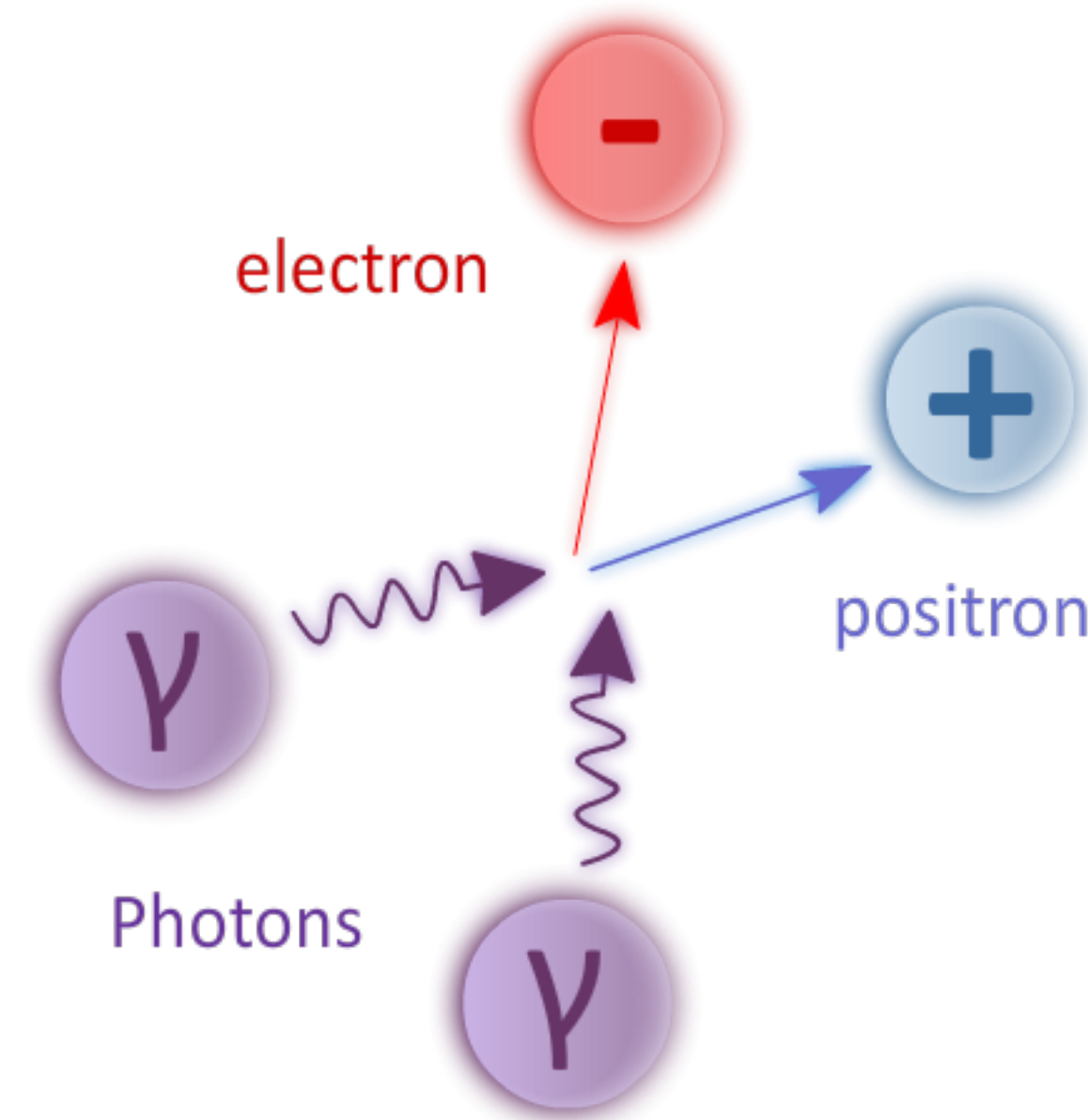
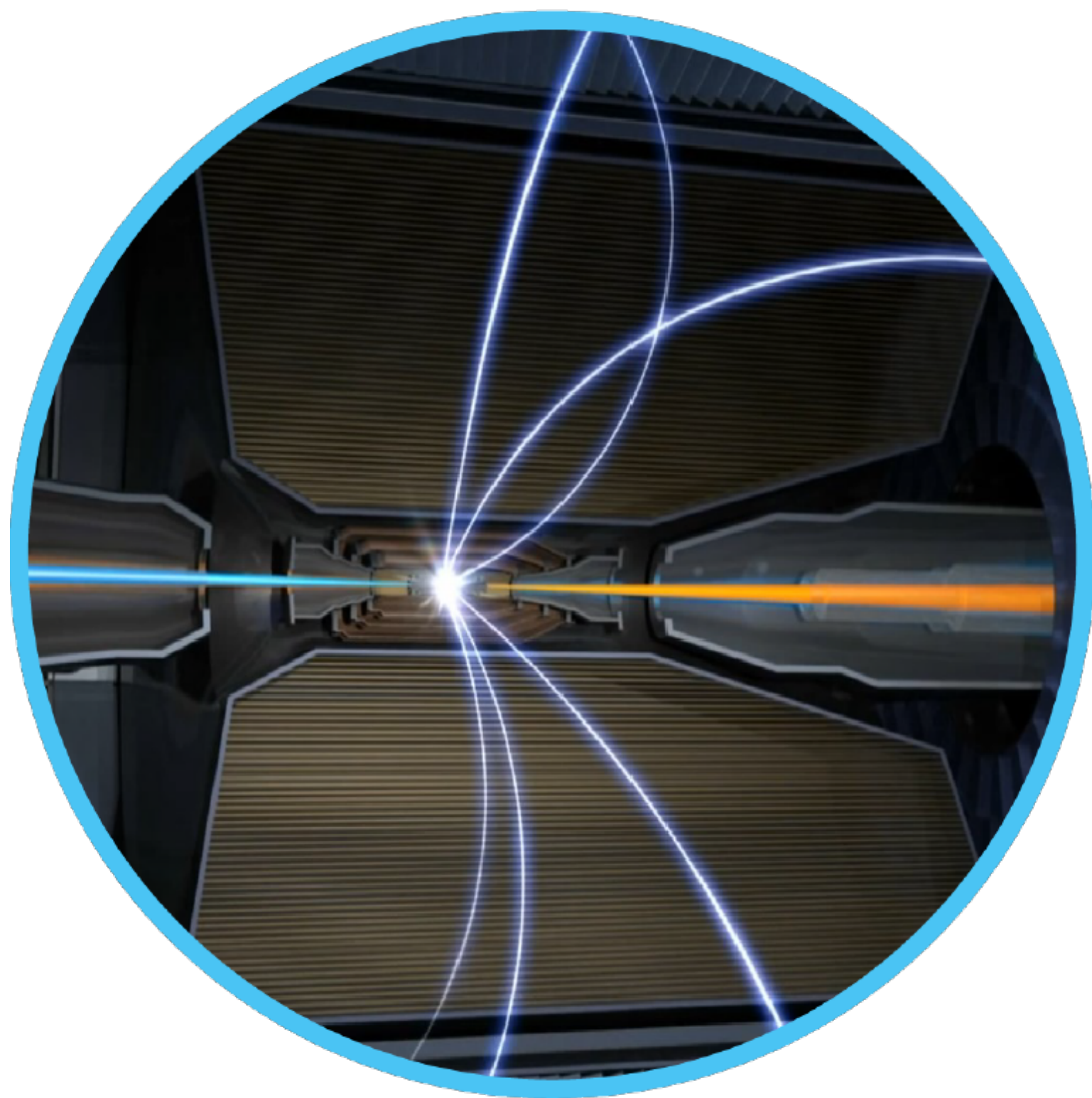


HIGHER ORDER ELECTROWEAK RADIATIVE CORRECTIONS IN PARITY VIOLATING ASYMMETRY USING COVARIANT APPROACH



Memorial
University of Newfoundland



MAHUMM GHAFFAR
MEMORIAL UNIVERSITY OF NEWFOUNDLAND
COLLABORATORS: A. ALEKSEJEVS AND S. BARKANOVA

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Investigators: Dr. Adrianna Tassone Dr. Eden Hennessey, Skye Hennessey, Dr. Shohini Ghose, Anastasia Smolina
Laurier Centre for Women in Science (WinS), Wilfrid Laurier University
University of Toronto
Dalhousie University

This study has been reviewed and approved by the Laurier Ethics Board (REB # 9615).



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- Motivation
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- Electroweak parity violating asymmetry for electron-positron (e^-e^+) process at Leading Order (LO), Next-to-Leading Order (NLO) and Next-to-Next-to-Leading Order (NNLO)
- Results
- Conclusion

MOTIVATION

- **Low energy precision physics** becomes important → provides indirect searches achieved through precise measurements of well-predicted SM observables.
- Precise measurement of electroweak Parity Violating Asymmetry (A_{PV}) → higher order corrections considered up to NNLO (α^4) using **Covariant approach**.

Our recent paper: Phys. Rev. D 113, 113002 (2026)

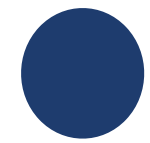
- The collisions between e^-e^+ at Chiral Belle experiment with the center-of-mass energy of 10.58 GeV provide clear kinematics, and missing momentum signatures can be fully reconstructed.
- Measurement of neutral current vector couplings to five fermion flavours (b, c, τ, μ, e) with precisions comparable to Z-pole measurements. **Phys. Rev. D 112, 013006 (2025)**

MOTIVATION

- A longitudinally polarized electron collides with an unpolarized positron target

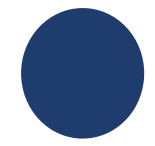
MOTIVATION

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MOTIVATION

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(m_1, k_1)

