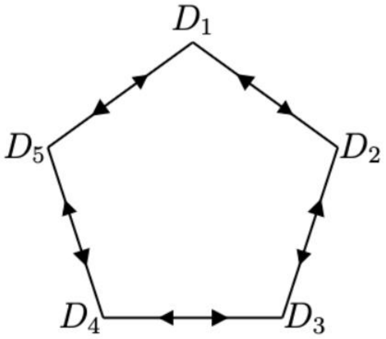
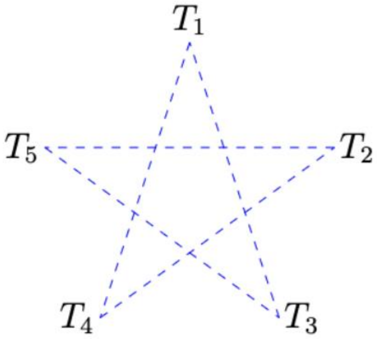
Entanglement summoning

from entanglement sharing schemes





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Background

Quantum information with relativistic constraints

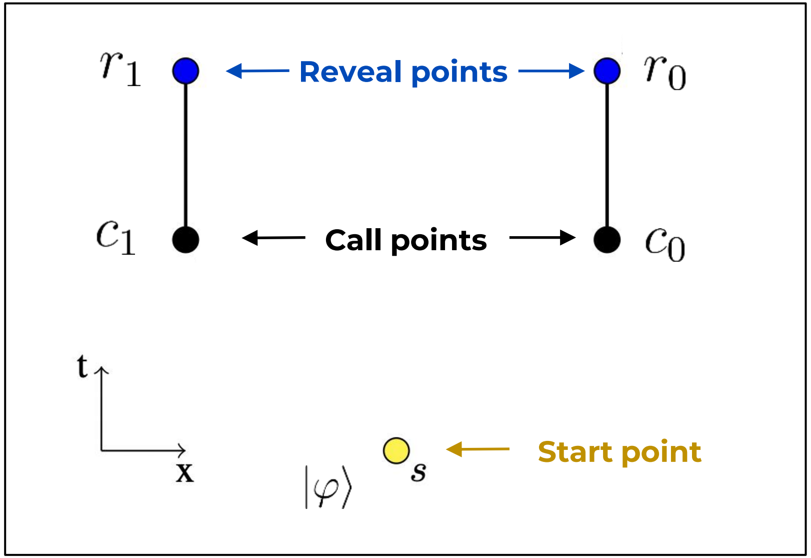
Basic question: What is physically possible when imposing certain quantum mechanical and relativistic constraints?

E.g.

- QM: No-cloning, monogamy of entanglement, etc.
- SR: No-signalling.

A no-summoning theorem

- Quantum state $|\varphi\rangle$ is initially localized at s
- A request will be made at a call point c_i at some point in the causal future of s
- If a request is made at c_i, then $|\varphi\rangle$ should be returned to the reveal point r_i .



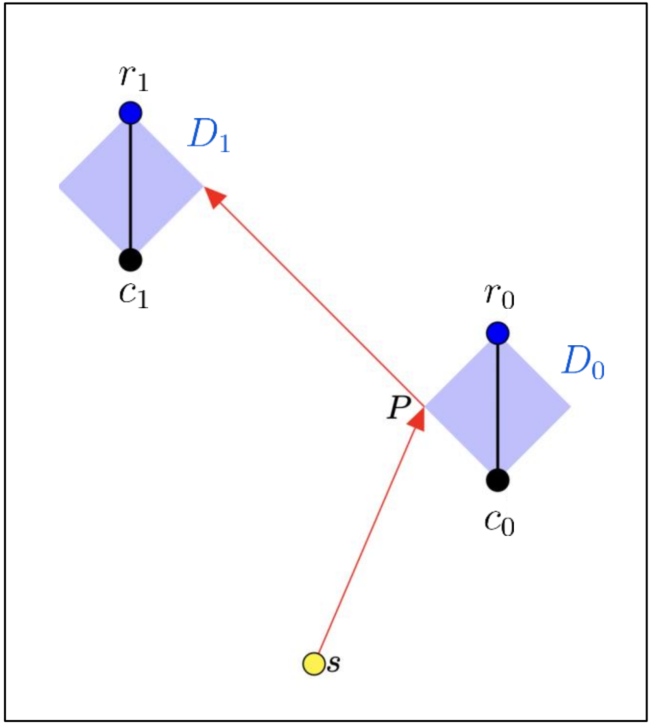
<u>No summoning:</u> The above task cannot be completed successfully 100% of the time due to the spacelike separation of the causal regions defined by $r_0\ c_0$ and $c_1\ r_1$

(Kent 2011, Hayden & May 2012)

When is summoning possible?

