Neutrino induced pion production

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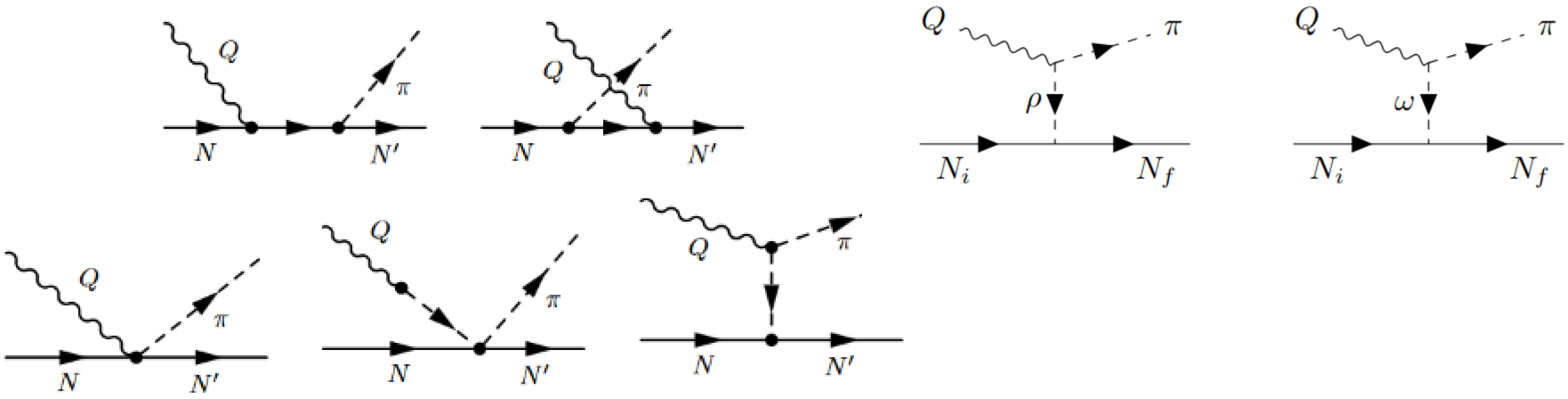
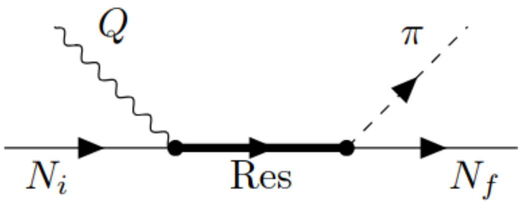
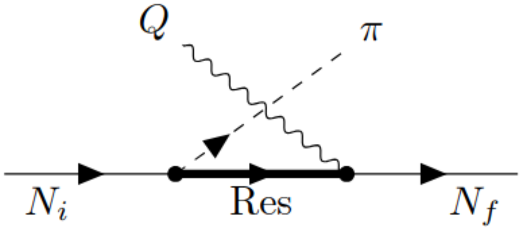
> Neutrino Theory Network (NTN) Fellow
> University of Washington

Hadronic modeling of SPP



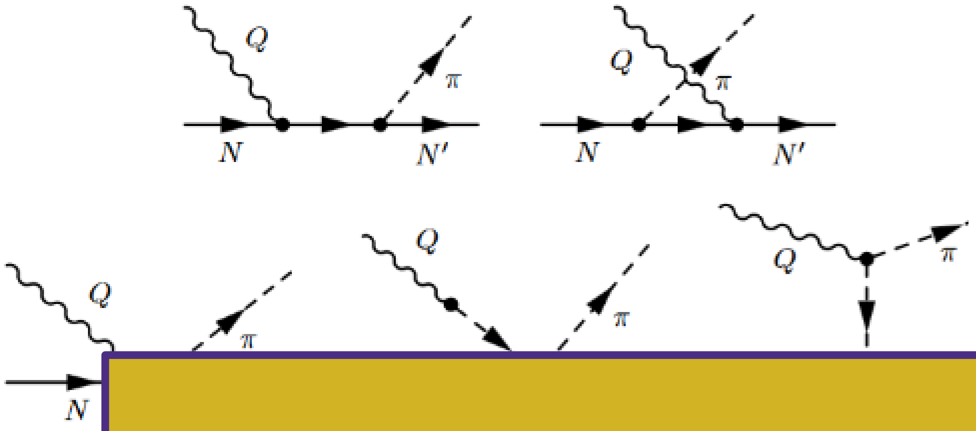
$+\Sigma_{\text{res}}$

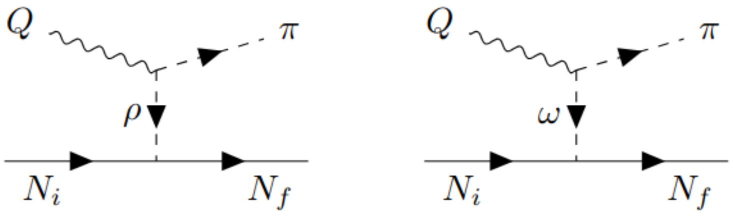




+ ...

Hadronic modeling of SPP

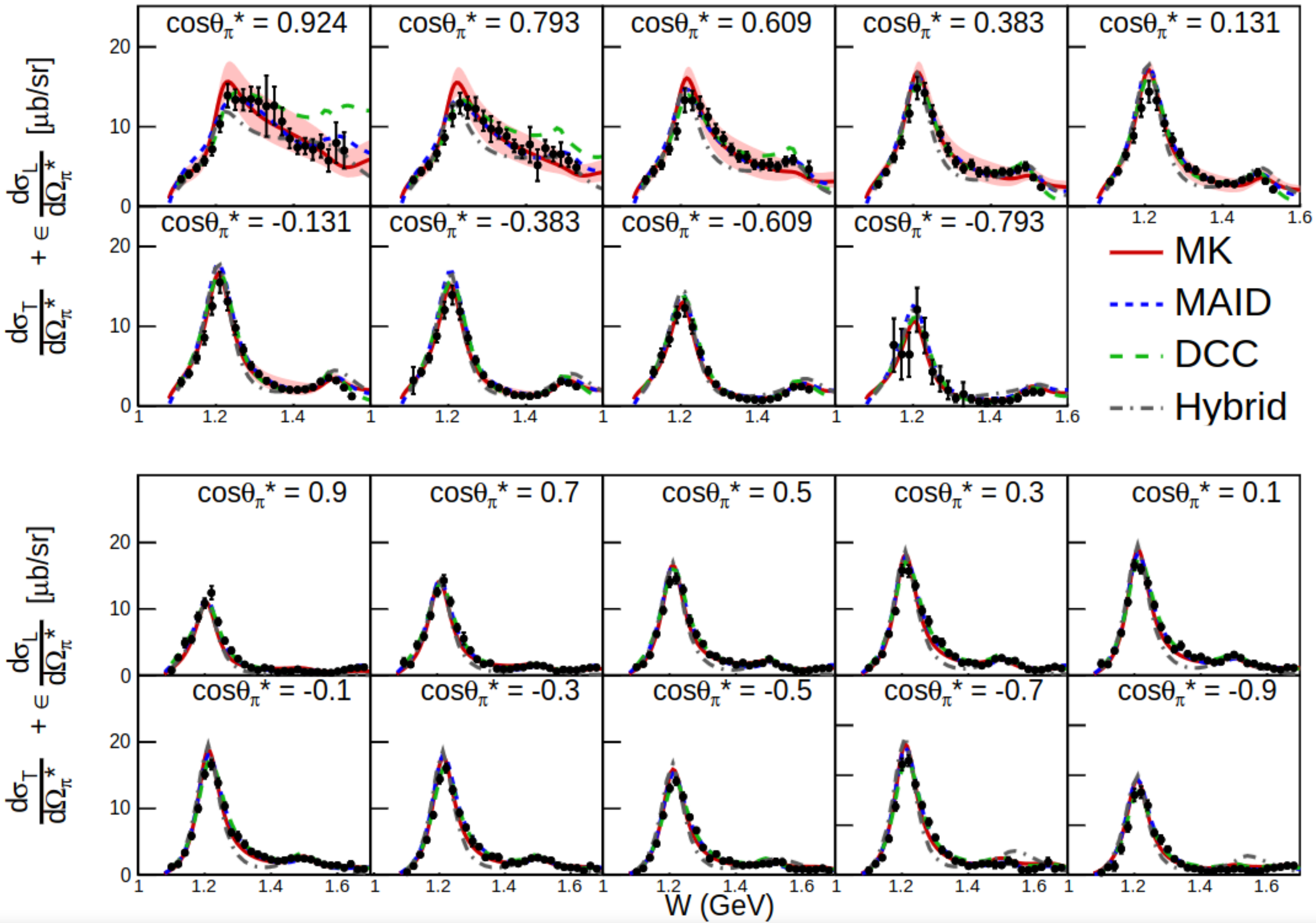




+Σ

A lot of *effective* couplings/propagators that need to be determined!

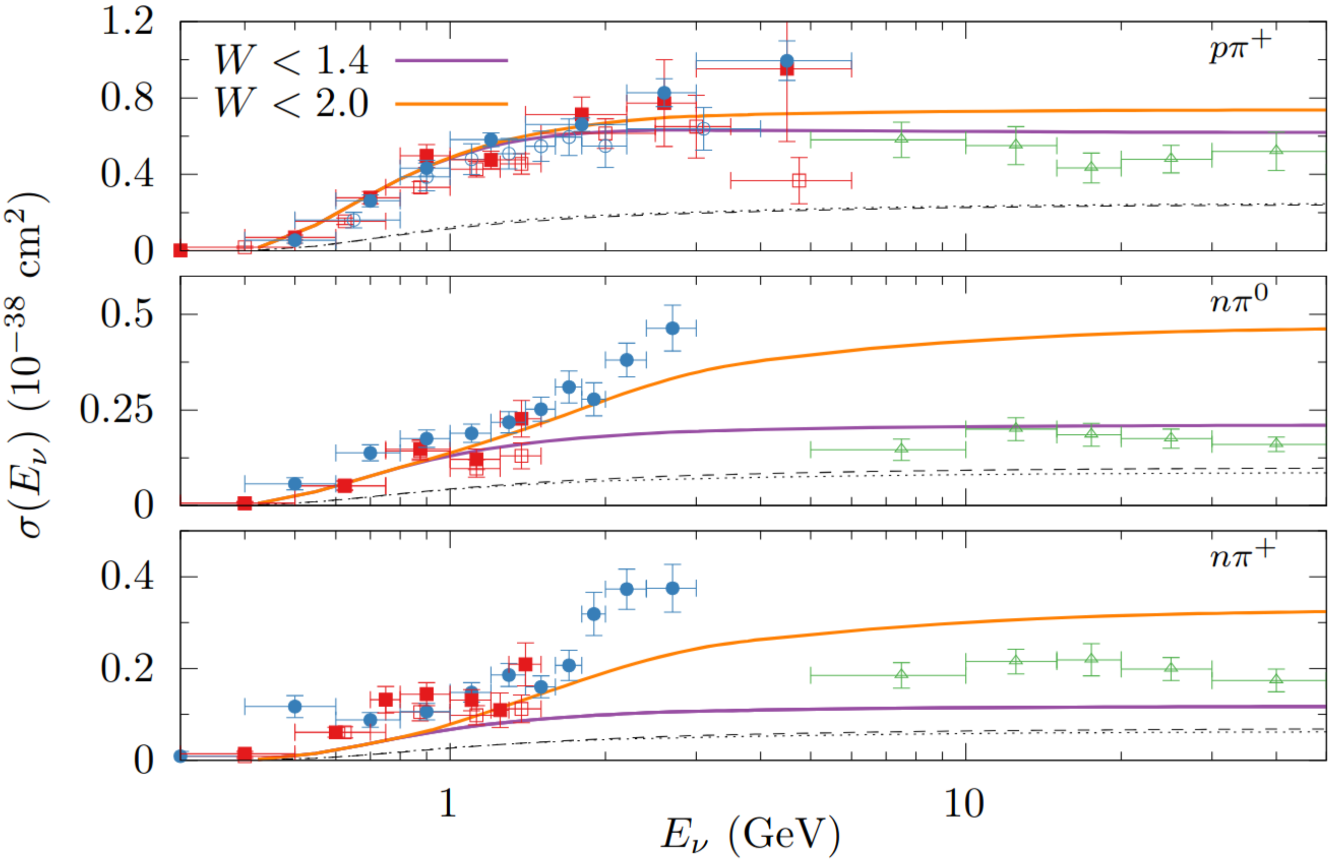
Lot of electromagnetic interaction data



From [Kabirnezhad, 2203.15594]