MOTIVATION

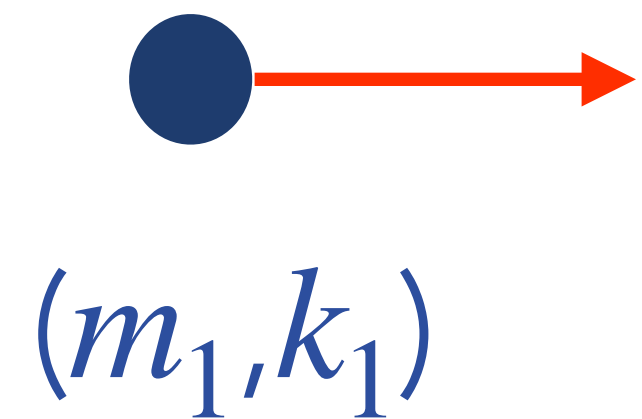
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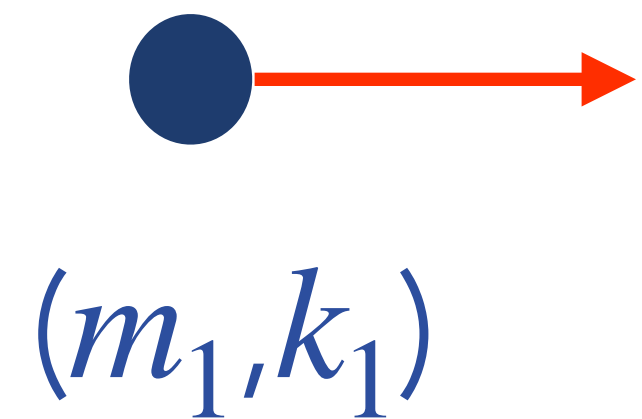
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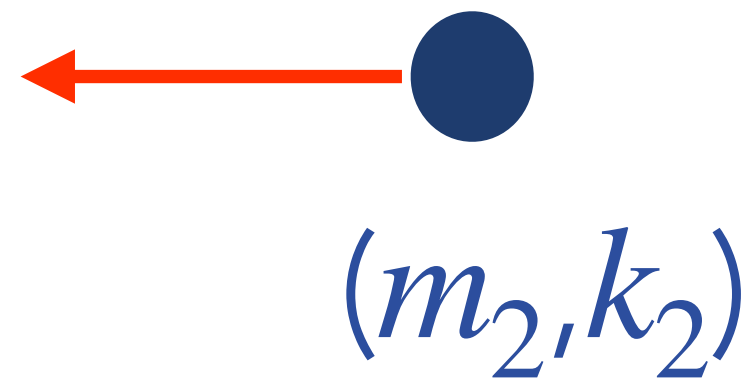
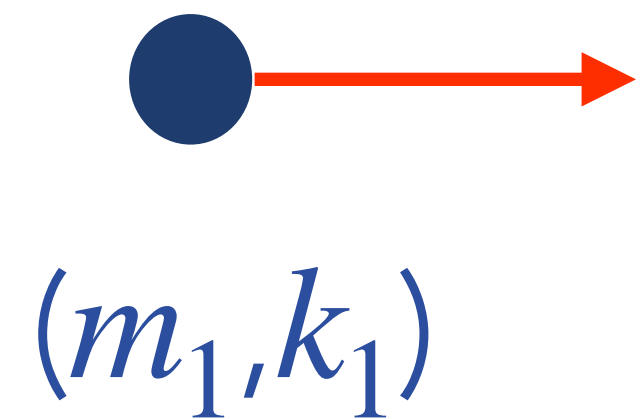
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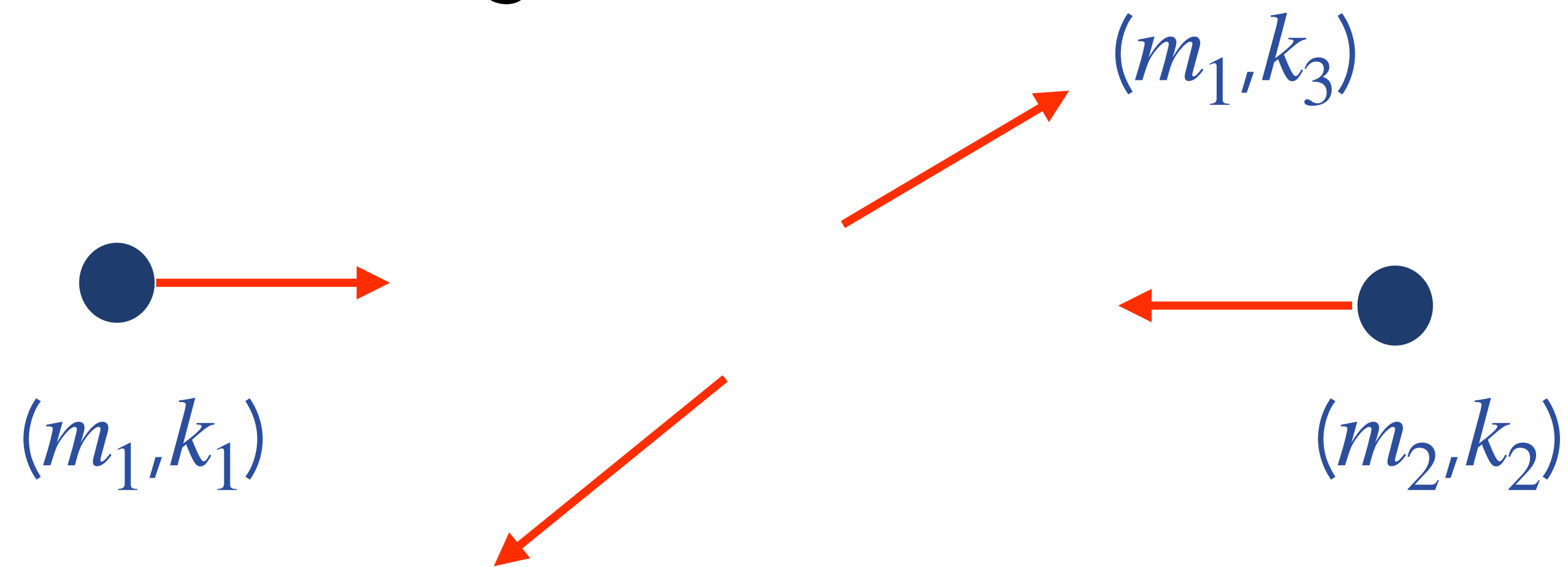
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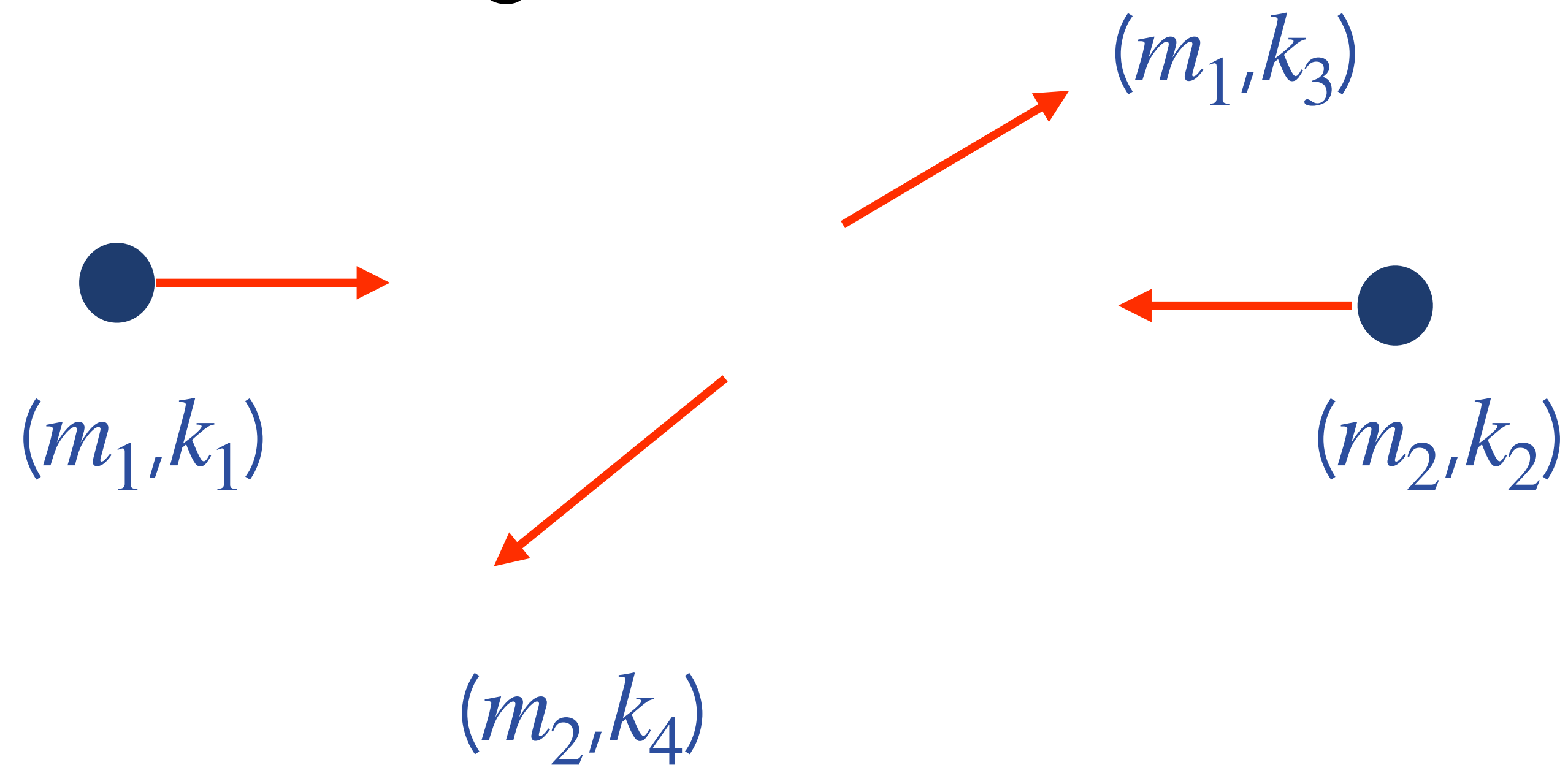
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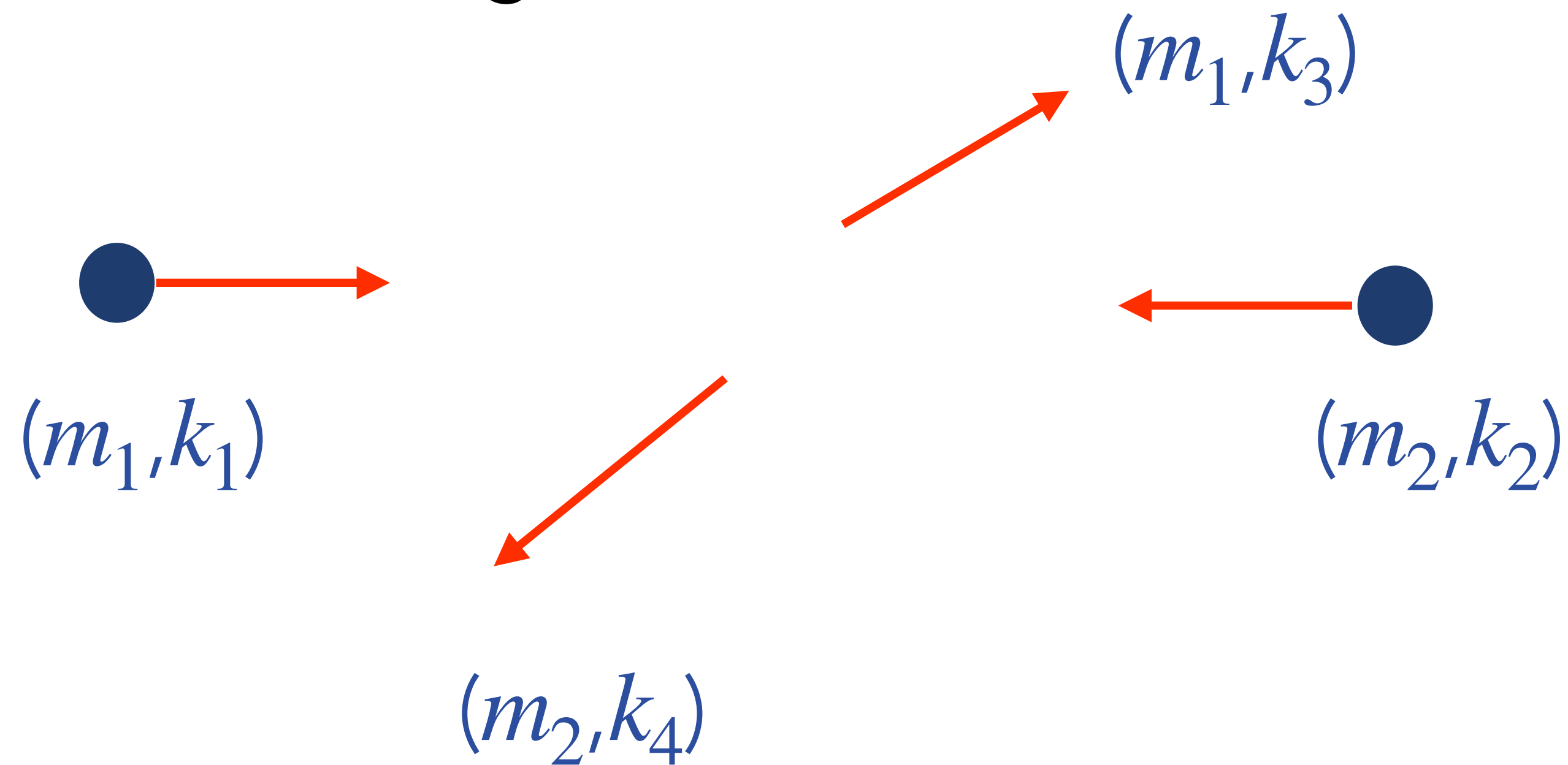
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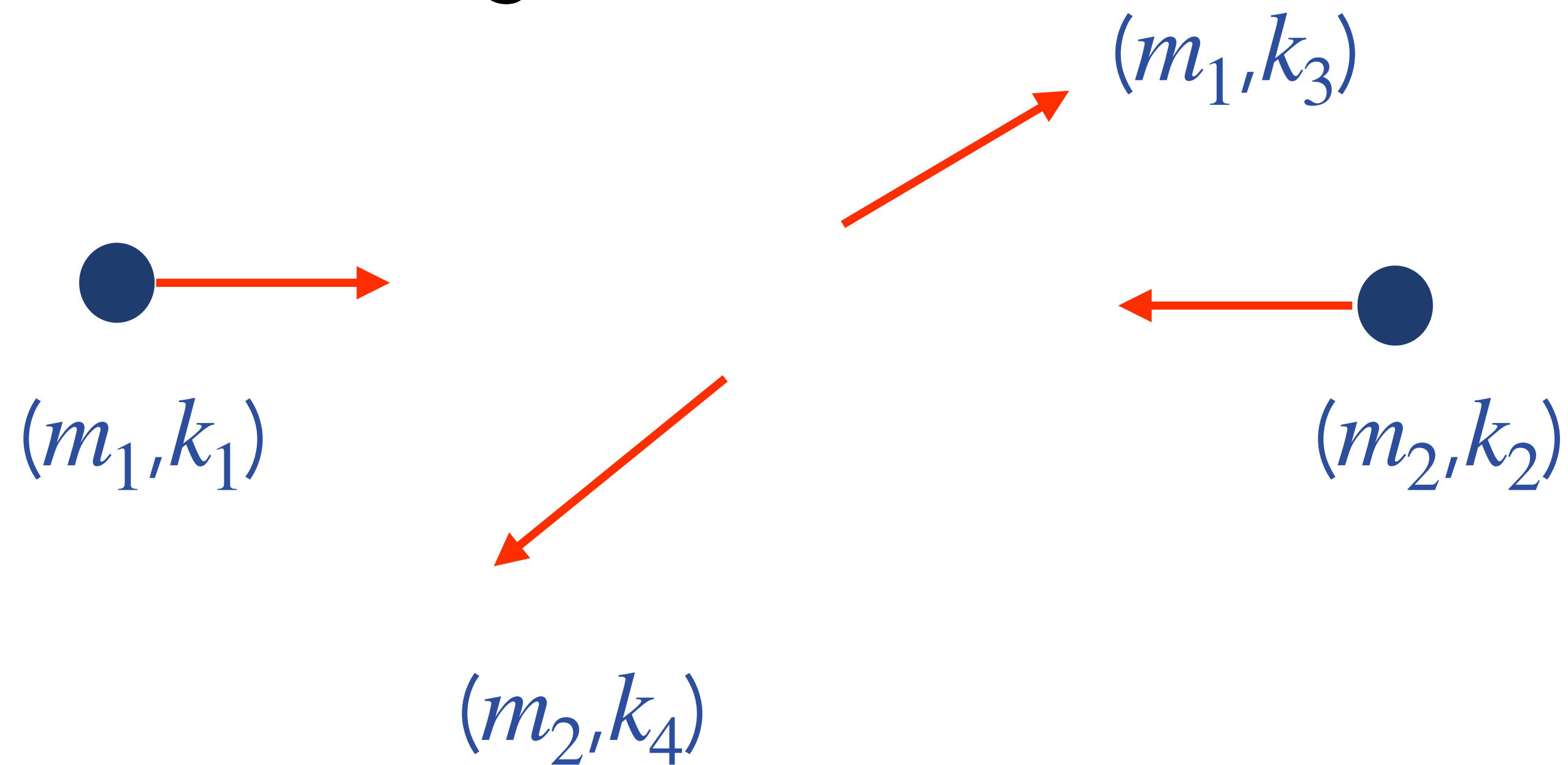
- A longitudinally polarized electron collides with an unpolarized positron target



Covariant Approach

MOTIVATION

- A longitudinally polarized electron collides with an unpolarized positron target