- Hayden & May established a theorem that characterizes all possible instances of single-state summoning.

- We define a ***causal diamond*** D_i to each call-and-reveal pair. This is the spacetime region that is the intersection of the causal *future* of and the causal c_i *past* of r_i



Single-state summoning: Summoning is possible if and only if

1) Every reveal point is in the causal future of s
2) For each pair (i, j), there is a causal connection between D_i and D_j

(Hayden & May 2012)

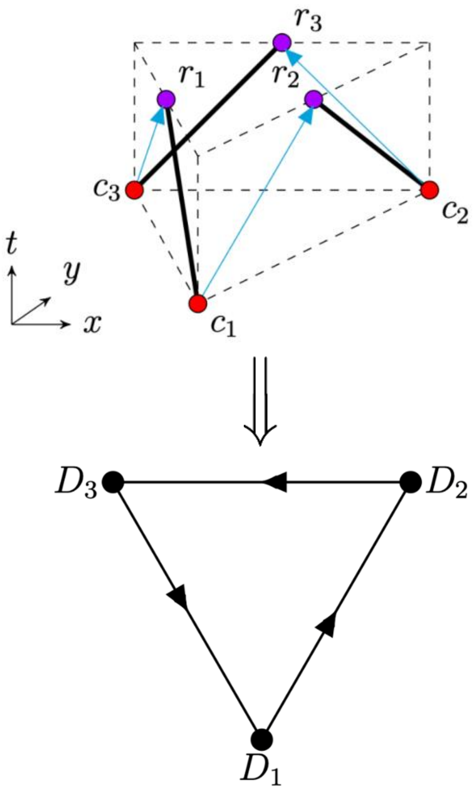
When is summoning possible?

Causal graphs

- Only care about causal connections between diamonds

- Geometry of summoning task reduced to a graph problem.

Theorem: Summoning is possible if and only if the causal graph is a tournament.



Entanglement summoning

Definition: An entanglement summoning task is specified by a set of causal diamonds $\{D_j\}_{i=1}^n$ each of which is defined by a call point c_j and return point r_j. The task involves two agencies, Alice and Bob, which perform the following:

At each c_j, Bob gives Alice a classical bit b_j. Alice is promised that $b_{j^*} = b_{k^*} = 1$ for exactly two j^*, k^*, but these are unknown to her beforehand.

We say the task is **achievable** if for any choice of j^*, k^* by Bob, Alice can return system A at r_{j^*} and B at r_{k^*} such that the state on AB is $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

(Adlam 2018, Dolev, May & Wan 2021)

This work

- ***Previously:*** Entanglement summoning only characterized for directed graphs (Dolev, May, Wan 2021)

- ***Our contribution:***
  - We fully characterize the bidirected entanglement summoning case

  - We partially characterize the most general case of entanglement summoning (both oriented and bidirected edges).

How?: By leveraging *entanglement sharing schemes.*

Entanglement sharing schemes (ESS)

- A dealer prepares a state $|\Psi\rangle_{S_1S_2\ldots S_n}$ and sends i share S_i to party

$$\mathcal{S} = (\mathcal{A}, \mathcal{U})$$

- We define an ***access-pair structure***

$\mathcal{A} = \{A_k\}_k$ authorized sets

$\mathcal{U} = \{U_l\}_l$ unauthorized sets

Each A_k and U_l is a pair $\{T_i, T_j\}$ where $T_i, T_j \subseteq \{S_i\}_{i=1}^{n}$

Each A_k should be able to, by means of local maps on subsystems in T_i and T_j to recover $|\Phi^+\rangle$

