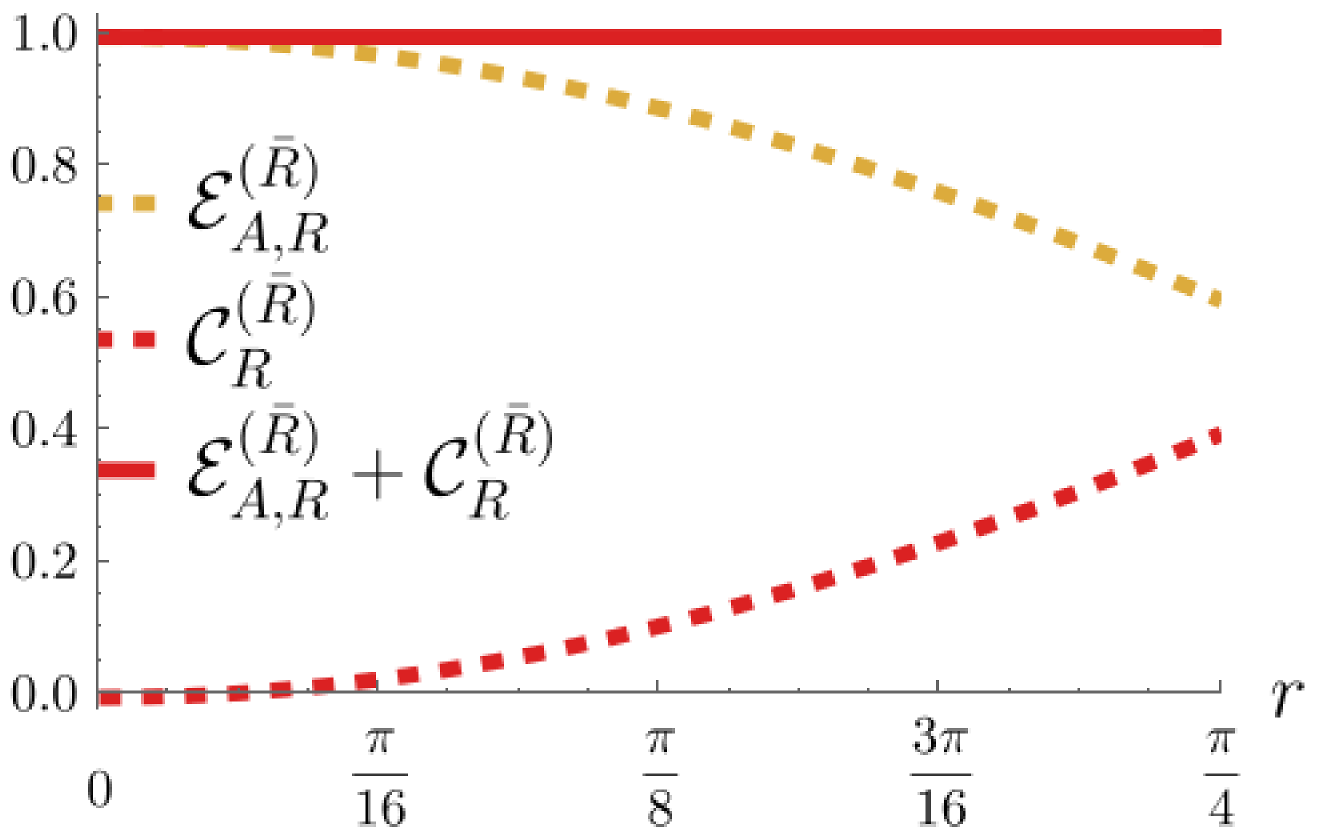
- What about coherence?

- Coherence increases with acceleration!

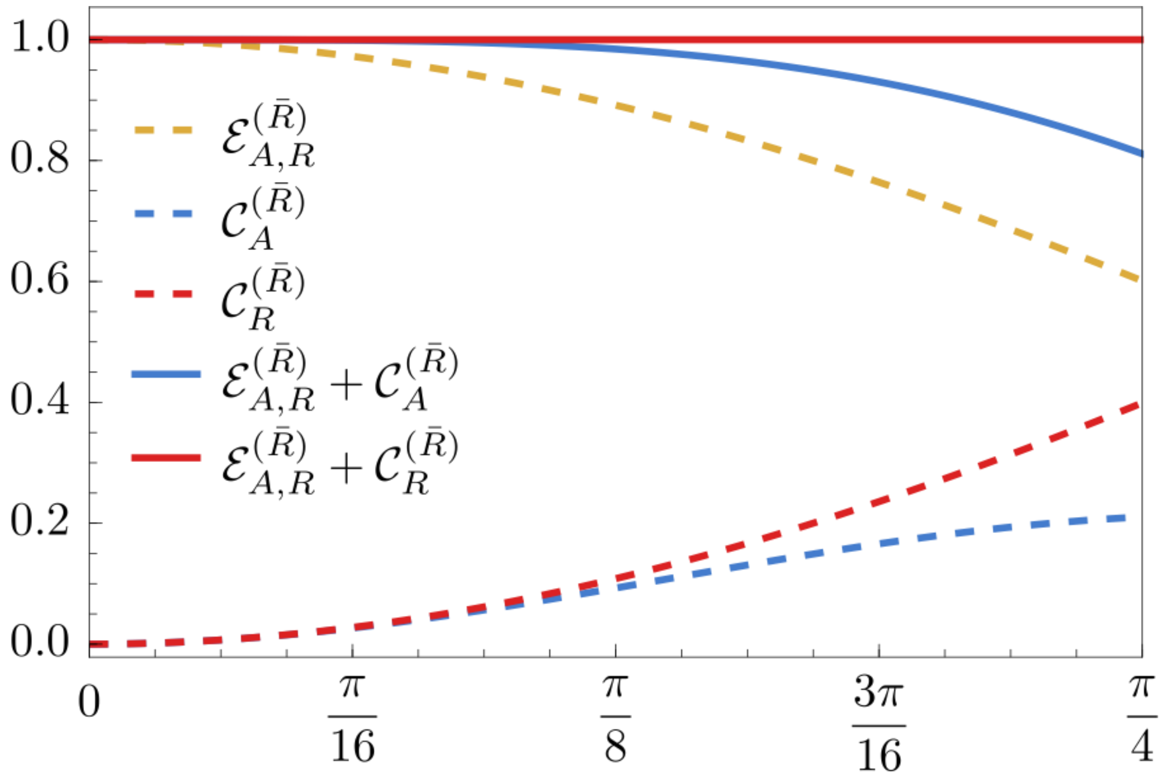


E+C for RQI

- What about coherence?
- Coherence increases with acceleration!
- AND exactly offsets the lost entanglement!
- Thermalization no longer a good explanation!



