Extension of superscaling approaches to semi-inclusive lepton-nucleus reactions and their implementation in generators

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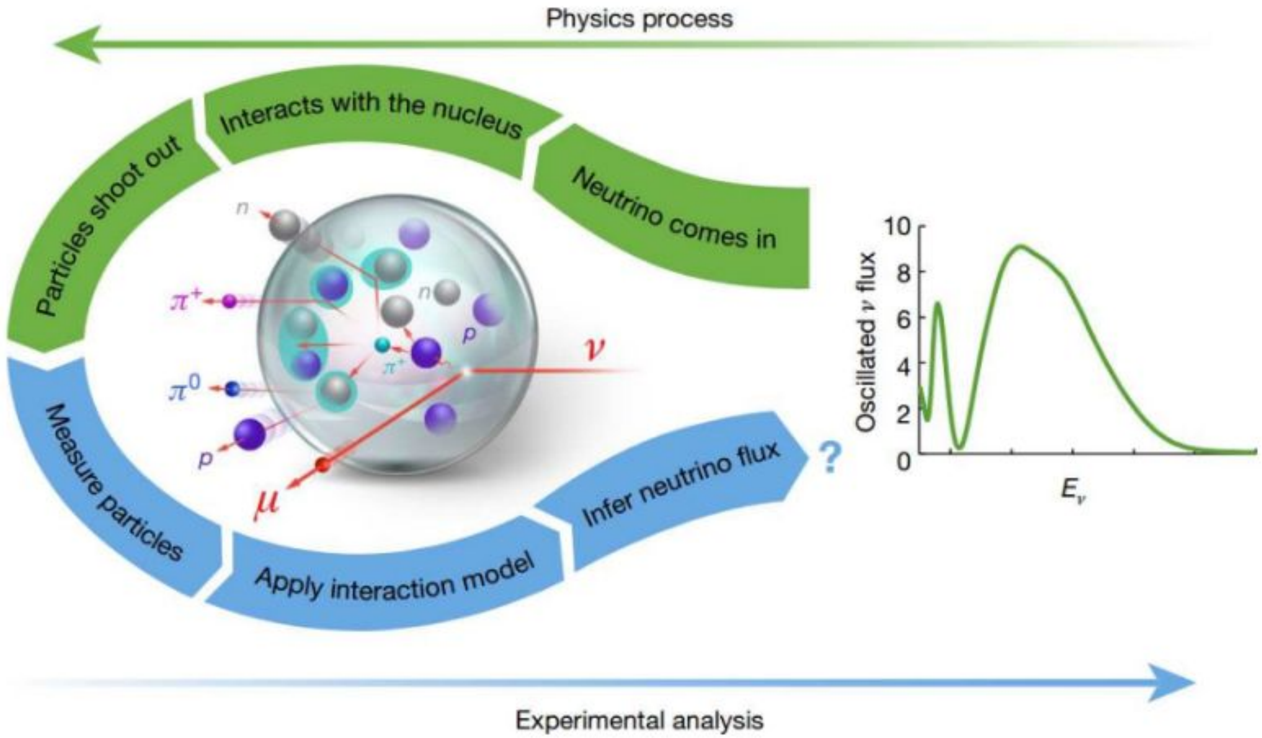
Event generators and cross-section models

Event generators:

- NEUT (SK, T2K, HK)
- GENIE (MicroBooNE, MINERvA, DUNE)
- Others: NuWro, GiBUU, Achilles, MARLEY, etc.



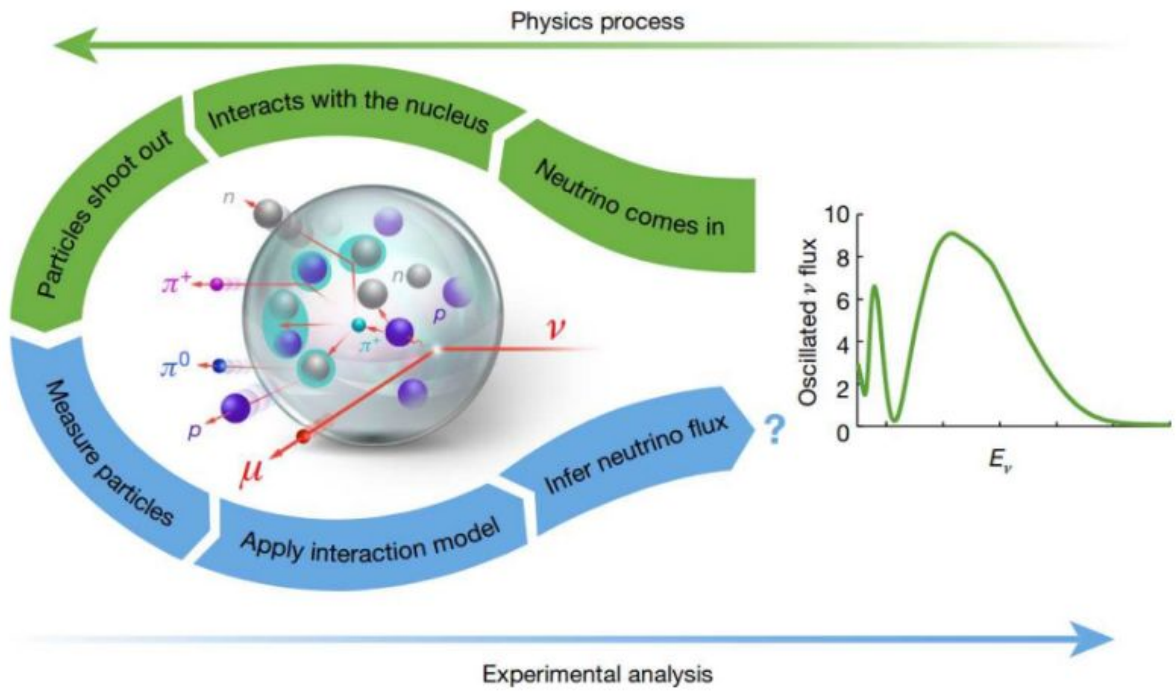
Nuclear models: RFG, LFG, SF, CRPA(HF), RMF, SuSA, etc.

Nuclear regimes (channels): QE, 2p2h, Resonance, DIS, SIS, interferences, etc.

Event generators and cross-section models

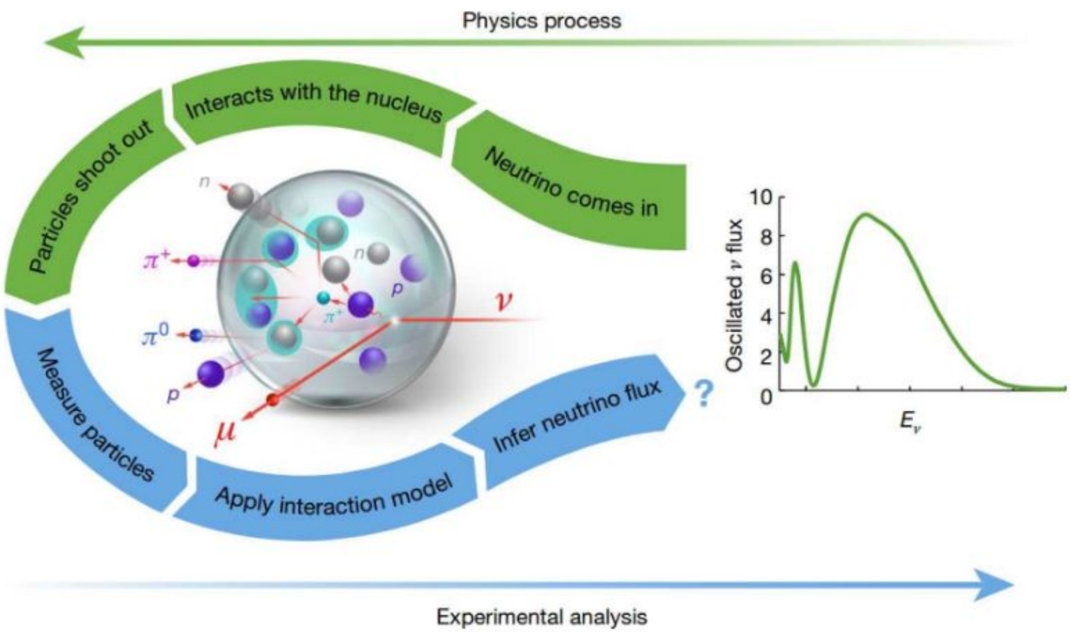
- Nuclear models and cross-section studies are the **starting point** of all neutrino event generators.
- Model constraints from experimental data:

- ***electron data*** for validation and improvement of nuclear models used in ***ν*** event generators (*n(p), SF profiles, E*$_b$*, Fermi motion, FSI, NN correlations,* etc.) and also for improvement of MC ***ν energy reconstruction*** methods. Better control of experimental conditions (monochromatic beam).
- ***ν data*** for axial part and nuclear effects.



Limitations:

- Some models are **specific** for an interaction **channel** or limited **phase-space**.
- Some models only predict lepton kinematics → assumptions for hadron kinematics (*factorization approaches*).
- **Model uncertainties** missing → ~~Reweighting of parameters~~.
- Simple models: FGs for 1p1h or 2p2h, Berger-Sehgal (RES) or PDFs in DIS.
- Most models ignore interferences between channels RES+DIS, 1body+2body, etc.



