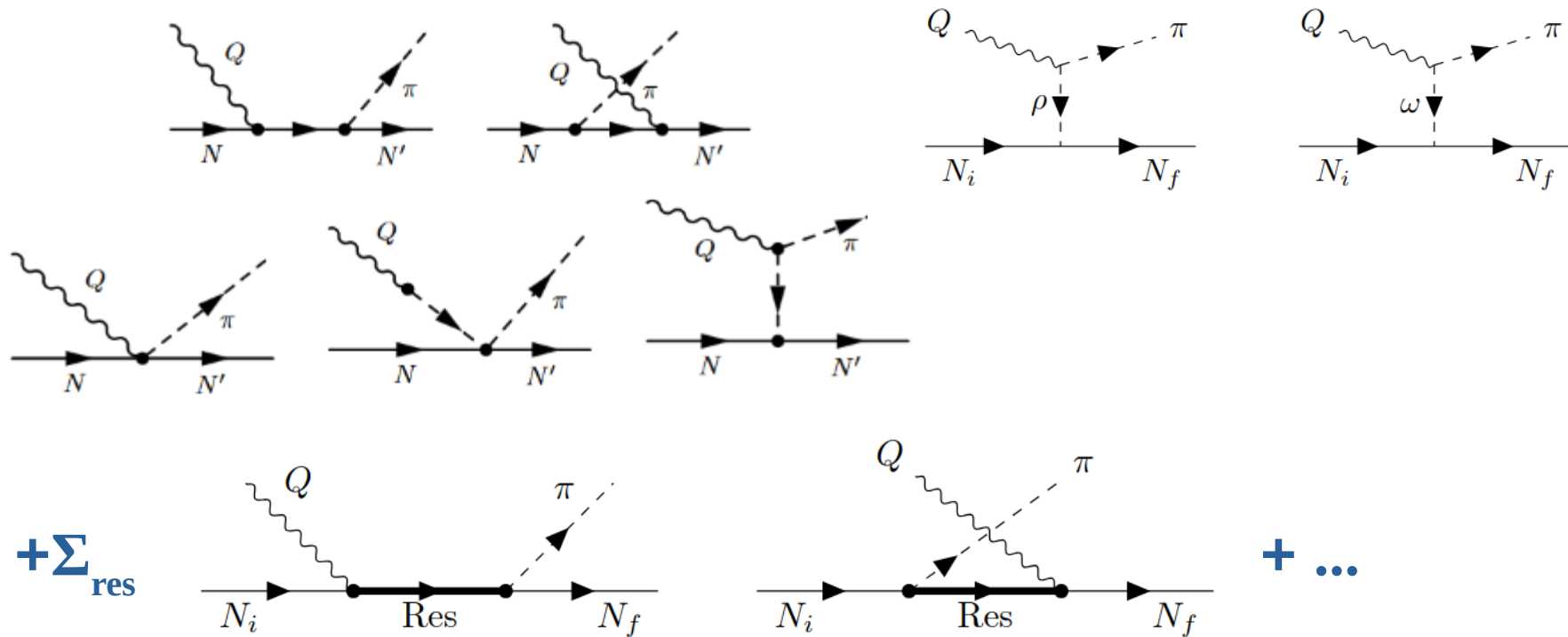


Neutrino induced pion production

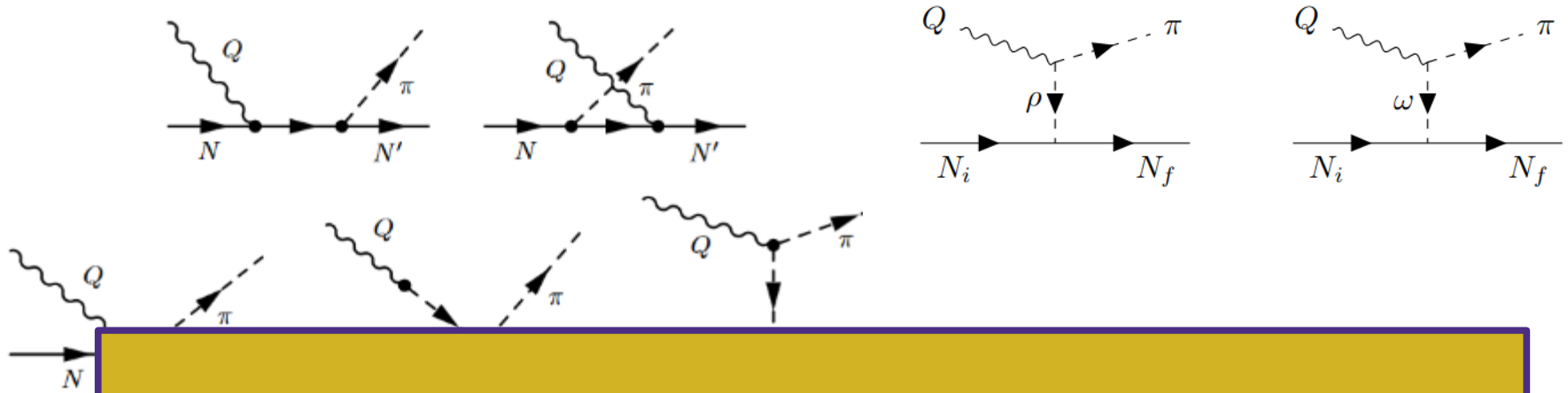
- > Alexis Nikolakopoulos
- > Neutrino Theory Network (NTN) Fellow
- > University of Washington



Hadronic modeling of SPP



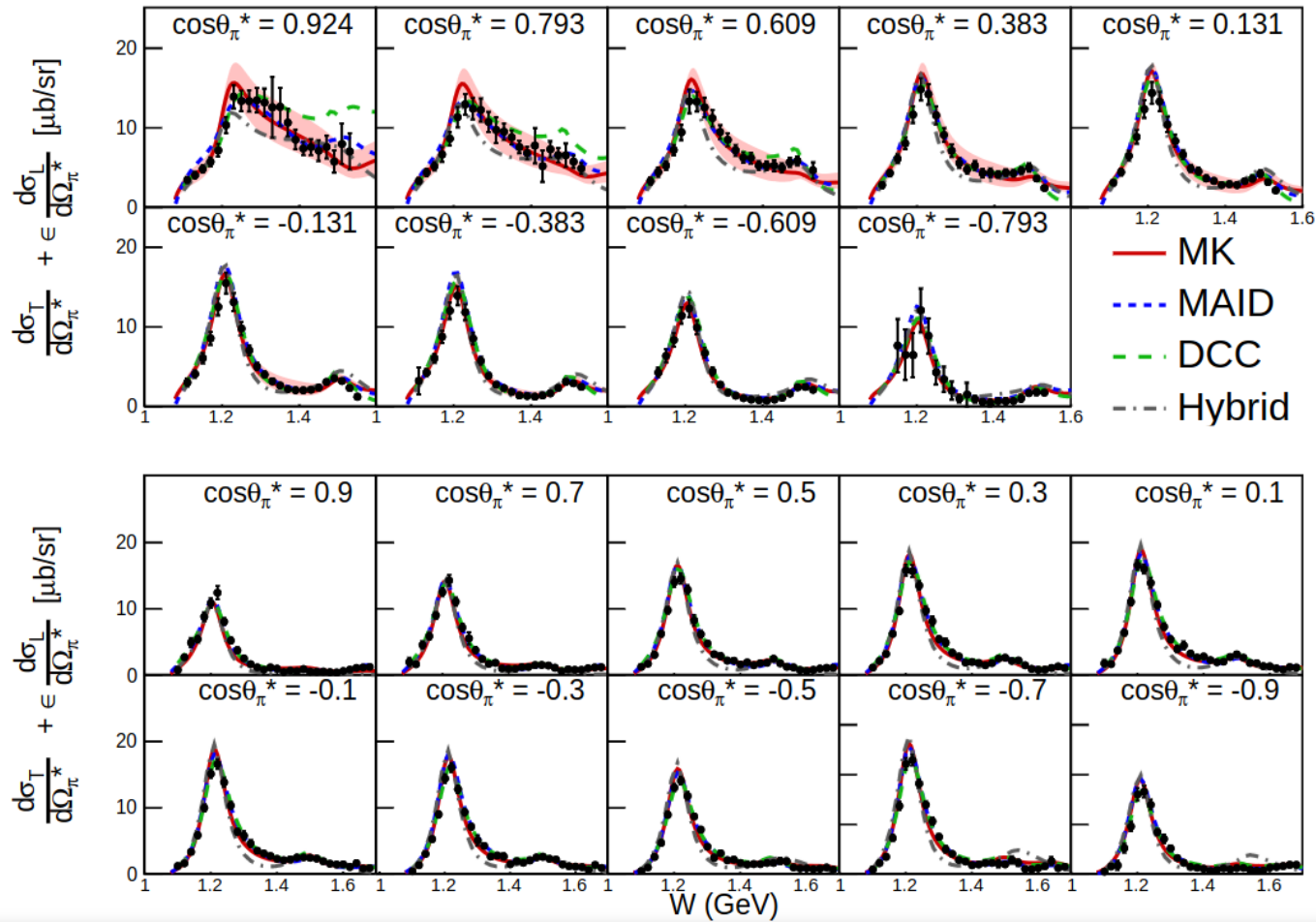
Hadronic modeling of SPP



A lot of *effective* couplings/propagators that need to be determined!

+Σ

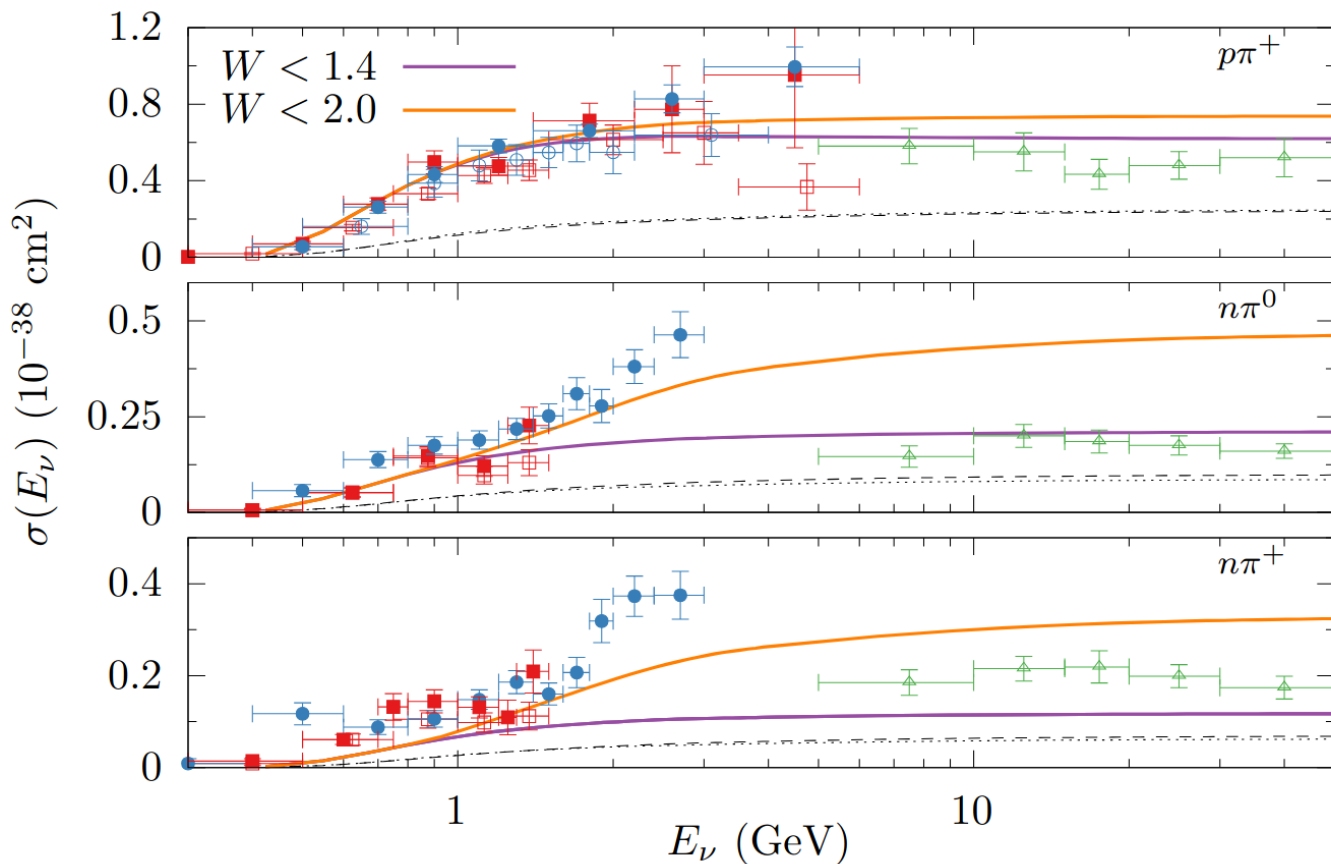
Lot of electromagnetic interaction data



From [Kabirnezhad,
2203.15594]