Neutrino data: much less abundant

Bubble chamber data on deuterium (proton) from ANL/BNL/BEBC

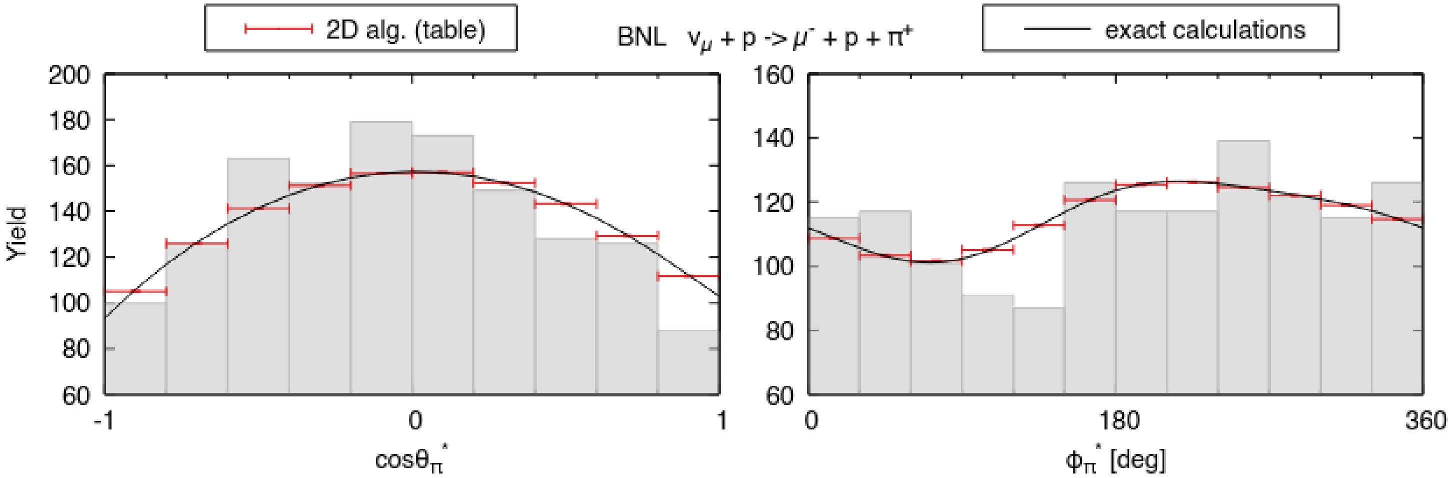


Neutrino interaction data

Bubble chamber data on deuterium (proton) from ANL/BNL/BEBC

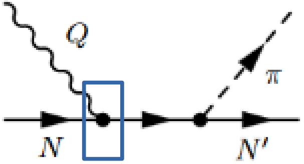
→ Observables integrated over large phase space

→ Sometimes shape-only



+ Data on nuclei (^{12}C, ^{16}O, ^{40}Ar) from Minerva, T2K, MiniBooNE, MicroBooNE, ...

Data determines couplings to resonances



$$\mathcal{A}_{1/2} = \sqrt{\frac{2\pi\alpha}{K}}\frac{1}{e}\langle S_{z,R} = \frac{1}{2}|\epsilon_\mu^{(+)}(q)J^\mu|S_{z,N} = -\frac{1}{2}\rangle$$

$$\mathcal{A}_{3/2} = \sqrt{\frac{2\pi\alpha}{K}}\frac{1}{e}\langle S_{z,R} = \frac{3}{2}|\epsilon_\mu^{(+)}(q)J^\mu|S_{z,N} = \frac{1}{2}\rangle$$

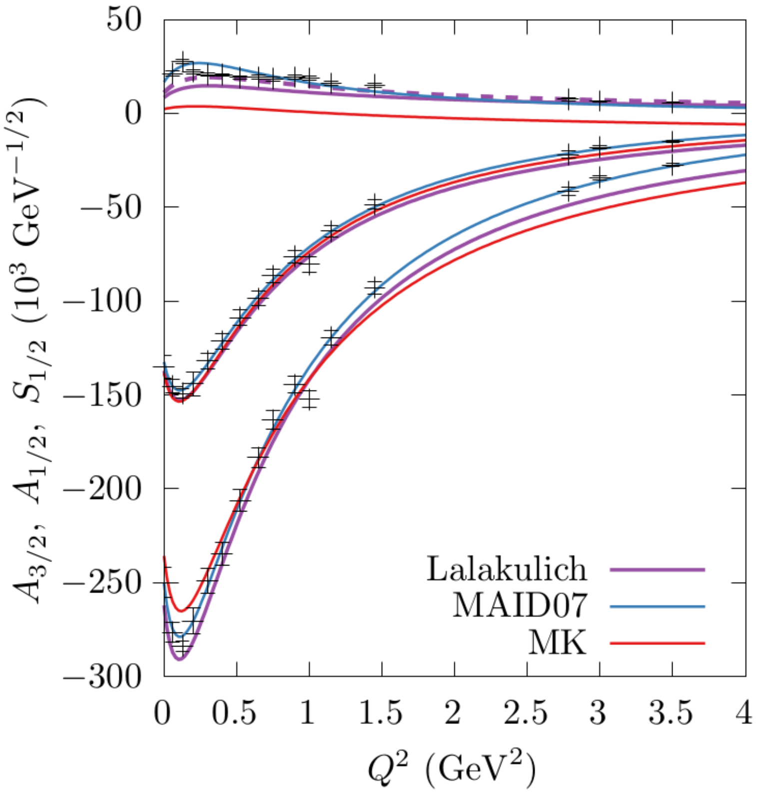
$$\mathcal{S}_{1/2} = \sqrt{\frac{2\pi\alpha}{K}}\frac{1}{e}\langle S_{z,R} = \frac{1}{2}|\frac{|\vec{q}|}{\sqrt{Q^2}}\epsilon_\mu^{(0)}(q)J^\mu|S_{z,N} = \frac{1}{2}\rangle$$

Reduced helicity amplitudes at W = M_R

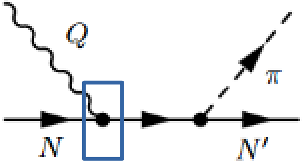
[userweb.jlab.org/~mokeev/resonance_electrocouplings]

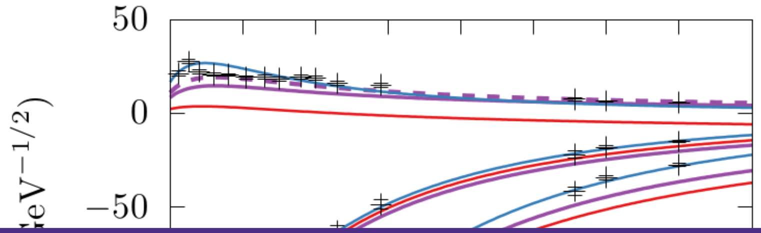
Helicity amplitudes from CLAS/MAID07 analyses used in:

- Lalakulich et al [Phys. Rev. D74, 014009 (2006)]
- Hernandez et al. [Phys Rev D 77 053009 (2008)]
- Nikolakopoulos et al. [arxiv:2210.12144 (2022)]
- ….



Data determines couplings to resonances





Neutrino-induced charged-current interactions:

1.) Isovector amplitudes

2) Axial amplitudes

Hernandez et al. [Phys Rev D 77 053009 (2008)]
Nikolakopoulos et al. [arxiv:2210.12144 (2022)]
….

Neutrino: Isovector form factors

$$J^\mu_{EM} = V^\mu_s + \mathbf{V}^\mu \qquad J^\mu_{CC\pm} = \mathbf{V}^\mu - \mathbf{A}^\mu$$

$$\langle \pi^+ n | J^\mu_{EM} | p \rangle = V^\mu_{3/2} - \sqrt{2}\boxed{\left(V^\mu_{1/2} + S^\mu\right)}$$

$$\langle \pi^- p | J^\mu_{EM} | n \rangle = V^\mu_{3/2} - \sqrt{2}\boxed{\left(V^\mu_{1/2} - S^\mu\right)}$$

$$\langle \pi^0 p | J^\mu_{EM} | p \rangle = \sqrt{2}V^\mu_{3/2} + \boxed{\left(V^\mu_{1/2} + S^\mu\right)}$$

$$\langle \pi^0 n | J^\mu_{EM} | n \rangle = \sqrt{2}V^\mu_{3/2} + \boxed{\left(V^\mu_{1/2} - S^\mu\right)}$$

$$\langle \pi^+ p | \, V^\mu_+ \, | p \rangle \; = 3V^\mu_{3/2},$$

$$\langle \pi^+ n | \, V^\mu_+ \, | n \rangle = V^\mu_{3/2} + 2\sqrt{2}V^\mu_{1/2},$$

$$\langle \pi^0 p | \, V^\mu_+ \, | n \rangle = -\sqrt{2}V^\mu_{3/2} + 2V^\mu_{1/2}.$$

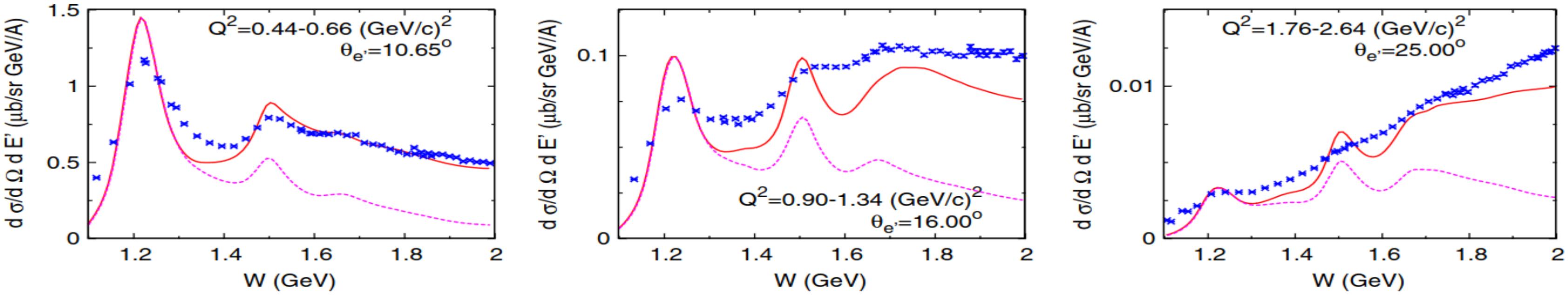
F_p $\quad$ F_n

$F_V = F_p - F_n$

Neutron (deuteron) target data much less abundant

Neutrino: Isovector form factors

ANL-Osaka Dynamic Coupled Channels (DCC) [PRD 92, 074024 (2015)]
→ Fit 'neutron' target exclusive data (deuteron)
→ Fit inclusive structure function on deuteron