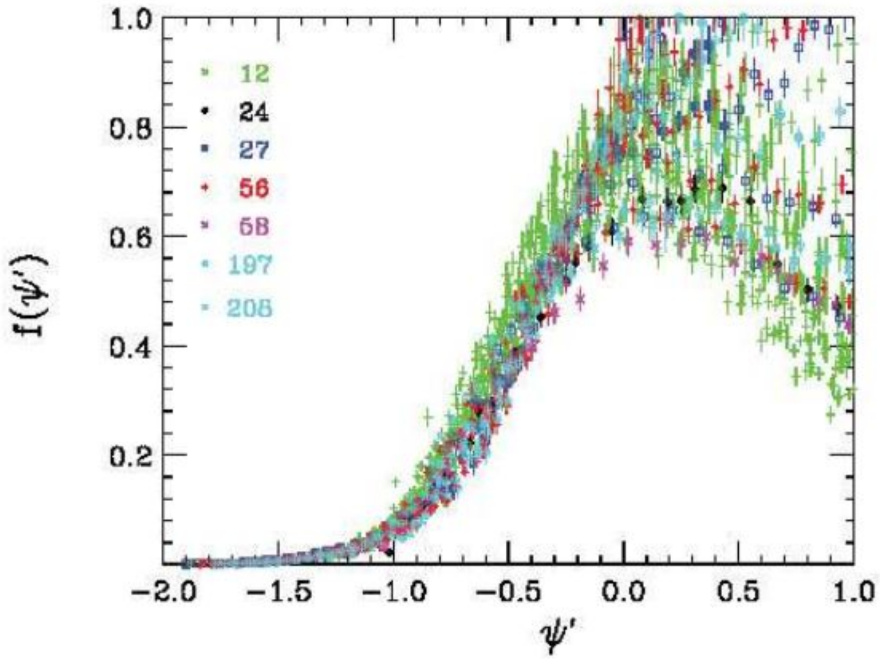
SuperScaling Approach (SuSA)

➠ The analysis of the large amount of existing (e, e') data at different kinematics is a solid benchmark to test the validity of theoretical models for neutrino reactions as well as to study the nuclear dynamics. The **SuperScaling Approach** exploits universal features of lepton-nucleus scattering to connect the two processes.

In inclusive QE scattering we can observe:

- ☆ Scaling of 1^{st} kind (independence on q)
- ☆ Scaling of 2^{nd} kind (independence on Z)

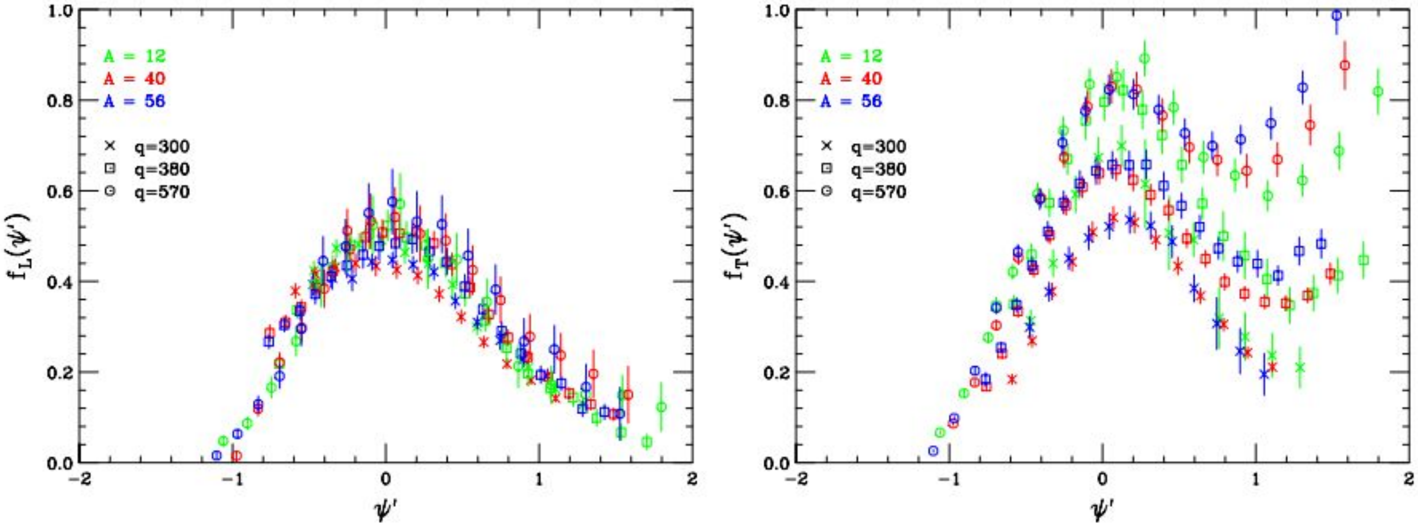
$\Longrightarrow$ **SuperScaling**



$$f(\psi) \equiv f(q,\omega) \sim \frac{\sigma_{QE}\left(\substack{\text{nuclear}\\\text{effects}}\right)}{\sigma_{\text{single nucleon}}\left(\substack{\text{no nuclear}\\\text{effects}}\right)}$$

$$f(\psi') = k_F \frac{\left(\frac{d^2\sigma}{d\Omega_e d\omega}\right)_{exp}}{\sigma_{Mott}(v_L G_L^{ee'} + v_T G_T^{ee'})}$$

Good superscaling behavior at $\psi' < 0$ (below QE peak). At higher kinematics (ψ'), other contributions beyond QE and IA (2p2h, Δ, etc.) can play an important role and scaling is broken.

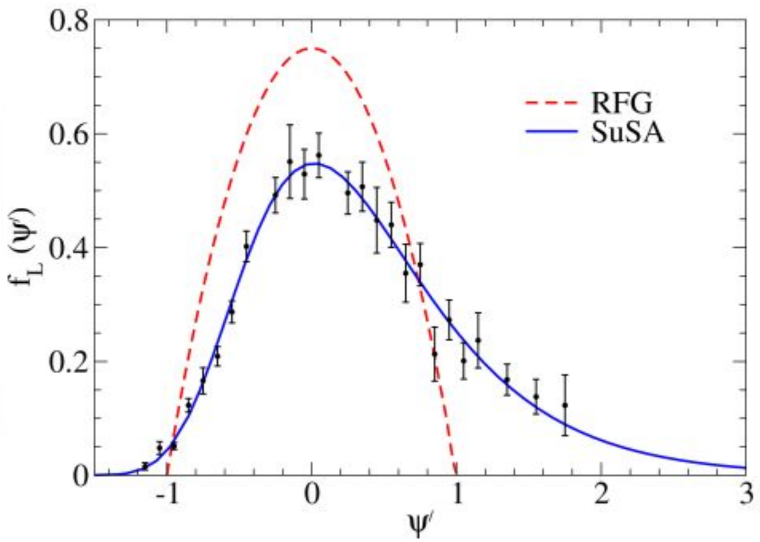


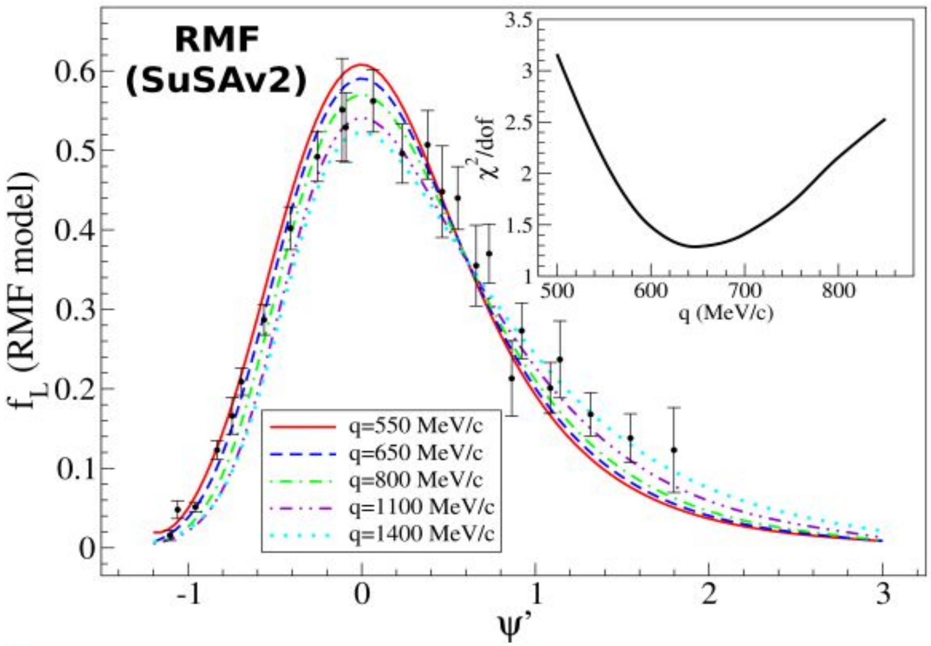
$$f_L = k_F R_L / G_L$$
$$f_T = k_F R_T / G_T$$