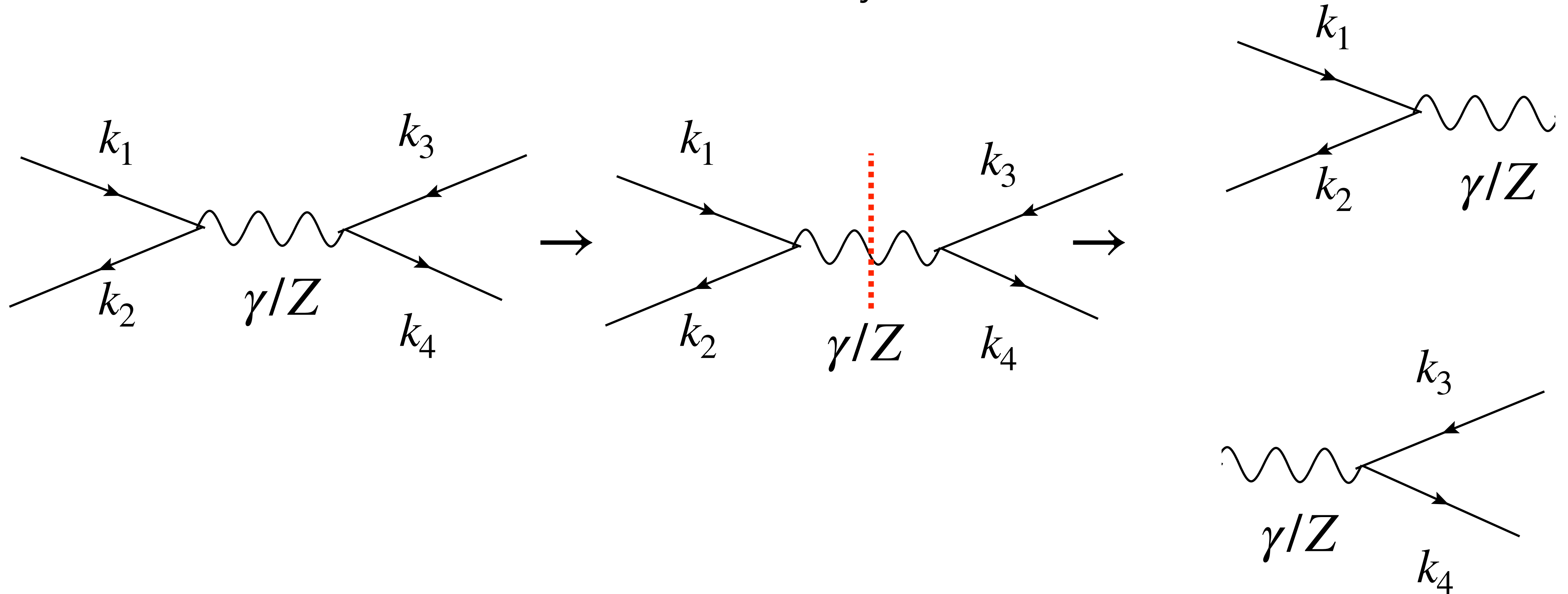


Covariant Approach

Next-to-Next-to Leading Order

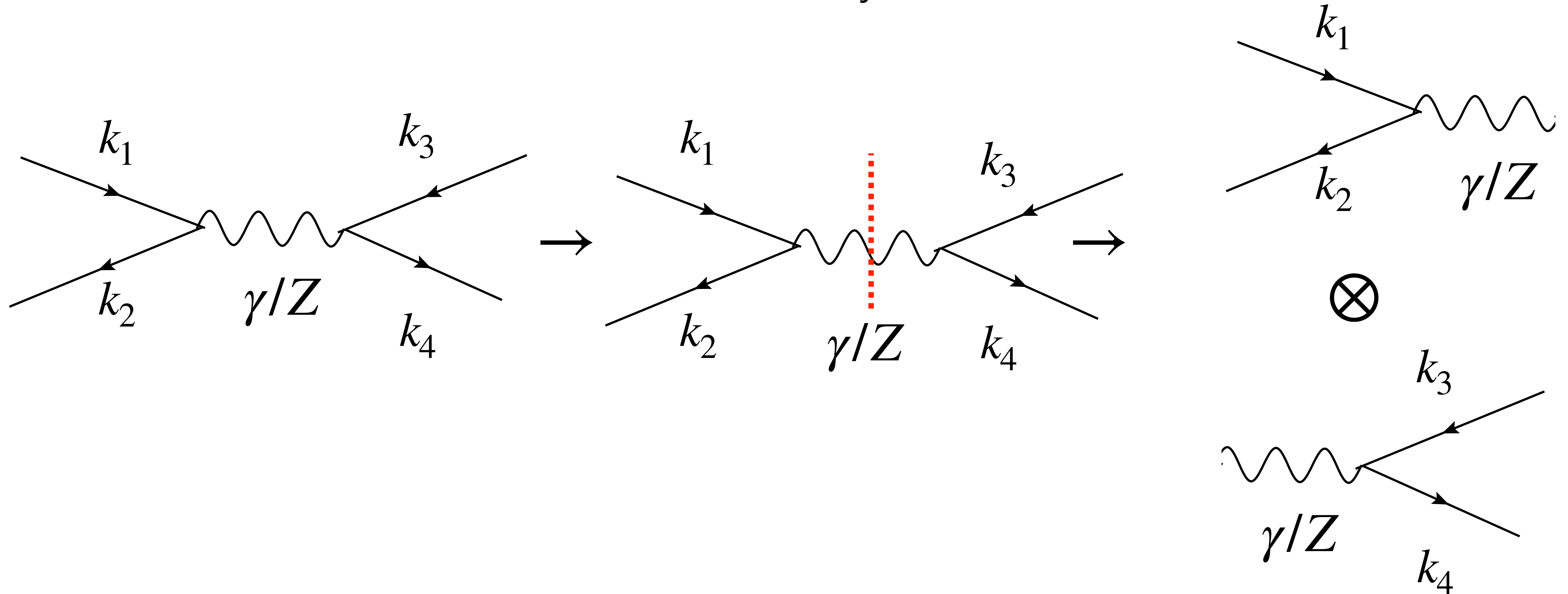
WHAT IS A COVARIANT APPROACH?

- Bardin and Shumeiko in 1976 (Nuclear Physics **B127**)



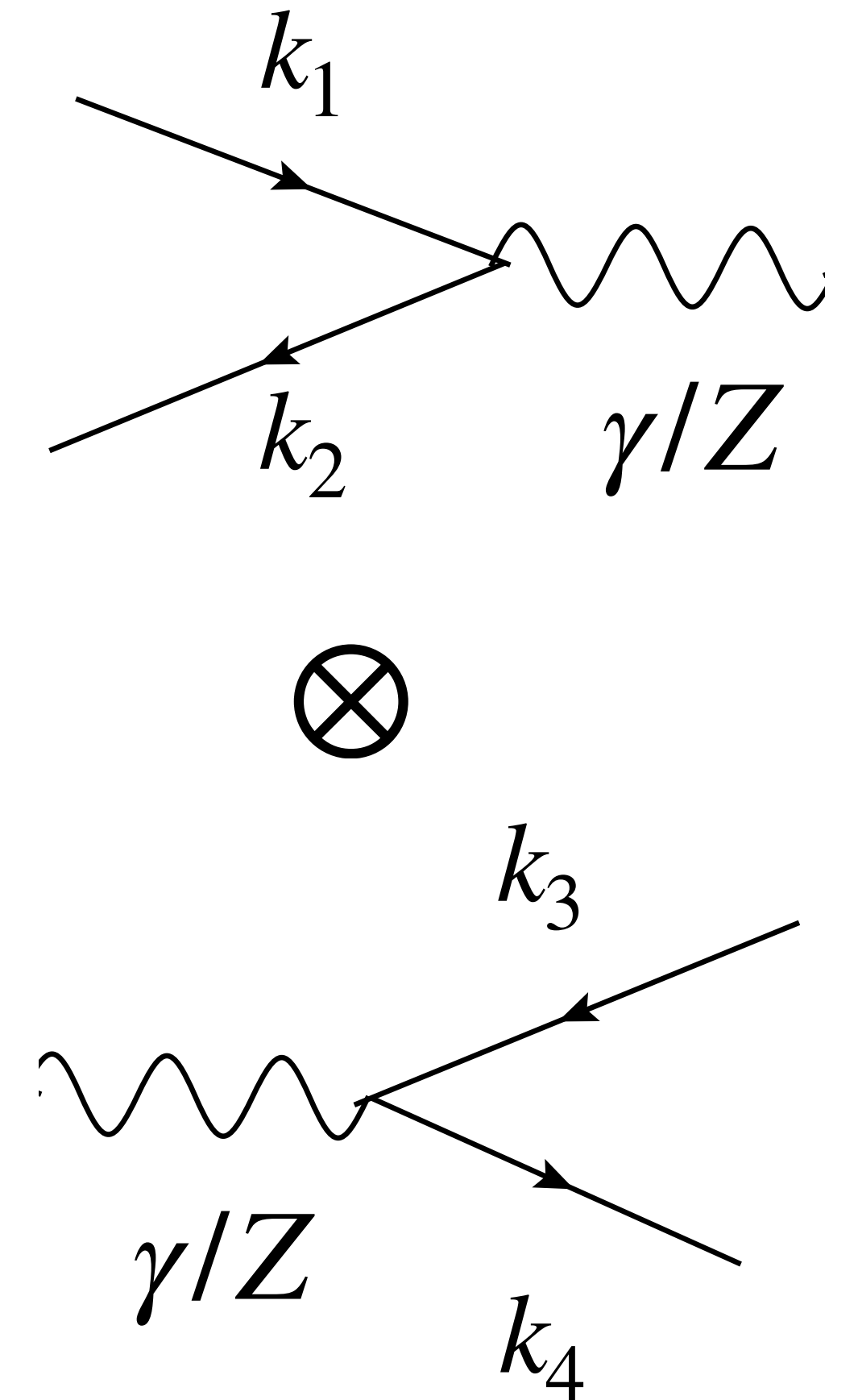
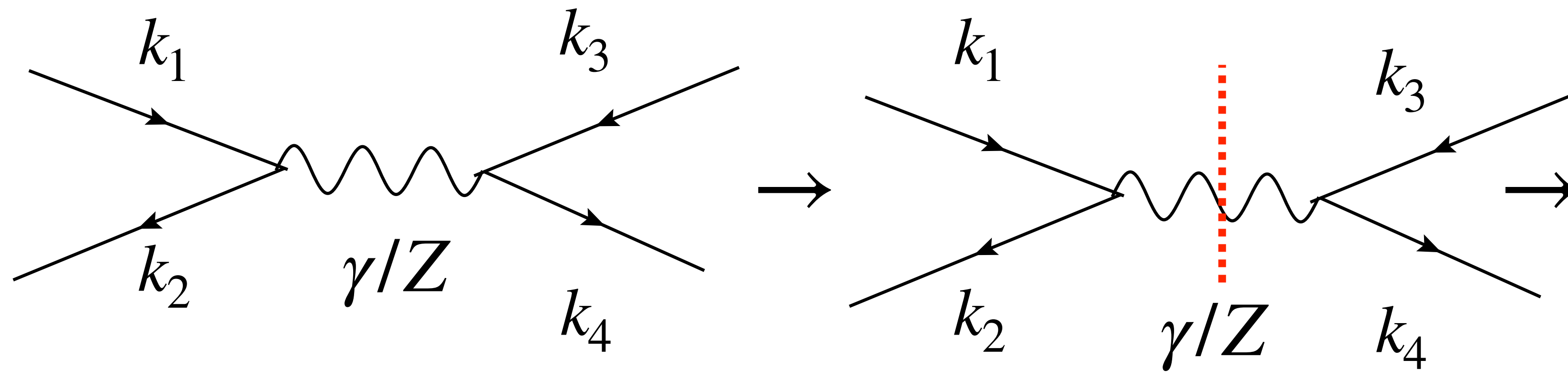
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WHAT IS A COVARIANT APPROACH?