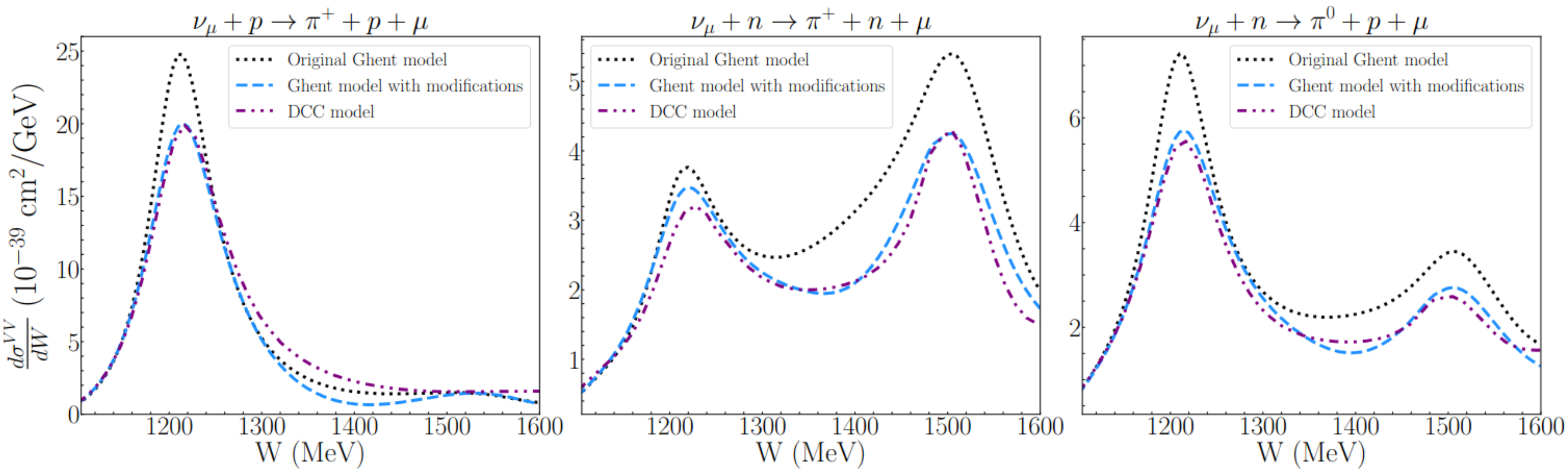


→ Progress on **deuteron FSI** e.g. [PRD 99 031301]
→ **data** e.g. CLAS [Phys. Rev. C 107, 015201 (2023)] over large W-region

VV contribution to neutrino scattering

$\sigma = \boldsymbol{\sigma_{VV}} + \sigma_{AA} + \sigma_{VA}$

→ **Averaged over BEBC experiment energy distribution:**



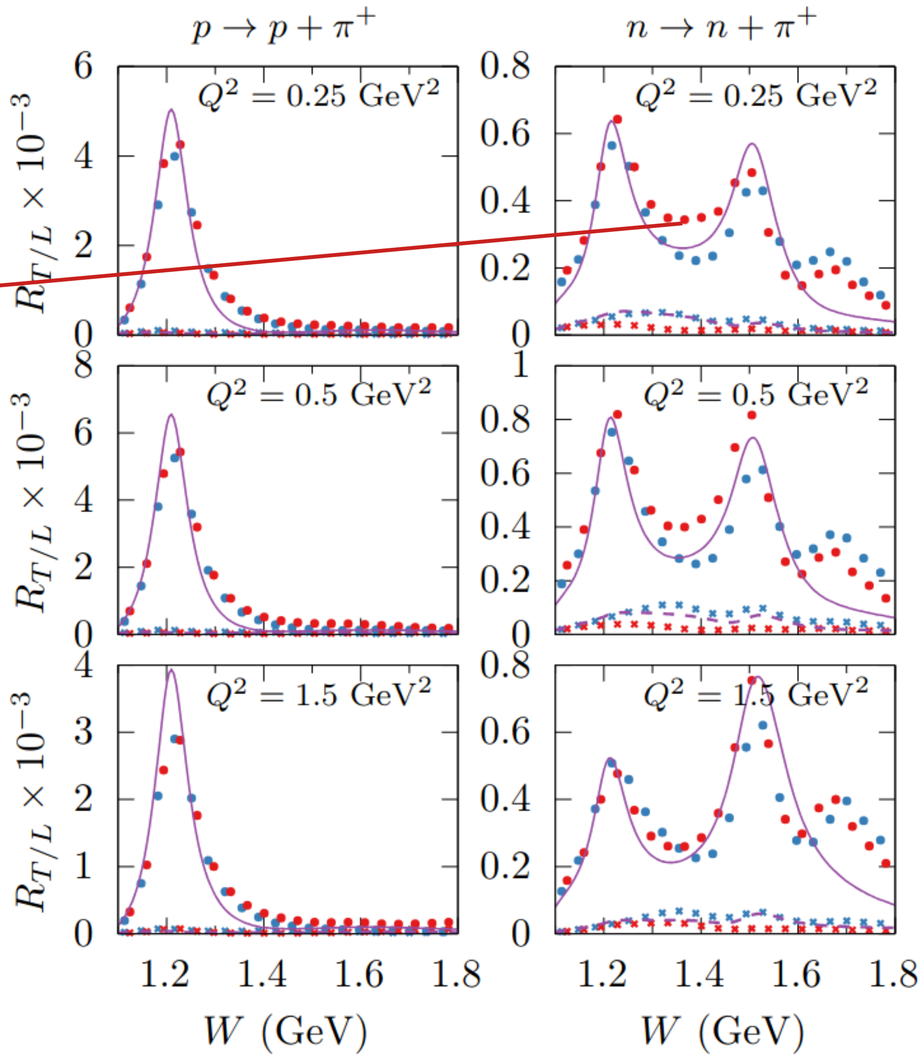
[M. Hooft et al. 2603.29486], comparison to ANL-Osaka model

VV contribution to neu

$\sigma = \boldsymbol{\sigma_{VV}} + \sigma_{AA} + \sigma_{VA}$

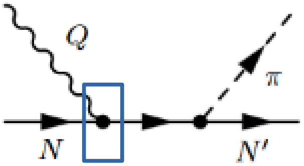
→ **Inclusive partial cross sections:**

- ANL-Osaka DCC
- MAID07



[A.N. et al. 2210.12144]

Axial couplings are poorly constrained



Spin-3/2 resonance axial couplings:

$$\Gamma_A^{\beta\mu} = \frac{C_3^A}{M}\left(g^{\beta\mu}\not{Q} - Q^\beta\gamma^\mu\right) + \frac{C_4^A}{M^2}\left(g^{\beta\mu}Q\cdot k_R - Q^\beta k_R^\mu\right) + C_5^A g^{\beta\mu} + \frac{C_6^A}{M^2}Q^\beta Q^\mu,$$

PCAC / π-pole dominance

$$C_A^5(0) = f_\pi I_{iso}\frac{\sqrt{2}f_{\pi NR}}{m_\pi\sqrt{3}}$$

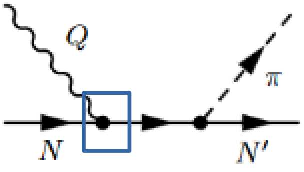
$$C_A^6(Q^2) = -M_N^2\frac{C_A^5(Q^2)}{Q^2+m_\pi}$$

-No strong constraints for Q^2 dependence

-No strong constraint on C_3 or C_4

-No high-quality data available!

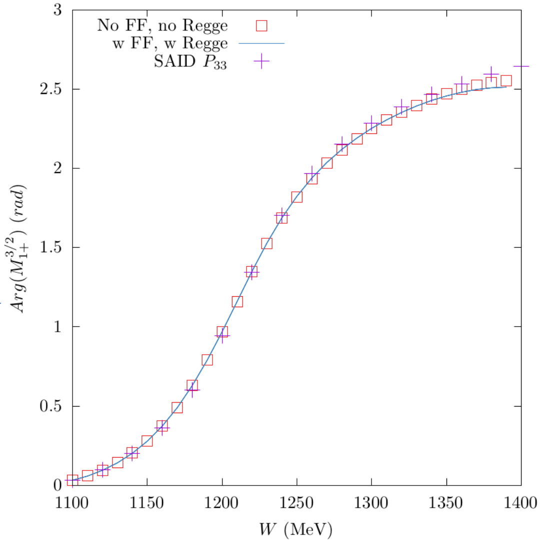
Axial coupling to delta from data

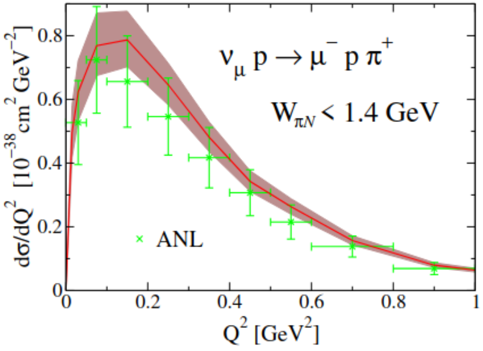


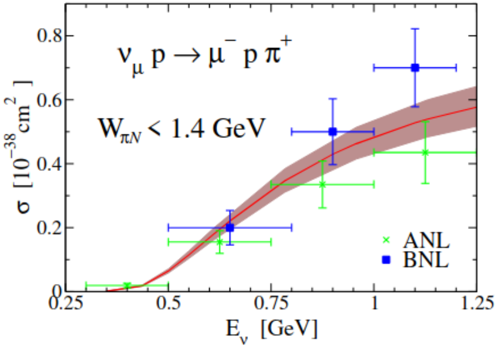
L. Alvarez-Ruso, E. Hernández, J. Nieves, M.J. Vicente Vacas [Phys. Rev. D93, 014016]

- ANL data
- Adler model for Delta
- → $C_4 = -C_5/4$, $C_3 = 0$
- → C_4 and C_3 contributions small

+Unitarity in *Δ* dominated
V and A multipoles





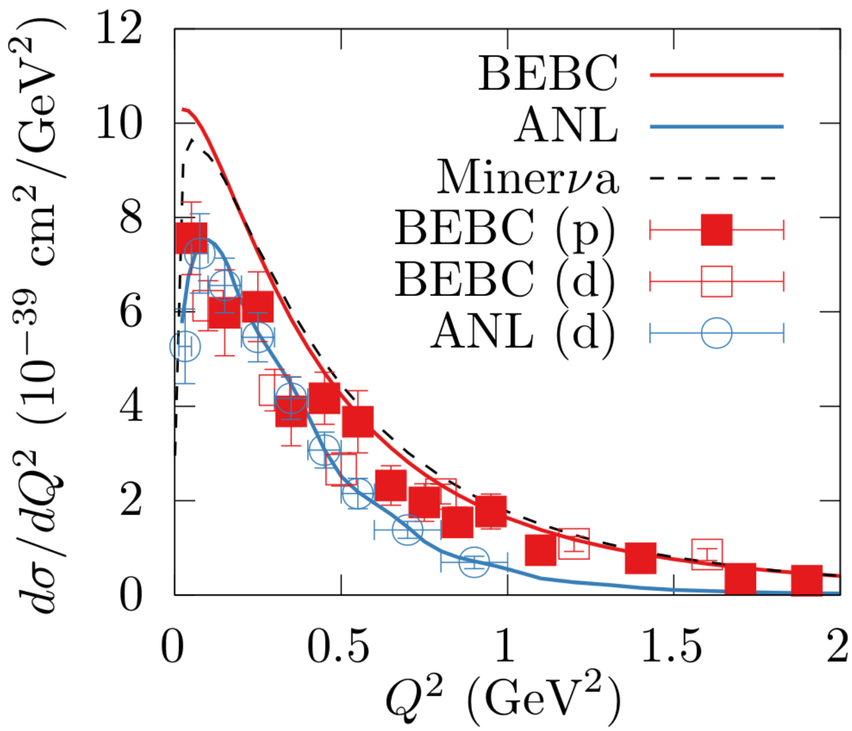


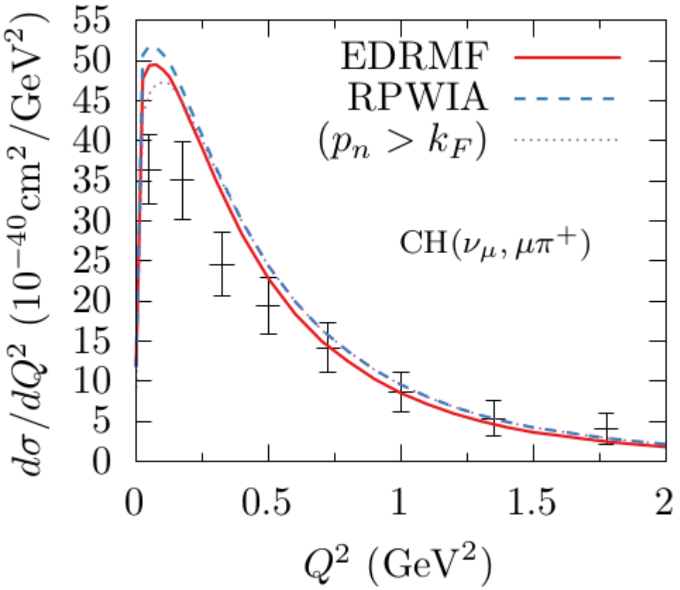
- Find $C_5(0)$ consistent with PCAC
- parametrize $C_5(Q^2)$