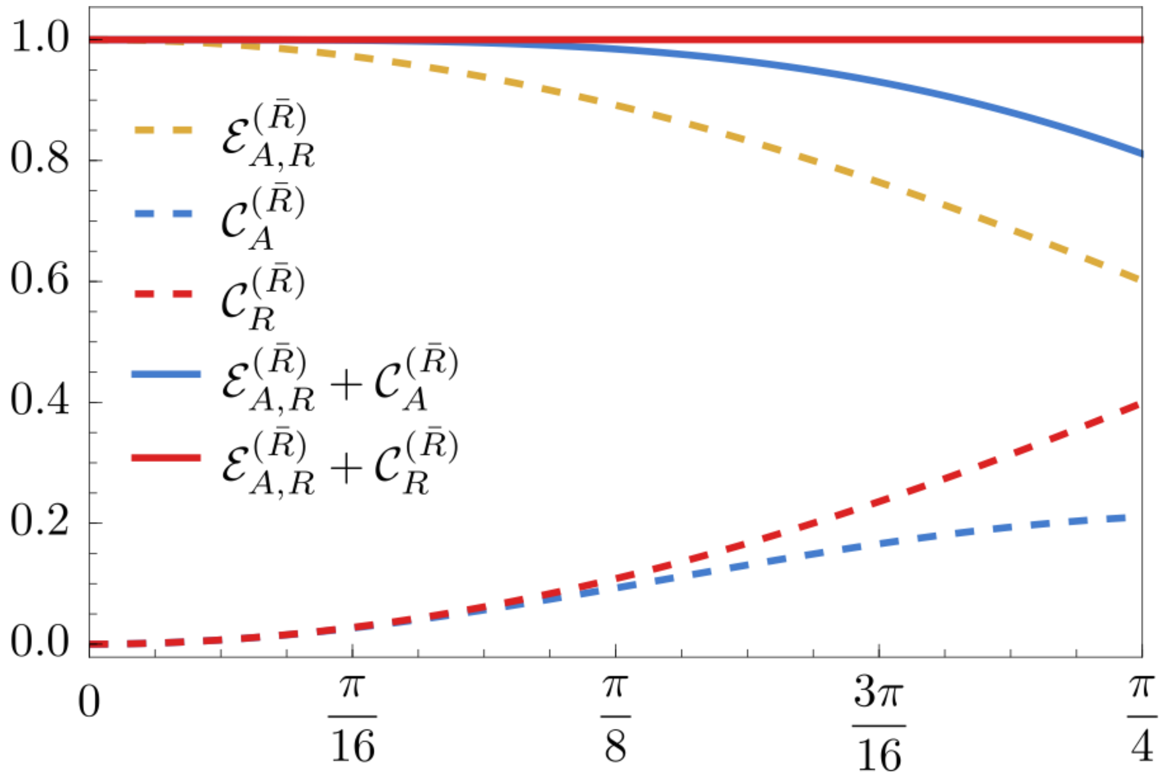
E+C for RQI



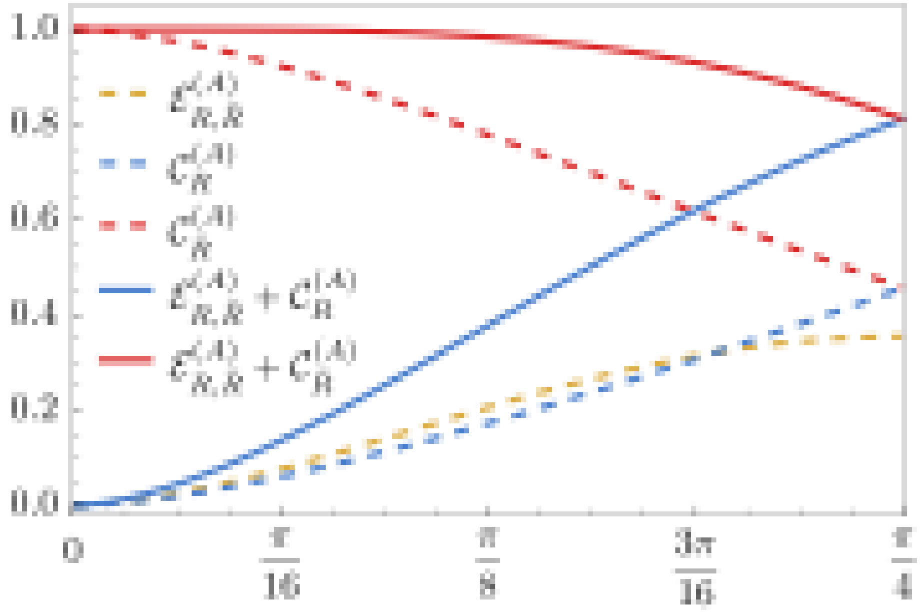
$(a)\ r\ (\mathcal{E}^{(\bar{R})}, \mathcal{C}^{(\bar{R})})$

- What about other subsystem coherence?
- E+C should be invariant for QRF transformations.