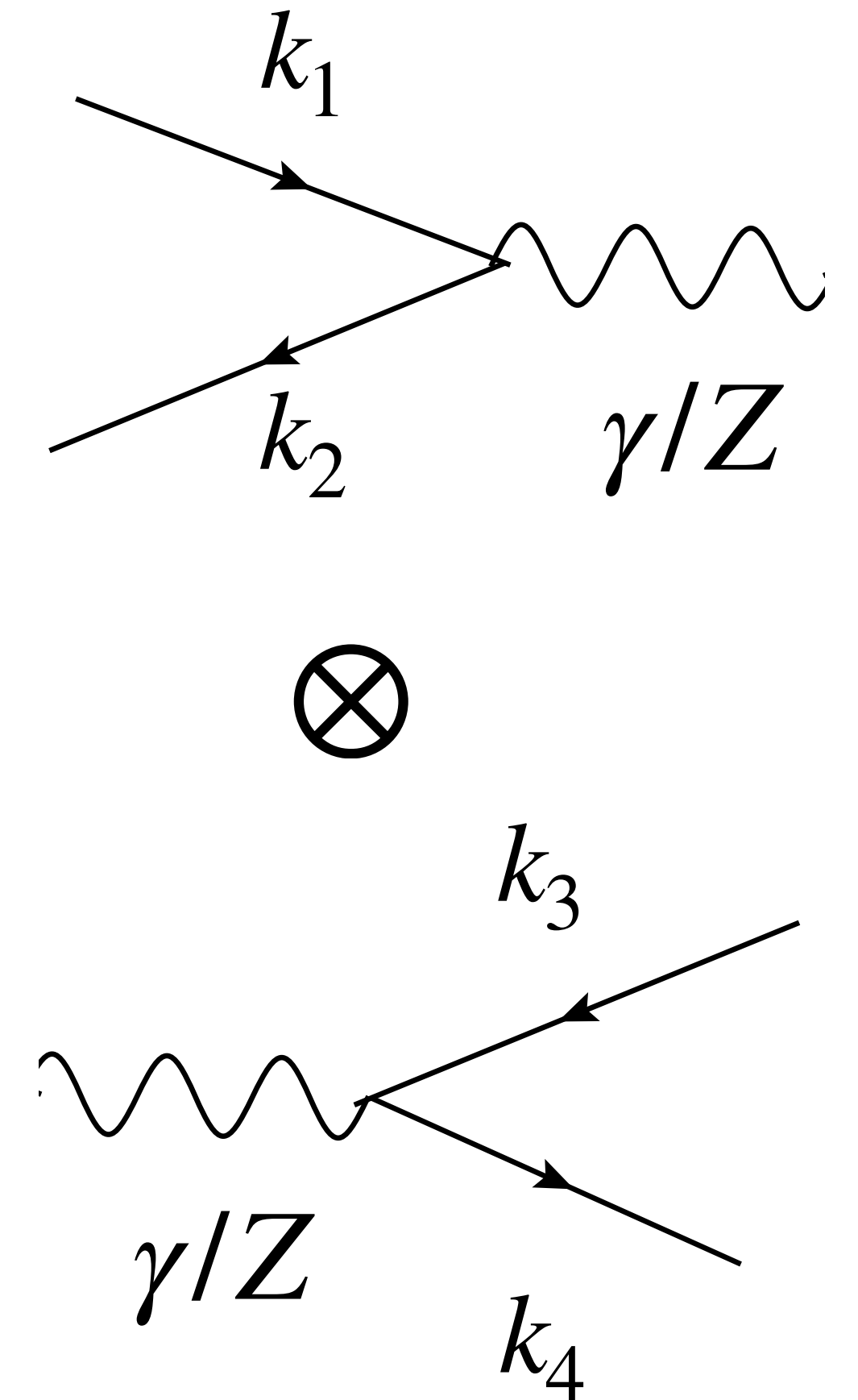
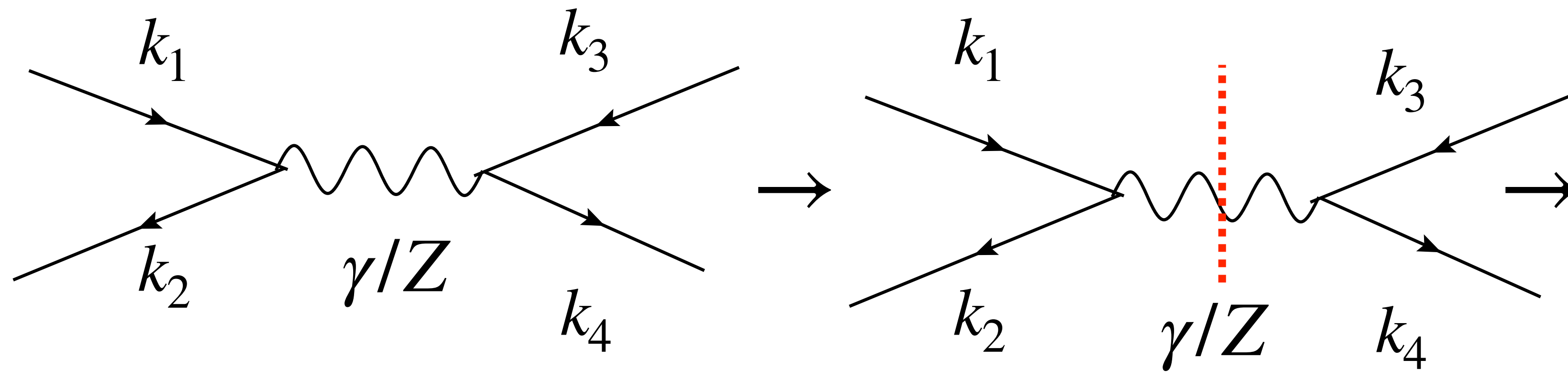
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$$d\sigma \sim L^{\mu\nu} L_{\mu\nu}$$

WHAT IS A COVARIANT APPROACH?

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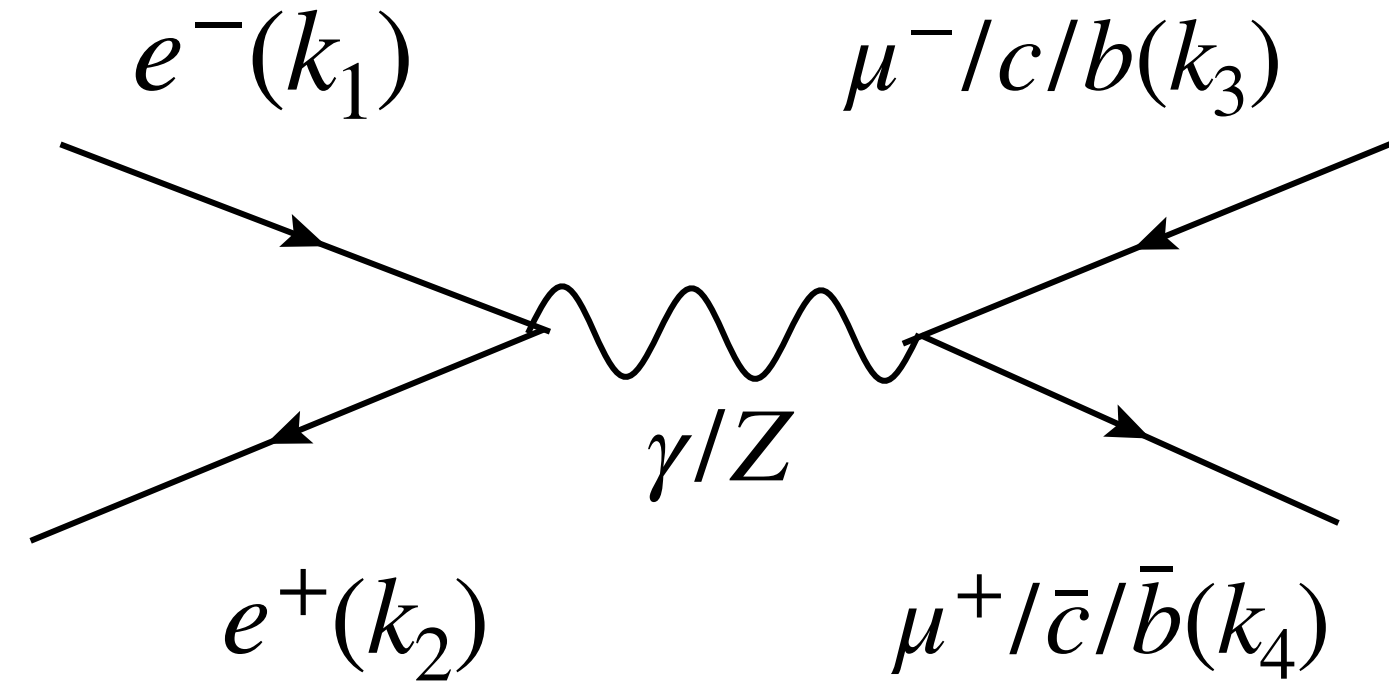
$$d\sigma \sim L^{\mu\nu} L_{\mu\nu} \quad \rightarrow \quad A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

PARITY VIOLATING ASYMMETRY

- Formula: $A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$, where: $\sigma_R \propto |\mathcal{M}_R|^2$ and $\sigma_L \propto |\mathcal{M}_L|^2$
- For QED, $|\mathcal{M}_{\gamma R}| = |\mathcal{M}_{\gamma L}|$, numerator contains just weak+electroweak cross terms.
- Denominator contains just QED terms as m_Z (90 GeV) $>$ m_{e^-} (0.5 MeV)

- $$A_{PV} = \frac{|\mathcal{M}_{ZZ}|_R^2 - |\mathcal{M}_{ZZ}|_L^2 + 2\Re(\mathcal{M}_\gamma \mathcal{M}_Z^\dagger)_R - 2\Re(\mathcal{M}_\gamma \mathcal{M}_Z^\dagger)_L}{|\mathcal{M}_{ZZ}|_{R+L}^2 + 2\Re(\mathcal{M}_\gamma \mathcal{M}_Z^\dagger)_{R+L} + |\mathcal{M}_{\gamma\gamma}|_{R+L}^2}$$

ELECTROMAGNETIC AND WEAK INTERACTIONS FOR ELECTRON-POSITRON SCATTERING



$$\mathcal{M}_\gamma = [\bar{v}(k_2)(-ie\gamma_\mu)u^{s(1)}(k_1)] \left(\frac{-i}{q^2} \right) \times [\bar{u}(k_4)(-ie\gamma^\mu(q^2))v(k_3)]$$

$$\mathcal{M}_Z = [\bar{v}(k_2)(-ie(a_V\gamma_\mu + a_{AV}\gamma_\mu\gamma_5))u^{s(1)}(k_1)] \times \left(\frac{-i}{q^2 - m_Z^2} \right) [\bar{u}(k_4)(-ie(a_V\gamma^\mu + a_{AV}\gamma^\mu\gamma_5))v(k_3)]$$

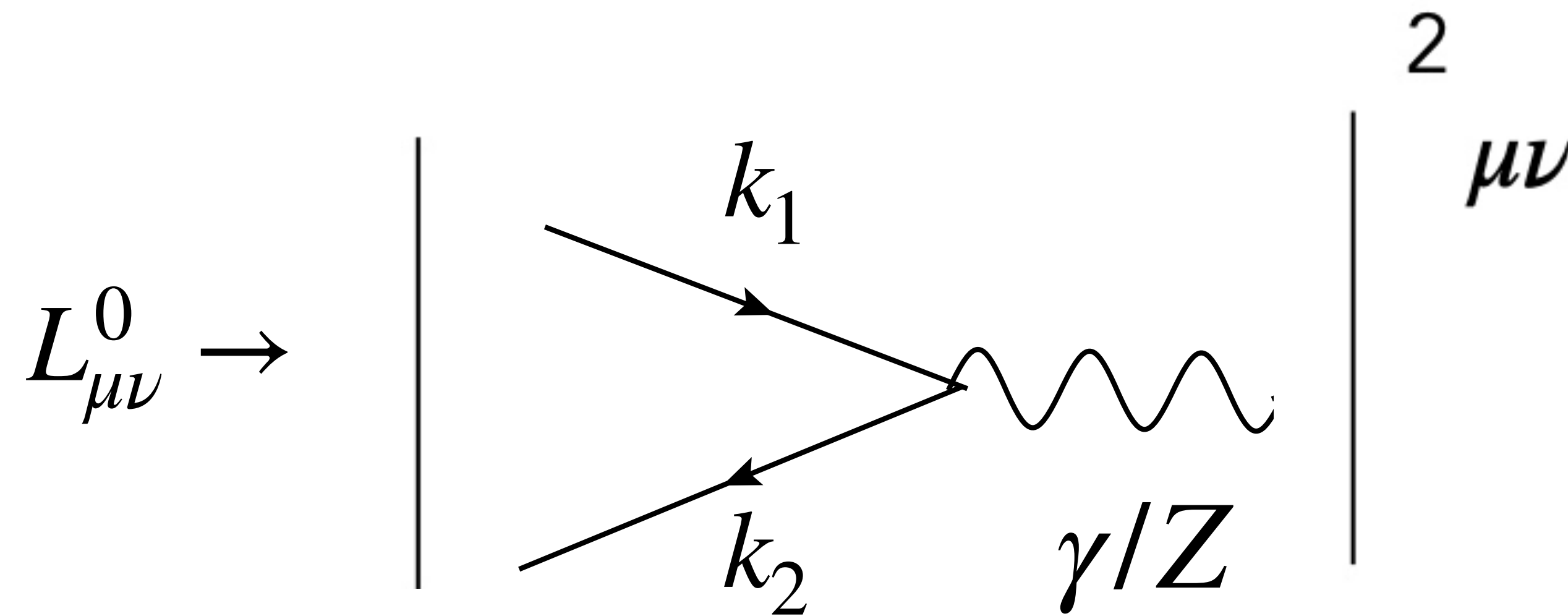
- The vector a_V and axial-vector a_{AV} couplings are defined as:

$$a_V = \frac{I_3 - 2 \sin^2 \theta_W Q_f}{2 \sin \theta_W \cos \theta_W}; \quad a_{AV} = \frac{I_3}{2 \sin \theta_W \cos \theta_W}$$

Where:

- $\sin \theta_W = \sqrt{1 - \frac{m_W^2}{m_Z^2}} \rightarrow$ sin of Weinberg mixing angle
- $Q_f = -1(e^-) \rightarrow$ electric charge
- $I_3 = -\frac{1}{2} \rightarrow$ weak isospin (for down-type left-handed fermions)

TREE-LEVEL LEPTONIC TENSOR (α -ORDER)