Axial coupling to delta from data

[A.N. et al PRD 107 , 053007]

- ANL + BEBC + Minerva comparison





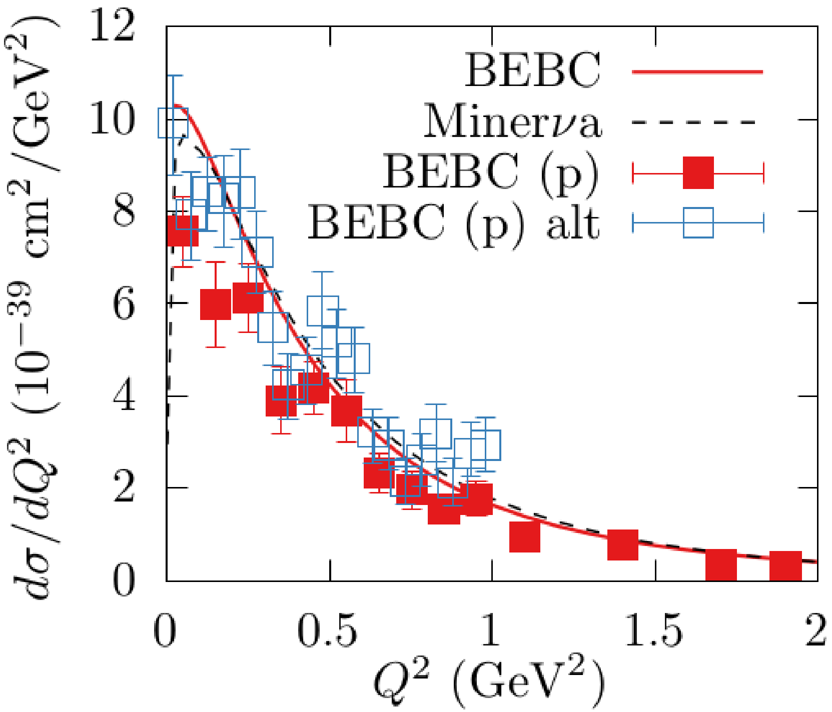
Large uncertainties in data

→ Form factor not well constrained?

Axial coupling to delta from data

[A.N. et al PRD 107 , 053007]

- ANL + BEBC + Minerva comparison



Data from BEBC:

[Z. Phys. C 43, 527–540 (1989)]

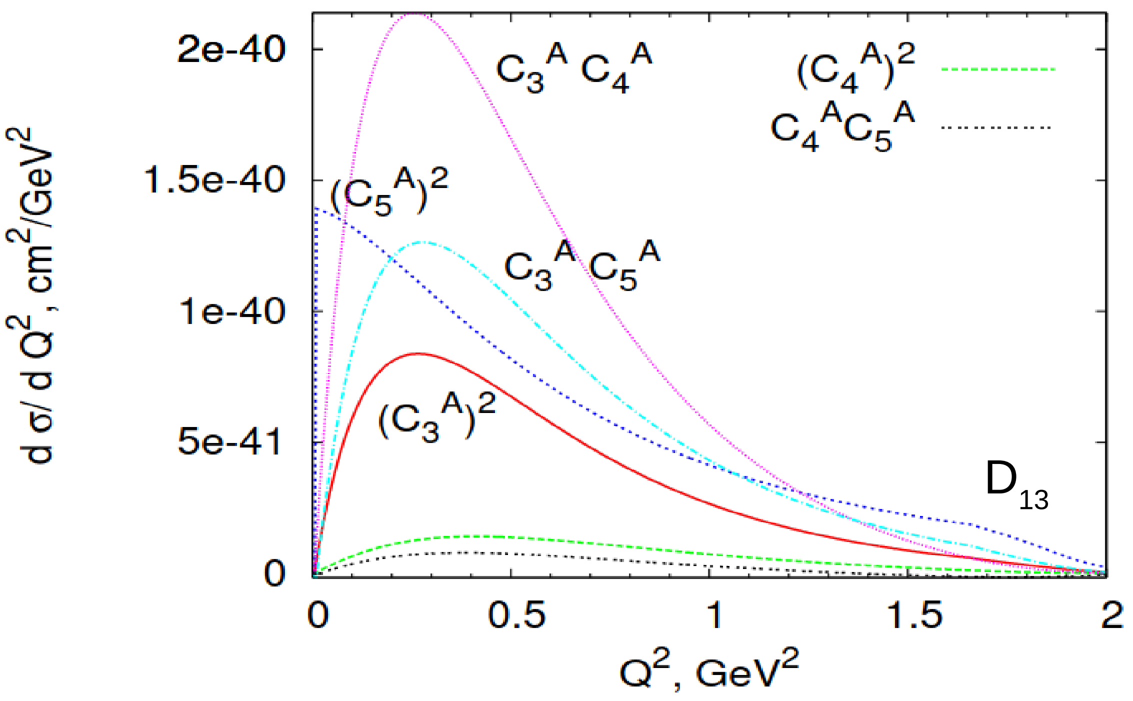
[Nucl.Phys.B 176 (1980) 269]

- related to low-p_N efficiency
- Unclear how correction is done
- Errors likely underestimated?

Axial couplings to higher mass resonances

Lalakulich et al.
[PRD 74, 014009 (2006)]

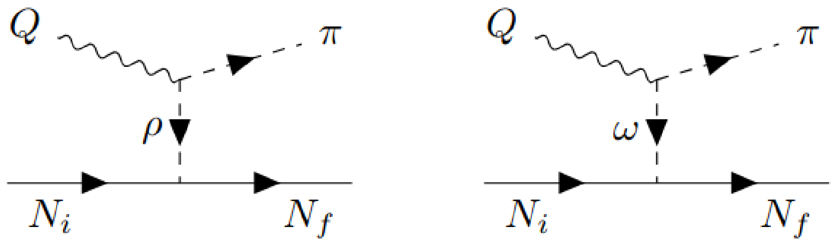
100%
Uncertainty ?



‘Unitarization’ in the delta region: [2603.29486]

M. Hooft,[1] A. Nikolakopoulos,[1,2] J. García-Marcos,[1,3] Y. De Backer,[1]
T. Franco-Munoz,[1] K. Niewczas,[1] R. González-Jiménez,[4] and N. Jachowicz[1]

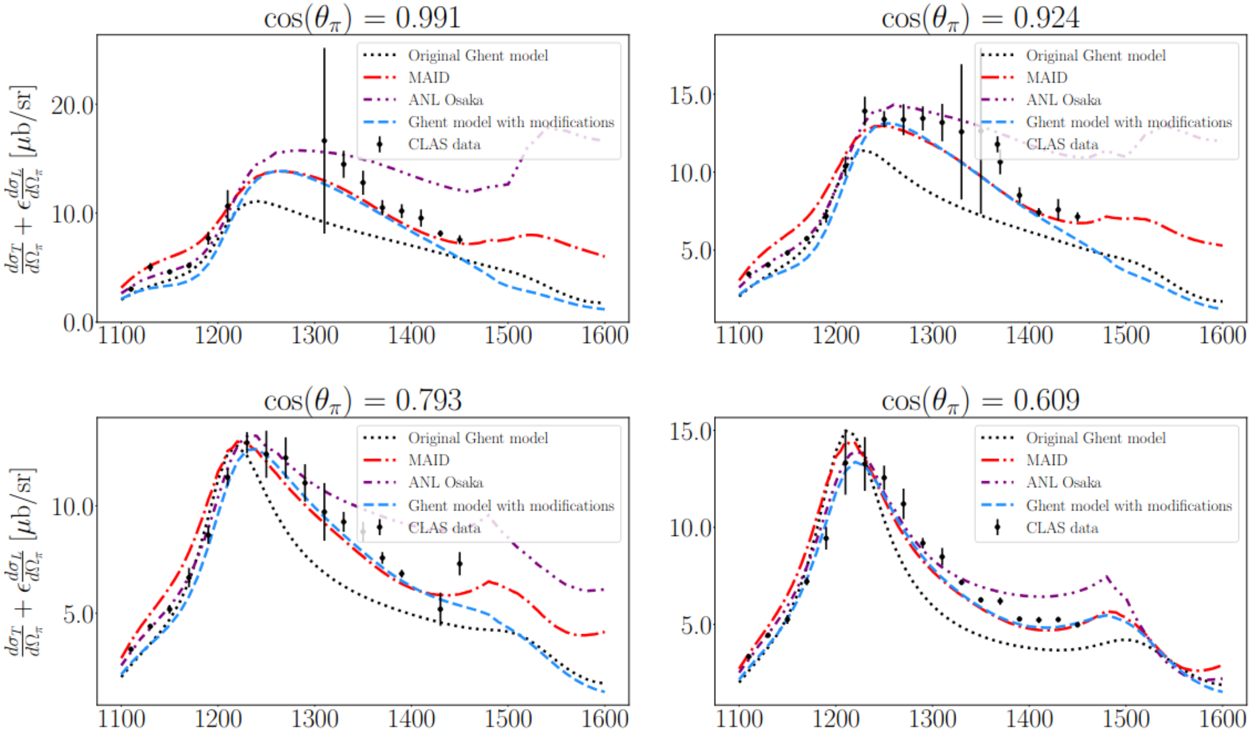
- Add vector meson exchanges:



- Update of delta form factors
- Unitarity constraints through K-matrix approach
- Results for vector current in [2603.29486]

‘Unitarization’ in the delta region: [2603.29486]

N ... ıcker,[1]
T. I ... ıchowicz[1]