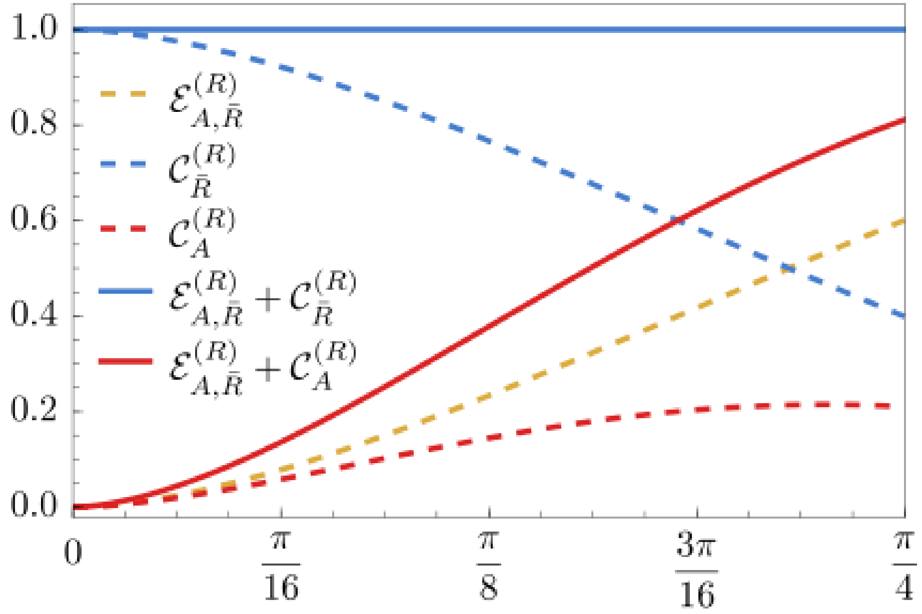
E+C for RQI



Anti-Rob $(a)\ r\ (\mathcal{E}^{(\bar{R})}, \mathcal{C}^{(\bar{R})})$

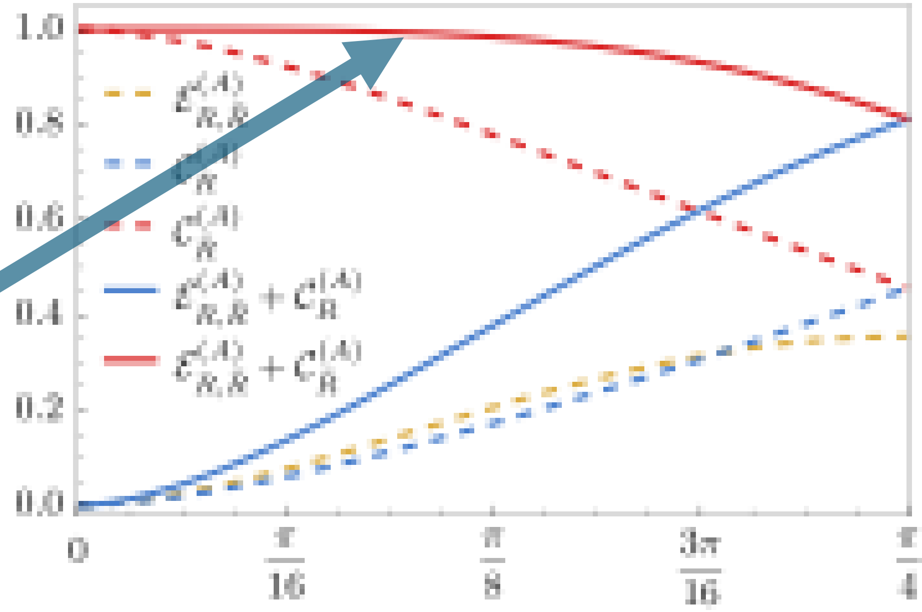


Alice $(c)\ r\ (\mathcal{E}^{(A)}, \mathcal{C}^{(A)})$

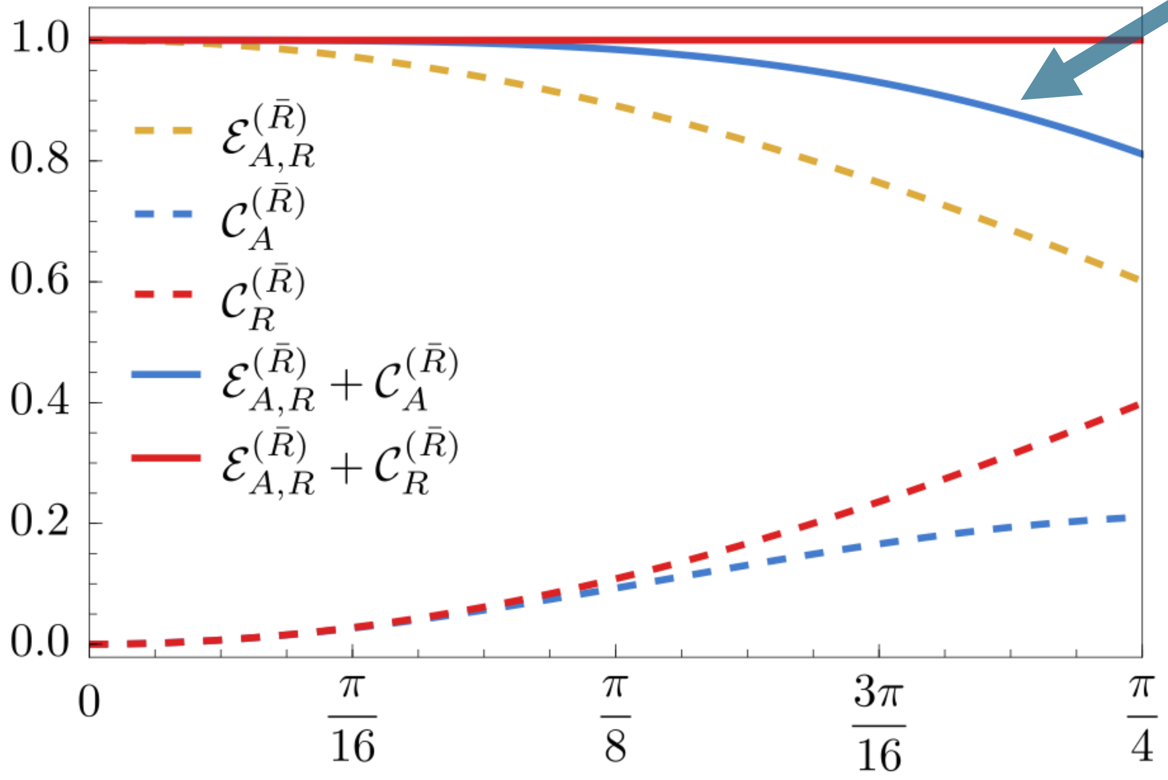


Rob $(b)\ r\ (\mathcal{E}^{(R)}, \mathcal{C}^{(R)})$

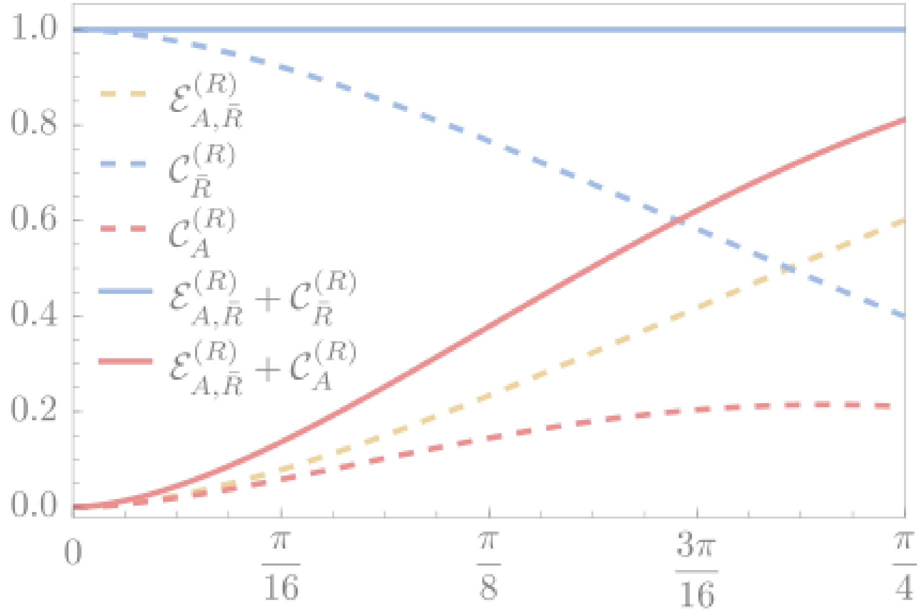