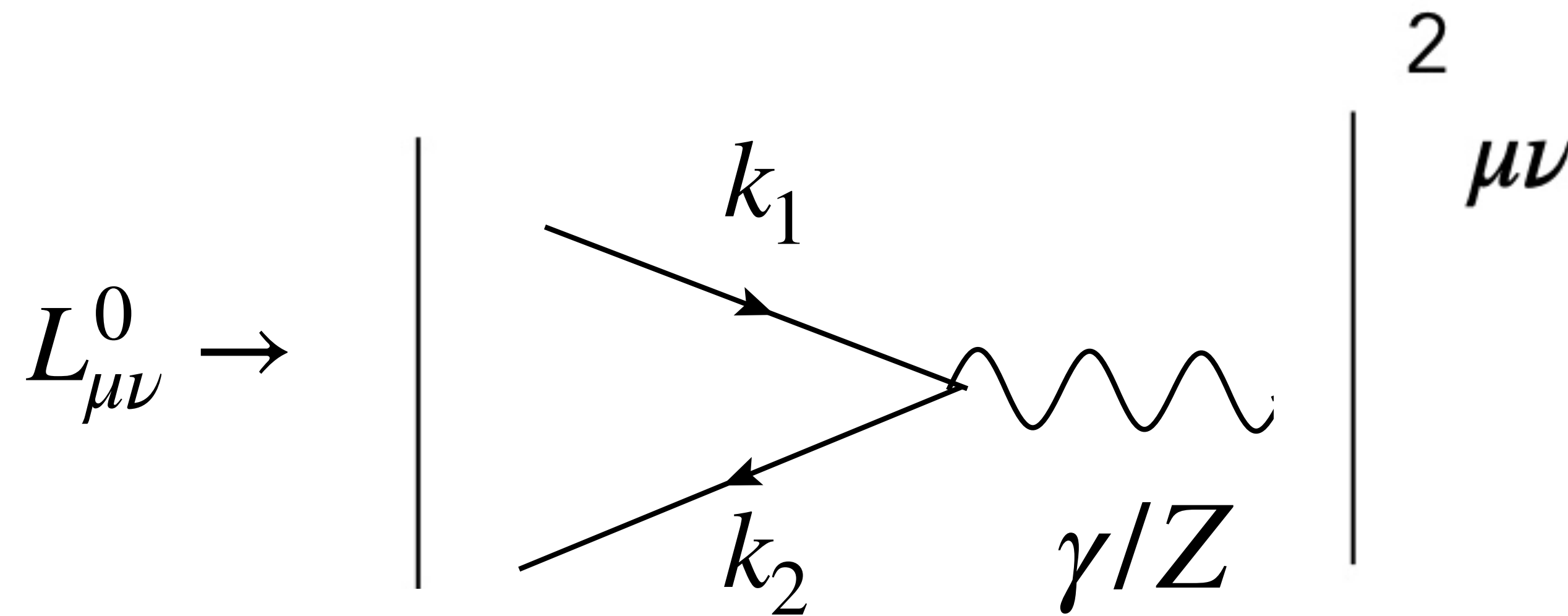


- Leading Order electroweak leptonic tensor:

$$L_{\mu\nu}^0 = 4\pi\alpha[l_1g_{\mu\nu} + l_2k_{2\mu}k_{1\nu} + l_3k_{1\mu}k_{2\nu} + l_4\epsilon_{s_1,\mu,\nu,k_1} + l_5\epsilon_{s_1,\mu,\nu,k_2} + l_6\epsilon_{\mu,\nu,k_1,k_2} + l_7k_{2\mu}s_{1\nu} + l_8k_{2\nu}s_{1\mu}]$$

where l_{1-8} are tree level leptonic tensor structure functions. $s_1 \rightarrow$ helicity reference vector of the incoming lepton.

TREE-LEVEL LEPTONIC TENSOR (α -ORDER)



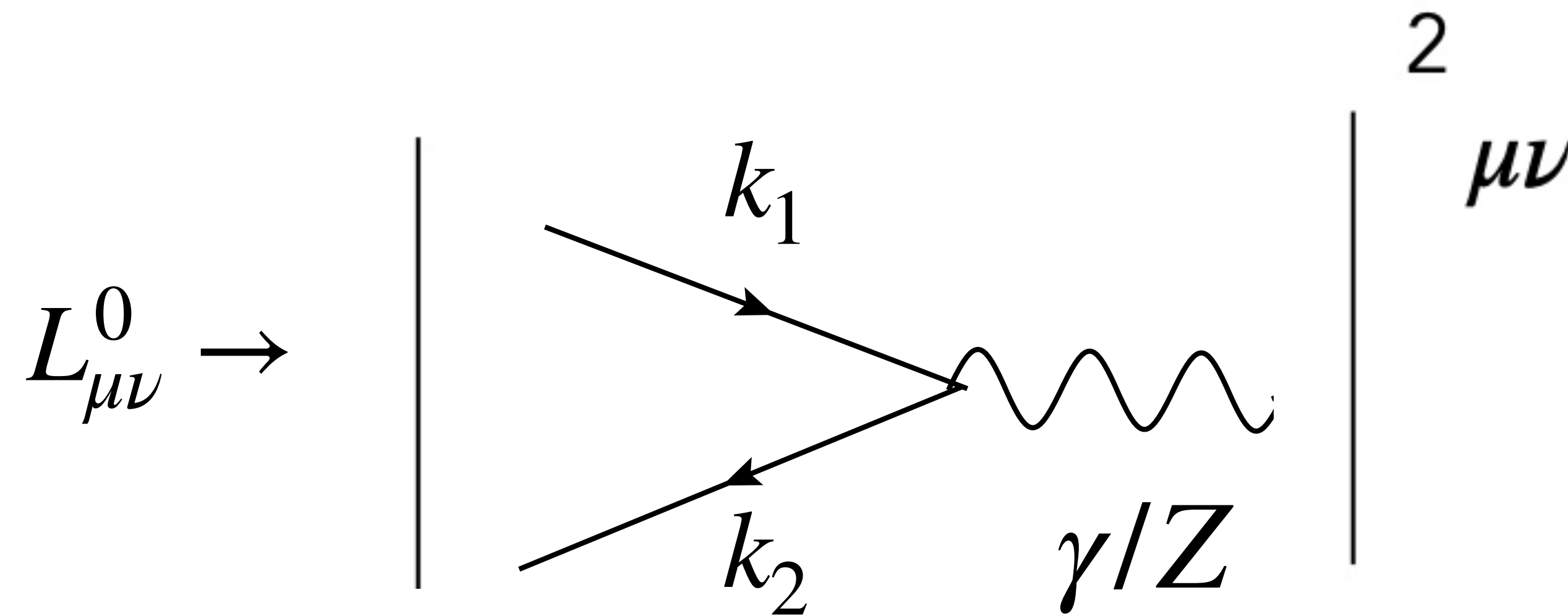
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$$s_1 = \frac{1}{m_{l/q}}(p, 0, 0, E_1),$$

TREE-LEVEL LEPTONIC TENSOR (α -ORDER)



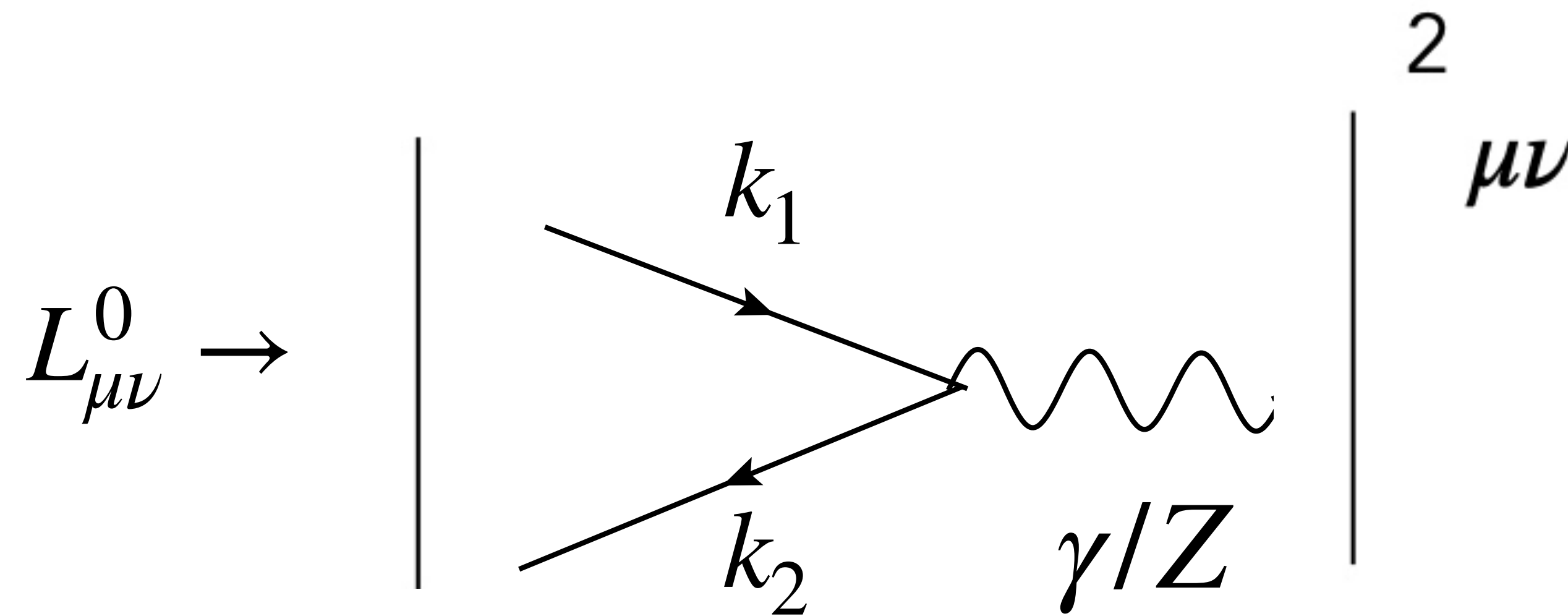
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where l_{1-8} are tree level leptonic tensor structure functions. $s_1 \rightarrow$ helicity reference vector of the incoming lepton.

$$s_1 = \frac{1}{m_{llq}}(p, 0, 0, E_1), \quad p = \sqrt{\frac{E_{CM}^2 + m_{llq}^2 - m_e^2}{4E_{CM}^2 - m_{llq}^2}},$$

TREE-LEVEL LEPTONIC TENSOR (α -ORDER)



- Leading Order electroweak leptonic tensor:

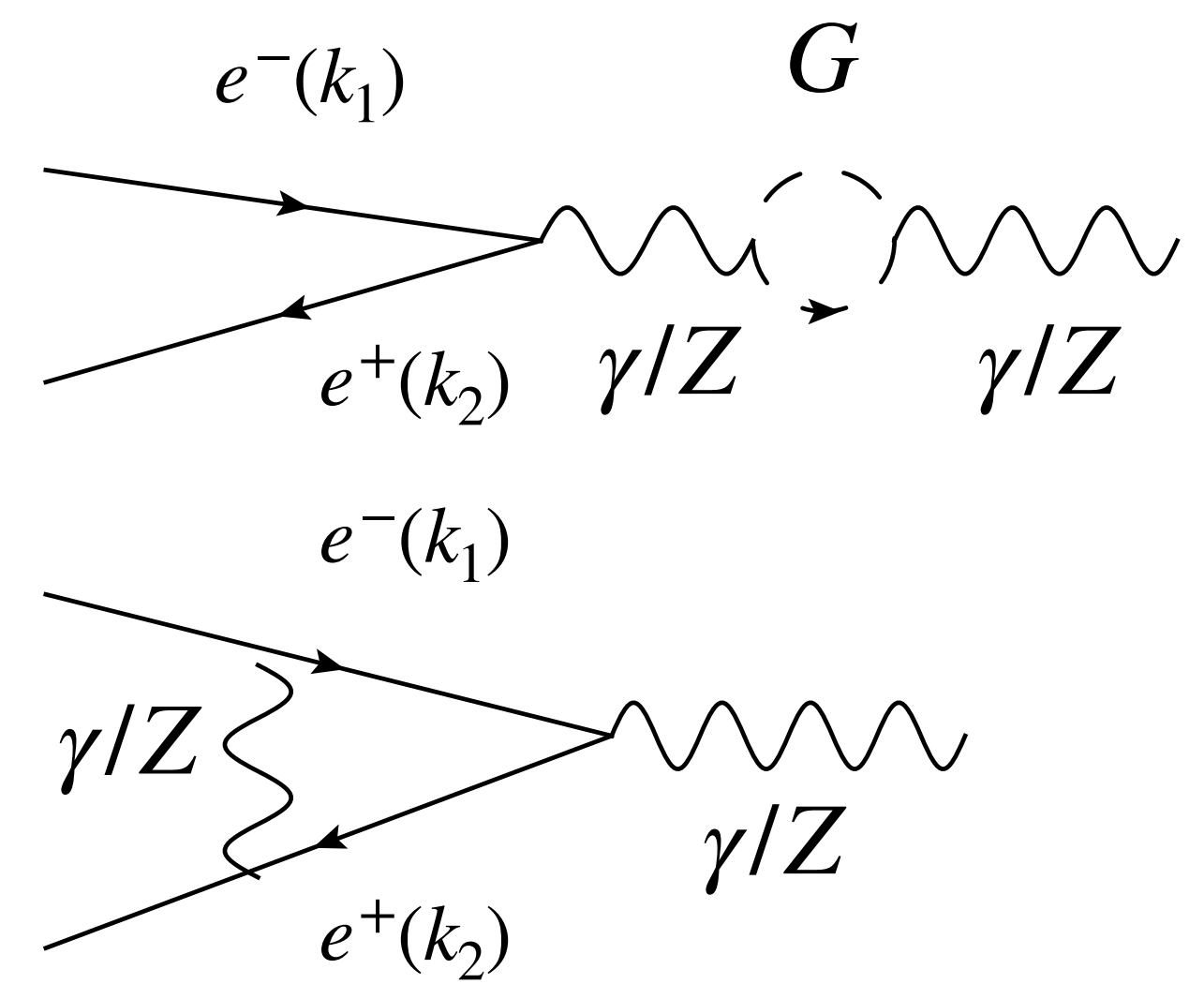
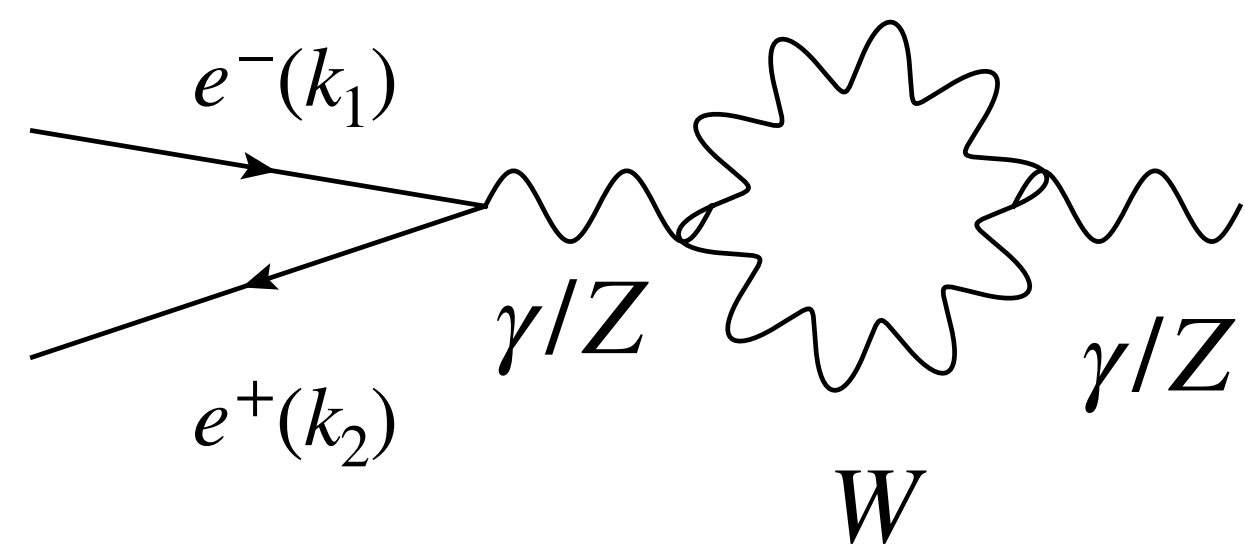
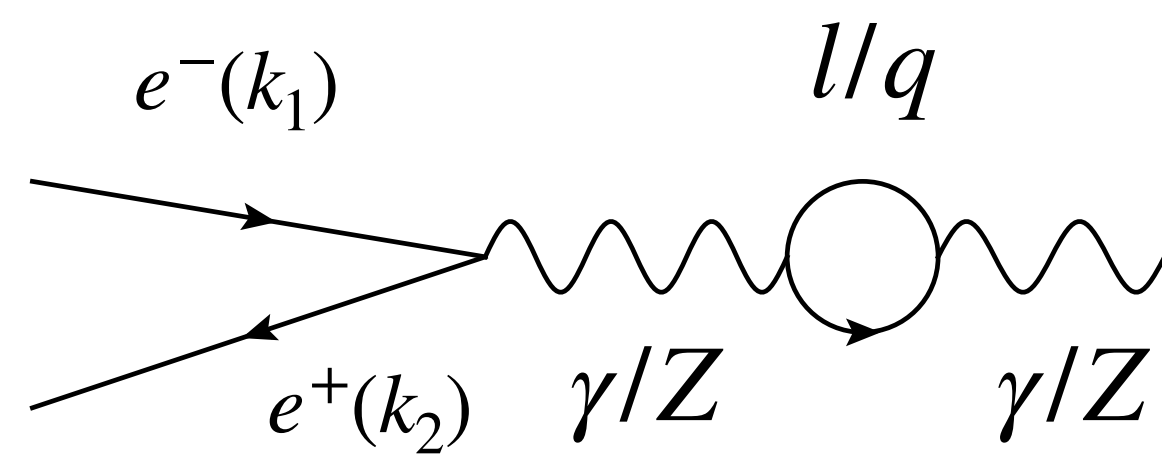
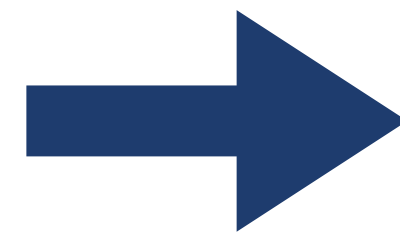
$$L_{\mu\nu}^0 = 4\pi\alpha[l_1 g_{\mu\nu} + l_2 k_{2\mu} k_{1\nu} + l_3 k_{1\mu} k_{2\nu} + l_4 \epsilon_{s_1, \mu, \nu, k_1} + l_5 \epsilon_{s_1, \mu, \nu, k_2} + l_6 \epsilon_{\mu, \nu, k_1, k_2} + l_7 k_{2\mu} s_{1\nu} + l_8 k_{2\nu} s_{1\mu}]$$