No U_l should be able to recover a state arbitrary close $|\Phi^+\rangle$ to

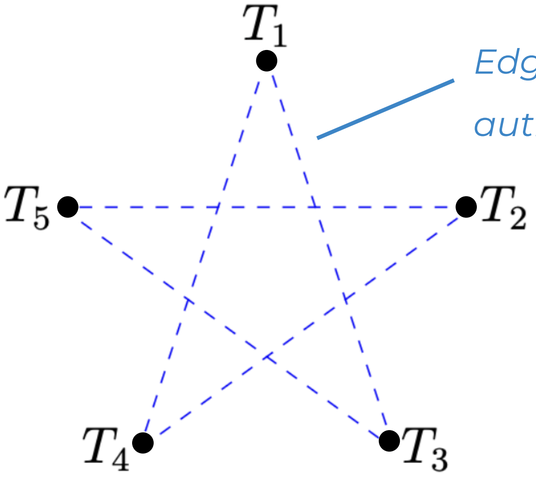
Entanglement sharing scheme = $|\Psi\rangle_{S_1S_2\ldots S_n}$ + $\mathcal{S}$

* This definition is for an ESS with an ***unknown partner***

(Khanian et al. 2025)

Entanglement sharing schemes

ESS can be described with an undirected graph. The “Access-Pair Graph”



Vertices are the subsets of $\{S_i\}_{i=1}^{n}$

Edges represent authorizations between pairs

(Khanian et al. 2025)

Entanglement sharing schemes

Special case of ESS: No unauthorized sets, $\mathcal{S} = (\mathcal{A}, \emptyset)$

Theorem: A pair access structure $\mathcal{S} = (\mathcal{A}, \emptyset)$ is realizable as an ESS if and only if the following condition is satisfied.

Monogamy: For every even-length path $\{T_1, T_2\}, \{T_2, T_3\}, \dots \{T_{m-1}, T_m\}$ in the access-pair graph of $\mathcal{S}$, it is the case that $T_1 \cap T_m \neq \emptyset$.

(Khanian et al. 2025)

Summoning from ESS

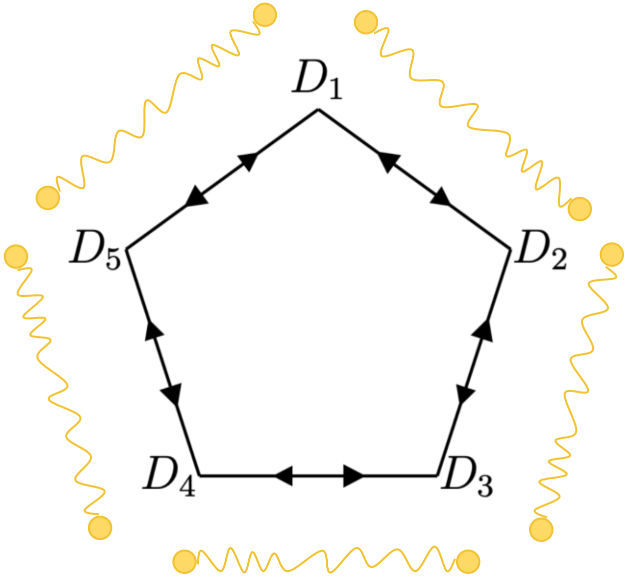
Main idea:

- Map entanglement summoning to ESS
- Use ESS characterization theorem to characterize summoning

Results

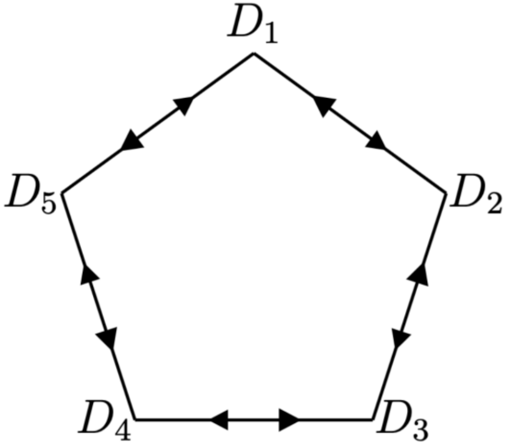
Bidirected summoning

We restrict our attention to the case where non-neighbouring diamonds receive calls; > we can handle this with a dedicated EPR state between each pair.