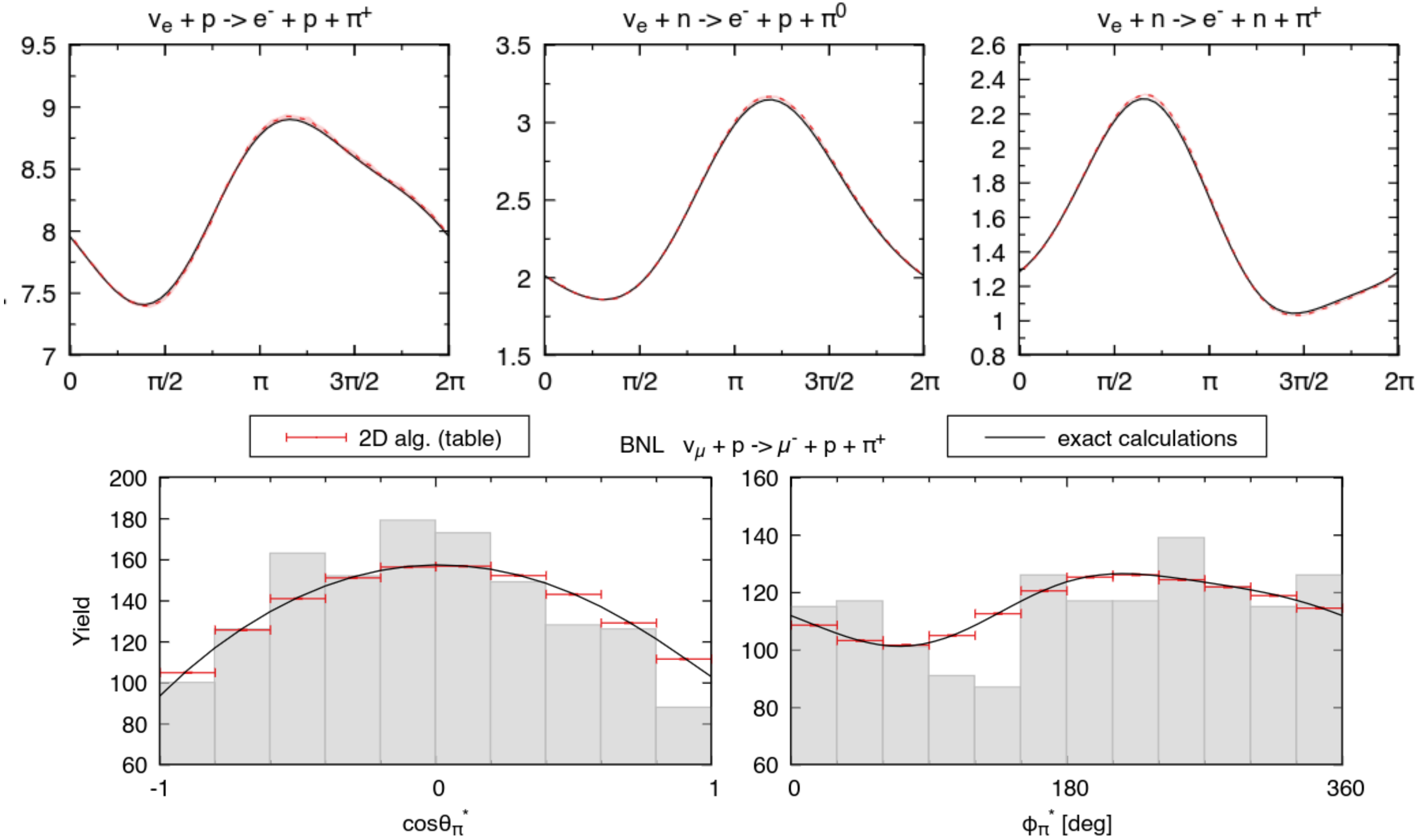


→ Neutrino results coming soon

SPP in event generators

Full implementation of cross section with all interference in NuWro : [Niewczas, PRD 103, 053003 (2021)]

Completely general: works for every model (see also : [Q. Yan et al, JHEP 12 141]



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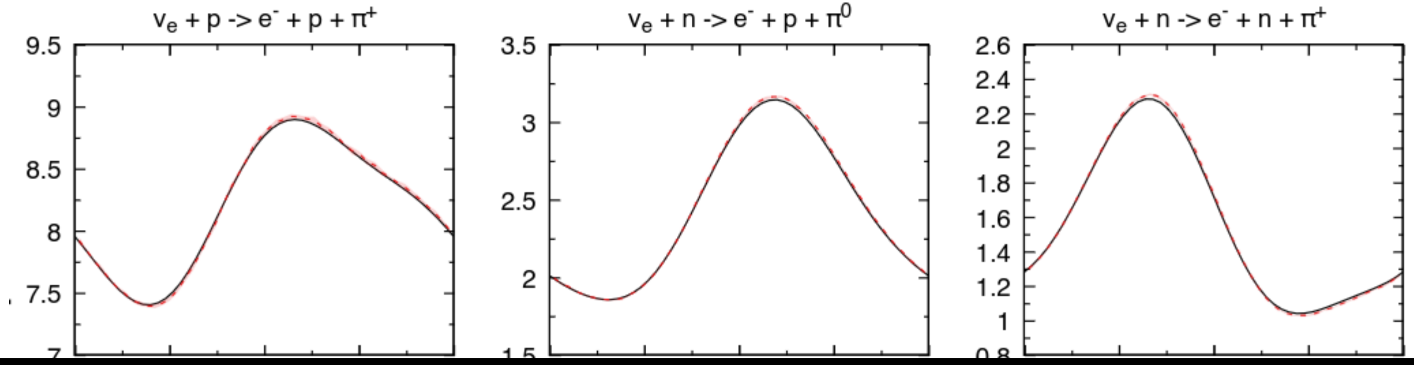
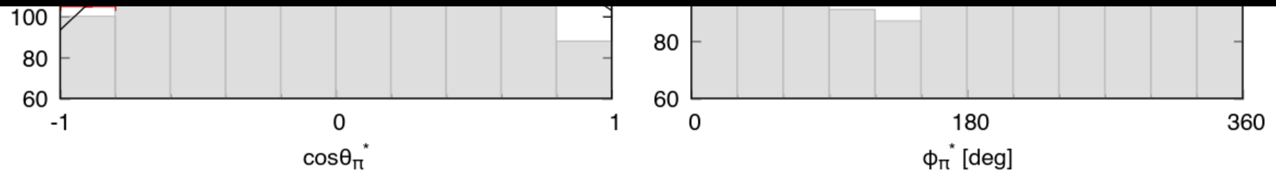


+ ACHILLES [J. Isaacson et al 2508.19213]:

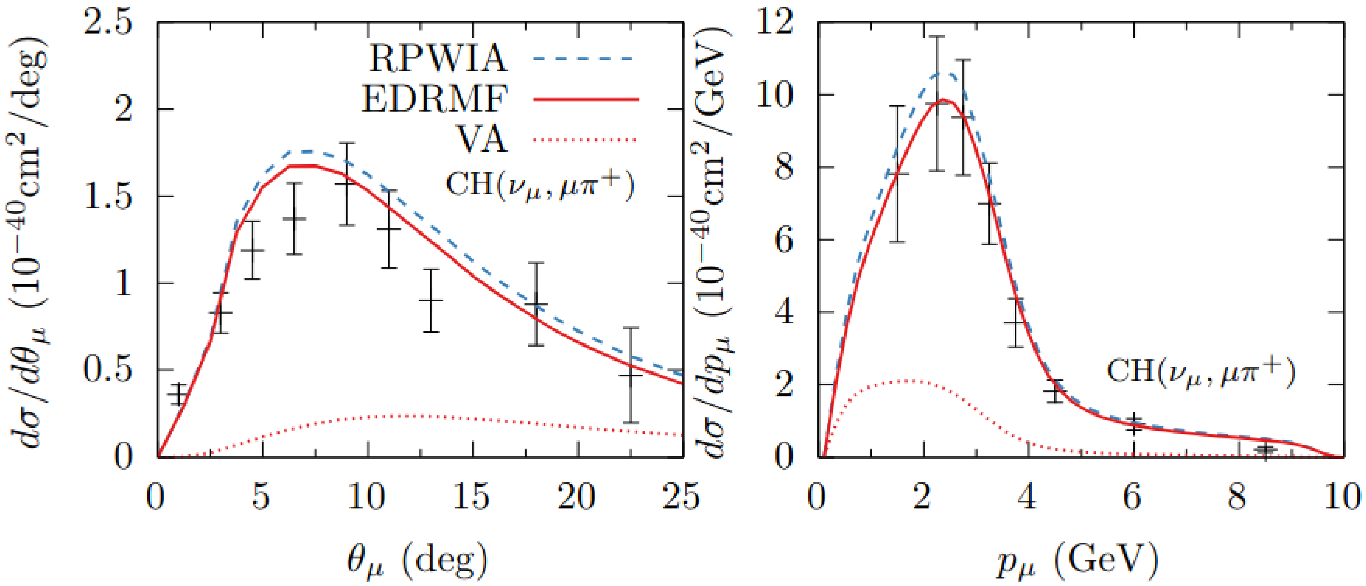
Include SPP fully (ANL-Osaka) + ANL-Osaka MB amplitudes for Cascade

(See Noemi Rocco's talk Friday)



SPP off nuclei

Relativistic mean field modeling: including nucleon final-state interactions



[A.N. et. al 2210.12144] : use 'asymptotic approximation'

[J. Garcia-Marcos et. al 2310.18056] : → include full momentum dependence

J. Garcia Marcos (in preparation) : → include pion final-state interactions

Factorized RDWIA calculations

Factorized distorted wave calculations for electron, neutrino and BSM processes

A. Nikolakopoulos[1,*] and R. González-Jiménez[2,†]

Factorization of **hadron tensor** in **PWIA**

$$H^{\mu\nu}_{PWIA} = h^{\mu\nu}_{s.n.} \times S(E_m, |\mathbf{p}_m|),$$

Does not occur in general

[arxiv:2512.22763]
github.com/alenikolak/Factorized_RDWIA

Factorized distorted wave calculations for electron, neutrino and BSM processes

A. Nikolakopoulos[1,*] and R. González-Jiménez[2,†]

A more general factorization of the amplitude

(still an approximation)

$$J^\mu(\kappa, s, m_j) = \mathrm{Tr}\left[\Gamma^\mu S(\mathbf{t}, \mathbf{k}_N; \kappa, s, m_j)\right]$$

'nucleon physics'

For example (e,e'):

$$\Gamma^\mu = F_1(Q^2)\gamma^\mu - \frac{F_2(Q^2)}{4M_N} Q_\nu \left[\gamma^\mu, \gamma^\nu\right]$$

You can put any Γ here: arbitrary couplings to N

[arxiv:2512.22763]

github.com/alenikolak/Factorized_RDWIA

Factorized distorted wave calculations for electron, neutrino and BSM processes

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(still an approximation)

$$J^\mu(\kappa, s, m_j) = \mathrm{Tr}\left[\Gamma^\mu S(\mathbf{t}, \mathbf{k}_N; \kappa, s, m_j)\right]$$

Nuclear overlaps

$$\left(s\mathbb{1} + p\gamma^5 + V^\mu\gamma^\mu + A^\mu\gamma^\mu\gamma^5 + T^{\mu\nu}\frac{1}{2}\left[\gamma^\mu, \gamma^\nu\right]\right)$$

We calculated and tabulated all overlaps

→ traces can be done analytically/automatically

[arxiv:2512.22763]

github.com/alenikolak/Factorized_RDWIA

Factorized distorted wave calculations for electron, neutrino and BSM processes

A. Nikolakopoulos[1,*] and R. González-Jiménez[2,†]

We calculated and tabulated all overlaps:

$$\left(s\mathbb{1} + p\gamma^5 + V^\mu\gamma^\mu + A^\mu\gamma^\mu\gamma^5 + T^{\mu\nu}\frac{1}{2}\left[\gamma^\mu,\gamma^\nu\right] \right)$$

Can be used to reproduce e.g.
- [R. G. J. et al 1904.10696]
- [A. N. et al. 2406.09244]
- [A. N. et al. 2202.01689]
- R.G.J et al. 2104.01701]
- ...

And much more

Extension to general processes (e.g SPP) underway

[arxiv:2512.22763]

github.com/alenikolak/Factorized_RDWIA

A caveat : operator ambiguities

The *ansatz* for relating free-nucleon matrix elements to nuclear matrix elements:

$$\langle\beta|\hat{J}_\mu(\mathbf{q})|\alpha\rangle \rightarrow \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\overline{\phi}_\beta(\mathbf{p}+\mathbf{q})\left[\sum_i F_i(Q^2)\Gamma_i^\mu\right]\phi_\alpha(\mathbf{p})$$

Lead to non-unique results depending on the arbitrary parametrization of Γ^μ

A r

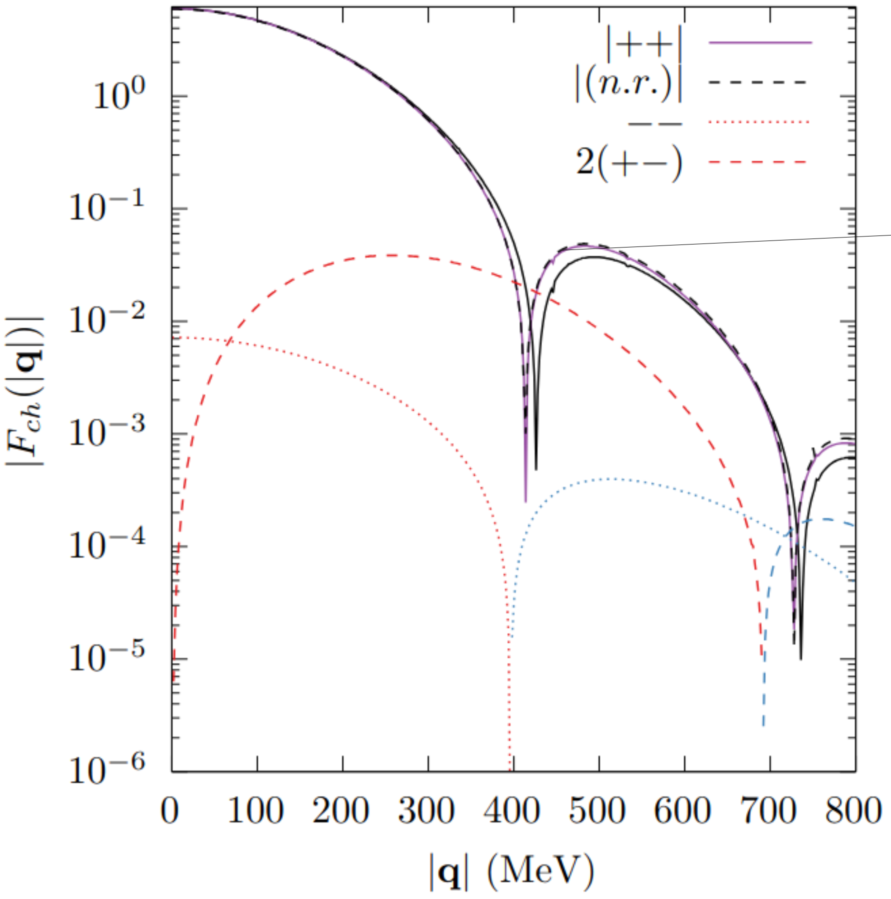
Explicitly on-shell currents in relativistic mean field models