E+C for RQI



Alice (c) $r\ (\mathcal{E}^{(A)}, \mathcal{C}^{(A)})$

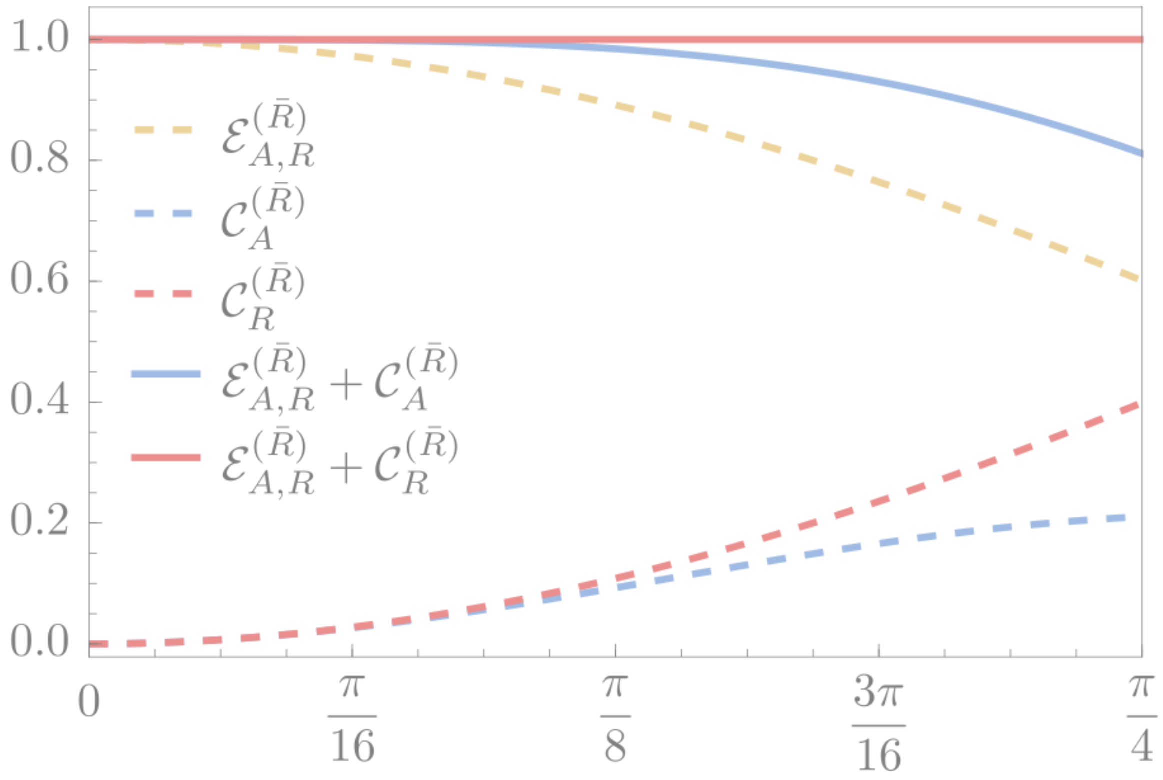


Anti-Rob (a) $r\ (\mathcal{E}^{(\bar{R})}, \mathcal{C}^{(\bar{R})})$

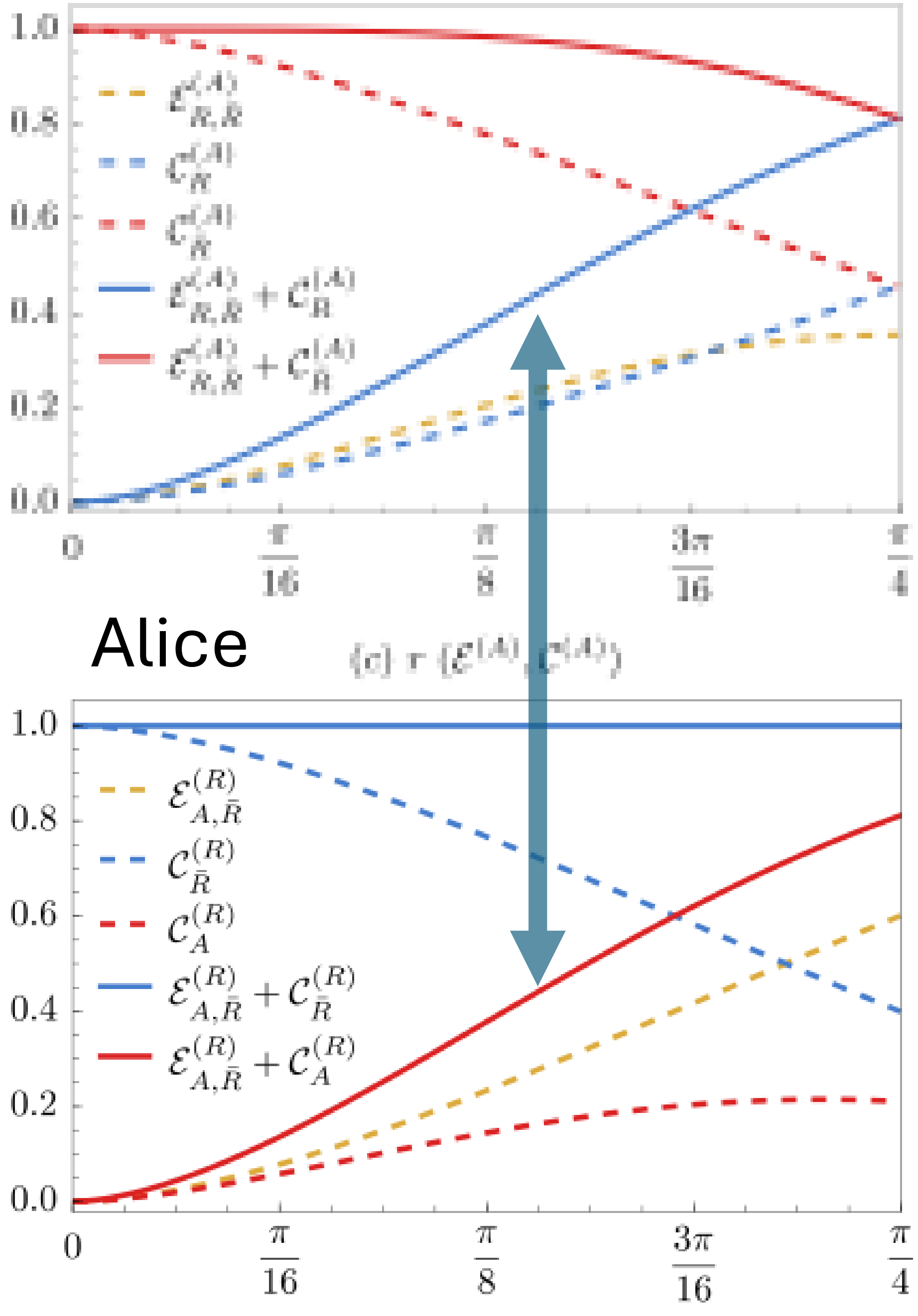


Rob (b) $r\ (\mathcal{E}^{(R)}, \mathcal{C}^{(R)})$

E+C for RQI

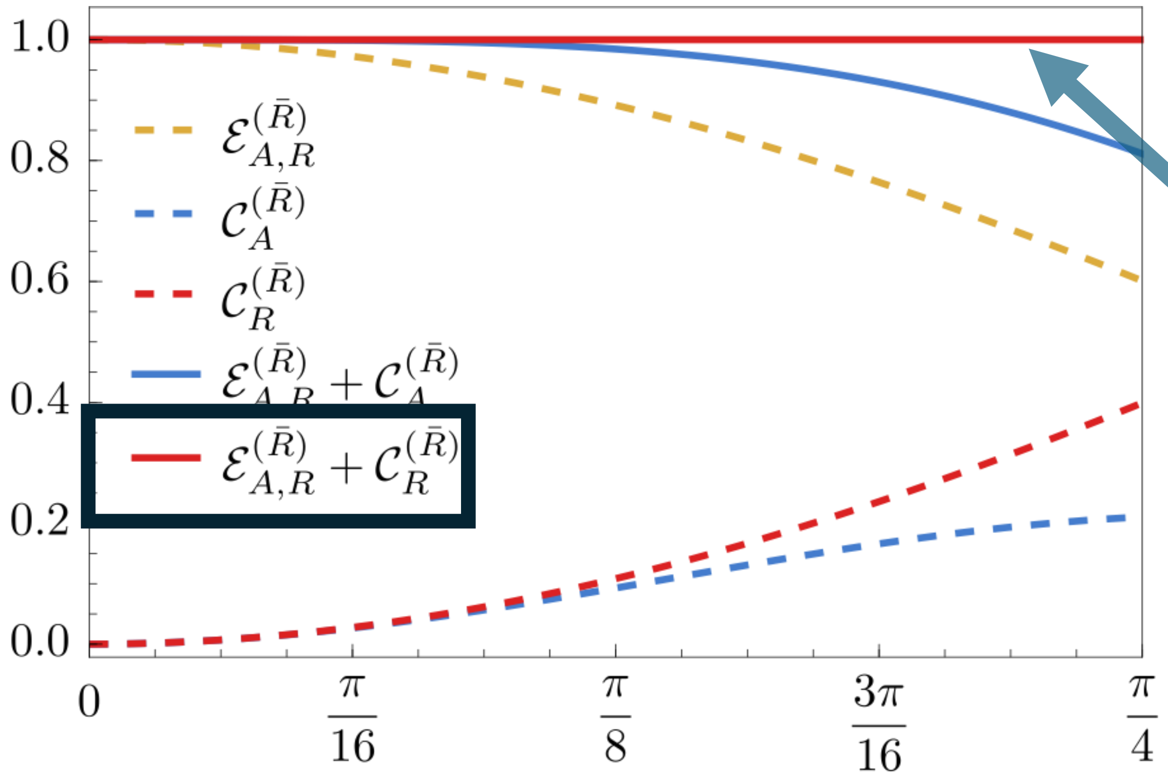