Neutrino data: much less abundant

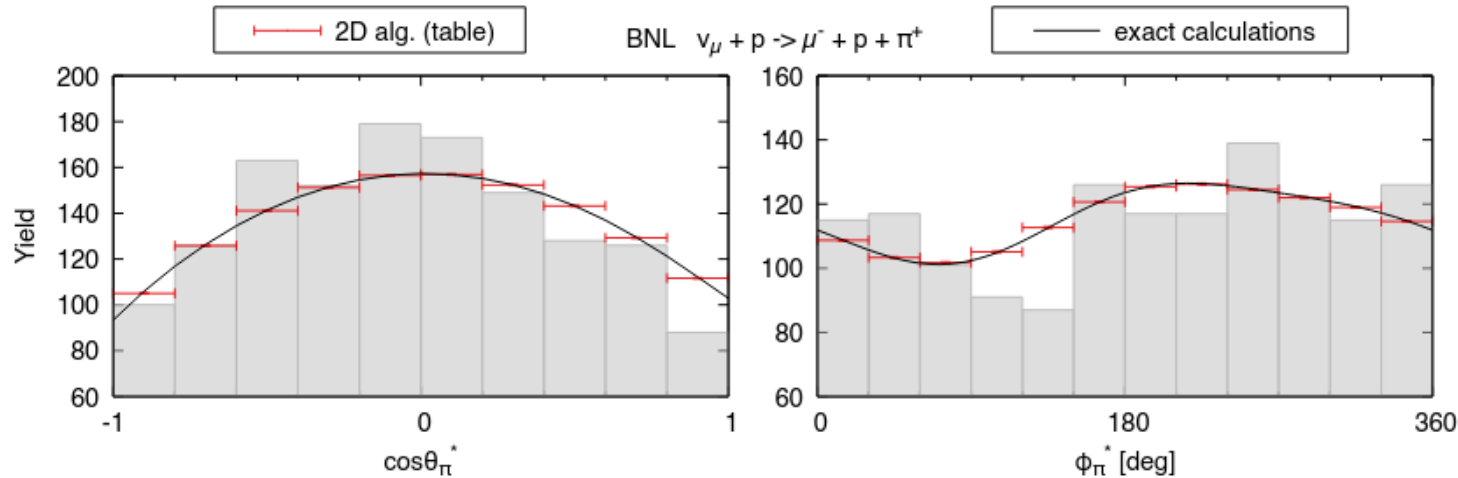
Bubble chamber data on deuterium (proton) from **ANL**/**BNL**/**BEBC**



Neutrino interaction data

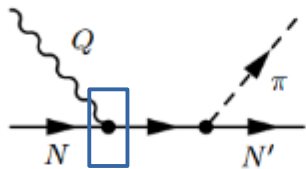
Bubble chamber data on deuterium (proton) from ANL/BNL/BEBC

- Observables integrated over large phase space
- Sometimes shape-only



+ Data on nuclei (^{12}C , ^{16}O , ^{40}Ar) from Minerva, T2K, MiniBooNE, MicroBooNE, ...

Data determines couplings to resonances



$$A_{1/2} = \sqrt{\frac{2\pi\alpha}{K}} \frac{1}{e} \langle S_{z,R} = \frac{1}{2} | \epsilon_{\mu}^{(+)}(q) J^{\mu} | S_{z,N} = -\frac{1}{2} \rangle$$

$$A_{3/2} = \sqrt{\frac{2\pi\alpha}{K}} \frac{1}{e} \langle S_{z,R} = \frac{3}{2} | \epsilon_{\mu}^{(+)}(q) J^{\mu} | S_{z,N} = \frac{1}{2} \rangle$$

$$S_{1/2} = \sqrt{\frac{2\pi\alpha}{K}} \frac{1}{e} \langle S_{z,R} = \frac{1}{2} | \frac{|\vec{q}|}{\sqrt{Q^2}} \epsilon_{\mu}^{(0)}(q) J^{\mu} | S_{z,N} = \frac{1}{2} \rangle$$

Reduced helicity amplitudes at $W = M_R$

[userweb.jlab.org/~mokeev/resonance_electrocouplings]

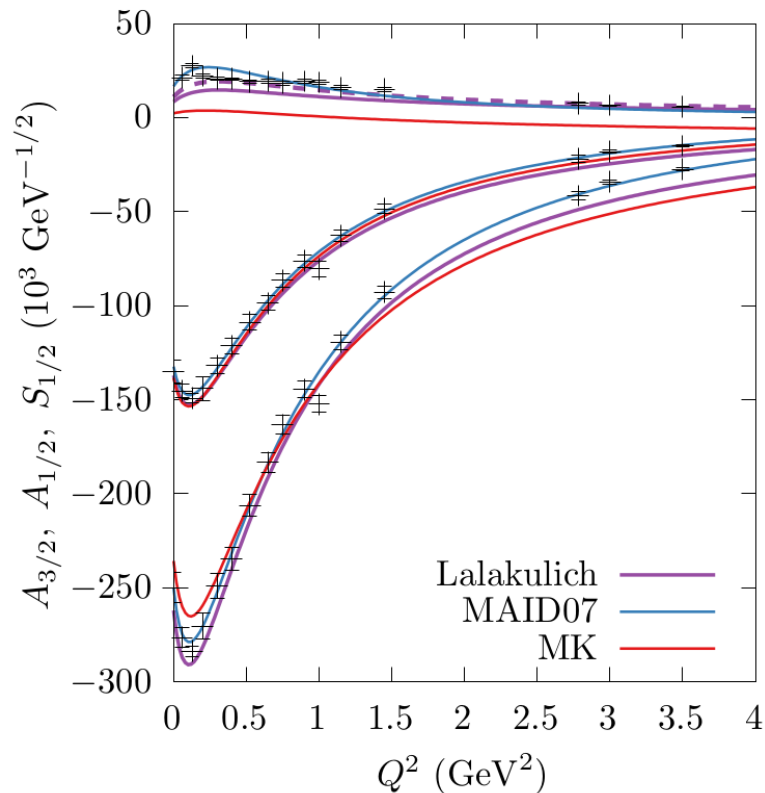
Helicity amplitudes from CLAS/MAID07 analyses used in:

Lalakulich et al [Phys. Rev. D74, 014009 (2006)]

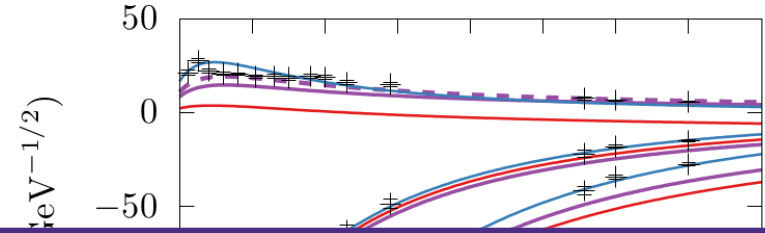
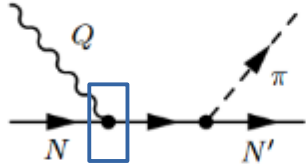
Hernandez et al. [Phys Rev D 77 053009 (2008)]

Nikolakopoulos et al. [arxiv:2210.12144 (2022)]

....



Data determines couplings to resonances



Neutrino-induced charged-current interactions:

1.) Isovector amplitudes

2) Axial amplitudes

Hernandez et al. [Phys Rev D 77 053009 (2008)]

Nikolakopoulos et al. [arxiv:2210.12144 (2022)]

....

Neutrino: Isovector form factors

$$J_{EM}^\mu = V_s^\mu + \mathbf{V}^\mu$$

$$J_{CC\pm}^\mu = \mathbf{V}^\mu - \mathbf{A}^\mu$$

$$\langle \pi^+ n | J_{EM}^\mu | p \rangle = V_{3/2}^\mu - \sqrt{2} \left(V_{1/2}^\mu + S^\mu \right)$$

$$\langle \pi^- p | J_{EM}^\mu | n \rangle = V_{3/2}^\mu - \sqrt{2} \left(V_{1/2}^\mu - S^\mu \right)$$

$$\langle \pi^0 p | J_{EM}^\mu | p \rangle = \sqrt{2} V_{3/2}^\mu + \left(V_{1/2}^\mu + S^\mu \right)$$

$$\langle \pi^0 n | J_{EM}^\mu | n \rangle = \sqrt{2} V_{3/2}^\mu + \left(V_{1/2}^\mu - S^\mu \right)$$

$\mathbf{F}_p$

$\mathbf{F}_n$

$$\langle \pi^+ p | V_+^\mu | p \rangle = 3V_{3/2}^\mu,$$

$$\langle \pi^+ n | V_+^\mu | n \rangle = V_{3/2}^\mu + 2\sqrt{2}V_{1/2}^\mu,$$

$$\langle \pi^0 p | V_+^\mu | n \rangle = -\sqrt{2}V_{3/2}^\mu + 2V_{1/2}^\mu.$$

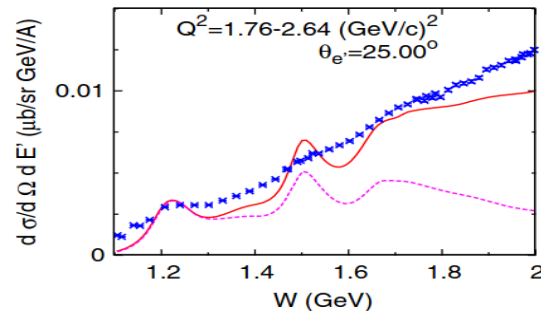
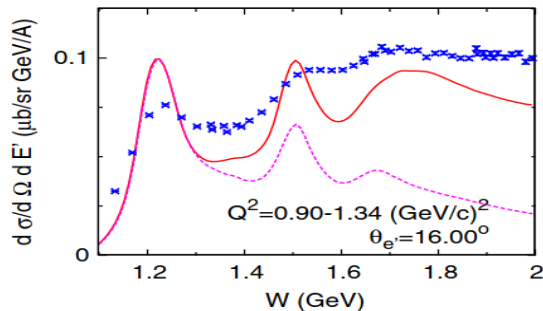
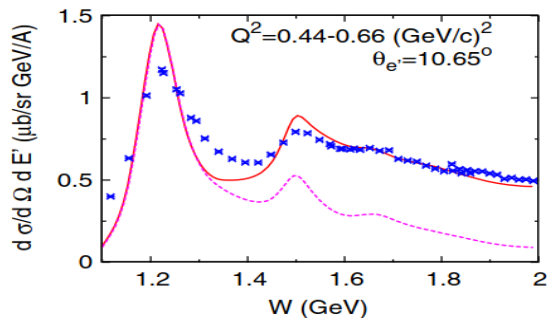
$$\mathbf{F}_v = \mathbf{F}_p - \mathbf{F}_n$$