Alexis Nikolakopoulos[1] and Ryan Plestid[2]

[1] *Department of Physics, University of Washington, Seattle, Washington 98195, USA*
[2] *CERN, 1 Esplanade des Particules, CH-1211 Geneva 23, Switzerland*

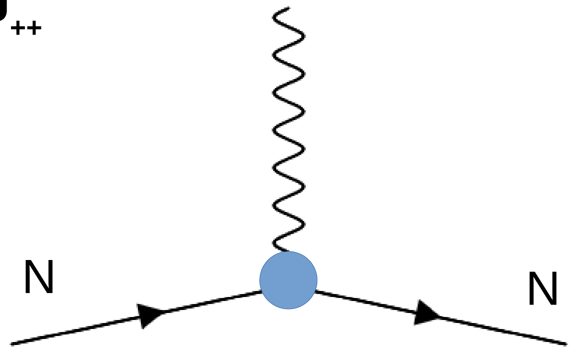
(Dated: August 3, 2026)

Elastic electron scattering [arxiv:2607.29634]



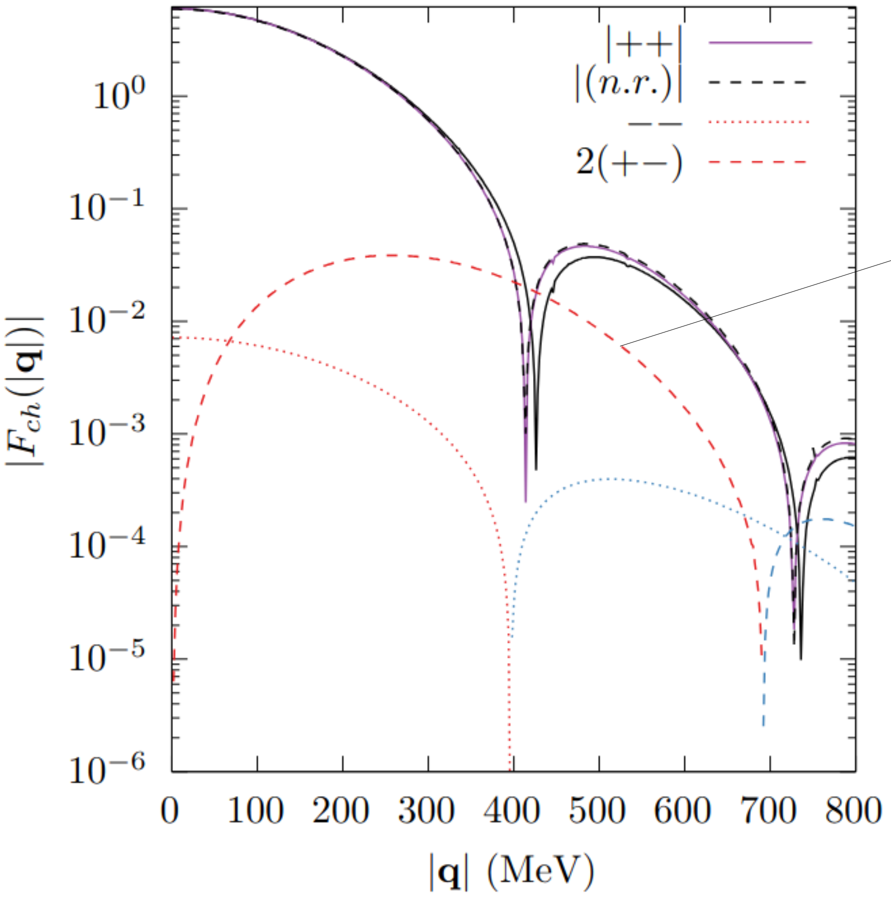
$$\rightarrow \int \frac{d^3\mathbf{p}}{(2\pi)^3} \bar{\phi}_\beta(\mathbf{p}+\mathbf{q}) \left[\sum_i F_i(Q^2)\Gamma_i^\mu \right] \phi_\alpha(\mathbf{p})$$

$\mathbf{J}_{++}$

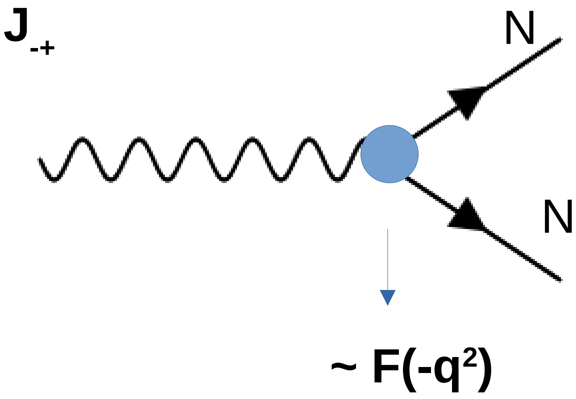


→ can be mapped to textbook non-relativistic result $J_{(n.r.)}$

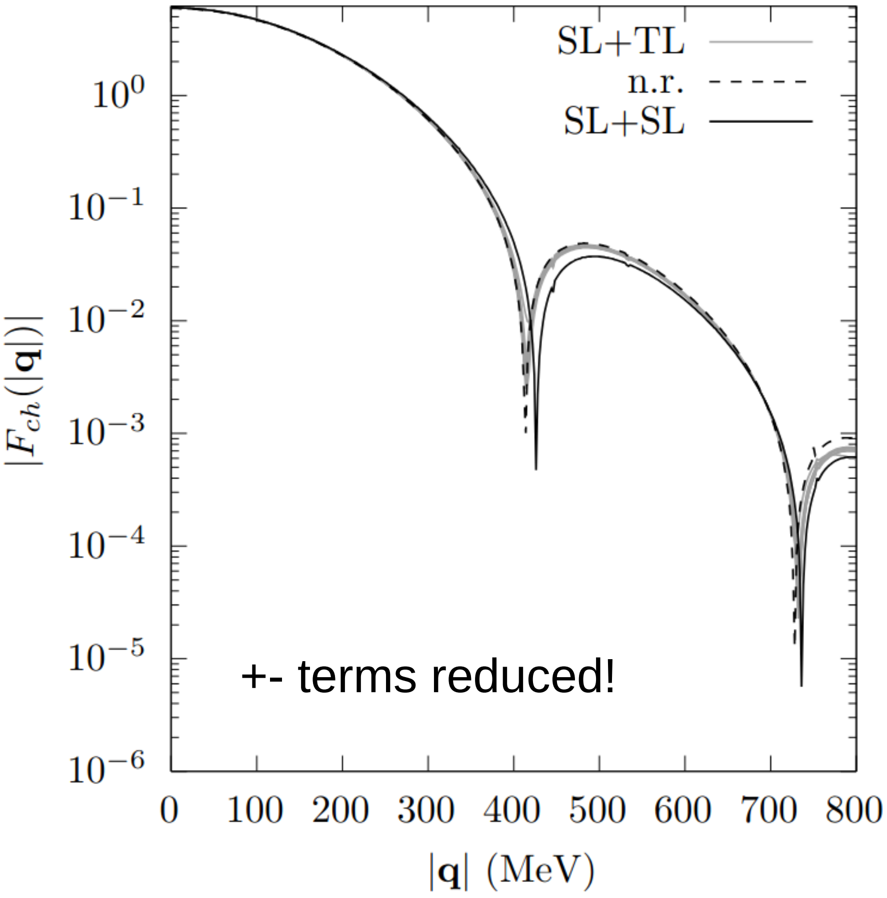
Elastic electron scattering [arxiv:2607.29634]



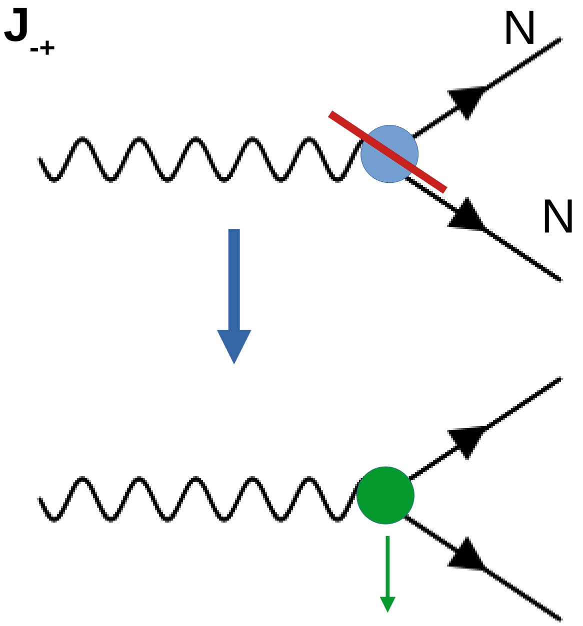
$$\to \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\,\bar{\phi}_\beta(\mathbf{p}+\mathbf{q})\left[\sum_i F_i(Q^2)\Gamma_i^\mu\right]\phi_\alpha(\mathbf{p})$$



Elastic electron scattering [arxiv:2607.29634]



$$\cancel{\rightarrow \int \frac{d^3\mathbf{p}}{(2\pi)^3}\bar{\phi}_\beta(\mathbf{p}+\mathbf{q})\left[\sum_i F_i(Q^2)\Gamma_i^\mu\right]\phi_\alpha(\mathbf{p})}$$

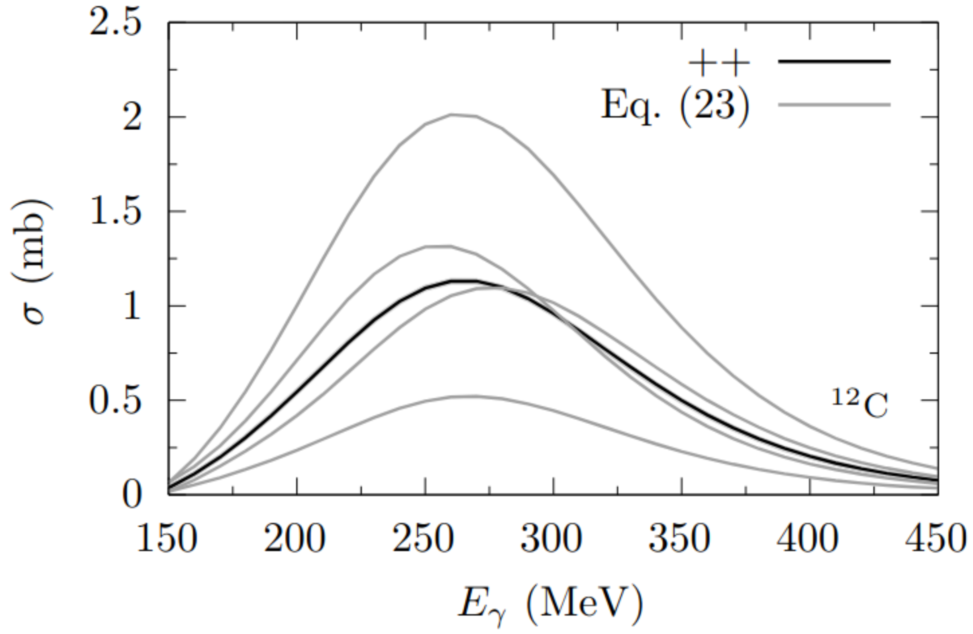


$\sim F(4M_N^2 + q^2)$

→ **timelike region**

Coherent pion production [arxiv:2607.29634]