Anti-Rob (a) $r\ (\mathcal{E}^{(\bar{R})}, \mathcal{C}^{(\bar{R})})$

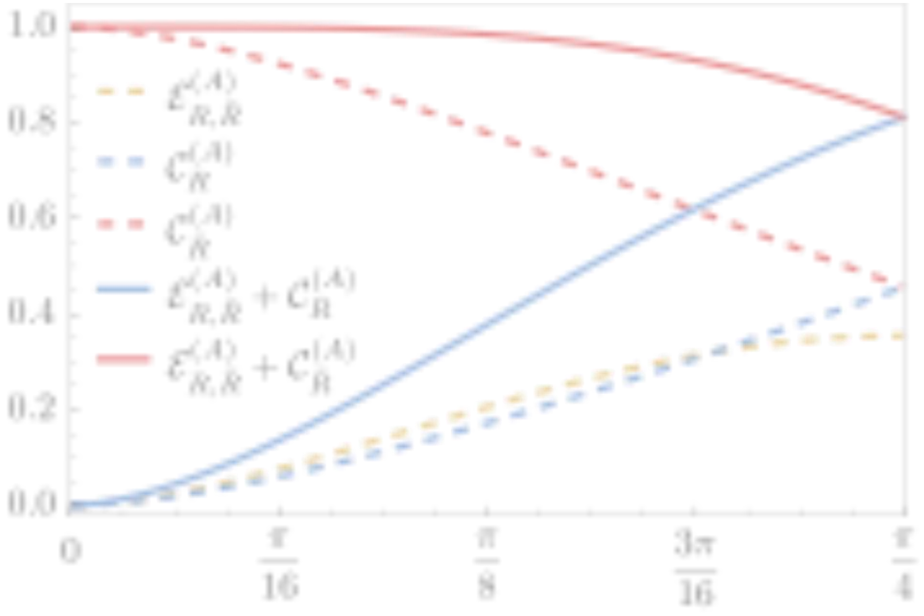


Rob (b) $r\ (\mathcal{E}^{(R)}, \mathcal{C}^{(R)})$

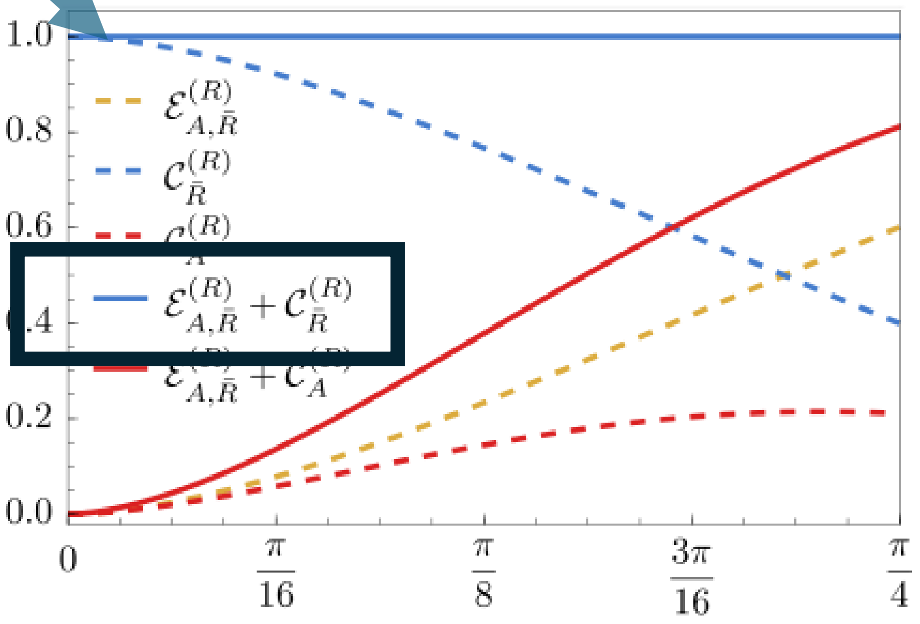
E+C for RQI



Anti-Rob (a) r $(\mathcal{E}^{(\bar{R})}, \mathcal{C}^{(\bar{R})})$

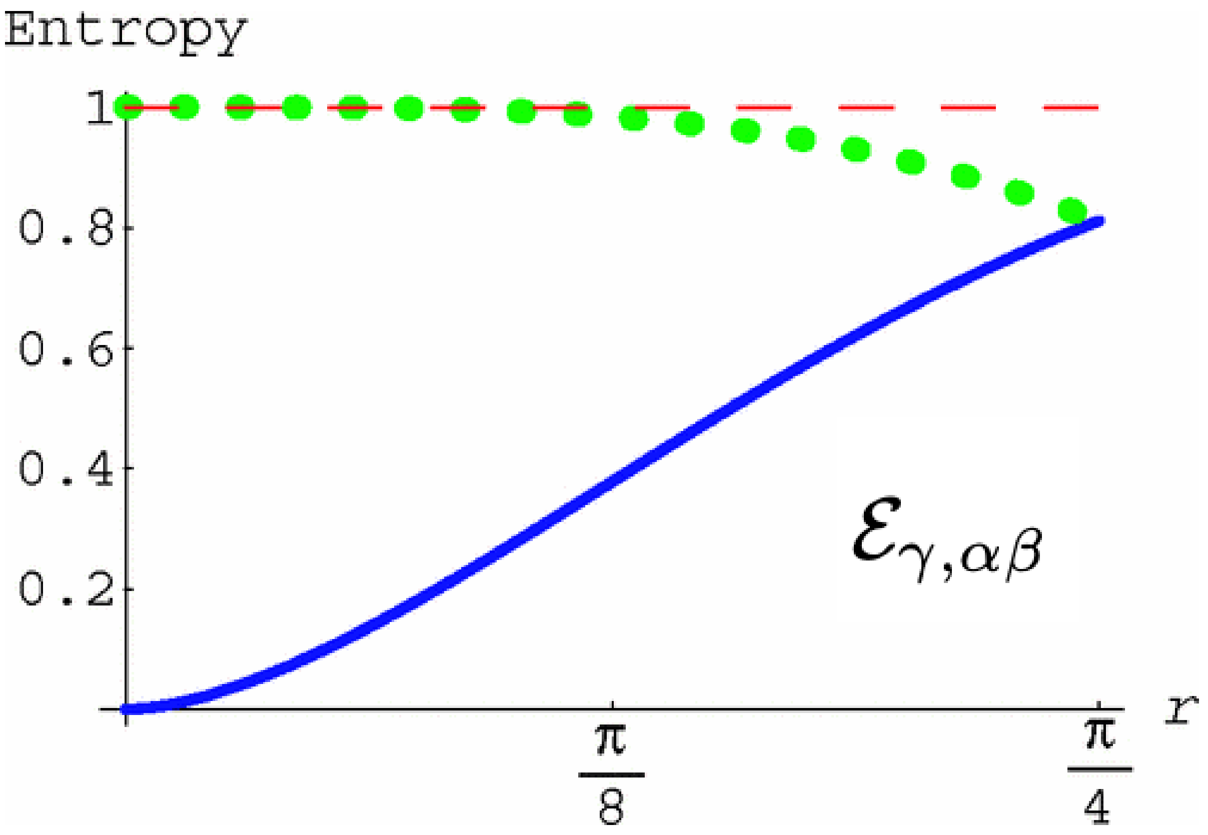