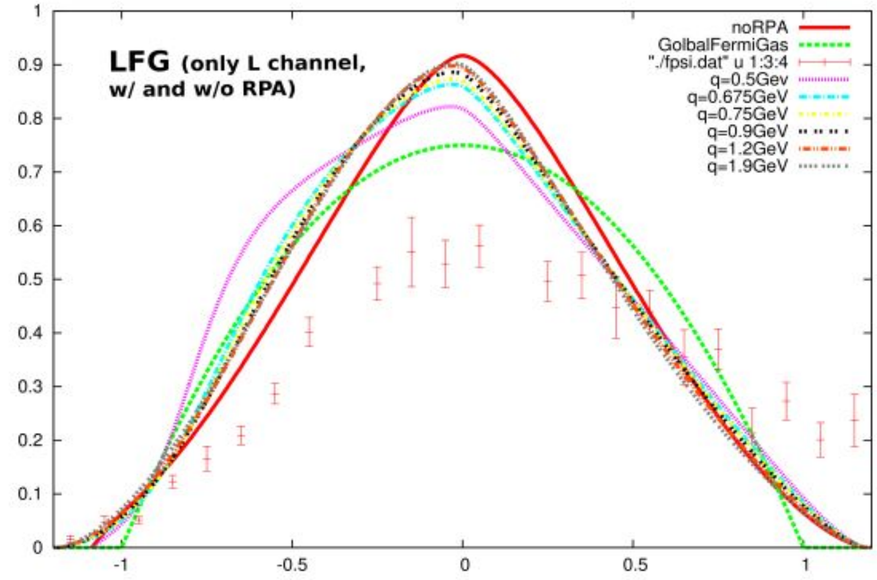
Scaling violations in the T channel ⇒ 2p-2h MEC, correlations, Δ-resonance ⇒ Mainly transverse

$$f(\psi) \equiv f(q,\omega) \sim \frac{\sigma_{QE}\left(\substack{\text{nuclear}\\ \text{effects}}\right)}{\sigma_{\text{single nucleon}}\left(\substack{\text{no nuclear}\\ \text{effects}}\right)}$$

$$f(\psi') = k_F \frac{\left(\frac{d^2\sigma}{d\Omega_e d\omega}\right)_{exp}}{\sigma_{Mott}(v_L G_L^{ee'} + v_T G_T^{ee'})}$$





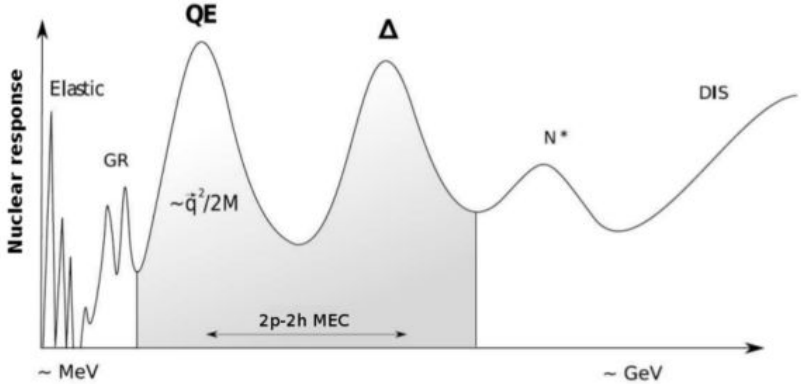


The SuSAv2 model *PRC90, 035501 (2014)* *PRD94, 013012 (2016)*

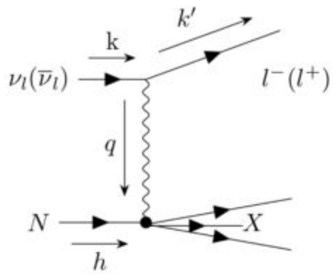
✪ **SuSAv2 model:** lepton-nucleus reactions adressed in the SuperScaling Approach and based on Relativistic Mean Field (RMF) theoretical scaling functions (FSI) to reproduce nuclear dynamics.

✪ RMF: Good description of the QE (e,e') data and superscaling properties $(f^{ee'}_{L,exp})$.
RMF predicts $f_T > f_L$ ($\sim$ 20%) as a pure relativistic effect (FSI with the residual nucleus).
Strong RMF potentials at high q_3 are corrected by RPWIA and q-dependent blending function.

➠ **Relativistic Mean Field (RMF, ED-RMF, SuSAv2)**: Fully relativistic shell model with accurate description of nuclear dynamics and FSI effects. Bound nucleons: self-consistent Dirac-Hartree solutions, derived within a RMF Lagrangian with local relativistic potentials (S+V) fitted to saturation properties of nuclear matter, radii and nuclear masses.

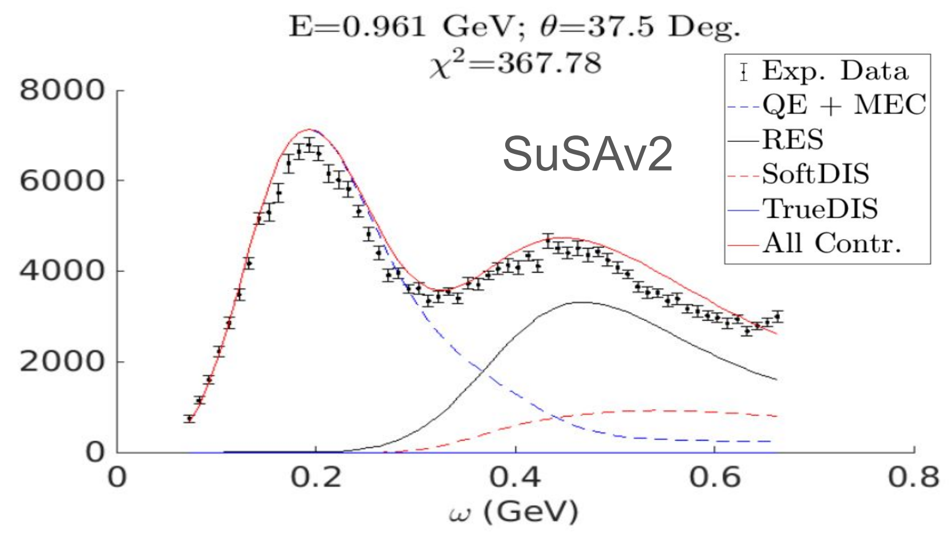


- Quasielastic region.
- 2p-2h excitations.
- **Δ** resonance, other resonances and DIS.



SuSAv2-inelastic model describes the full inelastic spectrum (Δ, other res. And DIS)[G. D. Megias, PhD Thesis (2017), M. B. Barbaro et al., Phys. Rev. C 69, 035502 (2004), J. Gonzalez-Rosa et al., Phys. Rev. D 105, 093009 (2022)]. Good agreement with (e,e') data.

$$R^K_{inel}(\kappa,\tau) = \frac{N}{\eta_F^2 \kappa}\xi_F \int_{\mu_X^{min}}^{\mu_X^{max}} d\mu_X f^{model}(\psi'_X) U^k$$



- TrueDIS (Deep inelastic scattering)

$W_x^{min} = 2.1\ GeV;\ \ W_x^{max} = m_N + \omega - E_s$

Bodek-Ritchie/ Bosted-Christy/ Parton Distribution Function

- RES (Resonances)

$W_x^{min} = m_N + m_\pi;\ \ W_x^{max} = 2.1\ GeV$

Dynamical Coupled Channels

- SoftDIS (Deep inelastic scattering in the resonance region)

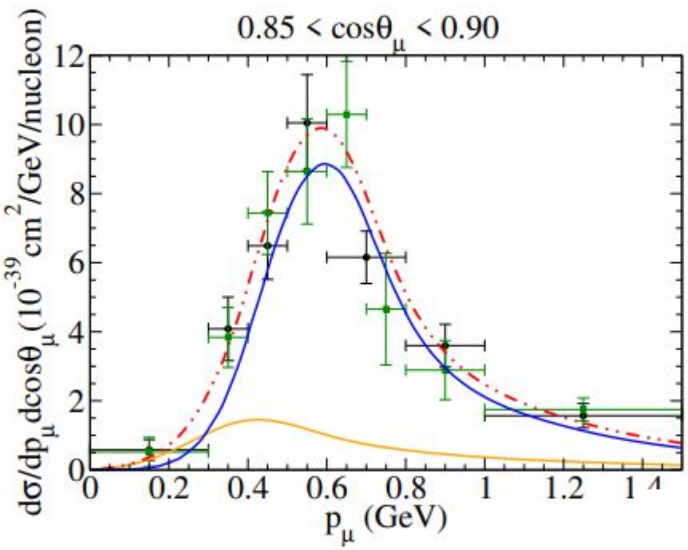
$W_x^{min} = m_N + m_\pi;\ \ W_x^{max} = 2.1\ GeV$

Dynamical Coupled Channels and Bodek-Ritchie/Bosted-Christy

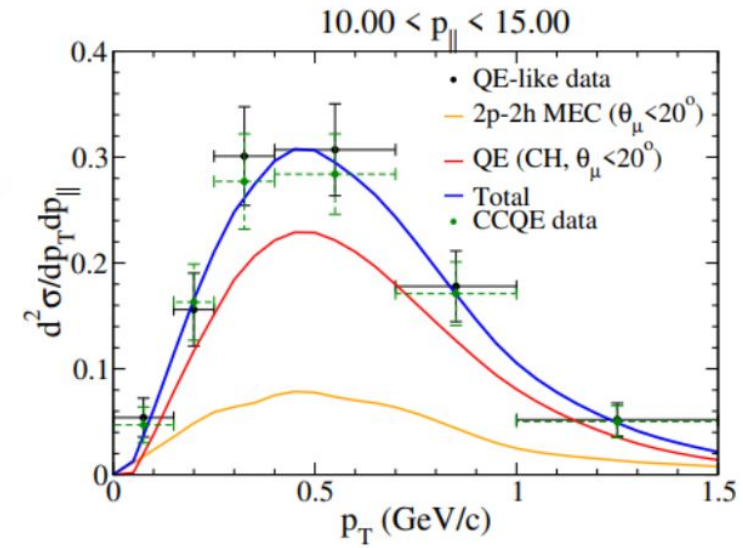
SuSAv2 (e,e') results: *PRD94, 013012 (2016); PRC99, 042501 (2019); PRD108, 113008 (2023)*