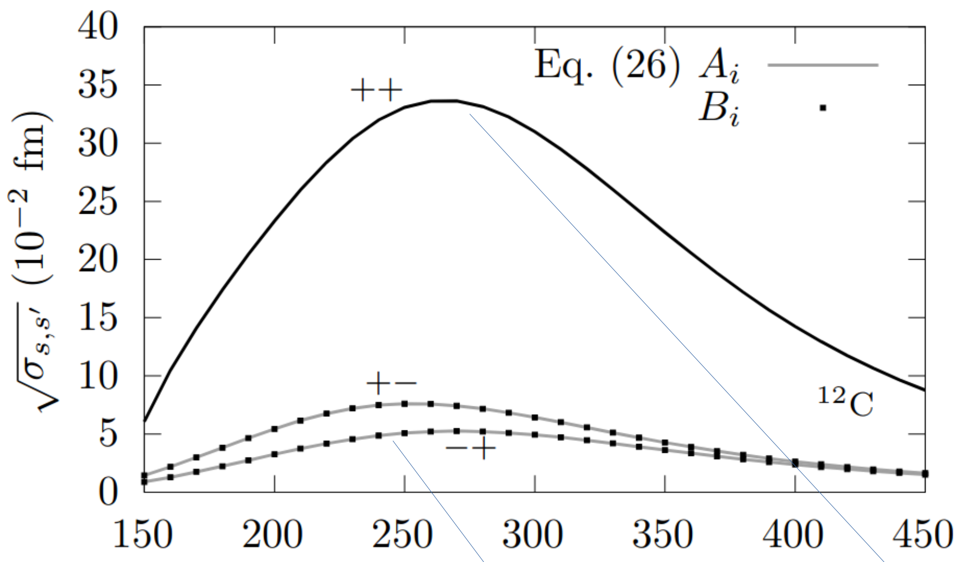
$$\gamma(\mathbf{q}) + A(\mathbf{0}) \to A(\mathbf{t} = \mathbf{k}_\pi - \mathbf{q}) + \pi(\mathbf{k}_\pi).$$



$$\int \frac{d^3\mathbf{p}}{(2\pi)^3} \bar{\phi}_\beta(\mathbf{p}+\mathbf{q}) \left[\sum_i F_i(Q^2) \Gamma_i^\mu \right] \phi_\alpha(\mathbf{p})$$

Large ambiguities!

Coherent pion production [arxiv:2607.29634]



$$\cancel{\int \frac{d^3\mathbf{p}}{(2\pi)^3}\bar{\phi}_\beta(\mathbf{p}+\mathbf{q})\left[\sum_i F_i(Q^2)\Gamma_i^\mu\right]\phi_\alpha(\mathbf{p})}$$

~~**Large ambiguities!**~~

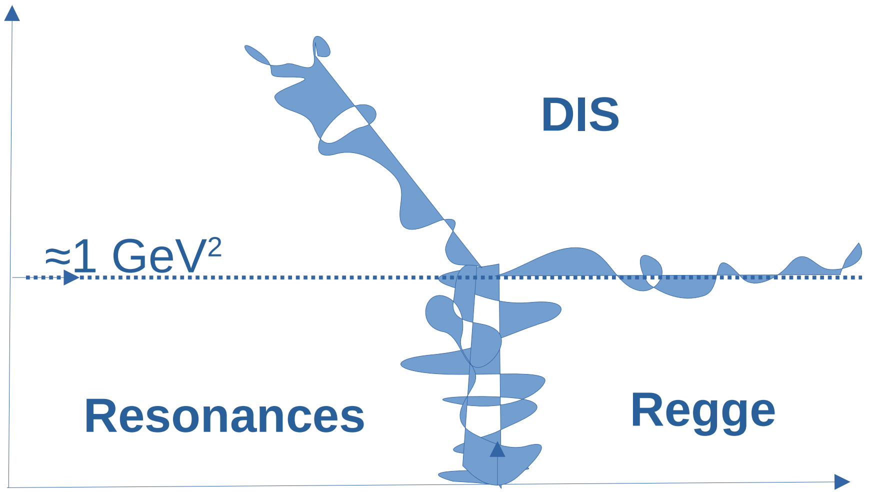
Unique results

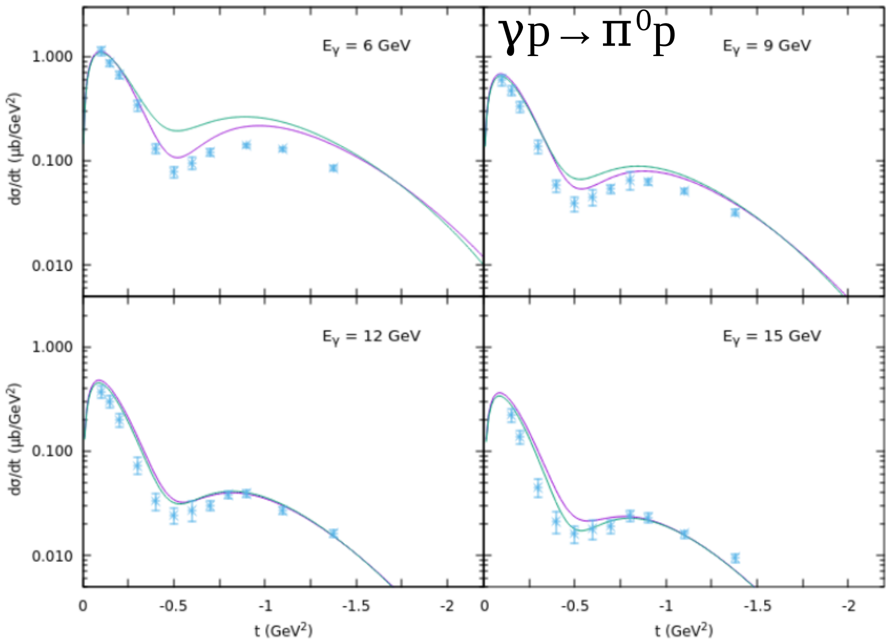
$$\langle\beta|\hat{J}^{(1)}_\mu|\alpha\rangle = \int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_p}\frac{d^3p'}{(2\pi)^3}\frac{1}{2E_{p'}}\tilde{\phi}^\dagger_\beta(\mathbf{p}')\Big[(\not{p}'+m)\Gamma^{(a)}_\mu(\mathbf{p}',\mathbf{p})(\not{p}+m)\gamma_0 + (\tilde{\not{p}}'-m)\Gamma^{(b)}_\mu(-\mathbf{p}',-\mathbf{p})(\tilde{\not{p}}-m)\gamma_0$$
$$+ (\tilde{\not{p}}'-m)\Gamma^{(c)}_\mu(-\mathbf{p}',\mathbf{p})(\not{p}+m)\gamma_0 + (\not{p}'+m)\Gamma^{(d)}_\mu(\mathbf{p}',-\mathbf{p})(\tilde{\not{p}}-m)\gamma_0\Big]\tilde{\phi}_\alpha(\mathbf{p})\ .$$

Thank you!

SPP in the Regge region

[H. Damien, Masters thesis, UGent (2020)]



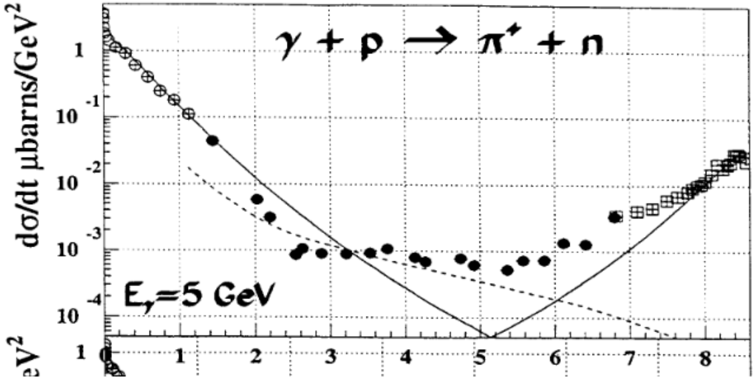


For high-s
Small -t

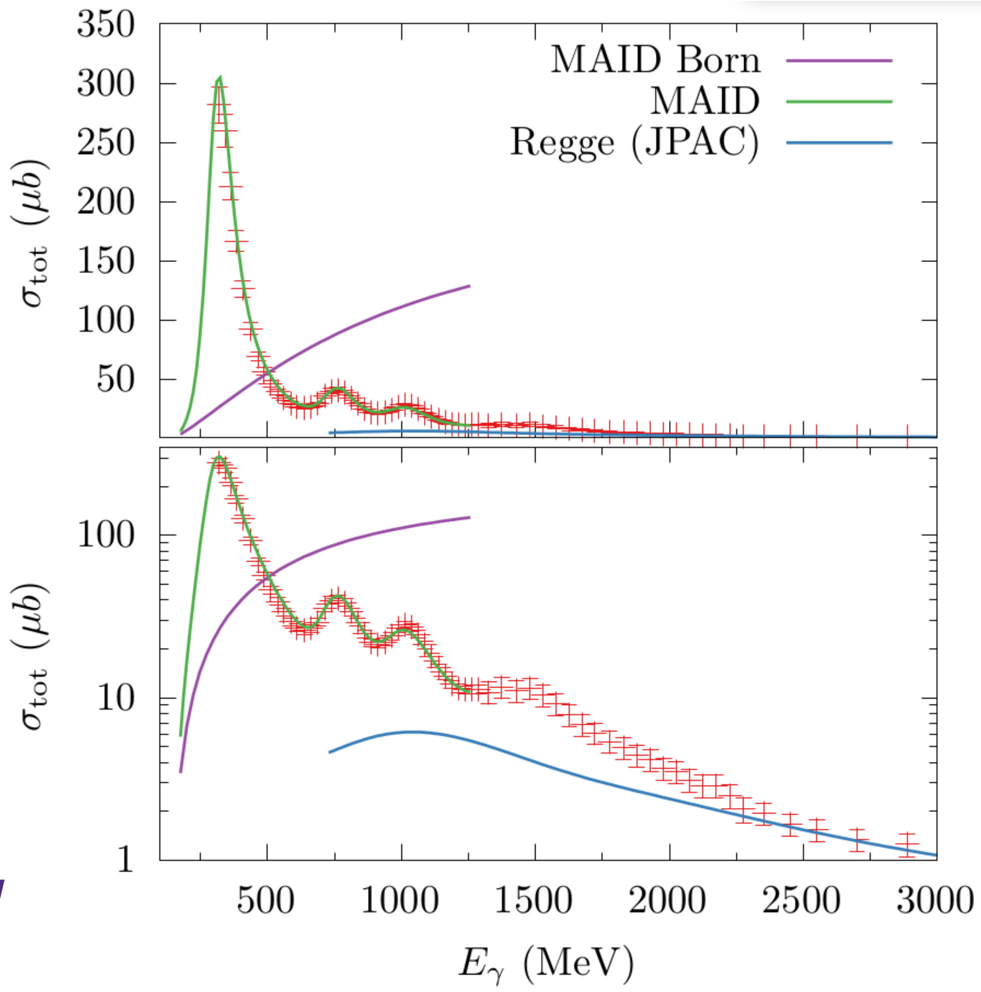
t-channel d.o.f

Peak at forward angles

Photoproduction



[Guidal, Laget, Vanderhaeghen]