Neutron (deuteron) target data much less abundant

Neutrino: Isovector form factors

ANL-Osaka Dynamic Coupled Channels (DCC) [PRD 92, 074024 (2015)]

- Fit 'neutron' target exclusive data (deuteron)
- Fit inclusive structure function on deuteron

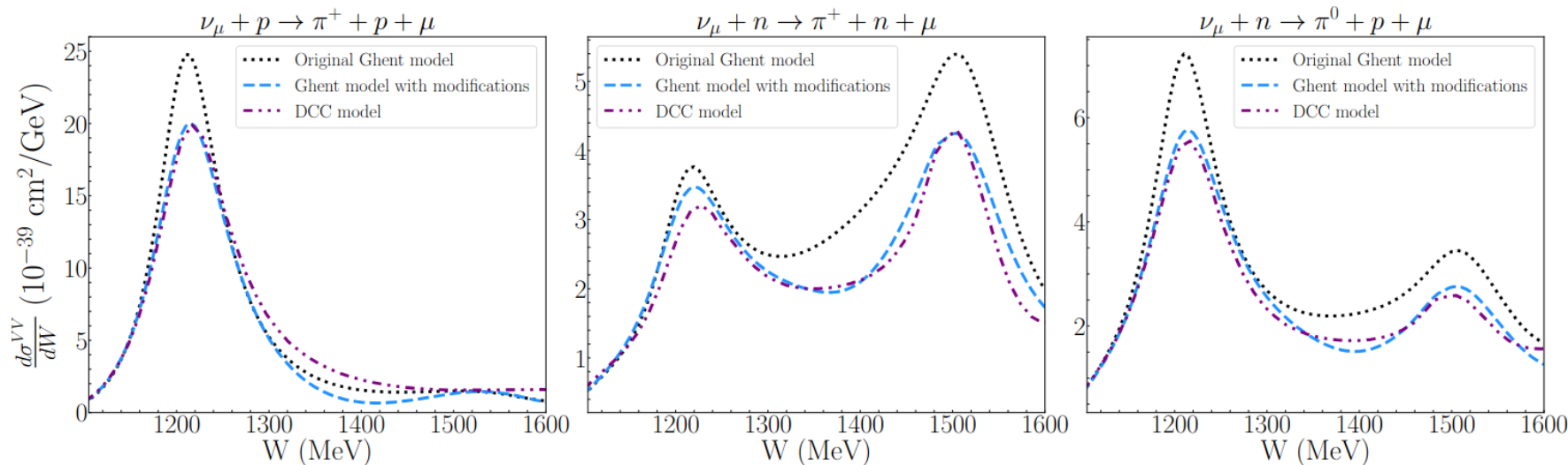


- Progress on **deuteron FSI** e.g. [PRD 99 031301]
- **data** e.g. CLAS [Phys. Rev. C 107, 015201 (2023)] over large W -region

VV contribution to neutrino scattering

$$\sigma = \sigma_{VV} + \sigma_{AA} + \sigma_{VA}$$

→ Averaged over BEBC experiment energy distribution:



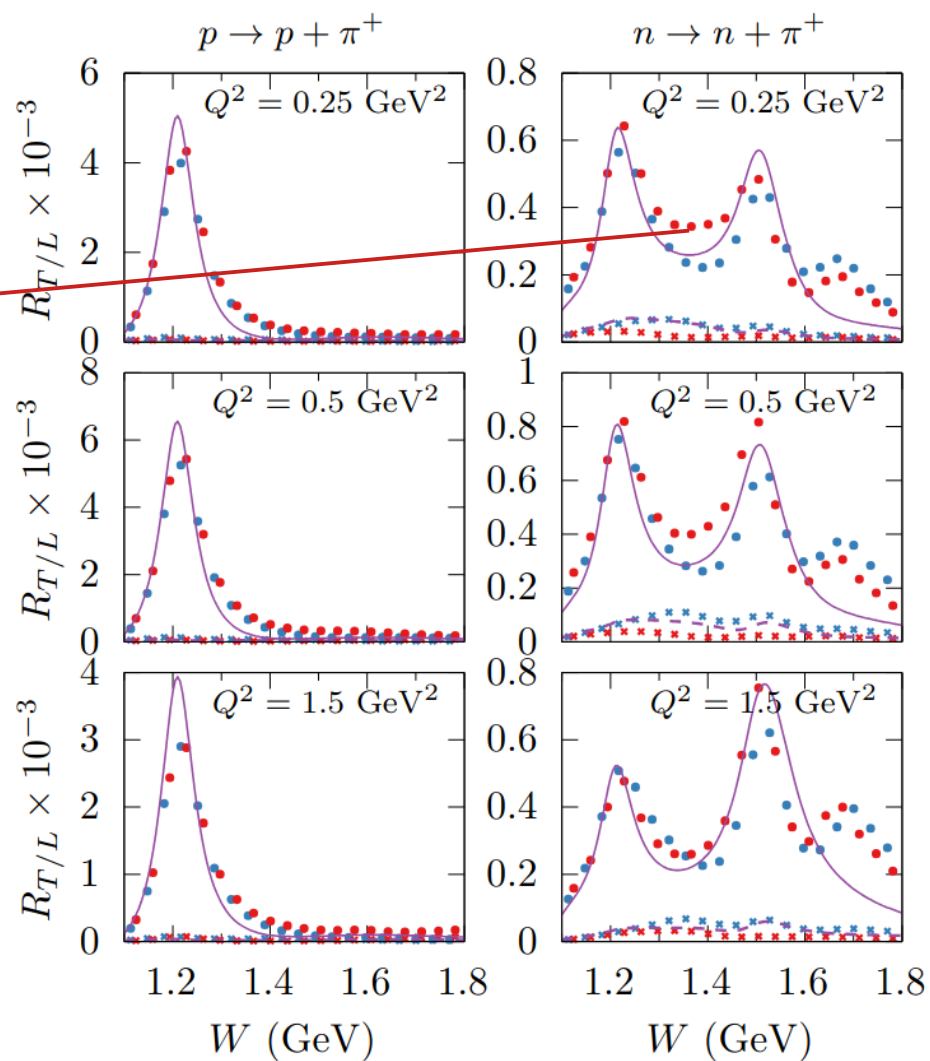
[M. Hooft et al. 2603.29486], comparison to ANL-Osaka model

VV contribution to neu

$$\sigma = \sigma_{VV} + \sigma_{AA} + \sigma_{VA}$$

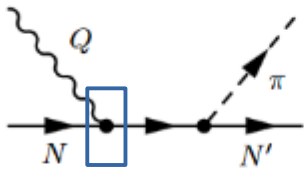
→ Inclusive partial cross sections:

- ANL-Osaka DCC
- MAID07



Axial couplings are poorly constrained

Spin-3/2 resonance axial couplings:



$$\Gamma_A^{\beta\mu} = \frac{C_3^A}{M} \left(g^{\beta\mu} \not{Q} - Q^\beta \gamma^\mu \right) + \frac{C_4^A}{M^2} \left(g^{\beta\mu} Q \cdot k_R - Q^\beta k_R^\mu \right) \\ + C_5^A g^{\beta\mu} + \frac{C_6^A}{M^2} Q^\beta Q^\mu,$$

PCAC / π -pole dominance

$$C_A^5(0) = f_\pi I_{iso} \frac{\sqrt{2} f_{\pi NR}}{m_\pi \sqrt{3}}$$

$$C_A^6(Q^2) = -M_N^2 \frac{C_A^5(Q^2)}{Q^2 + m_\pi^2}$$

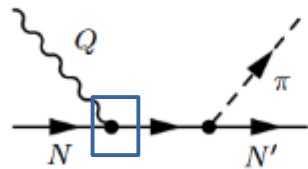
-No strong constraints for Q^2 dependence

-No strong constraint on C_3 or C_4

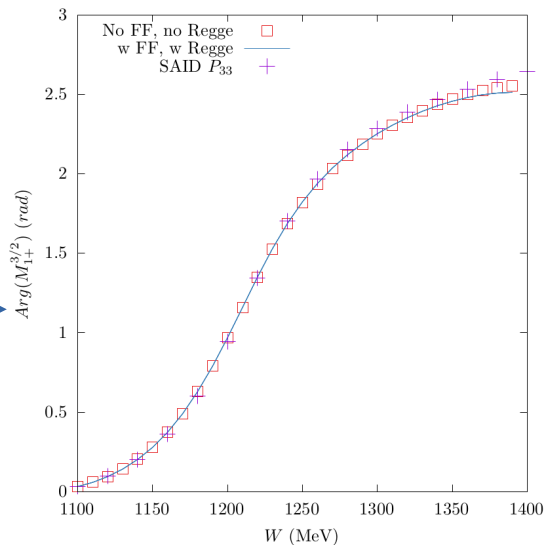
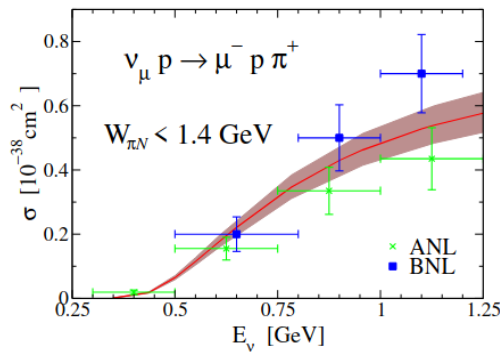
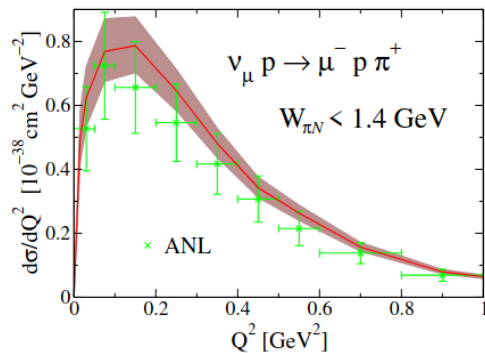
-No high-quality data available!

Axial coupling to delta from data

L. Alvarez-Ruso, E. Hernández, J. Nieves, M.J. Vicente Vacas [Phys. Rev. D93, 014016]



- ANL data
- Adler model for Delta
- $\rightarrow C_4 = -C_5/4$, $C_3 = 0$
- $\rightarrow C_4$ and C_3 contributions small
- +Unitarity in Δ dominated**
- V and A multipoles**