Comparison with CC0π ν_μ-nucleus data

T2K CC0π $\nu_\mu - C_8H_8$

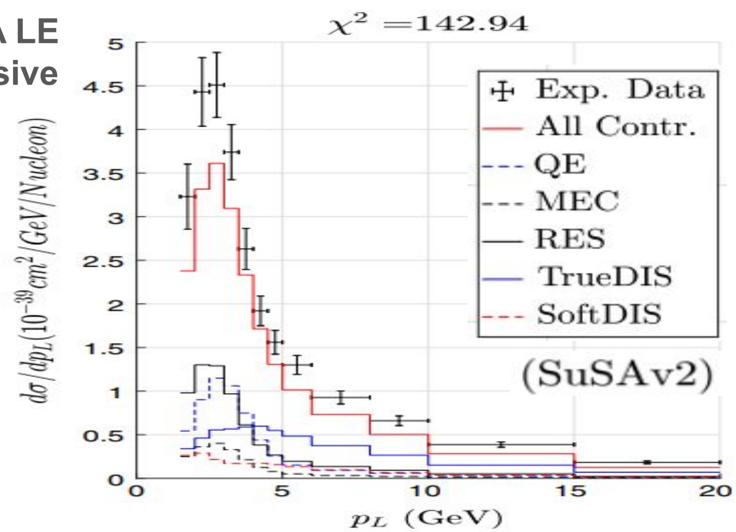


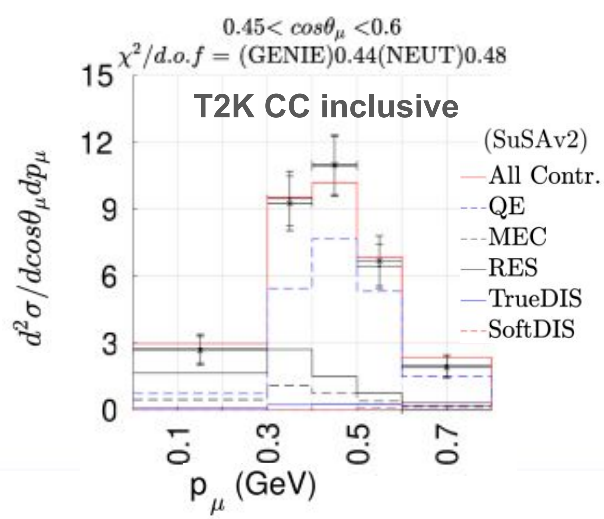
MINERνA $\bar{\nu}_\mu$-CH



Model uncertainties in RES and DIS models are the largest ones

MINERvA LE CC inclusive



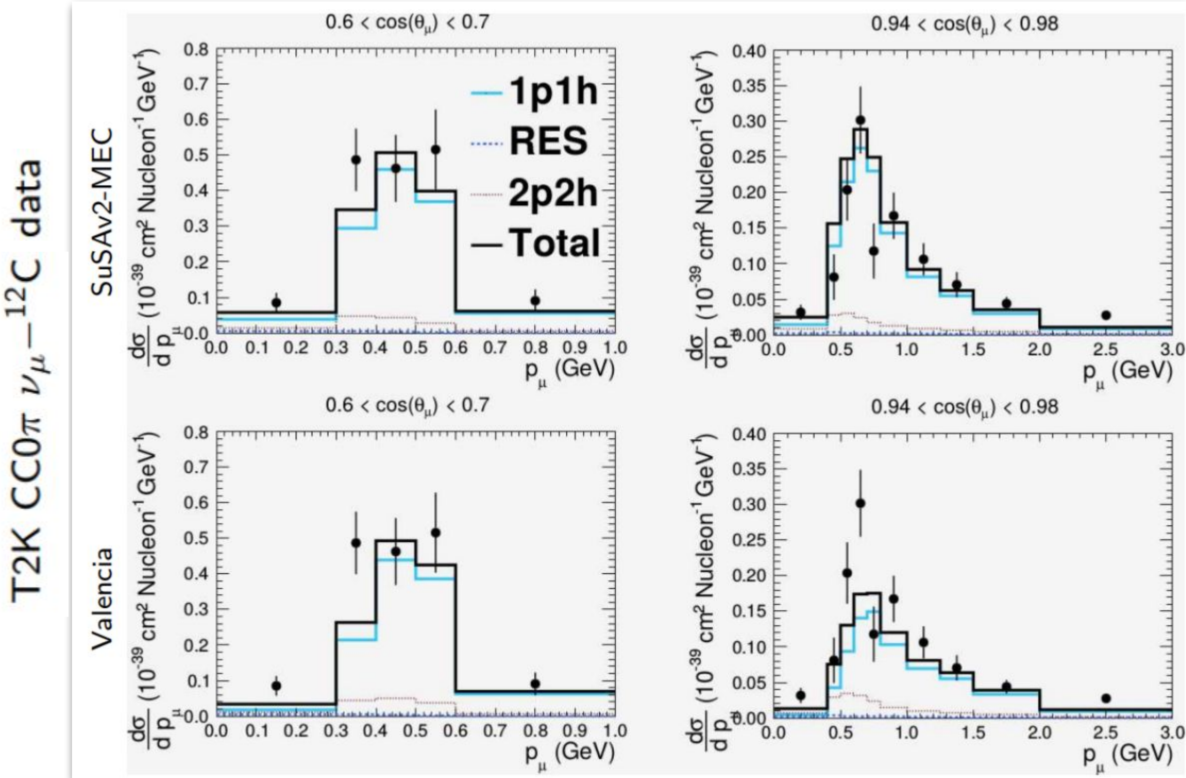


SuSAv2-RES is similar to RMF-1pi and Ghent Hybrid Model, unlike Berger-Sehgal GENIE. Comparison with NEUT-DCC is on the way.

SuSAv2 results: PRD108, 113008 (2023)

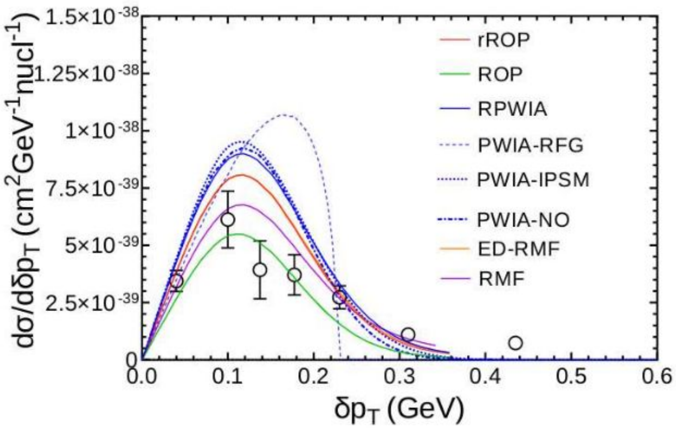
- **SuSAv2-QE** and **2p2h** are **implemented in GENIE** for both **(e,e')** and **CC ν_μ** reactions on **^{12}C** via precomputed **hadron tensors** in terms of **q** and **ω.** Easily **scale** to other **nuclei** (**^{16}O**, **^{40}Ar**, etc.).
- **SuSAv2-QE, RES and inelastic** codes are not computationally demanding and can be fully implemented for **reweighting** of parameters (M_A^{QE}, p_F, E_b, etc.). Their full implementation in MC event generators can also speed up calculations.

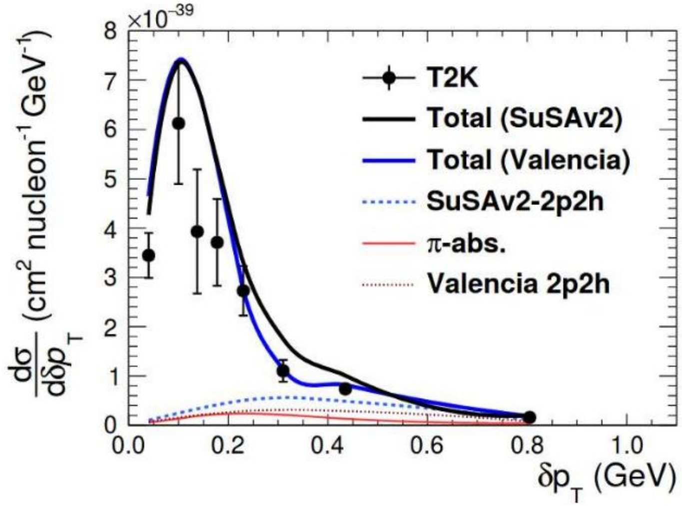
- **RMF** models are implemented by now using precomputed tables. Complex codes.
- SuSAv2 models only predict lepton kinematics but can predict hadron kinematics via *n(p)* from RMF theory under factorization approach (work in progress). Now in generators: *SuSAv2(lepton) + n(p) from LFG(hadron)*
- *RMF and SuSAv2 models produce equivalent results for lepton kinematics.*
- ***Work in progress to implement SuSAv2-RES (DCC) and SuSAv2-DIS in GENIE and NEUT.***

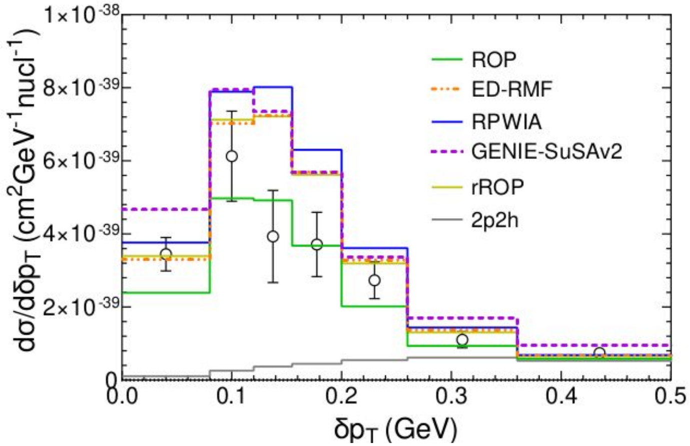


SuSAv2-QE and 2p2h implementation in GENIE (PRD101, 033003 (2020), PRD103, 113003 (2021)) and NEUT (in progress)

- **Factorization approach:** Inclusive (lepton kinematics) models + nucleon momentum distribution → "Semi-inclusive" but less accurate. SuSAv2-GENIE + LFG. Factorization approach can fail (mainly at **low kinematics**).