An example

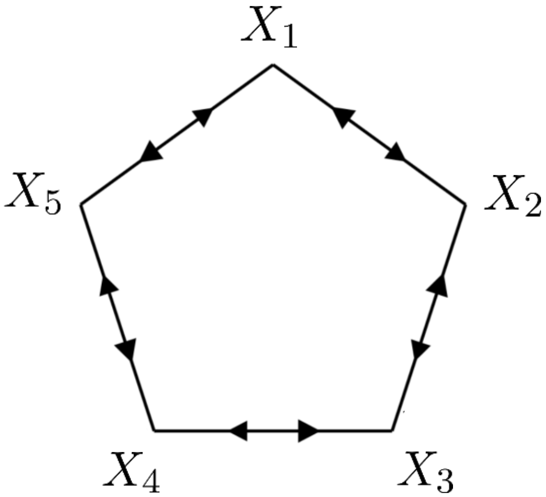
0) Prepare resource



$$|\Psi\rangle_{X_1X_2X_3X_4X_5}$$

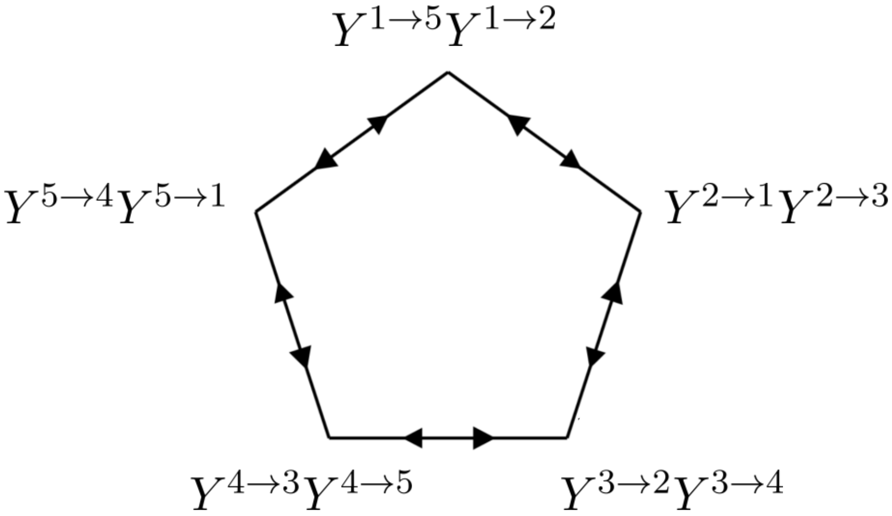
An example

1) Share the resource state.



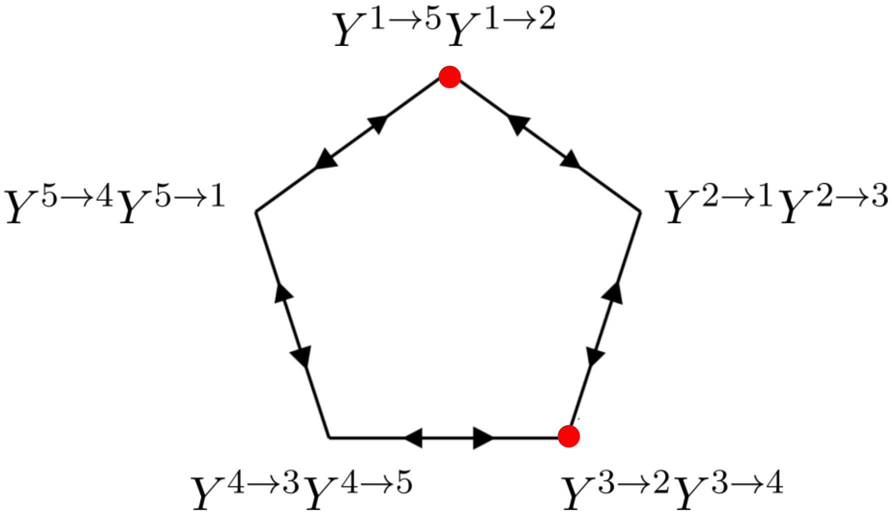
An example

2) Apply map.