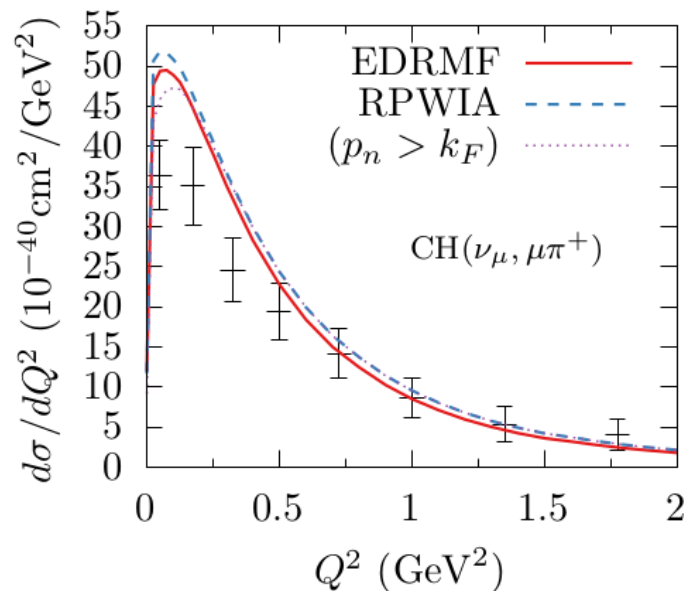
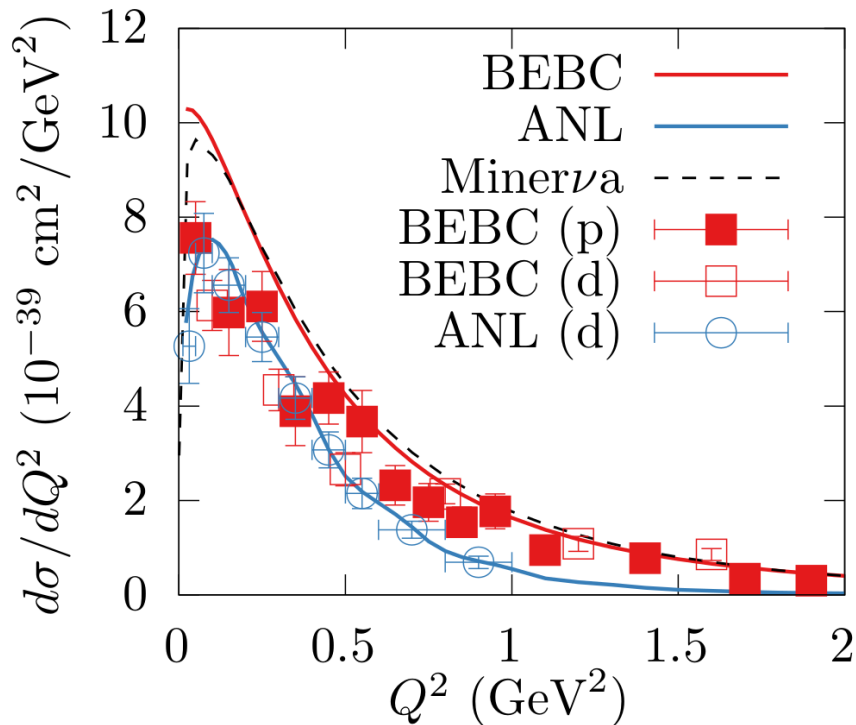


- Find $C_5(0)$ consistent with PCAC
- parametrize $C_5(Q^2)$

Axial coupling to delta from data

[A.N. et al PRD 107 , 053007]

- ANL + BEBC + Minerva comparison



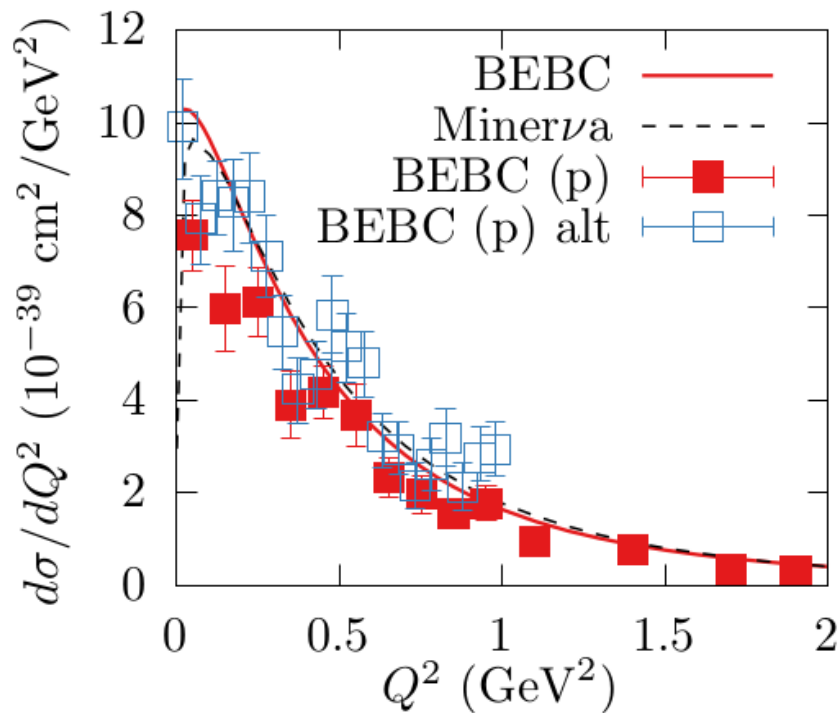
Large uncertainties in data

→ Form factor not well constrained?

Axial coupling to delta from data

[A.N. et al PRD 107 , 053007]

- ANL + BEBC + Minerva comparison



Data from BEBC:

[Z. Phys. C 43, 527–540 (1989)]

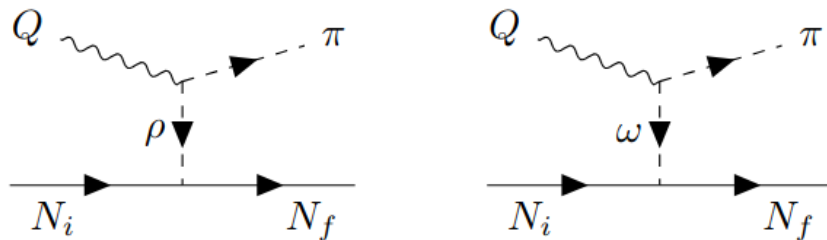
[Nucl.Phys.B 176 (1980) 269]

- related to low- p_N efficiency
- Unclear how correction is done
- Errors likely underestimated?

'Unitarization' in the delta region: [2603.29486]

M. Hooft,¹ A. Nikolakopoulos,^{1,2} J. García-Marcos,^{1,3} Y. De Backer,¹
T. Franco-Munoz,¹ K. Niewczas,¹ R. González-Jiménez,⁴ and N. Jachowicz¹

- Add vector meson exchanges:

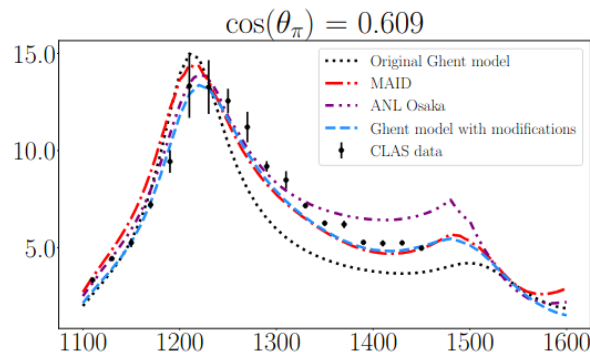
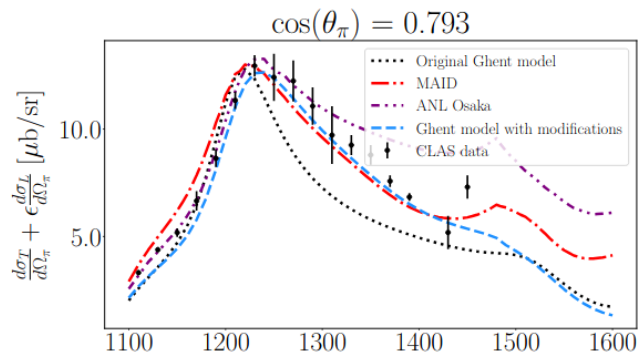
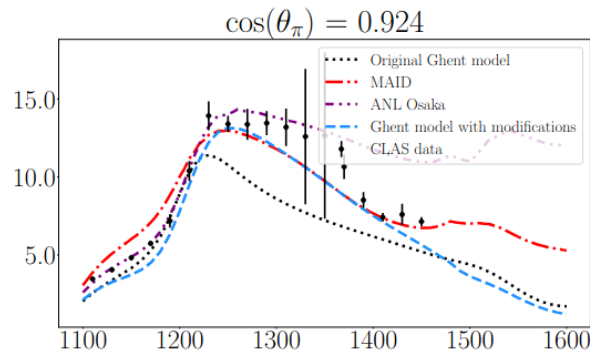
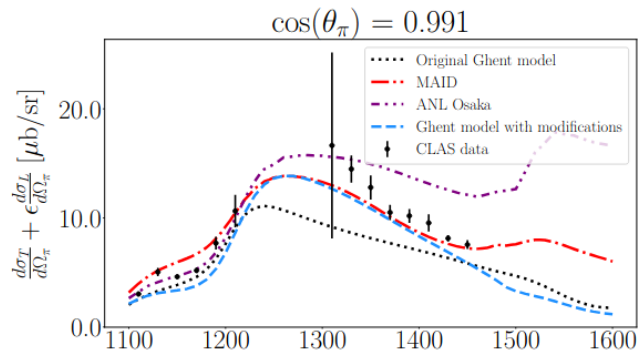


- Update of delta form factors
- Unitarity constraints through K-matrix approach
- Results for vector current in [2603.29486]

'Unitarization' in the delta region: [2603.29486]

N
T. 1

cker,¹
chowicz¹

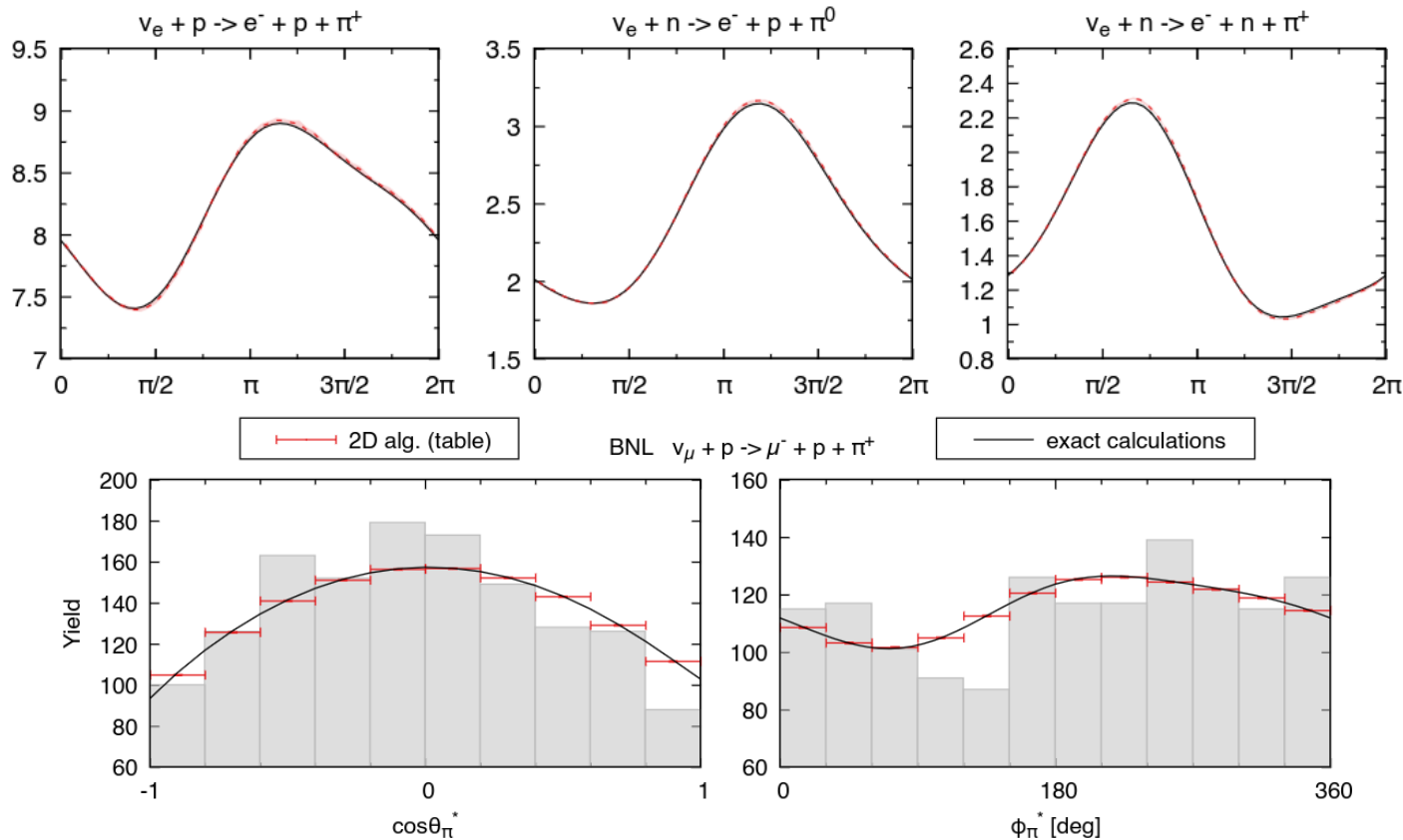


→ Neutrino results coming soon

SPP in event generators

Full implementation of cross section with all interference in NuWro : [Niewczas, PRD 103, 053003 (2021)]