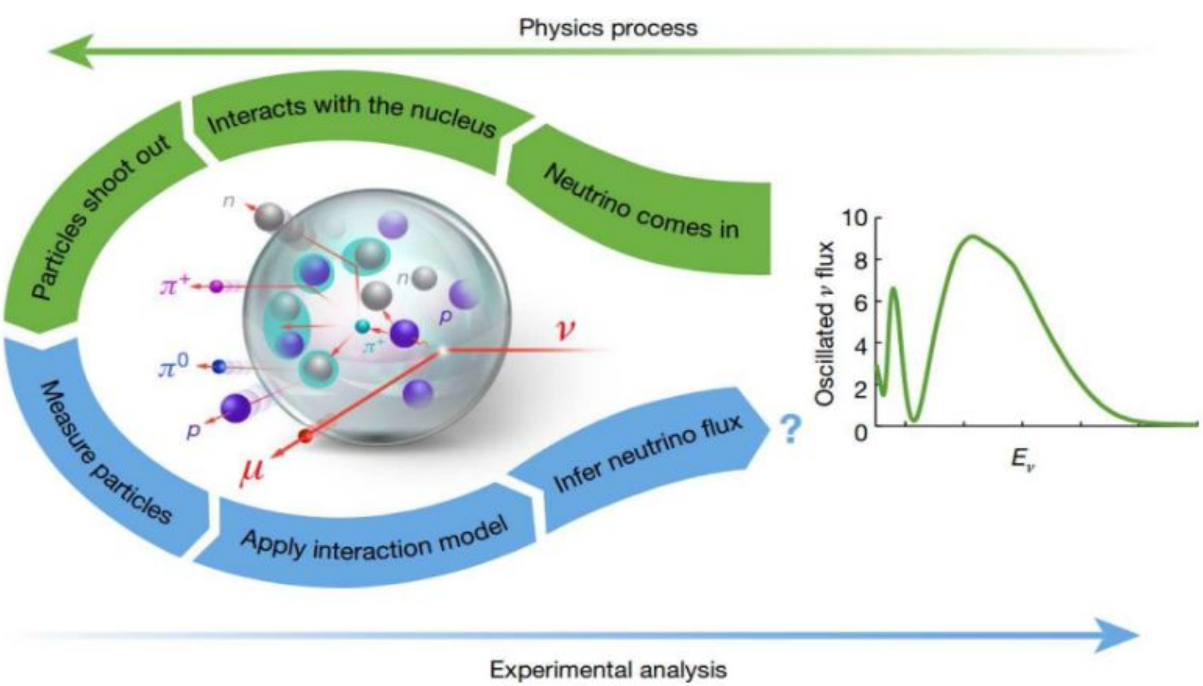




Phys. Rev. D **106**, 113005 (2022),
Phys. Rev. D **109**, 013004 (2024).

- **RPA and FSI effects** can introduce important differences in the experimental analyses.
  - These differences affect OA: E dependence of the CS, ν vs. $\bar{\nu}$, ν_e/ν_μ, C/O.
  - Largest uncertainty in semi-inclusive neutrino CS (lepton+hadron kinematics).

Do energy reconstruction methods and model implementation work properly?



ν-**oscillation experiments cannot directly measure E**$_\nu$ $\Rightarrow$ interpretation of results strongly relies on **MC simulations** and $\nu - \mathbf{A}$ interaction models to infer $\mathbf{E}_\nu$ $\Rightarrow$ Exploiting similarity of $\mathbf{e} - \mathbf{A}$ and $\nu - \mathbf{A}$ reactions and using **e-scattering data** with known beam energies allow **to test E**$_{\mathbf{reconst}}$ **methods and interaction models**. Even in simple interactions where no pions are detected, **only a small fraction of events reconstruct to the true incident energy.**

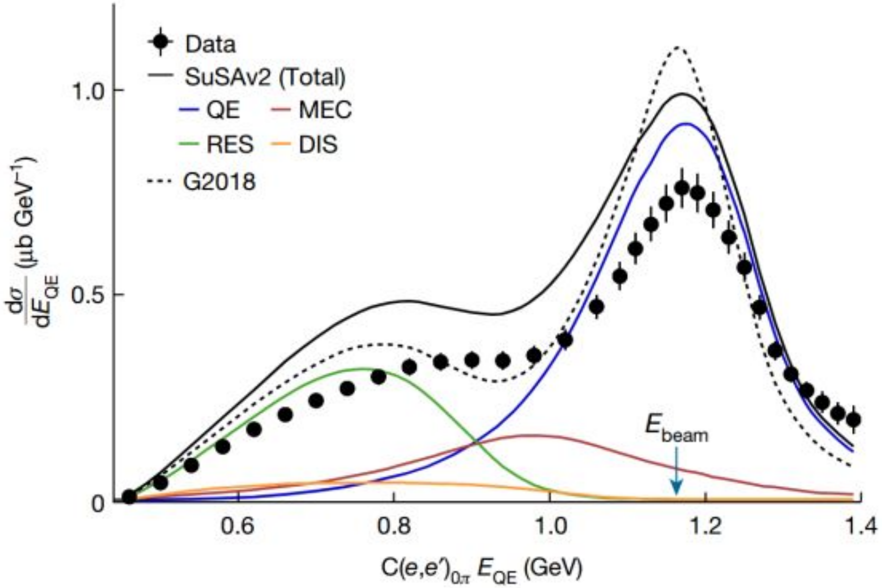
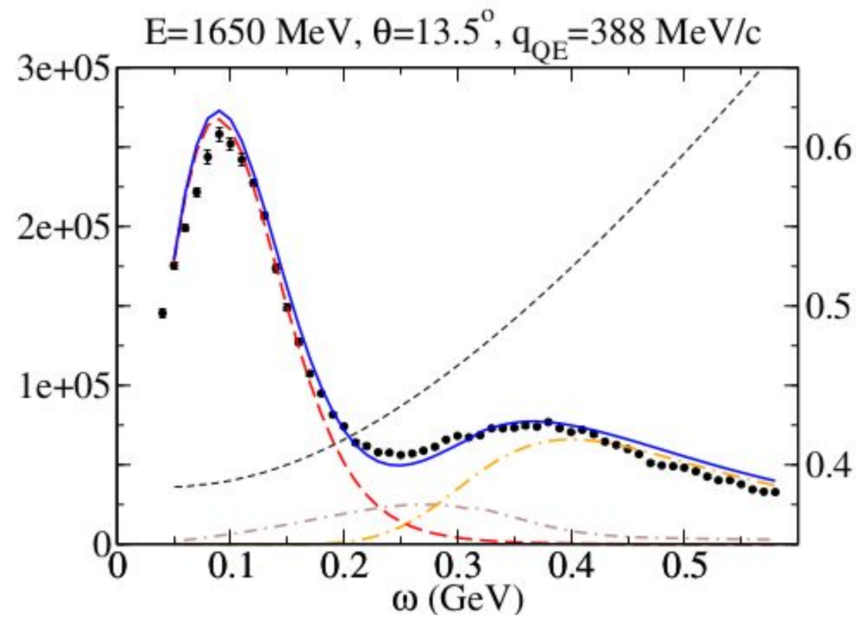
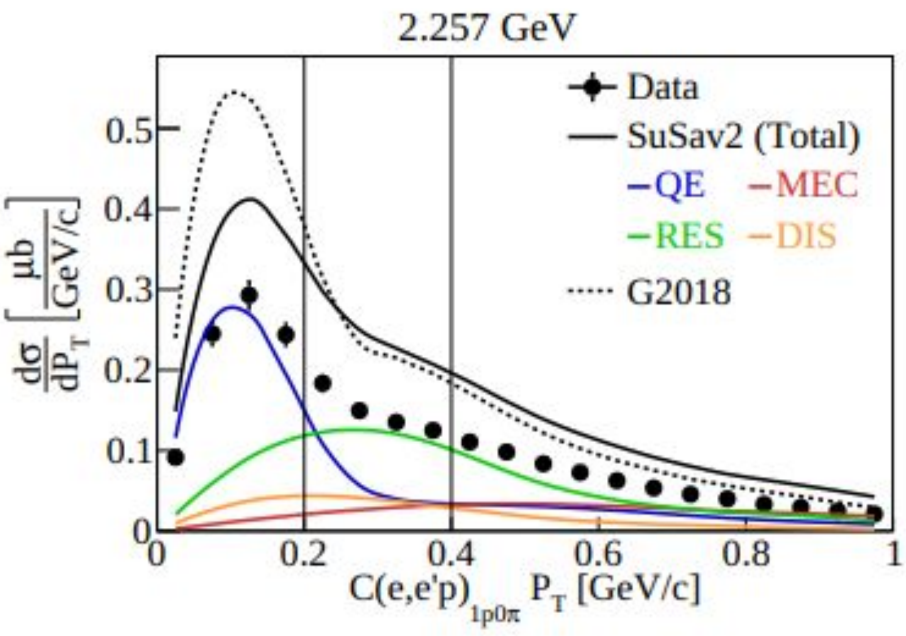


Fig. 2 | Quasi-elastic reconstructed energy. The 1.159 GeV C(*e*, *e*′)$_{0\pi}$ cross-section plotted as a function of the reconstructed energy E_{QE} for data (black points), GENIE SuSAv2 (solid black curve) and GENIE G2018 (dotted black curve). The coloured lines show the contributions of different processes to the