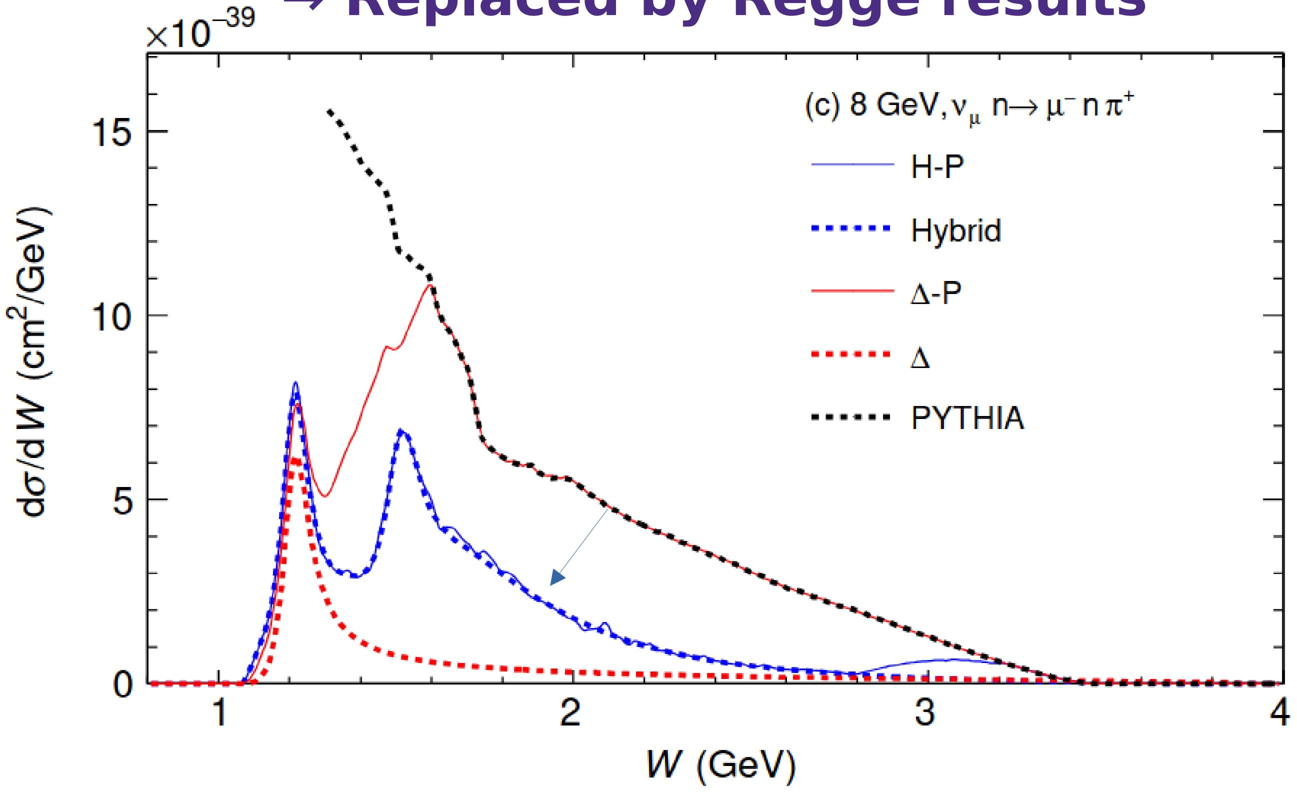


Peak dominates at Large W, low Q^2

→ Saturates cross section at large W

Hybrid model in NuWro: [Q. Yan et al., JHEP 12 141]

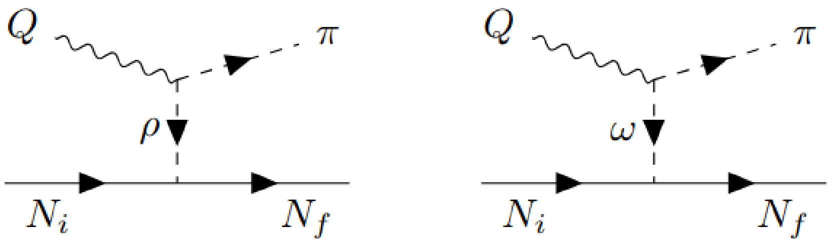
Remove the PYTHIA contribution in 1pion channel
→ Replaced by Regge results



'Unitarization' in the delta region: [2603.29486]

M. Hooft,[1] A. Nikolakopoulos,[1,2] J. García-Marcos,[1,3] Y. De Backer,[1]
T. Franco-Munoz,[1] K. Niewczas,[1] R. González-Jiménez,[4] and N. Jachowicz[1]

Recent work from Matthias Hooft (Phd student Ghent):



- Add vector meson exchanges:
- Update of delta form factors
- **Unitarity constraints through K-matrix approach**
- Results for vector current and formalism in [2603.29486]

→ Neutrino results coming soon

Unitarity constraints

$$\int d\Omega' \sum_{\lambda',\alpha} {}_F\langle \Omega_F, \lambda_F | \tilde{T} | \Omega', \lambda' \rangle^*_{\alpha} \, {}_{\alpha}\langle \Omega', \lambda' | \tilde{T} | \Omega_I, \lambda_I \rangle_I = -\,\mathrm{Im}\left\{ {}_F\langle \Omega_F, \lambda_F | \tilde{T} | \Omega_I, \lambda_I \rangle_I \right\} \in \mathbb{R}.$$

Below 2π threshold:
Watson's theorem

$$ {}_{\pi N}\langle J, l, s, I | \tilde{T} | J, l, s, I \rangle^*_{\pi N} \times {}_{\pi N}\langle J, l, s, I | \tilde{T} | J, L, S, I \rangle_{\gamma N} \in \mathbb{R}.$$

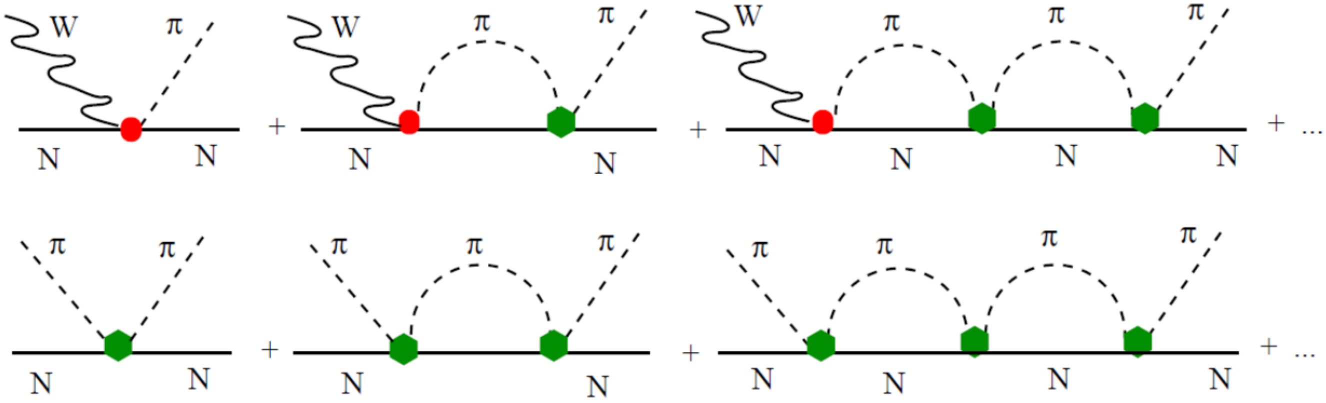


Figure from Talk J. Nieves 2017, KEK

Unitarity constraints and K-matrix

$$(\tilde{T}^{-1} + i\mathbb{1})^{\dagger} = (\tilde{T}^{-1} + i\mathbb{1}). \longrightarrow \tilde{K}^{-1} \equiv \tilde{T}^{-1} + i\mathbb{1}$$

Below 2π threshold

$$T_{\gamma\pi}^{J,l,I} = K_{\gamma\pi}^{J,l,I}(1 + i\rho_{\pi} T_{\pi\pi}^{J,l,I}).$$

Unitarity constraints and K-matrix

$$(\tilde{T}^{-1} + i\mathbb{1})^\dagger = (\tilde{T}^{-1} + i\mathbb{1}). \longrightarrow \tilde{K}^{-1} \equiv \tilde{T}^{-1} + i\mathbb{1}$$

Below 2π threshold

$$T_{\gamma\pi}^{J,l,I} = K_{\gamma\pi}^{J,l,I}(1 + i\rho_\pi T_{\pi\pi}^{J,l,I}).$$

Modeling (BG):

$$\approx T_{0,\mathrm{tree}}^{J,l,I}$$

From πN scattering
(we used JuBo analysis)

Note: this doesn't introduce 'parameters'

T_0 \approx K

$$T_{\pi\gamma}(k_\pi, q; W) = T^0_{\pi\gamma}(k_\pi, q; W) + \int dp \, \frac{p^2 T_{\pi\pi}(k_\pi, p; W) T^0_{\pi\gamma}(p, q; W)}{W - E_\pi(p) - E_N(p) + i\epsilon},$$

$\downarrow$ Neglect principal value integral

$$T_{\pi\gamma} = T^0_{\pi\gamma} + i\rho_\pi T_{\pi\pi} T^0_{\pi\gamma},$$

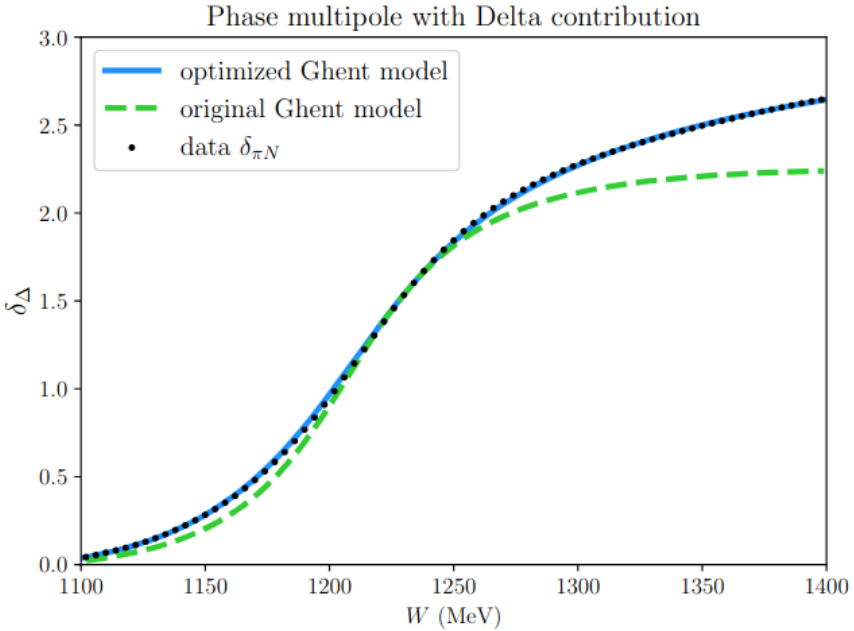
$$T_{\pi\gamma} = K_{\pi\gamma}\left(1 + i\rho_\pi T_{\pi\pi}\right),$$

Unitarity for width and phase of Delta

$$S_{\Delta,\alpha\beta} = \frac{\not{k}_\Delta + M_\Delta}{k_\Delta^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta(W)}\mathcal{P}_{\alpha\beta},$$

Width determined by πN amplitudes

$$\Gamma_\Delta(W) = \frac{M_\Delta^2 - s}{M_\Delta}\tan(\delta_{\pi N}(W)),$$



Note: this removes 'parameters'

Gives correct form of T-matrix

But RES-BG separation is model-dependent

$$K_{\gamma\pi}^{\Delta} = \frac{A_\Delta}{W^2 - M_\Delta^2} + B_\Delta,$$

$$K_{\pi\pi}^{\Delta} = \frac{C_\Delta}{W^2 - M_\Delta^2} + D_\Delta,$$

$$T_{\gamma\pi}^{\Delta} = \left(\frac{A_\Delta}{W^2 - M_\Delta^2} + B_\Delta \right) \times \frac{(W^2 - M_\Delta^2)}{W^2 - M_\Delta^2 - i[C_\Delta + D_\Delta(W^2 - M_\Delta^2)]}.$$