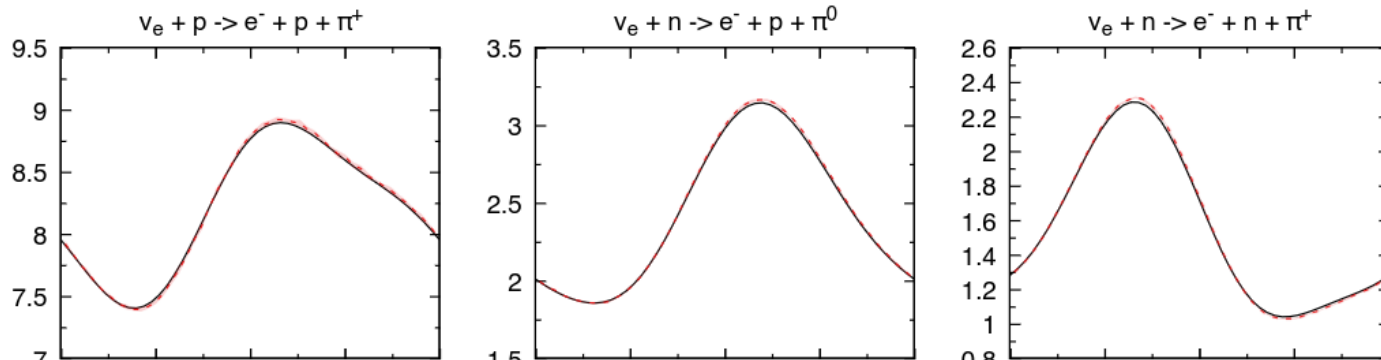
Completely general: works for every model (see also : [Q. Yan et al, JHEP 12 141])



SPP in event generators

Full implementation of cross section with all interference in NuWro : [Niewczas, PRD 103, 053003 (2021)]

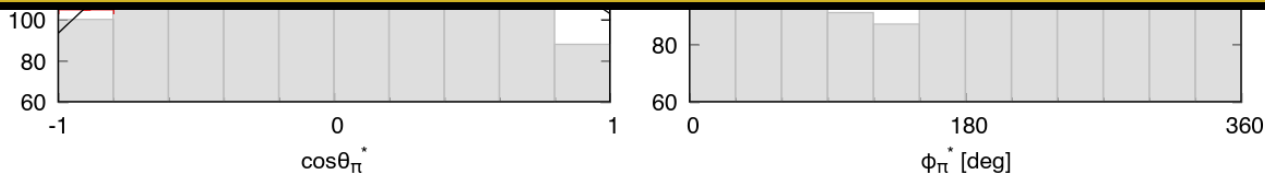
Completely general: works for every model (see also : [Q. Yan et al, JHEP 12 141])



+ ACHILLES [J. Isaacson et al 2508.19213]:

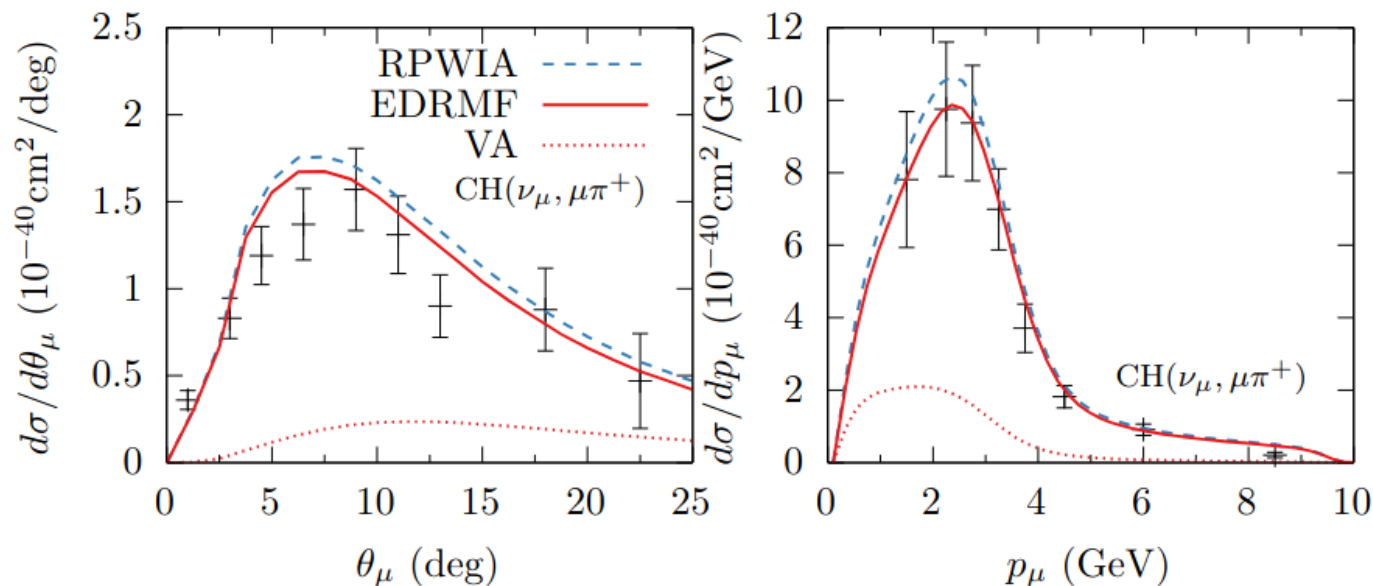
Include SPP fully (ANL-Osaka) + ANL-Osaka MB amplitudes for Cascade

(See Noemi Rocco's talk Friday)



SPP off nuclei

Relativistic mean field modeling: including nucleon final-state interactions



[A.N. et. al 2210.12144] : use 'asymptotic approximation'

[J. Garcia-Marcos et. al 2310.18056] : → include full momentum dependence

J. Garcia Marcos (in preparation) : → include pion final-state interactions

Factorized RDWIA calculations

Factorized distorted wave calculations for electron, neutrino and BSM processes

A. Nikolakopoulos^{1,*} and R. González-Jiménez^{2,†}

Factorization of **hadron tensor** in **PWIA**

$$H_{PWIA}^{\mu\nu} = h_{s.n.}^{\mu\nu} \times S(E_m, |\mathbf{p}_m|),$$

Does not occur in general

[[arxiv:2512.22763](https://arxiv.org/abs/2512.22763)]

github.com/alениkolak/Factorized_RDWIA

Factorized distorted wave calculations for electron, neutrino and BSM processes

A. Nikolakopoulos^{1,*} and R. González-Jiménez^{2,†}

A more general factorization of the amplitude

(still an approximation)

$$J^\mu(\kappa, s, m_j) = \text{Tr} [\Gamma^\mu S(\mathbf{t}, \mathbf{k}_N; \kappa, s, m_j)]$$



‘nucleon physics’

For example (e,e’):

$$\Gamma^\mu = F_1(Q^2)\gamma^\mu - \frac{F_2(Q^2)}{4M_N}Q_\nu [\gamma^\mu, \gamma^\nu]$$

You can put any Γ here: arbitrary couplings to N

[[arxiv:2512.22763](https://arxiv.org/abs/2512.22763)]

github.com/alениkolak/Factorized_RDWIA

Factorized distorted wave calculations for electron, neutrino and BSM processes

A. Nikolakopoulos^{1,*} and R. González-Jiménez^{2,†}

A more general factorization of the amplitude

(still an approximation)

$$J^\mu(\kappa, s, m_j) = \text{Tr} [\Gamma^\mu S(\mathbf{t}, \mathbf{k}_N; \kappa, s, m_j)]$$



Nuclear overlaps

$$\left(s\mathbb{1} + p\gamma^5 + V^\mu\gamma^\mu + A^\mu\gamma^\mu\gamma^5 + T^{\mu\nu}\frac{1}{2}[\gamma^\mu, \gamma^\nu] \right)$$

We calculated and tabulated all overlaps

→ traces can be done analytically/automatically

[[arxiv:2512.22763](https://arxiv.org/abs/2512.22763)]

github.com/alениkolak/Factorized_RDWIA

Factorized distorted wave calculations for electron, neutrino and BSM processes

A. Nikolakopoulos^{1,*} and R. González-Jiménez^{2,†}

We calculated and tabulated all overlaps:

$$\left(s\mathbb{1} + p\gamma^5 + V^\mu\gamma^\mu + A^\mu\gamma^\mu\gamma^5 + T^{\mu\nu}\frac{1}{2}[\gamma^\mu, \gamma^\nu] \right)$$

Can be used to reproduce e.g. - [R. G. J. et al [1904.10696](#)]

- [A. N. et al. [2406.09244](#)]

- [A. N. et al. [2202.01689](#)]

- R.G.J et al. [2104.01701](#)]

- ...

And much more

Extension to general processes (e.g SPP) underway

[[arxiv:2512.22763](#)]

github.com/alnikolak/Factorized_RDWIA

A caveat : operator ambiguities

The *ansatz* for relating free-nucleon matrix elements to nuclear matrix elements:

$$\langle \beta | \hat{J}_\mu(\mathbf{q}) | \alpha \rangle \rightarrow \int \frac{d^3\mathbf{p}}{(2\pi)^3} \bar{\phi}_\beta(\mathbf{p} + \mathbf{q}) \left[\sum_i F_i(Q^2) \Gamma_i^\mu \right] \phi_\alpha(\mathbf{p})$$

Lead to non-unique results depending on the arbitrary parametrization of Γ^μ

A resolution [A.N. & R.P. 2607.29634]:

Explicitly on-shell currents in relativistic mean field models

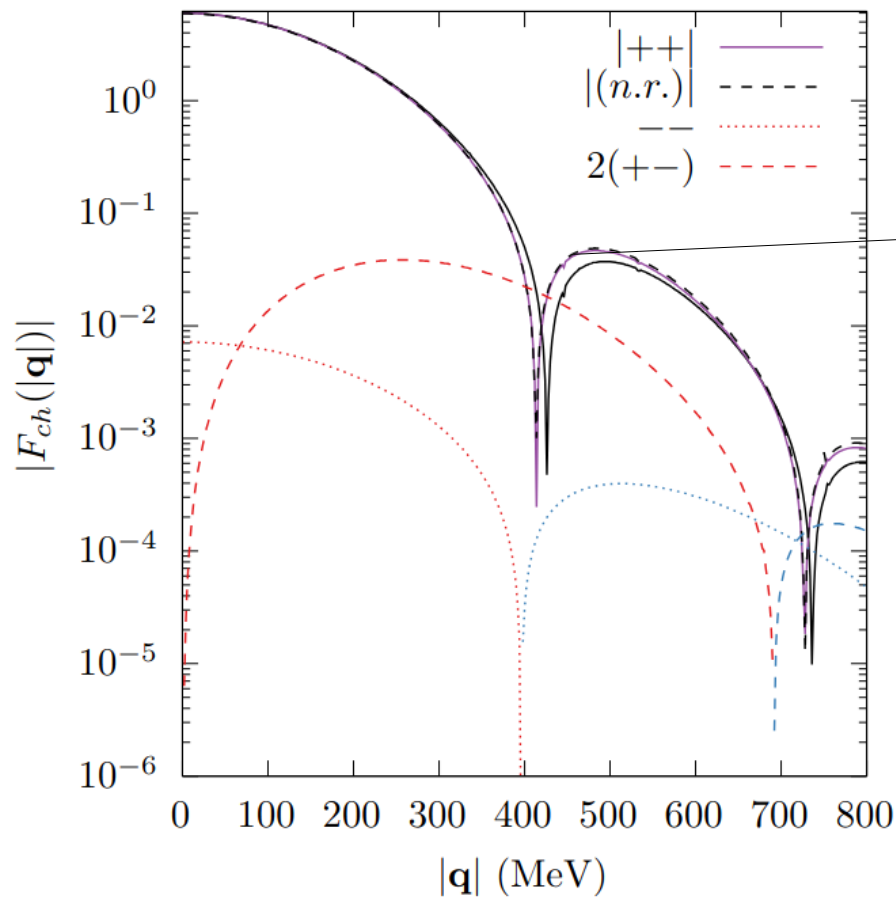
Alexis Nikolakopoulos¹ and Ryan Plestid²