Alice (c) r $(\mathcal{E}^{(A)}, \mathcal{C}^{(A)})$



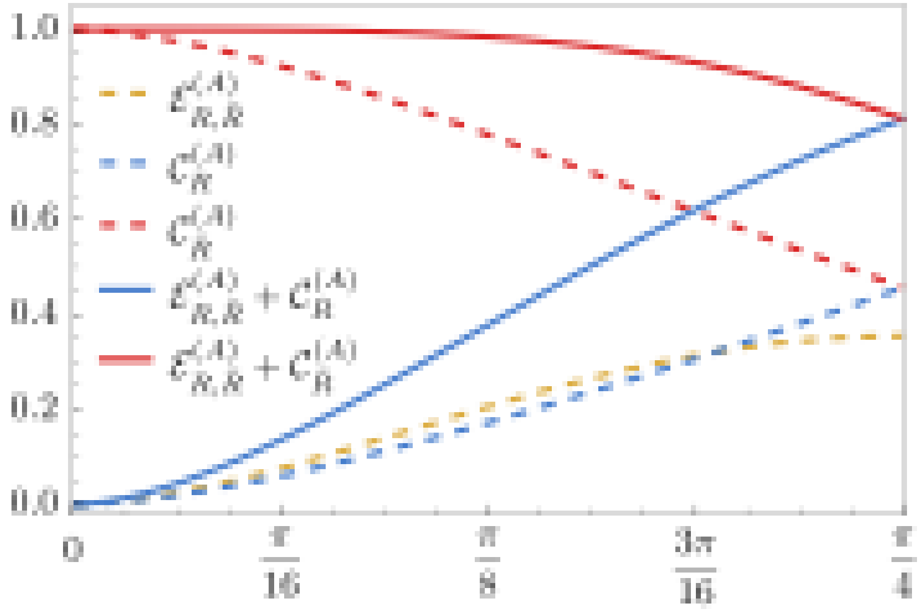
Rob (b) r $(\mathcal{E}^{(R)}, \mathcal{C}^{(R)})$

E+C for RQI



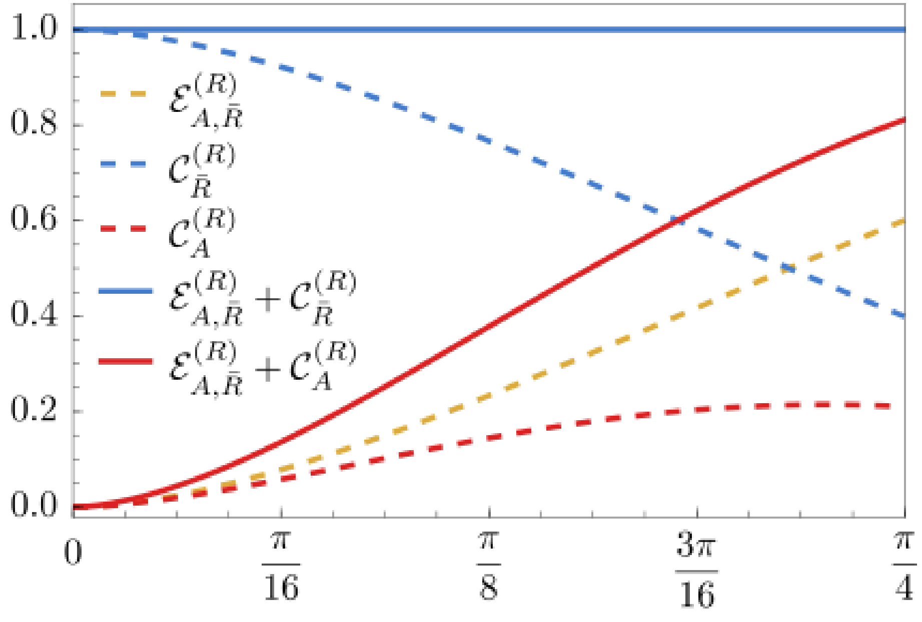
Plot from Alsing, *et al.* 2006 PRA

$$|\phi_r\rangle_{AR\overline{R}} = \frac{1}{\sqrt{2}}(\cos r|000\rangle + \sin r|011\rangle + |110\rangle),$$