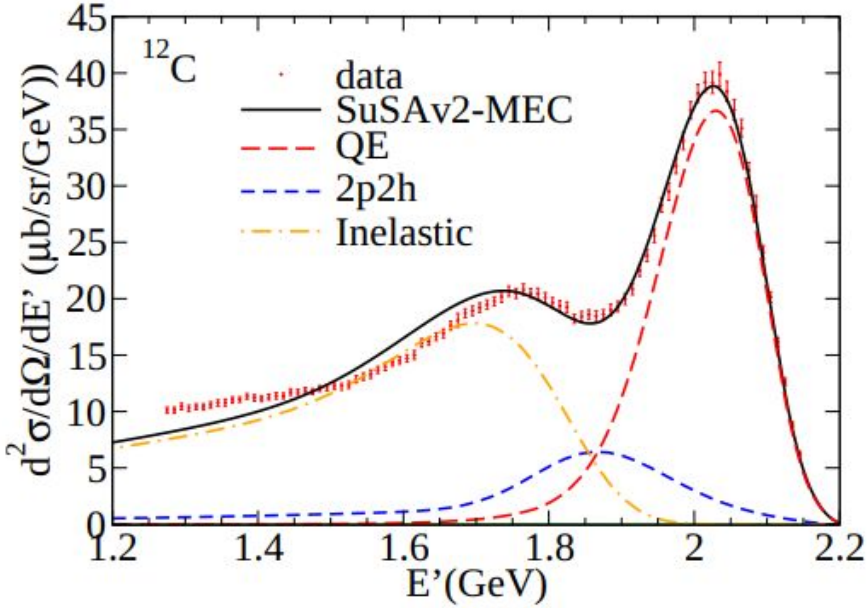


According to previous extensive **SuSAv2** studies, the model should not overestimate the inclusive data.

EDRMF could improve data agreement at very low kinematics.



SuSAv2 comparisons with electron reactions on 12C:
Phys. Rev. D 94, 013012 (2016), Phys. Rev. C 99, 042501 (2019)



Left: **GENIE-SuSAv2(QE+2p2h)+GENIE-Inelastic** at E_i**=2.257 GeV** with **CLAS semi-inclusive** data on ^{12}C

vs.

Right: **SuSAv2(QE+2p2h+Inelastic)** at E_i**=2.222 GeV** and θ=15.541° with **JLab inclusive** data on ^{12}C.

Overestimation for semi-inclusive could be related to **different issues**:

implementation, data analysis, factorization approach (SuSAv2-inclusive + LFG n(p)), etc.

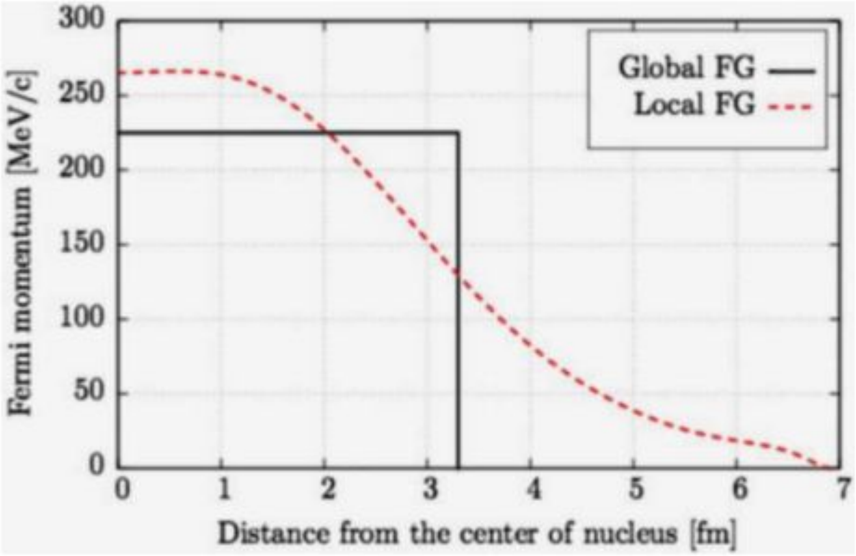
Towards a more realistic semi-inclusive SuSAv2 model in GENIE: 1st step (QE)

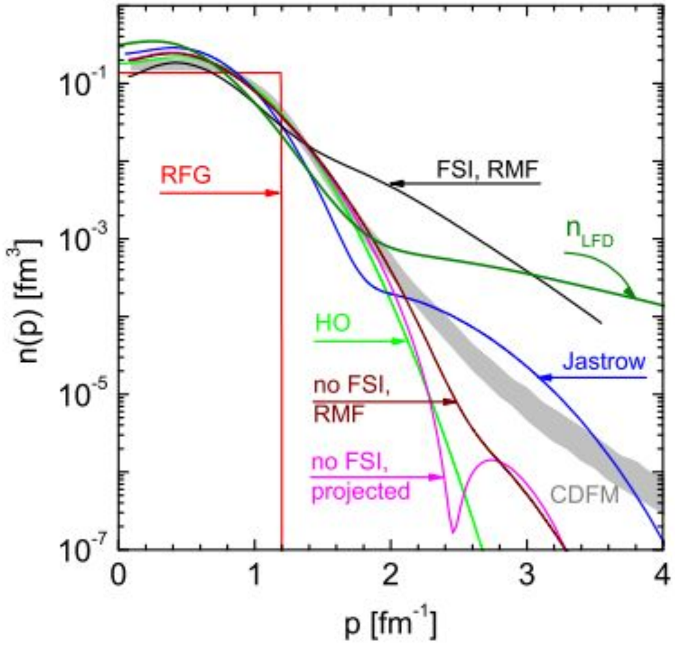
1. Current **SuSAv2-QE(GENIE) semi-inclusive** results depend on factorization approaches and information about **hadron kinematics from LFG n(p)**.

2. Replace **LFG n(p)** **by** **RMF/EDRMF n(p)**.

3. Produce **SuSAv2-QE(GENIE) semi-inclusive** results and compare with **original EDRMF-QE**. Check consistency and possible discrepancies at some kinematics (*expected at the **low-energy** region where nuclear models are "less factorizable"*), **quantify** them, and try to **reduce** them.

4. But **SuSAv2-QE semi-inclusive is not necessary!** **We already have EDRMF-QE**. Yes, but SuSAv2-QE is simpler, less computationally-demanding and easier to implement and modify/reweight parameters on the fly.



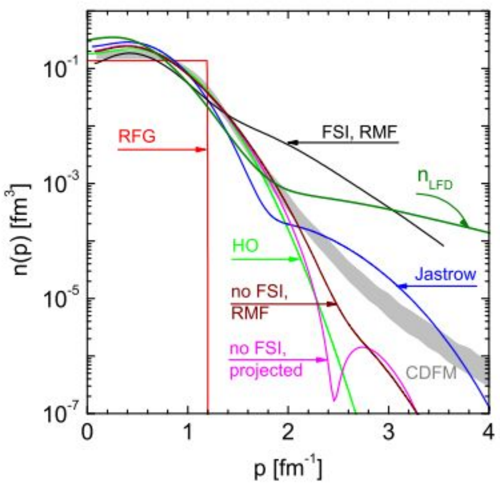


In a **RMF** framework, **n(p)** can be obtained via initial-nucleon distorted wave functions (no FSI):

$$n^{RMF}(p) = \sum_{\text{shells}} n_{\text{shell}}(p) \qquad n_{\text{shell}}(p) = |\varphi_{\text{shell}}(p)|^2$$

or with **elastic-FSI** (but no cascade effects): *PRC81, 055502 (2010), PRC83, 045504 (2011)*, **assuming factorization** (**PWIA semi-inclusive CS factorizes**):

$$\left[\frac{d\sigma}{d\epsilon' d\Omega' dp_N d\Omega_N}\right]^{PWIA}_{(e,e'N)} = K\sigma^{eN}(q,\omega;p,\mathcal{E},\phi_N)S(p,\mathcal{E})$$



$$F(q,\omega) = 2\pi \iint_{\Sigma(q,\omega)} p\,dp\,d\mathcal{E}\,S(p,\mathcal{E}) \qquad F(q,\omega) \cong \frac{[d\sigma/d\epsilon' d\Omega']_{(e,e')}}{\overline{\sigma}^{eN}(q,\omega;p=|y|,\mathcal{E}=0)}$$

$$n(p) = \int_0^\infty d\mathcal{E}\,S(p,\mathcal{E})$$

$$f_{\text{exp}}(q,\omega) \equiv k_A F_{\text{exp}}(q,\omega); \quad f^{L(T)}_{\text{exp}}(q,\omega) \equiv k_A F^{L(T)}_{\text{exp}}(q,\omega)$$

Then, **assuming factorization** also **in RDWIA** (RPWIA, RMF, EDRMF), we can connect $n_{RMF}(p)$ with $f_{SuSAv2\text{-}RMF}(\psi)$ and **semi-inclusive CS** (EDRMF) with **inclusive ones** (SuSAv2):

$$\left.\frac{d\sigma}{d\Omega' d\epsilon' d\Omega_N}\right|_{\text{PWIA}} = K\sigma^{eN}\tilde{n}_{\text{shell}}(p)$$

$$\rho(p) = n^{\text{dist}}(p) = \frac{\left[\frac{d\sigma}{d\Omega' d\epsilon' d\Omega_N}\right]_{\text{FSI}}}{K\sigma^{eN}}$$

$$f^{\text{dist}}(\psi) = k_F \frac{\left[\frac{d\sigma}{d\epsilon' d\Omega'}\right]^{\text{RMF+FSI}}_{(e,e')}}{\overline{\sigma}^{eN}(q,\omega;p=|y|,\mathcal{E}=0)}$$

For validation, semi-inclusive CS SuSAv2-QE(GENIE) + $n_{RMF}(p)$ + cascade need to be **compared with** EDRMF + cascade **or with** real+imaginary OP in RDWIA (see A. Nikolakopoulos et al. *Phys.Sci.Forum 8 (2023), PRC105, 054603 (2022)*).