¹Department of Physics, University of Washington, Seattle, Washington 98195, USA

²CERN, 1 Esplanade des Particules, CH-1211 Geneva 23, Switzerland

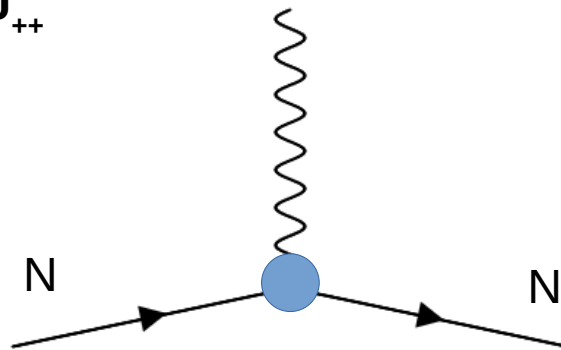
(Dated: August 3, 2026)

Elastic electron scattering [arxiv:2607.29634]



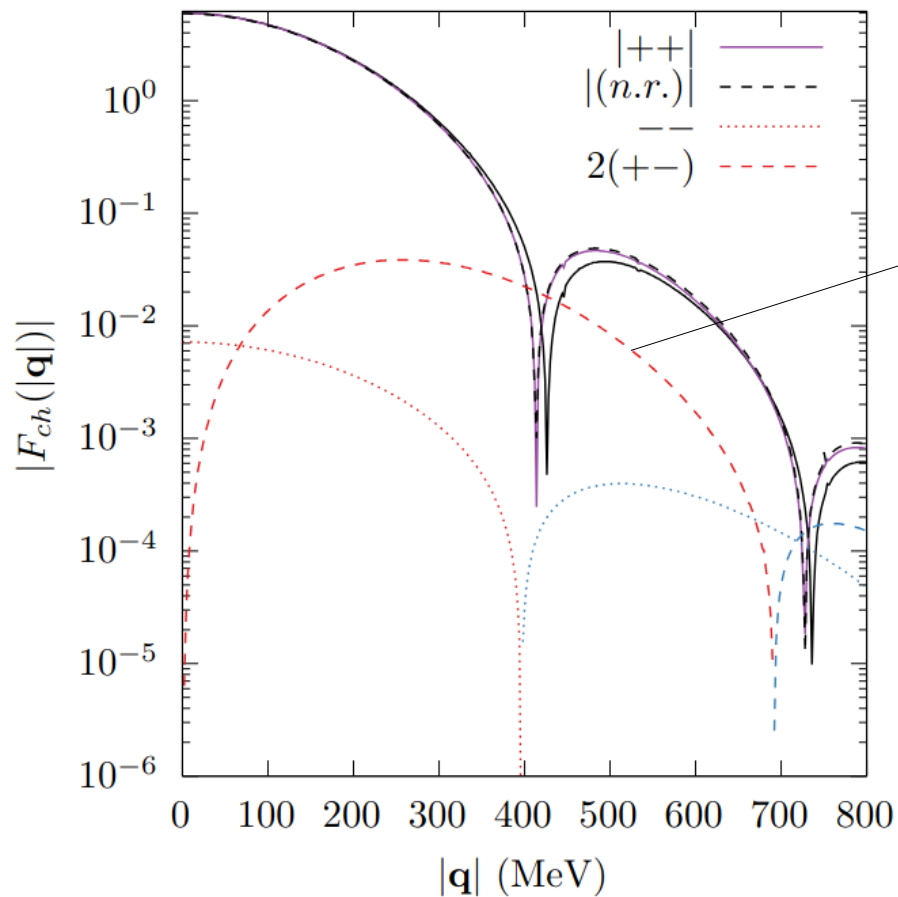
$$\rightarrow \int \frac{d^3\mathbf{p}}{(2\pi)^3} \bar{\phi}_\beta(\mathbf{p} + \mathbf{q}) \left[\sum_i F_i(Q^2) \Gamma_i^\mu \right] \phi_\alpha(\mathbf{p})$$

$\mathbf{J}_{++}$

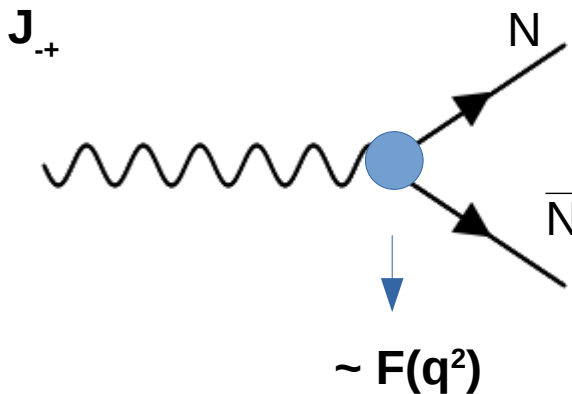


$\rightarrow$ can be mapped to textbook
non-relativistic result $\mathbf{J}_{(n.r.)}$

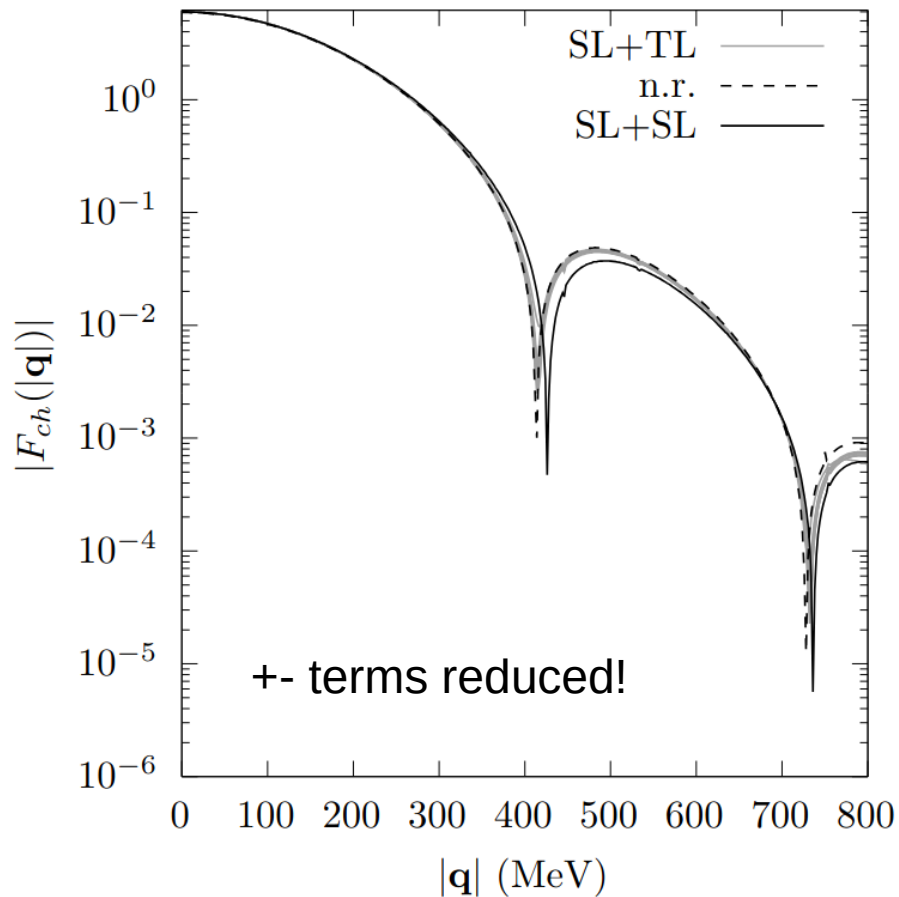
Elastic electron scattering [arxiv:2607.29634]



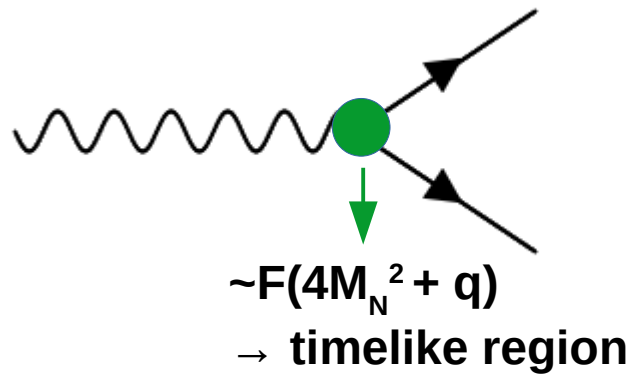
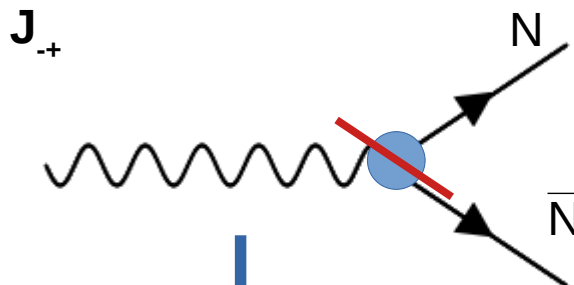
$$\rightarrow \int \frac{d^3\mathbf{p}}{(2\pi)^3} \bar{\phi}_\beta(\mathbf{p} + \mathbf{q}) \left[\sum_i F_i(Q^2) \Gamma_i^\mu \right] \phi_\alpha(\mathbf{p})$$



Elastic electron scattering [arxiv:2607.29634]

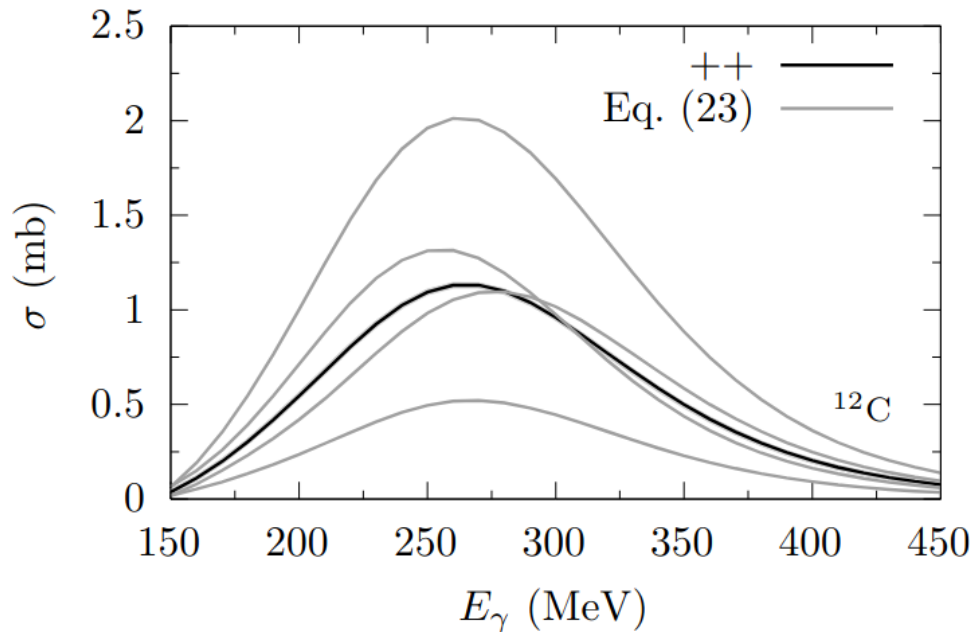


~~$$\rightarrow \int \frac{d^3\mathbf{p}}{(2\pi)^3} \bar{\phi}_\beta(\mathbf{p} + \mathbf{q}) \left[\sum_i F_i(Q^2) \Gamma_i^\mu \right] \phi_\alpha(\mathbf{p})$$~~