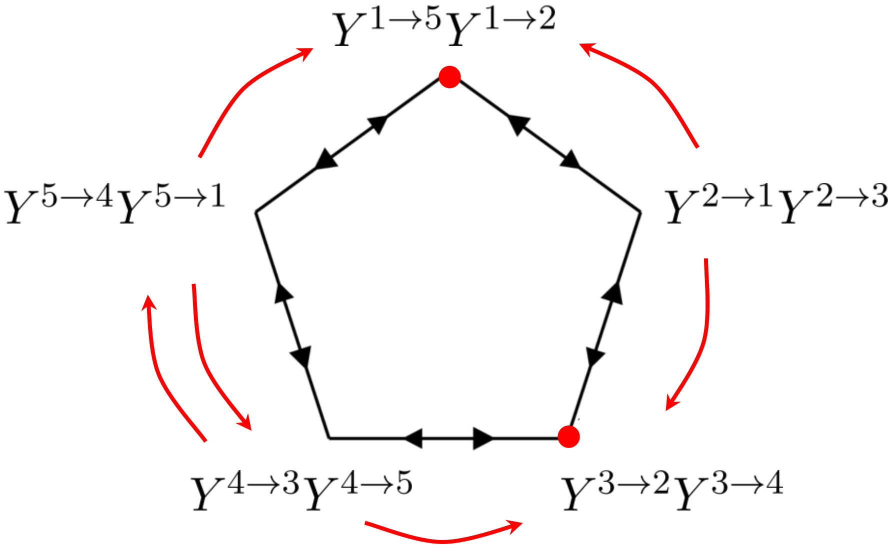
An example

Suppose 1 and 3 get calls.

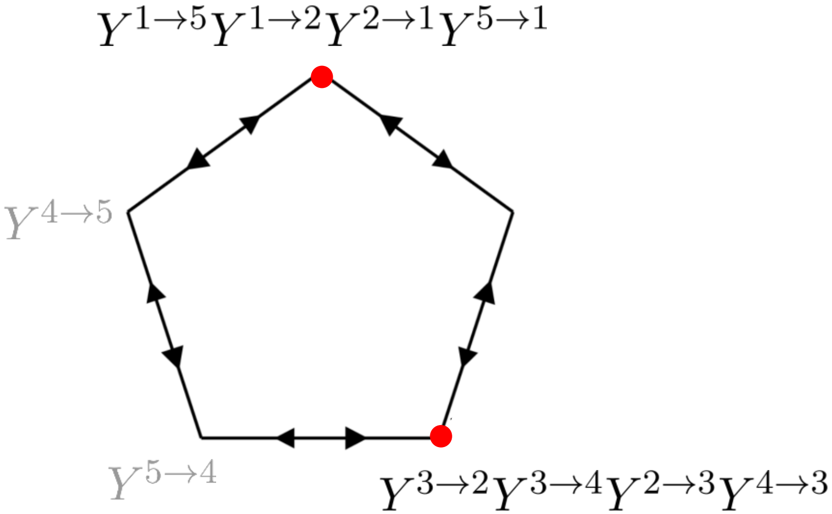


An example

3) 1 and 3 receives their neighbours' shares, all other diamonds send out their shares.

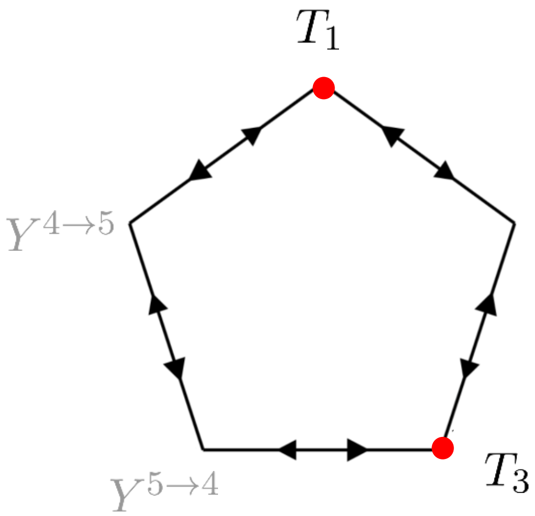


An example


$Y^{1\to5}Y^{1\to2}Y^{2\to1}Y^{5\to1}$
$Y^{4\to5}$
$Y^{5\to4}$
$Y^{3\to2}Y^{3\to4}Y^{2\to3}Y^{4\to3}$

An example

The shares currently at 1 and 3 should have enough information to recover a Bell pair.
They form an authorized pair.