where l_{1-8} are tree level leptonic tensor structure functions. $s_1 \rightarrow$ helicity reference vector of the incoming lepton.

$$s_1 = \frac{1}{m_{llq}}(p, 0, 0, E_1), \quad p = \sqrt{\frac{E_{CM}^2 + m_{llq}^2 - m_e^2}{4E_{CM}^2 - m_{llq}^2}}, \quad E_1 = \sqrt{p^2 + m_e^2}$$

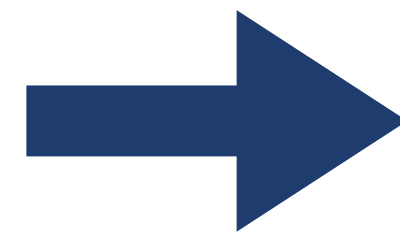
NLO Graphs

One loop level
Examples

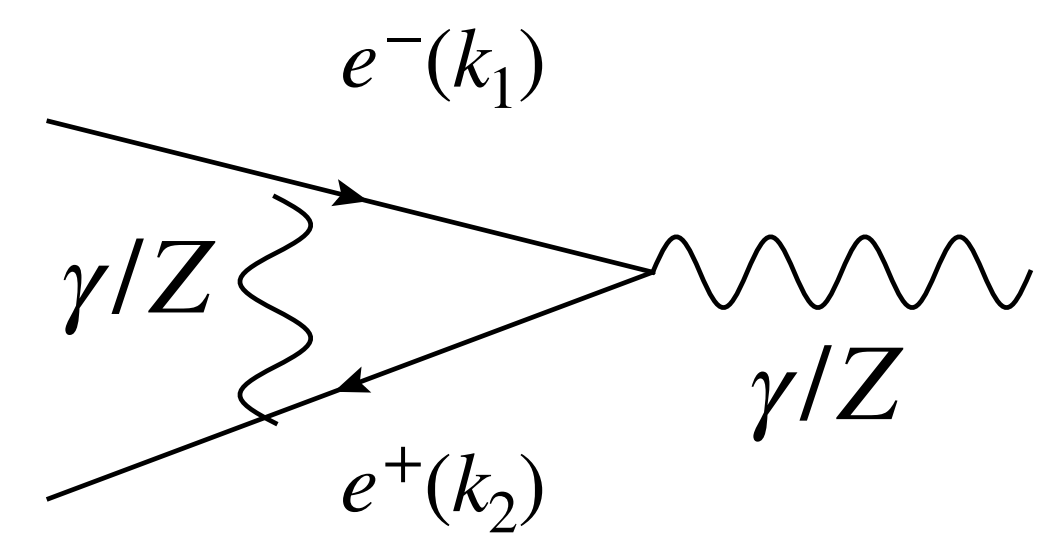
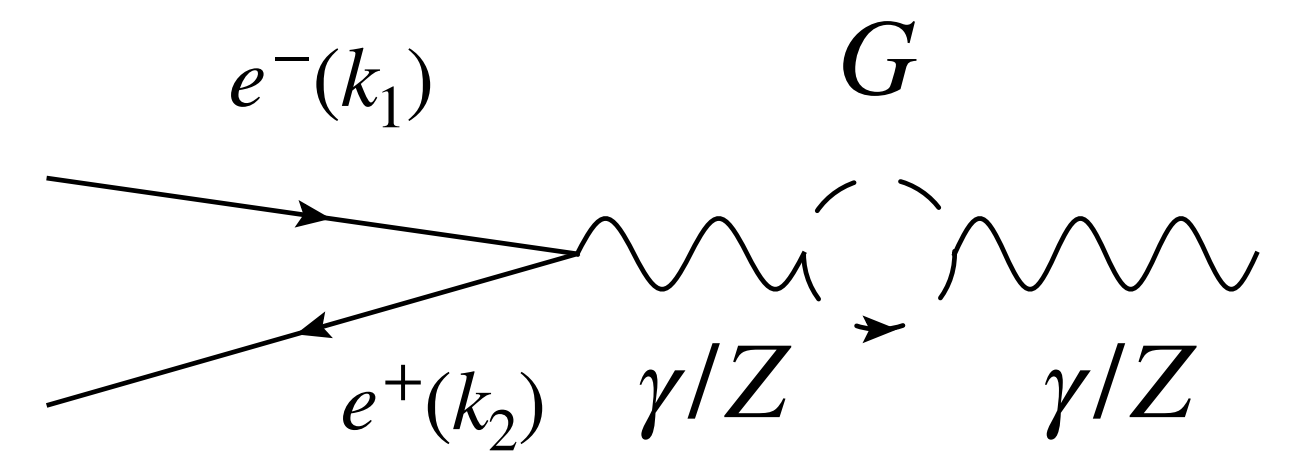
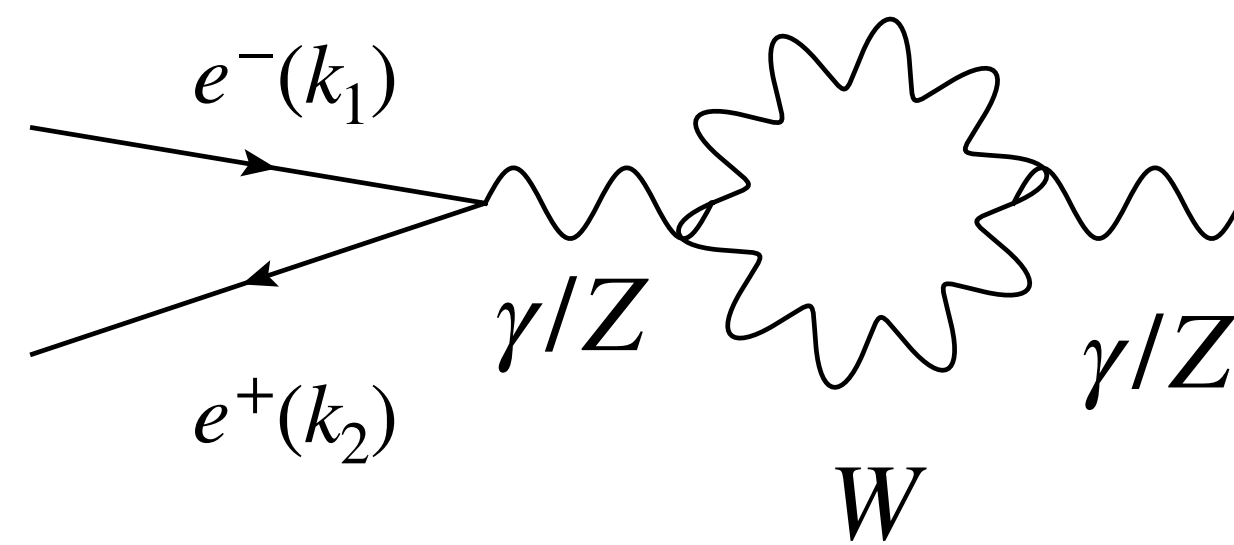
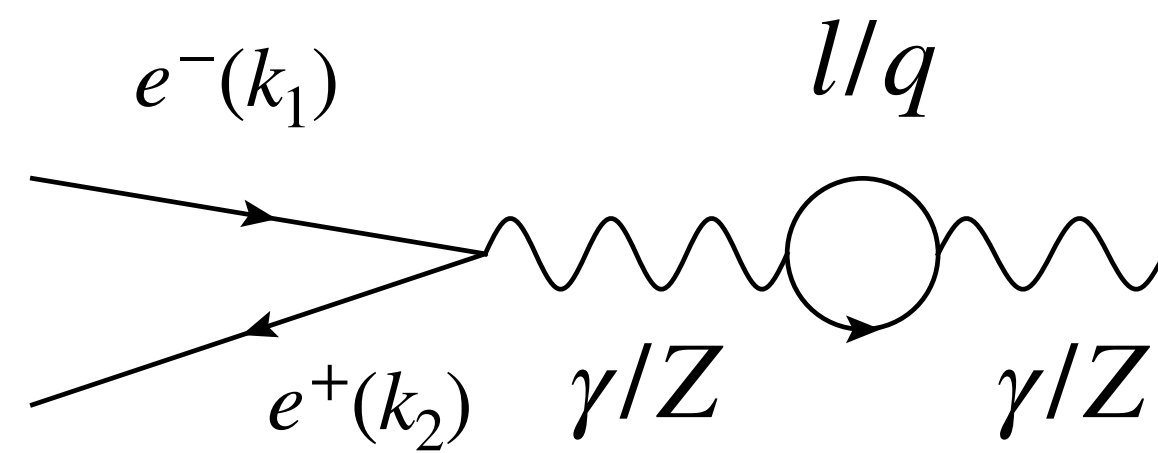