Coherent pion production [arxiv:2607.29634]

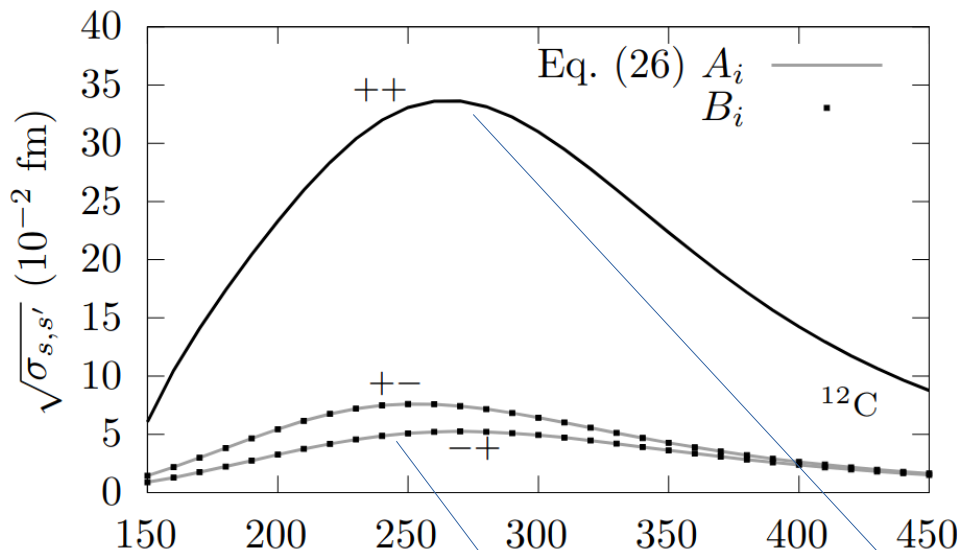
$$\gamma(\mathbf{q}) + A(0) \rightarrow A(\mathbf{t} = \mathbf{k}_\pi - \mathbf{q}) + \pi(\mathbf{k}_\pi).$$



$$\int \frac{d^3\mathbf{p}}{(2\pi)^3} \bar{\phi}_\beta(\mathbf{p} + \mathbf{q}) \left[\sum_i F_i(Q^2) \Gamma_i^\mu \right] \phi_\alpha(\mathbf{p})$$

Large ambiguities!

Coherent pion production [arxiv:2607.29634]



$$\int \frac{d^3\mathbf{p}}{(2\pi)^3} \bar{\phi}_\beta(\mathbf{p} + \mathbf{q}) \left[\sum_i F_i(Q^2) \Gamma_i^\mu \right] \phi_\alpha(\mathbf{p})$$

Large ambiguities!



Unique results

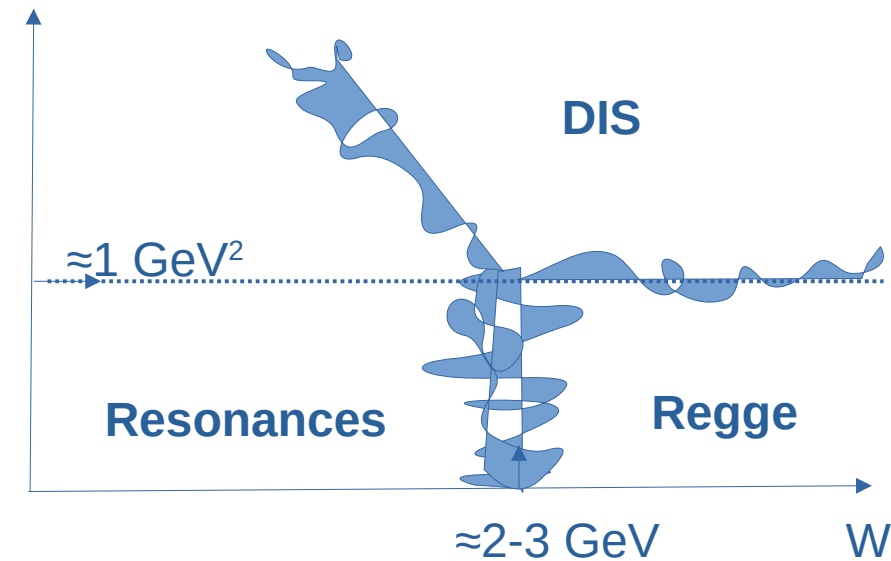
$$\langle \beta | \hat{J}_\mu^{(1)} | \alpha \rangle = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} \frac{d^3p'}{(2\pi)^3} \frac{1}{2E_{p'}} \tilde{\phi}_\beta^\dagger(\mathbf{p}') \left[(\not{p}' + m) \Gamma_\mu^{(a)}(\mathbf{p}', \mathbf{p}) (\not{p} + m) \gamma_0 + (\not{\tilde{p}}' - m) \Gamma_\mu^{(b)}(-\mathbf{p}', -\mathbf{p}) (\not{\tilde{p}} - m) \gamma_0 \right. \\ \left. + (\not{\tilde{p}}' - m) \Gamma_\mu^{(c)}(-\mathbf{p}', \mathbf{p}) (\not{p} + m) \gamma_0 + (\not{p}' + m) \Gamma_\mu^{(d)}(\mathbf{p}', -\mathbf{p}) (\not{\tilde{p}} - m) \gamma_0 \right] \tilde{\phi}_\alpha(\mathbf{p}) .$$



Thank you!

SPP in the Regge region

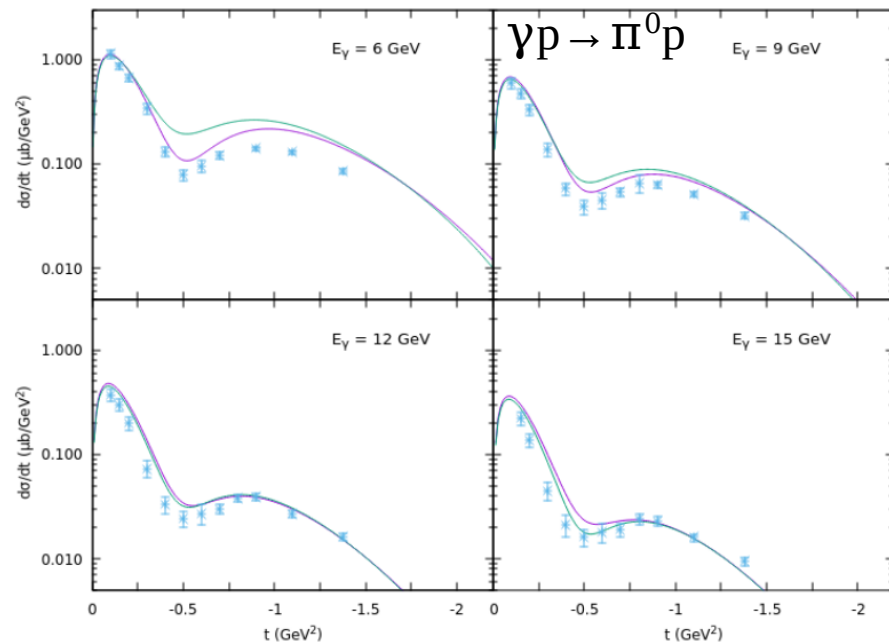
[H. Damien, Masters thesis, UGent (2020)]



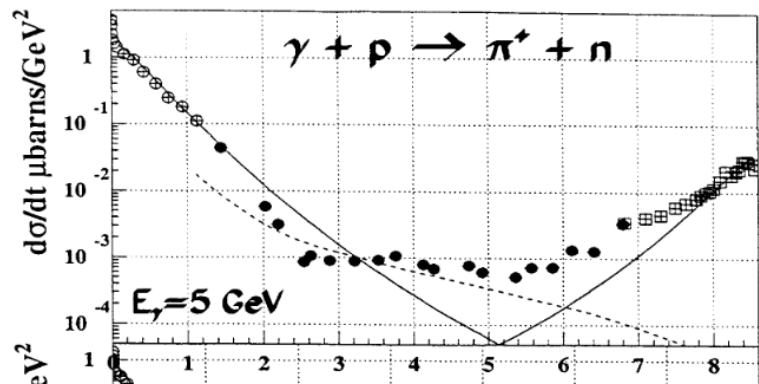
For high- s
Small $-t$

t -channel d.o.f

→ Peak at forward angles



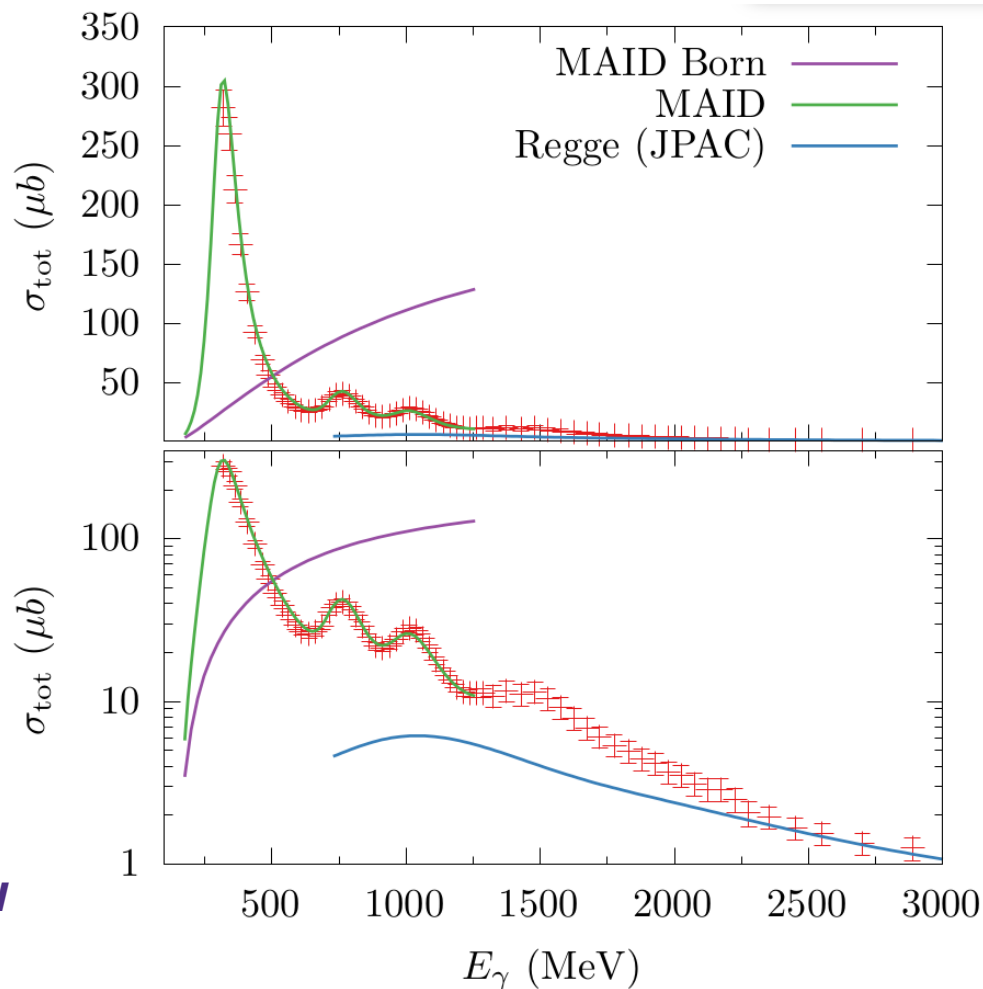
Photoproduction



[Guidal, Laget, Vanderhaeghen]