Alice

(c) r $(\mathcal{E}^{(A)}, \mathcal{C}^{(A)})$



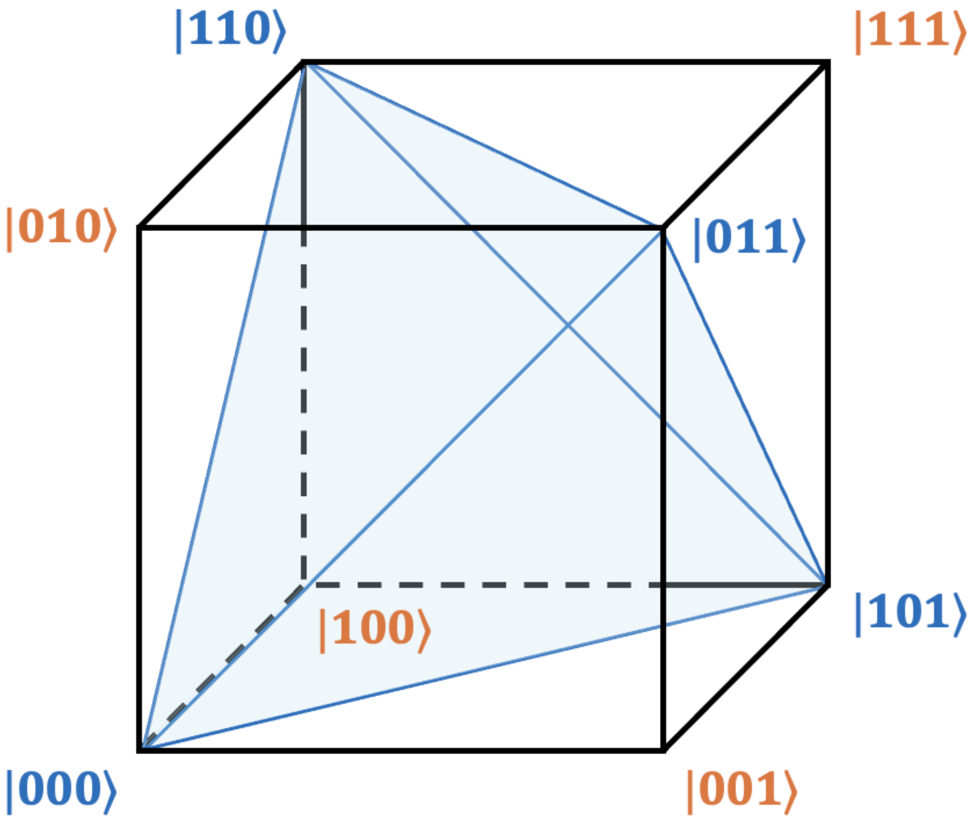
Rob

(b) r $(\mathcal{E}^{(R)}, \mathcal{C}^{(R)})$

Entanglement Transference

$$\mathcal{E}_{B,C}^{(A)} + \mathcal{C}_{B}^{(A)} = \mathcal{E}_{C,AB}$$

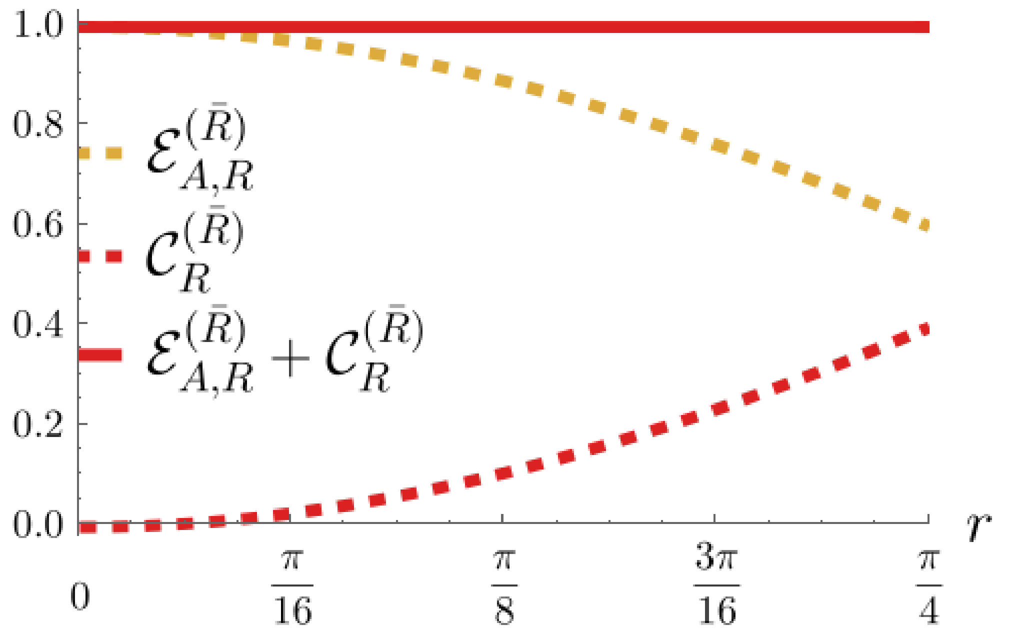
- Connects perspectival and global quantum resources!
- Restricted to “Parity States”
- Because of perspective assignment procedure



$$|E\rangle = E_1\,|000\rangle + E_2\,|011\rangle + E_3\,|101\rangle + E_4\,|110\rangle$$

$$|O\rangle = O_1\,|001\rangle + O_2\,|010\rangle + O_3\,|100\rangle + O_4\,|111\rangle$$

Concluding Remarks



- QRFs seem interesting and useful

$$\mathcal{E}_{B,C}^{(A)} + \mathcal{C}_{B}^{(A)} = \mathcal{E}_{C,AB}$$

Open questions include:

- Other resources?
- Apply QRFs to the 'non-inertial frames'
- QRFs insights for you?

arXiv:
2603.23601

https://arxiv.org/abs/2603.23601

UNIVERSITY OF WATERLOO | FACULTY OF SCIENCE

Supplementary Materials

Perspective Assignment

- Describe the 5-step procedure
- Comment on the reduction of state space and compression of information

$$|\psi\rangle_{ABC} = \sum_{i,j,k \in \{0,1\}} a_{ijk} |ijk\rangle$$

$$|\psi\rangle_{BC}^{(A)} = (a_{000}^2 + a_{111}^2)^{\frac{1}{2}} |00\rangle + (a_{001}^2 + a_{110}^2)^{\frac{1}{2}} |01\rangle + (a_{010}^2 + a_{101}^2)^{\frac{1}{2}} |10\rangle + (a_{011}^2 + a_{100}^2)^{\frac{1}{2}} |11\rangle$$

Perspective Assignment

Given an N-qubit pure state, the perspective assigning operation $\mathcal{R}_P : (\mathbb{C}^2)^{\otimes N} \rightarrow (\mathbb{C}^2)^{\otimes N-1}$ takes an initial N-qubit state to the $(N-1)$-qubit state from the perspective of one of the qubits, P. This operation can be described in 5 steps:

1. Express the N-qubit pure state as a density matrix, for a preferred basis.
2. Apply a generalized dephasing channel to the N-qubit system.
3. Apply the (non-unitary) *perspective operator* for the P-th qubit.
4. Trace out the P-th qubit, to recover an $(N-1)$-qubit system.
5. Purify the resulting perspectival state and project over the ancilla degrees of freedom, to recover the $(N-1)$-qubit system as a pure state.