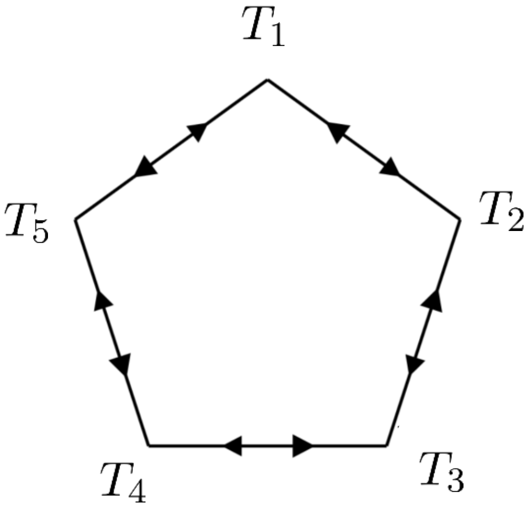
We define

$$T_1 = Y^{1\to5}Y^{1\to2}Y^{2\to1}Y^{5\to1}$$

$$T_3 = Y^{3\to2}Y^{3\to4}Y^{2\to3}Y^{4\to3}$$

An example

Similarly, for the other diamonds we have



$$T_1 = Y^{1\to 5}Y^{1\to 2}Y^{2\to 1}Y^{5\to 1}$$

$$T_2 = Y^{2\to 1}Y^{2\to 3}Y^{1\to 2}Y^{3\to 2}$$

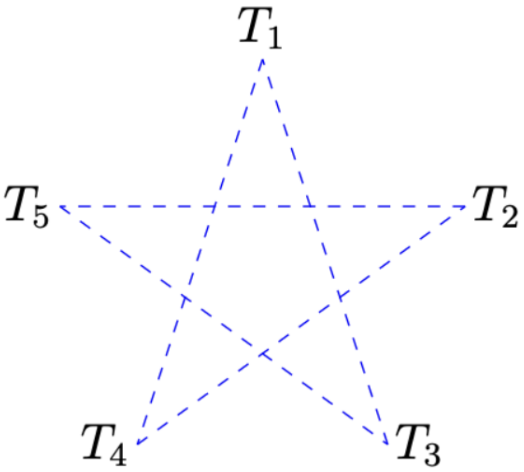
$$T_3 = Y^{3\to 2}Y^{3\to 4}Y^{2\to 3}Y^{4\to 3}$$

$$T_4 = Y^{4\to 3}Y^{4\to 5}Y^{3\to 4}Y^{5\to 4}$$

$$T_5 = Y^{5\to 4}Y^{5\to 1}Y^{4\to 5}Y^{1\to 5}$$

An example

Drawing the access-pair graph, we have



The state $|\Psi'\rangle_{Y^{1\to2}Y^{1\to5}\dots Y^{5\to1}}$ along with the access structure of the above defines an ESS.

An example

Recalling the characterization of realizable ESSs,

Theorem: A pair access structure $\mathcal{S} = (\mathcal{A}, \emptyset)$ is realizable as an ESS if and only if the following condition is satisfied.

Monogamy: For every even-length path $\{T_1, T_2\}, \{T_2, T_3\}, \dots \{T_{m-1}, T_m\}$ in the access-pair graph of $\mathcal{S}$, it is the case that $T_1 \cap T_m \neq \emptyset$.

$$\Updownarrow$$

For ESS derived from a summoning task

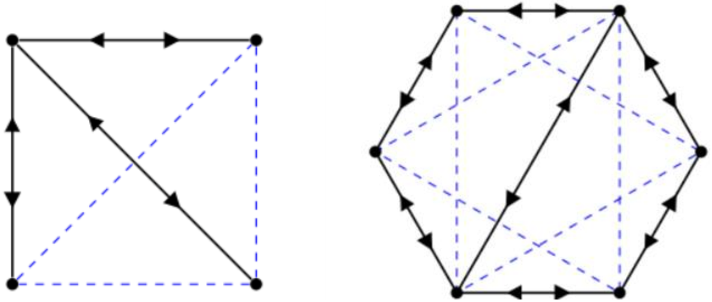
No odd cycles: There are no odd cycles in the access-pair graph of $\mathcal{S}$

Bidirected summoning

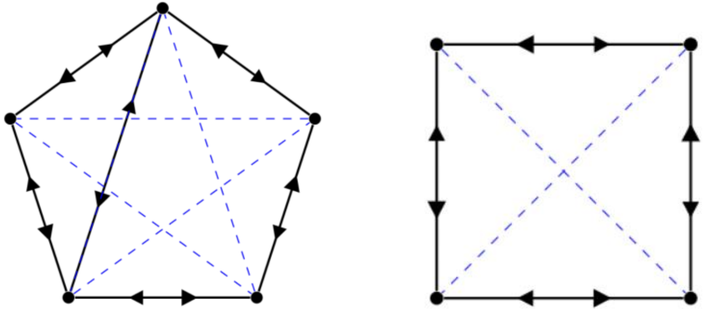
The previous analysis can be generalized to come up with the following theorem that characterizes all achievable entanglement summoning tasks.