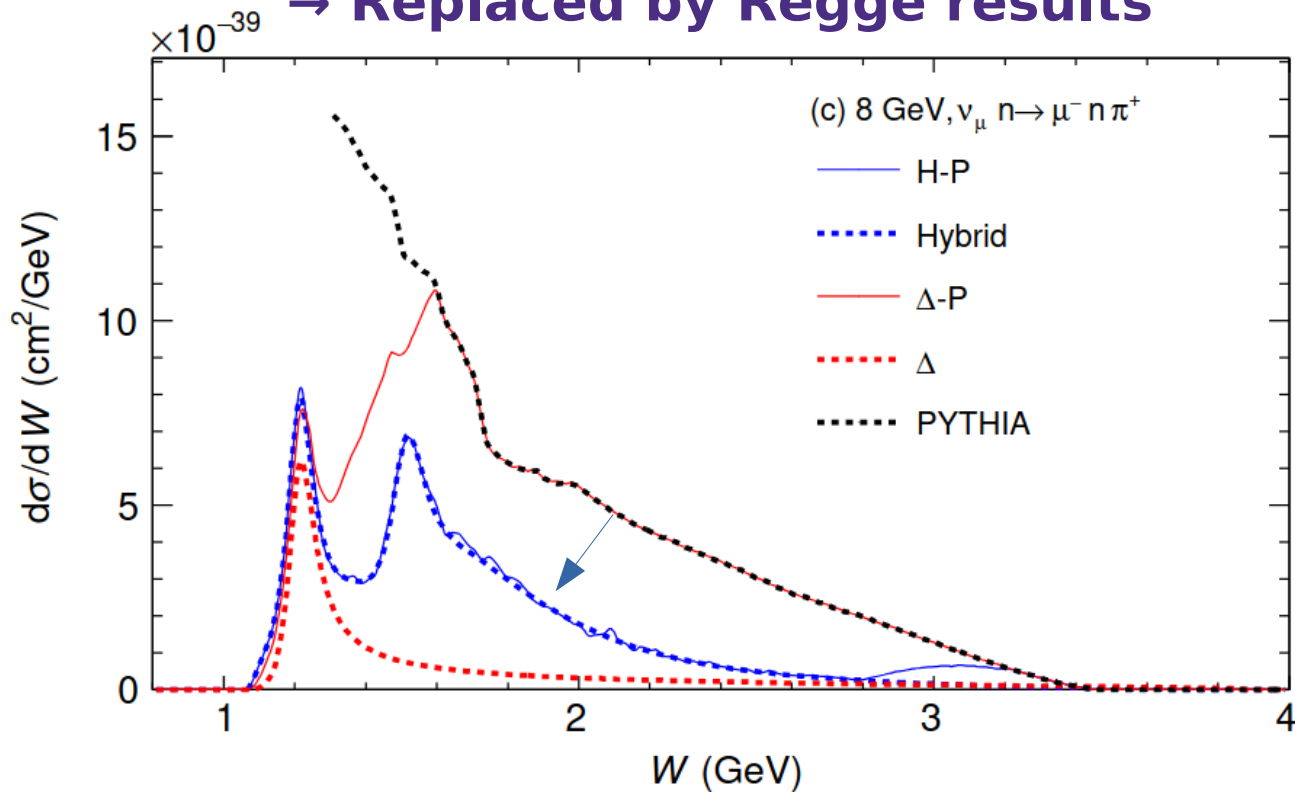
**Peak dominates at
Large W , low Q^2**

→ Saturates cross section at large W



Hybrid model in NuWro: [Q. Yan et al., JHEP 12 141]

Remove the PYTHIA contribution in 1pion channel
→ Replaced by Regge results

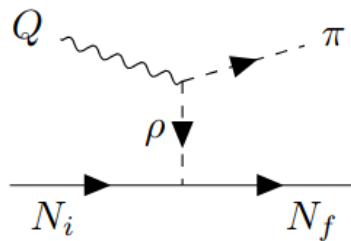


'Unitarization' in the delta region: [2603.29486]

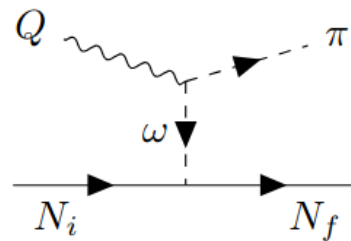
M. Hooft,¹ A. Nikolakopoulos,^{1,2} J. García-Marcos,^{1,3} Y. De Backer,¹
T. Franco-Munoz,¹ K. Niewczas,¹ R. González-Jiménez,⁴ and N. Jachowicz¹

Recent work from Matthias Hooft (Phd student Ghent):

- Add vector meson exchanges:



- Update of delta form factors



- Unitarity constraints through K-matrix approach

- Results for vector current and formalism in [2603.29486]
→ **Neutrino results coming soon**

Unitarity constraints

$$\int d\Omega' \sum_{\lambda', \alpha} {}_F \langle \Omega_F, \lambda_F | \tilde{T} | \Omega', \lambda' \rangle_{\alpha}^* \langle \Omega', \lambda' | \tilde{T} | \Omega_I, \lambda_I \rangle_I = -\text{Im} \left\{ {}_F \langle \Omega_F, \lambda_F | \tilde{T} | \Omega_I, \lambda_I \rangle_I \right\} \in \mathbb{R}.$$



Below 2π threshold:
Watson's theorem

$$\begin{aligned} & {}_{\pi N} \langle J, l, s, I | \tilde{T} | J, l, s, I \rangle_{\pi N}^* \\ & \times {}_{\pi N} \langle J, l, s, I | \tilde{T} | J, L, S, I \rangle_{\gamma N} \in \mathbb{R}. \end{aligned}$$

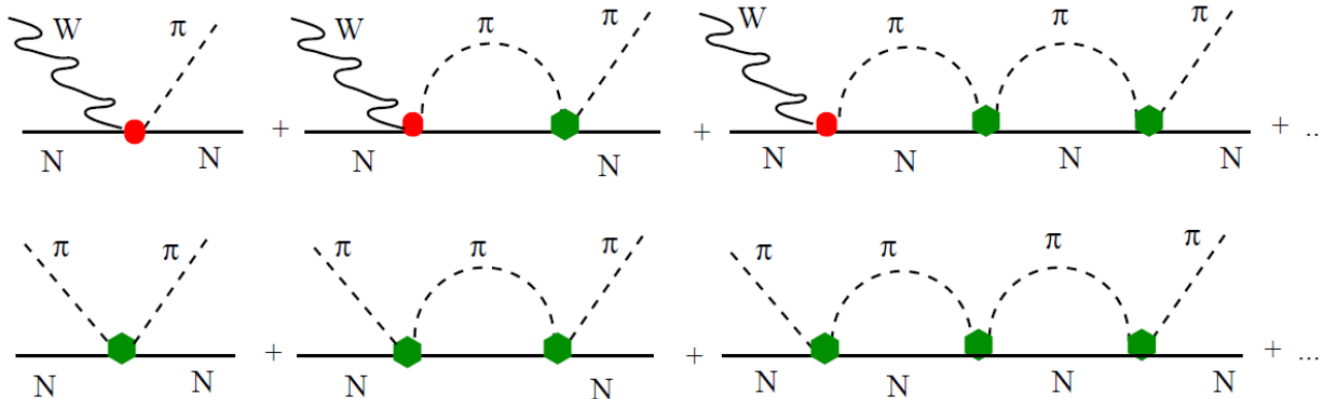


Figure from [Talk](#)
J. Nieves 2017,
KEK

Unitarity constraints and K-matrix

$$(\tilde{T}^{-1} + i\mathbb{1})^\dagger = (\tilde{T}^{-1} + i\mathbb{1}). \longrightarrow \tilde{K}^{-1} \equiv \tilde{T}^{-1} + i\mathbb{1}$$

↓ Below 2π threshold

$$T_{\gamma\pi}^{J,l,I} = K_{\gamma\pi}^{J,l,I} (1 + i\rho_\pi T_{\pi\pi}^{J,l,I}).$$

Unitarity constraints and K-matrix

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Modeling (BG):

↓

$$\approx T_{0,\text{tree}}^{J,l,I}$$

↘ From πN scattering
(we used JuBo analysis)

Note: this doesn't introduce 'parameters'

T₀ \approx K

$$T_{\pi\gamma}(k_\pi, q; W) = T_{\pi\gamma}^0(k_\pi, q; W) + \int dp \frac{p^2 T_{\pi\pi}(k_\pi, p; W) T_{\pi\gamma}^0(p, q; W)}{W - E_\pi(p) - E_N(p) + i\epsilon},$$



Neglect principal value integral

$$T_{\pi\gamma} = T_{\pi\gamma}^0 + i\rho_\pi T_{\pi\pi} T_{\pi\gamma}^0,$$

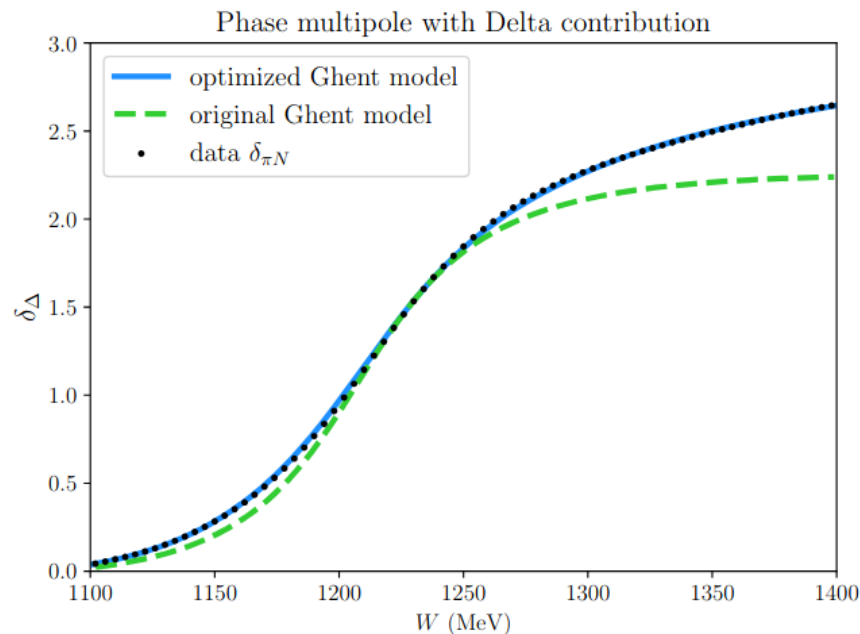
$$T_{\pi\gamma} = K_{\pi\gamma} (1 + i\rho_\pi T_{\pi\pi}),$$

Unitarity for width and phase of Delta

$$S_{\Delta,\alpha\beta} = \frac{k_{\Delta} + M_{\Delta}}{k_{\Delta}^2 - M_{\Delta}^2 + iM_{\Delta}\Gamma_{\Delta}(W)} \mathcal{P}_{\alpha\beta},$$

Width determined by πN amplitudes

$$\Gamma_{\Delta}(W) = \frac{M_{\Delta}^2 - s}{M_{\Delta}} \tan(\delta_{\pi N}(W)),$$



Note: this removes 'parameters'

Gives correct form of T-matrix

But RES-BG separation is model-dependent

$$K_{\gamma\pi}^{\Delta} = \frac{A_{\Delta}}{W^2 - M_{\Delta}^2} + B_{\Delta},$$

$$K_{\pi\pi}^{\Delta} = \frac{C_{\Delta}}{W^2 - M_{\Delta}^2} + D_{\Delta},$$

$$T_{\gamma\pi}^{\Delta} = \left(\frac{A_{\Delta}}{W^2 - M_{\Delta}^2} + B_{\Delta} \right) \times \frac{(W^2 - M_{\Delta}^2)}{W^2 - M_{\Delta}^2 - i[C_{\Delta} + D_{\Delta}(W^2 - M_{\Delta}^2)]}.$$