NLO Graphs

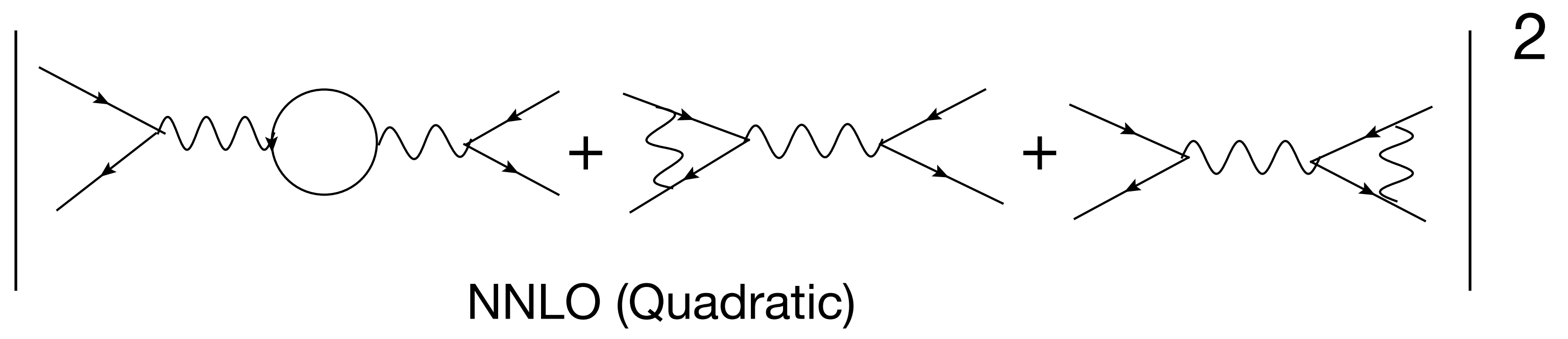
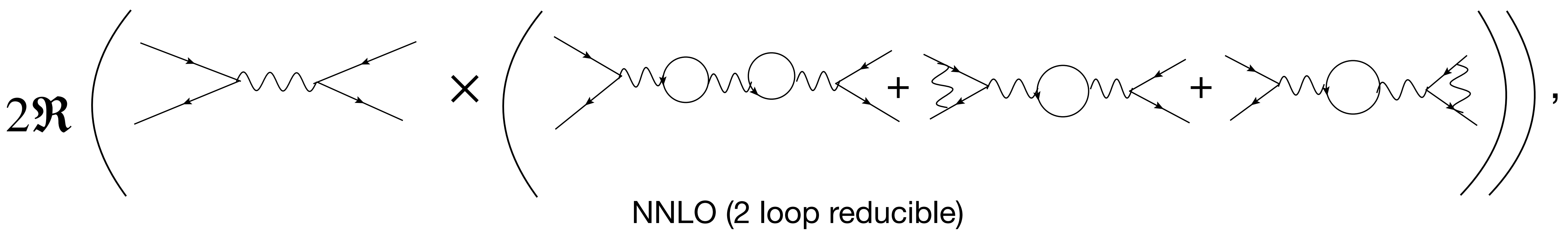
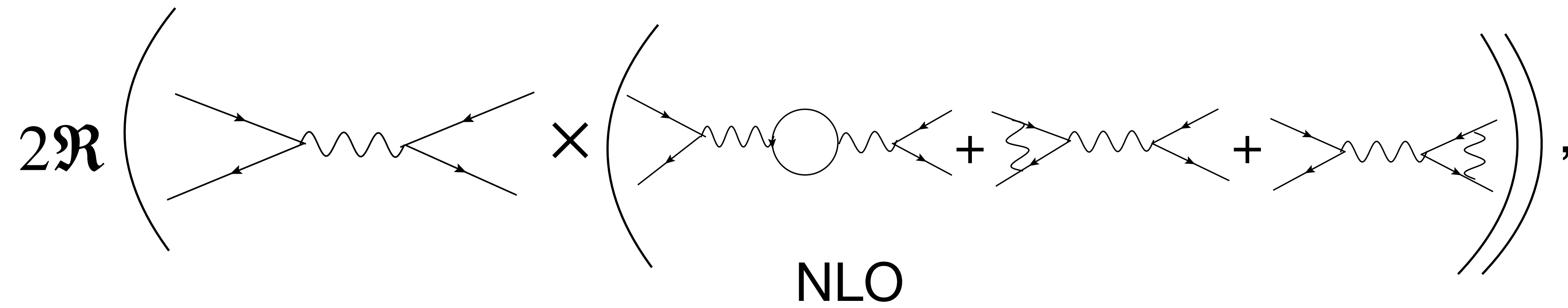
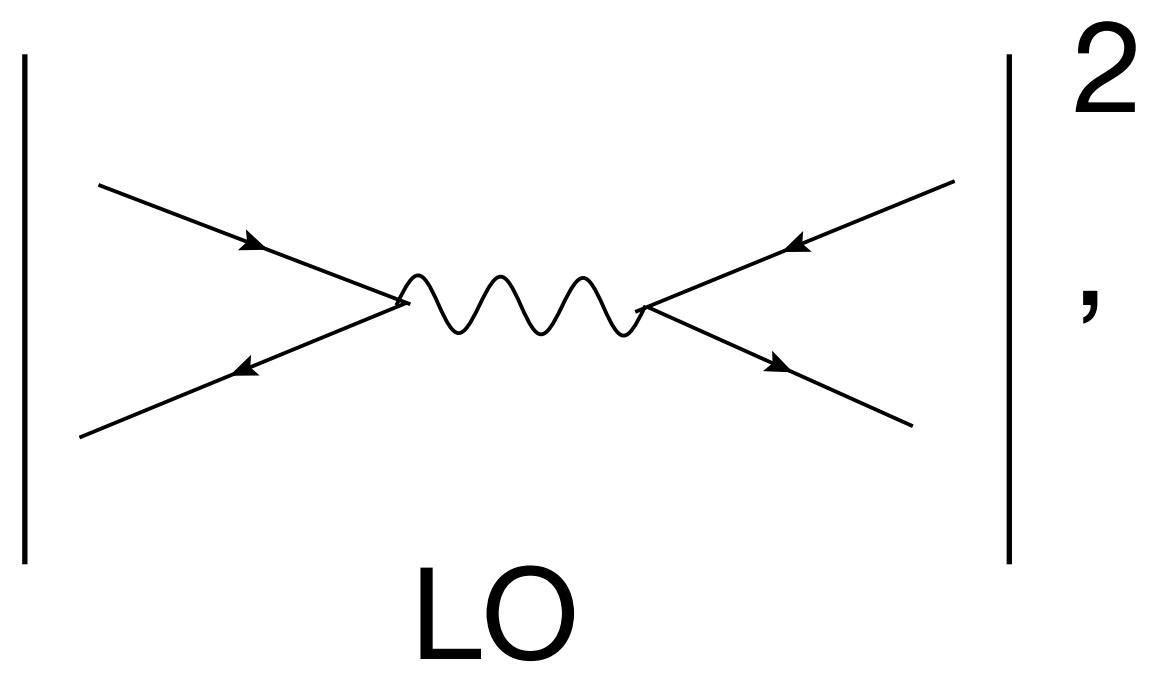
One loop level
Examples



307 graphs



Topology Graphs up to NNLO



NLO LEPTONIC TENSOR

$$\begin{aligned}
 L_{\mu\nu}^{NLO} = & r_1 g_{\mu\nu} + r_2 k_{2\mu} k_{1\nu} + r_3 k_{1\mu} k_{2\nu} + r_4 \epsilon_{s_1, \mu, \nu, k_1} + r_5 \epsilon_{s_1, \mu, \nu, k_2} + r_6 \epsilon_{\mu, \nu, k_1, k_2} + \\
 & r_7 k_{2\mu} s_{1\nu} + r_8 k_{2\nu} s_{1\mu} + r_9 k_{2\mu} k_{2\nu} + r_{10} \epsilon_{s_1, \mu, k_1, k_2} k_{2, \nu} + r_{11} \epsilon_{s_1, \nu, k_1, k_2} k_{2\mu} + r_{12} k_{1\nu} k_{1\mu} + \\
 & r_{13} \epsilon_{\mu, \nu, k_2, k_1} + r_{14} s_{1\nu} k_{1\mu} + r_{15} s_{1\mu} k_{1\nu} + r_{16} \epsilon_{s_1, \mu, k_1, k_2} k_{1\nu} + r_{17} \epsilon_{s_1, \mu, k_2, k_1} k_{2\nu} + \\
 & r_{18} \epsilon_{s_1, \nu, k_2, k_1} k_{1\mu} + r_{19} \epsilon_{s_1, \nu, k_2, k_1} k_{2\mu}.
 \end{aligned}$$

- r_{1-19} are leptonic structure functions, of α^2 order, depend on the momentum transfer (q^2) and written in terms of Passarino-Veltman (Pa-Ve) integral functions.

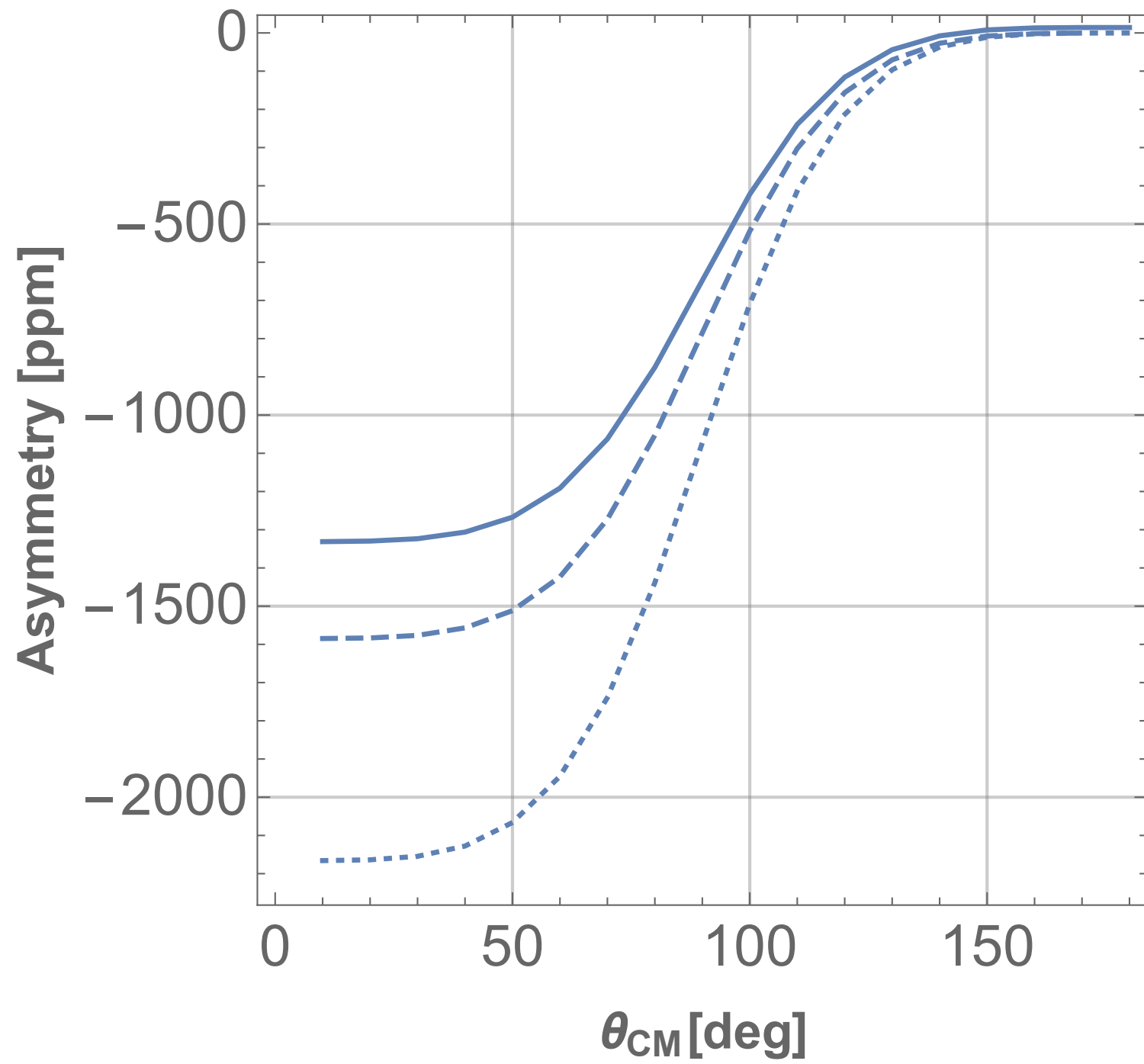
NNLO QUADRATIC LEPTONIC TENSOR

$$\begin{aligned} L_{\mu\nu}^{QD} = & n_1 g_{\mu\nu} + n_2 k_{2\mu} k_{1\nu} + n_3 k_{1\mu} k_{2\nu} + n_4 \epsilon_{s_1, \mu, \nu, k_1} + n_5 \epsilon_{s_1, \mu, \nu, k_2} + n_6 \epsilon_{\mu, \nu, k_1, k_2} + \\ & \cdot n_7 k_{2\mu} s_{1\nu} + n_8 k_{2\nu} s_{1\mu} + n_9 k_{2\mu} k_{2\nu} + n_{10} \epsilon_{s_1, \mu, k_1, k_2} k_{2, \nu} + n_{11} \epsilon_{s_1, \nu, k_1, k_2} k_{2\mu} + n_{12} k_{1\nu} k_{1\mu} + \\ & n_{13} \epsilon_{\mu, \nu, k_2, k_1} + n_{14} s_{1\nu} k_{1\mu} + n_{15} s_{1\mu} k_{1\nu} + n_{16} \epsilon_{s_1, \mu, k_1, k_2} k_{1\nu} + n_{17} \epsilon_{s_1, \mu, k_2, k_1} k_{2\nu} + \\ & n_{18} \epsilon_{s_1, \nu, k_2, k_1} k_{1\mu} + n_{19} \epsilon_{s_1, \nu, k_2, k_1} k_{2\mu} + n_{20} \epsilon_{s_1, \mu, k_2, k_1} k_{1\nu} + n_{21} \epsilon_{s_1, \nu, k_1, k_2} k_{1\mu}. \end{aligned}$$