Theorem: An entanglement sharing scheme whose causal graph is bidirected is achievable if and only if the complement of the causal graph contains no odd cycles.

E.g. unachievable



E.g. achievable



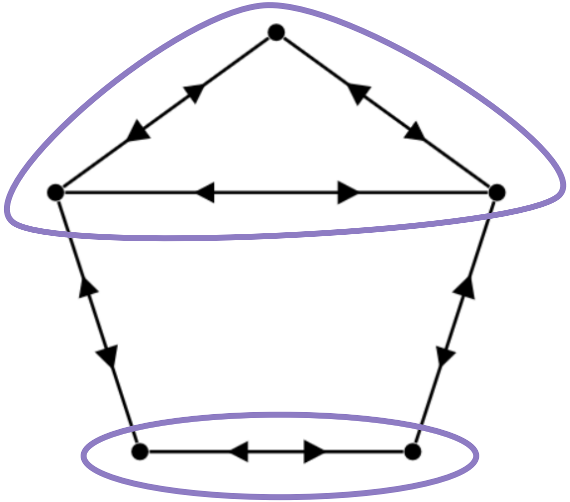
Bidirected summoning

We can also make use of some graph theory to get a natural characterization in terms of the original graph.

We call a graph whose complement has no odd cycles ***co-bipartite***.

Theorem: A graph G is co-bipartite if and only if it admits a two-clique partition.

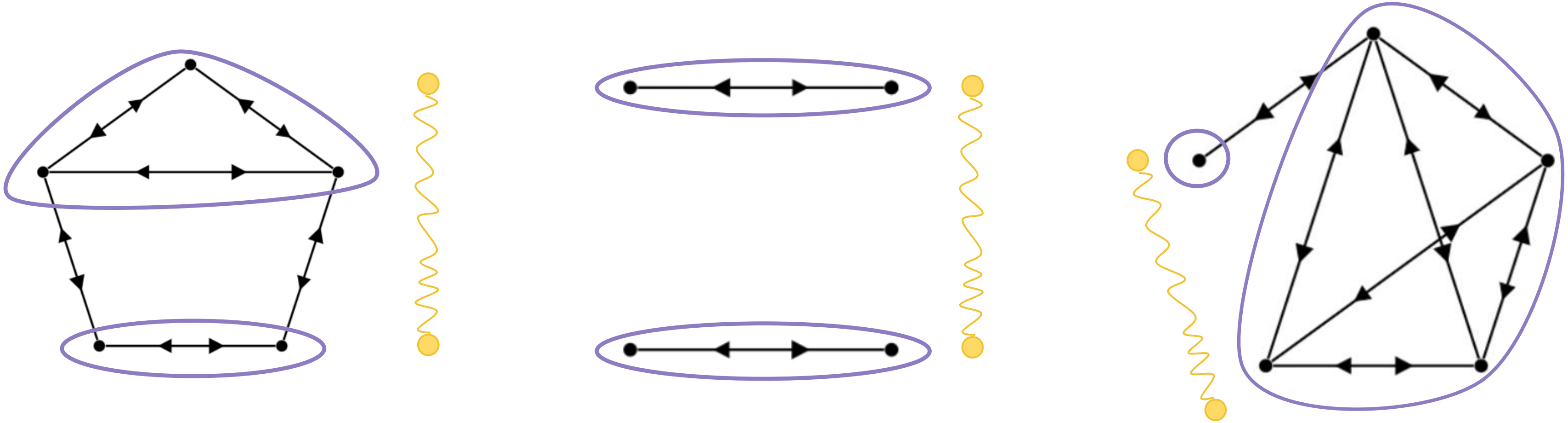
Theorem: An entanglement summoning task whose causal graph contains only bidirect edges is possible if and only if the causal graph admits a two-clique partition.



Bidirected summoning

Recall that single-state summoning requires a tournament.

We can interpret the theorem as telling us that bidirected summoning needs to admit a partitioning to summon half a Bell pair through each clique.



Thanks!