Perspective Assignment (1-3)

$$|\psi\rangle_{ABC} = \sum_{i,j,k \in \{0,1\}} a_{ijk} |ijk\rangle ,$$

(1.) **Density Matrix.** We can express the state as a density matrix $\rho_{ABC} = |\psi\rangle_{ABC\ ABC}\langle\psi|$.

(2.) **Dephasing.** We then apply the dephasing operator to our initial state. The single-qubit dephasing channel has Kraus operators $K_0 = \sqrt{1-p}\ \mathbf{1}$ and $K_1 = \sqrt{p}\ \sigma^z$, where p is the dephasing parameter. Here, we choose $p = 1/2$ which corresponds to maximal dephasing. For 3-qubit dephasing, we define the joint Kraus operators as $K_{ijk} = K_i \otimes K_j \otimes K_k$.

The 3-qubit dephasing operator can then be expressed as $D(\rho_{ABC}) = \sum_{ijk} K_{ijk} \rho_{ABC} K^{\dagger}_{ijk}$. The resulting density matrix has the square of the amplitudes a^2_{ijk} along the diagonal and vanishing off-diagonal entries, i.e., the dephased state has no coherence (in the preferred basis).

(3.) **Perspective Operator** $N^{(P)}$. Once we have the dephased state, we apply the *perspective operator*

$$N^{(P)} = |0\rangle\langle 0|_P \otimes^{N-1} \mathbf{1}_2 + |0\rangle\langle 1|_P \otimes^{N-1} \sigma^x. \tag{31}$$

Concretely, if we wanted to implement the perspective of subsystem A to our 3-qubit state, we would have the perspective operator

$$N^{(A)} = |0\rangle\langle 0|_A \otimes \mathbf{1}_B \otimes \mathbf{1}_C + |0\rangle\langle 1|_A \otimes \sigma^x_B \otimes \sigma^x_C. \tag{32}$$

This results in the state

$$\tilde{\rho}^{(A)}_{ABC} = (a^2_{000} + a^2_{111})\,|000\rangle\langle 000| + (a^2_{001} + a^2_{110})\,|001\rangle\langle 001| + (a^2_{010} + a^2_{101})\,|010\rangle\langle 010| + (a^2_{011} + a^2_{100})\,|011\rangle\langle 011|\,.$$

Perspective Assignment (4-5)

$$|\psi\rangle_{ABC} = \sum_{i,j,k\in\{0,1\}} a_{ijk} |ijk\rangle ,$$

(4.) **Trace out the p-th qubit.** We now apply a partial trace on the perspective of the system that has been taken. Taking the partial trace will leave us with the reduced density matrix with diagonal entries equal to $a^2_{ijk} + a^2_{(1-i)(1-j)(1-k)}$:

$$\rho^{(A)}_{BC} = (a^2_{000} + a^2_{111}) |00\rangle\langle 00| + (a^2_{001} + a^2_{110}) |01\rangle\langle 01| + (a^2_{010} + a^2_{101}) |10\rangle\langle 10| + (a^2_{011} + a^2_{100}) |11\rangle\langle 11| . \quad (34)$$

(5.) **Purification.** Finally, we apply quantum state purification and project over the ancilla degrees of freedom, leaving us with the desired final state.

Let $|\Psi\rangle^{(A)}_{BC(DE)}$ be the purification given by

$$|\Psi\rangle^{(A)}_{BC(DE)} = (a^2_{000}+a^2_{111})^{\frac{1}{2}} |00\rangle |00\rangle+(a^2_{001}+a^2_{110})^{\frac{1}{2}} |01\rangle |01\rangle+(a^2_{010}+a^2_{101})^{\frac{1}{2}} |10\rangle |10\rangle+(a^2_{011}+a^2_{100})^{\frac{1}{2}} |11\rangle |11\rangle , \quad (35)$$

with ancilla qubit systems D and E.

Project over the ancilla qubit systems to recover the desired perspectival state

$$|\psi\rangle^{(A)}_{BC} = (a^2_{000} + a^2_{111})^{\frac{1}{2}} |00\rangle + (a^2_{001} + a^2_{110})^{\frac{1}{2}} |01\rangle + (a^2_{010} + a^2_{101})^{\frac{1}{2}} |10\rangle + (a^2_{011} + a^2_{100})^{\frac{1}{2}} |11\rangle . \quad (36)$$

Ent. Deg. Set-Up (RQI)

- Include figure of set-up
- Mention Dirac (fermionic) fields
- Bogoliubov transformation
- Single-Mode Approximation

$$|0_k\rangle^+ = \cos r |0_k\rangle_R^+ |0_{-k}\rangle_{\overline{R}}^- + \sin r |1_k\rangle_R^+ |1_{-k}\rangle_{\overline{R}}^-,$$

$$|1_k\rangle^+ = |1_k\rangle_R^+ |0_{-k}\rangle_{\overline{R}}^-,$$

QI Definitions

- Entanglement Entropy

$$\mathcal{E}(|\psi\rangle_{AB}) \;=\; \mathcal{S}(\rho_A),$$

- Relative Entropy of Coherence

$$\mathcal{C}(\rho) \;=\; \mathcal{S}(\rho) - \mathcal{S}(\rho_d),$$

- Entanglement of Formation
- Linear Entropy
- L_2-norm of Coherence

In particular, if we quantify the entanglement and subsystem coherence using the measures of entanglement entropy defined as $\mathcal{E}(|\psi\rangle_{AB}) = \mathcal{S}(\rho_A)$, where $\rho_A = \mathrm{Tr}_B(|\psi\rangle_{AB\,AB}\langle\psi|)$ is the reduced density matrix of system A and $\mathcal{S}(\rho) = -\mathrm{Tr}(\rho \log \rho)$ is the von Neumann entropy, and relative entropy of subsystem coherence defined as $\mathcal{C}(\rho) = \mathcal{S}(\rho) - \mathcal{S}(\rho_d)$, where ρ_d is the state consisting of only the diagonal elements of ρ in the group element basis $\{|g\rangle\}_{g\in G}$. Then their sum $\mathcal{E}+\mathcal{C}$, when appropriately defined on the subsystems, is invariant under QRF transformations[2].