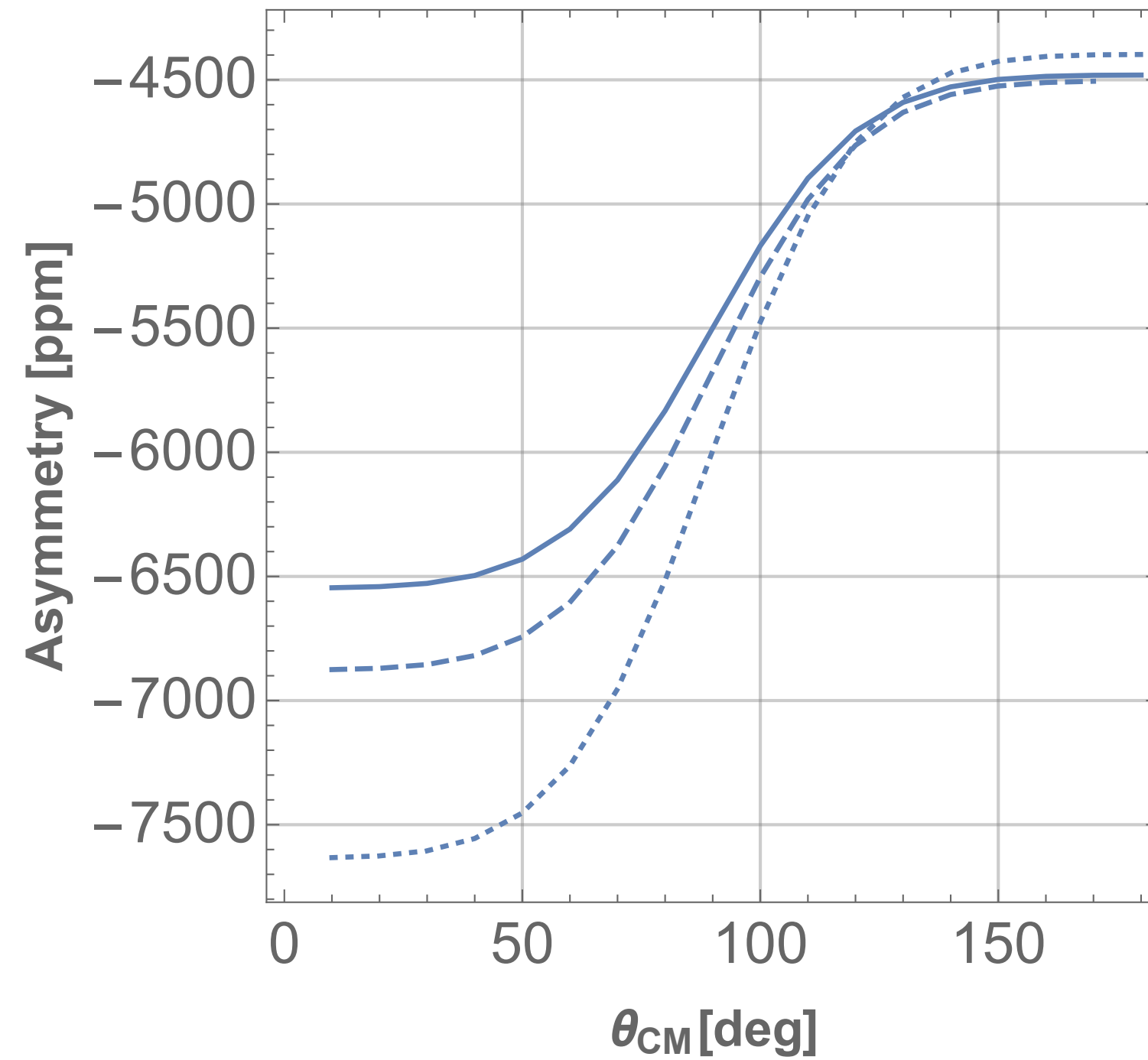
- n_{1-21} are leptonic structure functions, of α^3 order, depend on the momentum transfer (q^2) and written in terms of Passarino-Veltman (Pa-Ve) integral functions.

Corrected A_{PV} Graphs

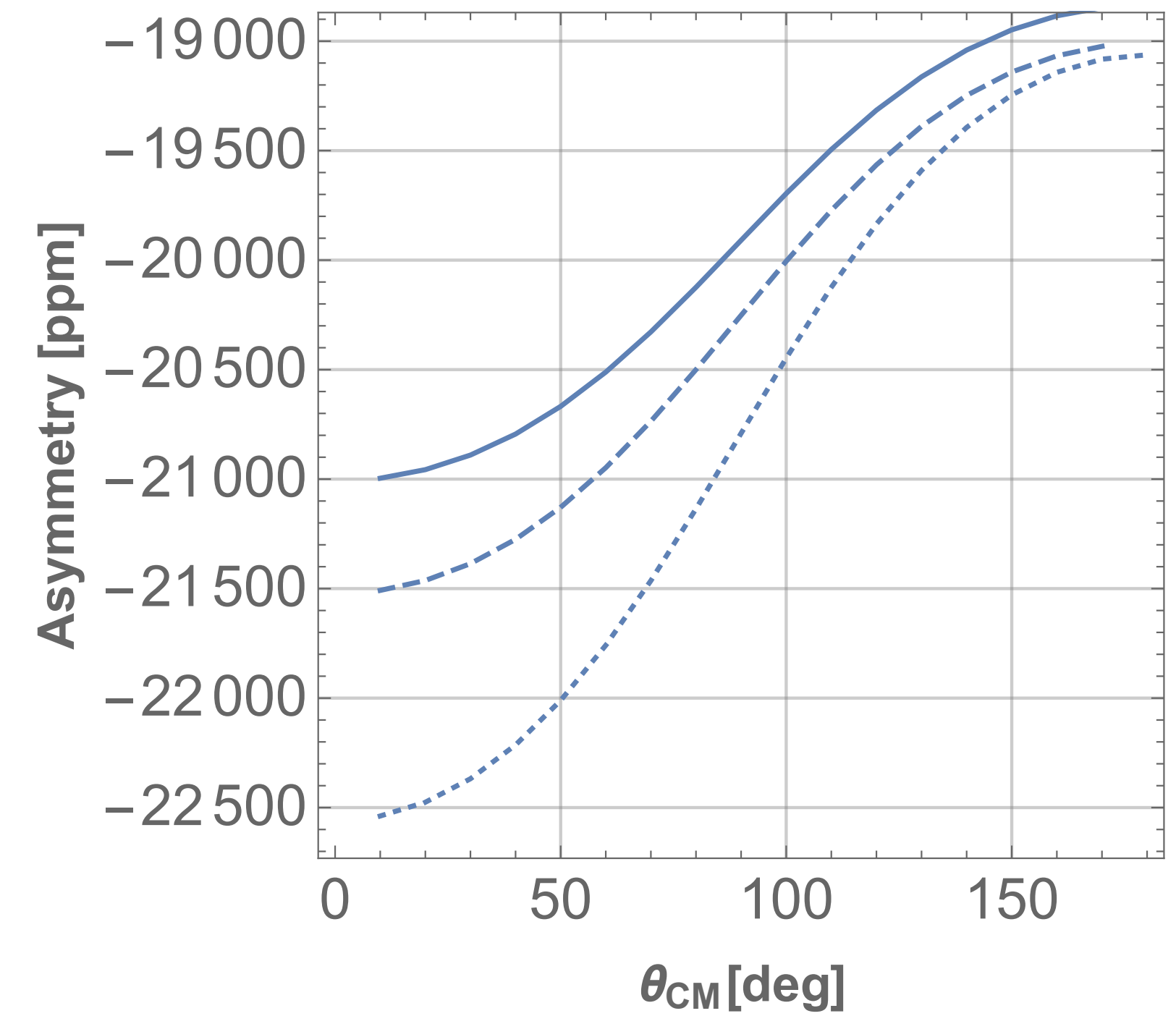
$$e^- + e^+ \rightarrow \mu^- + \mu^+$$



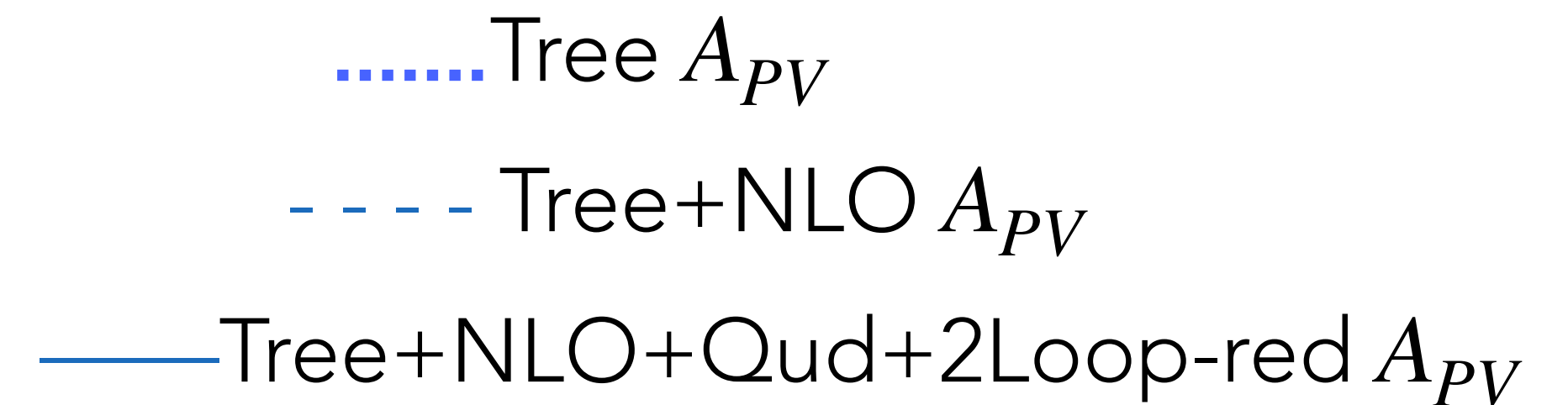
$$e^- + e^+ \rightarrow c + \bar{c}$$



$$e^- + e^+ \rightarrow b + \bar{b}$$



- A_{PV} obtained using covariant approach
- Box diagrams and bremsstrahlung not included
- On-shell renormalization is used



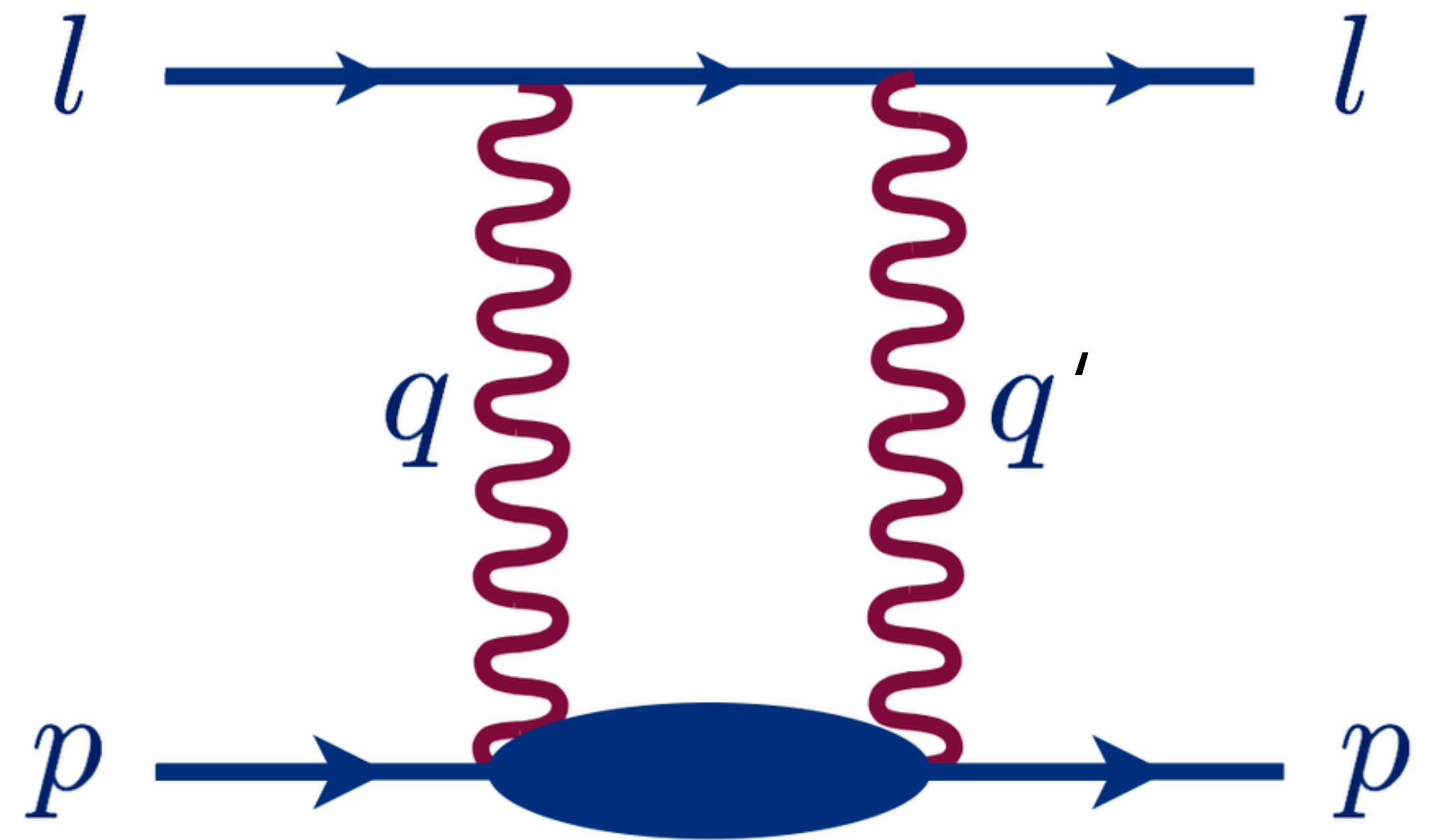
SUMMARY:

- We presented our preliminary results up to NNLO (quadratic + reducible two-loop) for electroweak A_{PV} using the kinematics of Chiral Belle at CM energy of 10.58 GeV.
- These calculations will be updated by adding the soft-photon and the hard-photon bremsstrahlung processes along with electroweak box diagrams.
- We have employed various techniques in this work, which have been checked with the higher-order QED corrections in A_{PV} .
- Our analytical and computer-based algebra routines show considerable promise for extension and applications towards the experiments, which are searching for physics beyond the Standard Model.

REFERENCES FOR BOX DIAGRAMS

[1] Peter G. Blunden et al., Physical Review Letters 91(14)

[2] M. Gorchtein, Phys. Rev. C **73**, 055201 (2006)



Thank you!!