Towards a more realistic semi-inclusive SuSAv2 model in GENIE: 2nd step (RES - CC1pi)

1. Once consistency of **semi-inclusive SuSAv2-QE(GENIE)** is validated with **EDRMF-QE**, we can explore other nuclear regimes and channels.

2. Using again **EDRMF n(p)** either without FSI (*same n(p) as EDRMF-QE no FSI*) or with FSI (*EDRMF n(p) for CC1N1pi*), we can implement **SuSAv2-1pi(GENIE)** and combine with **EDRMF n(p)**.

3. Check consistency between SuSAv2-CC1pi(GENIE) + $n_{RMF}(p)$ + cascade and EDRMF-1pi + cascade for inclusive and semi-inclusive cross sections.

   EDRMF and original SuSAv2-1pi inclusive results have been successfully compared in previous works. *See also Jesus' talk on Wednesday on SuSAv2-1pi and inelastic models.*

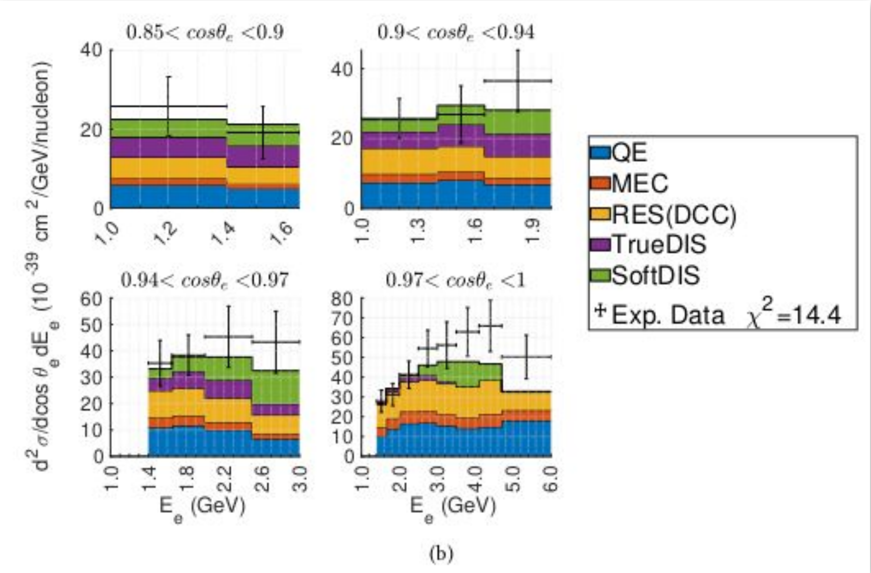
4. But **SuSAv2-1pi semi-inclusive is not necessary!** **We already have EDRMF-1pi**. Yes, but SuSAv2-1pi is simpler, easier to implement and less computationally demanding.

1. There is **no RMF** approach for the full inelastic regime or DIS. However, **SuSAv2-inelastic** reproduce very well (*e,e'*) inclusive data in the **full inelastic regime** and it also works well for *neutrinos* (but *there are some model-vs-data tensions, as for other models*).

2. The SuSAv2-inelastic framework allow to **separate** the results for different inelastic channels (**RES+SoftDis+TrueDIS**). *See also Jesus' talk on Wednesday about SuSAv2-inelastic models and status of inclusive implementation in GENIE.*

3. After checking that SuSAv2 QE and 1pi extensions to semi-inclusive are working properly, we can assume that SuSAv2-inelastic semi-inclusive should also work well.

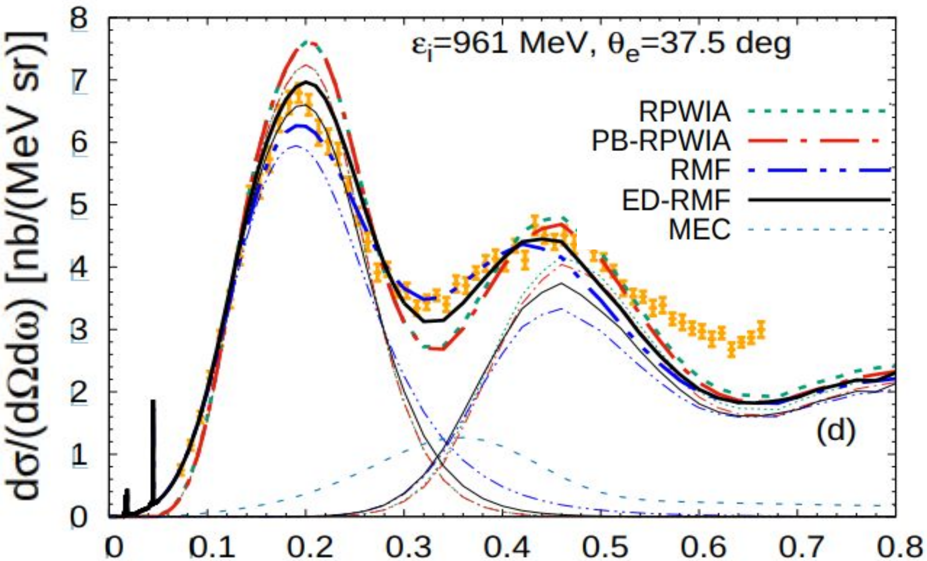
4. We cannot compare with EDRMF + cascade, but we could also **validate** SuSAv2-inelastic(GENIE) with **semi-inclusive data** from electron and neutrino reactions.

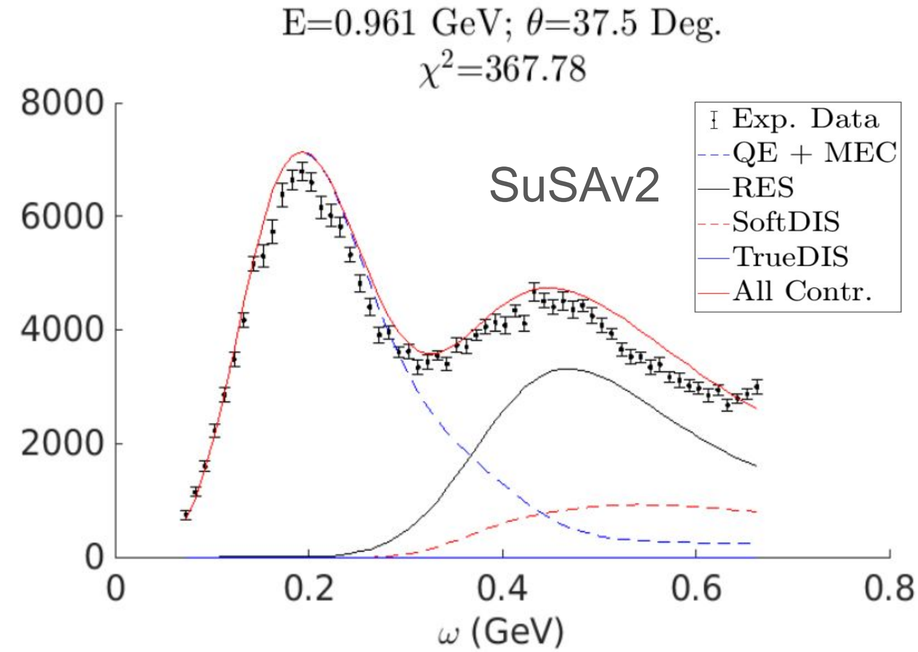


NOνA CC ν_e inclusive flux-averaged double-differential cross section vs. SuSAv2 [J. Gonzalez-Rosa et al., Phys. Rev. D 111, 073002 (2025)]

BACKUP SLIDES:

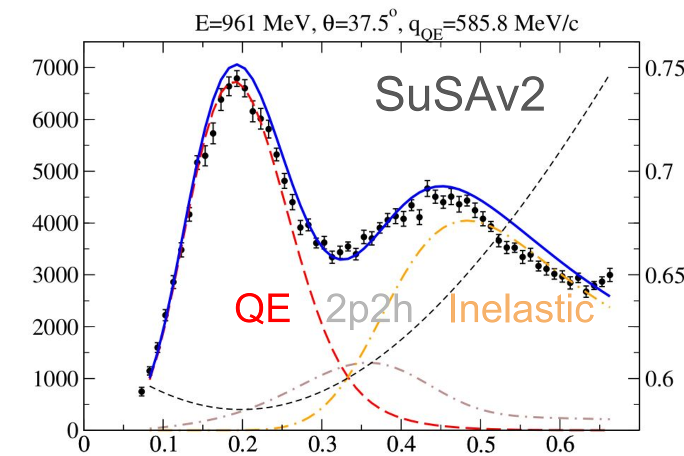
SuSAv2 and RMF models vs. (e,e') data

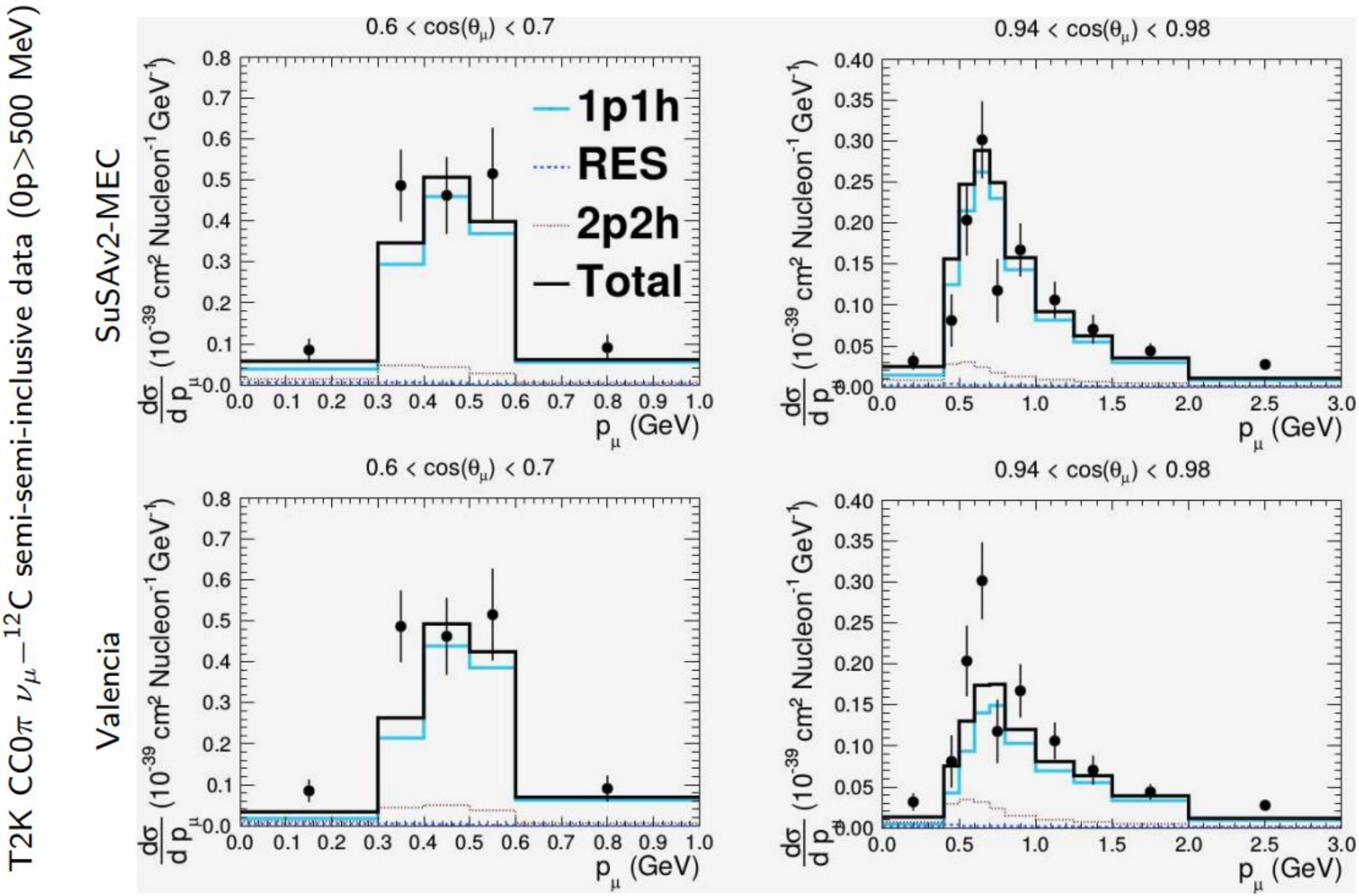




- **SuSAv2-QE:** parameterization ot **RMF** scaling functions and reduction of RMF nuclear potential's strength at high kinematics **~ ED-RMF**. *Similar approach for the RES and DIS regimes*.
- **SuSAv2-MEC (2p2h)**: **RFG** approach based on Granada model.
- **SuSAv2-RES(DCC)**: SuSAv2-inelastic for W<2.1 GeV + DCC.
- **SuSAv2-TrueDIS**: SuSAv2-inelastic(Bosted-Christy) at W>2.1 GeV.
- **SuSAv2-SoftDIS**: "*SuSAv2-inelastic - SuSAv2-RES*" at W<2.1 GeV.
- **RMF-QE** and **RMF-1pi** are currently available. *Work in progress for 2p2h. Different nuclear potentials available (different treatment of FSI).*

SuSAv2 (e,e') results: *PRD94, 013012 (2016); PRC99, 042501 (2019); PRD108, 113008 (2023)*




T2K CC0π $\nu_\mu-^{12}$C semi-semi-inclusive data (0p>500 MeV)
SuSAv2-MEC
Valencia
0.6 < cos(θ_μ) < 0.7
0.94 < cos(θ_μ) < 0.98
1p1h
RES
2p2h
Total
dσ/dp_μ (10^-39 cm² Nucleon^-1 GeV^-1)
p_μ (GeV)

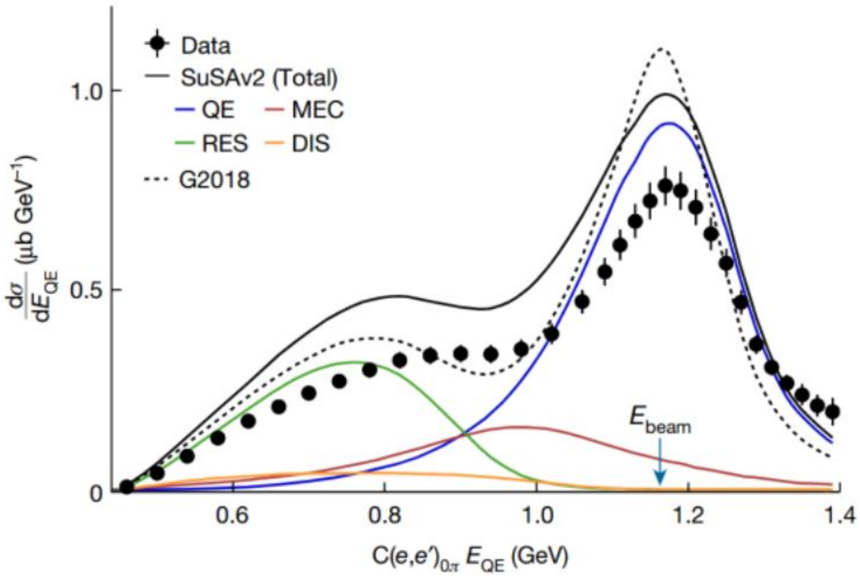
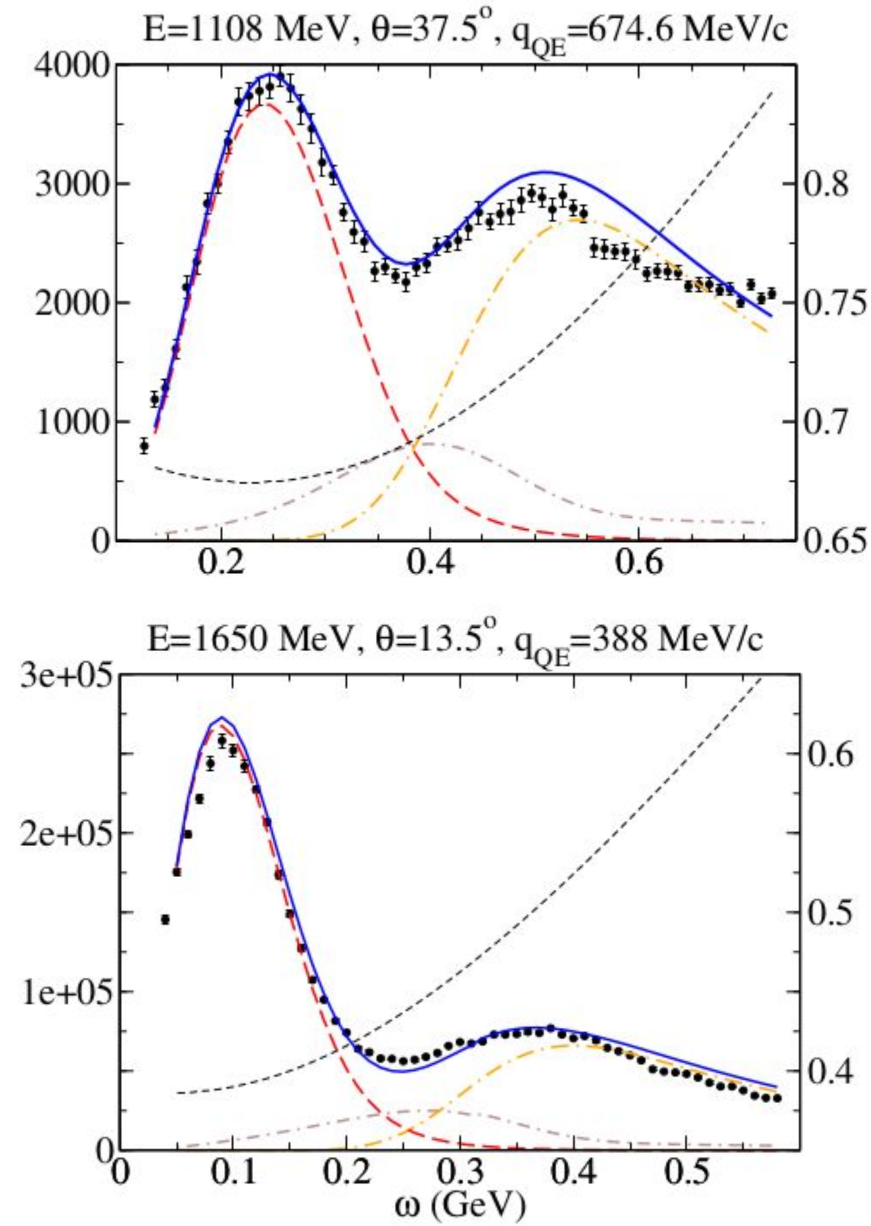


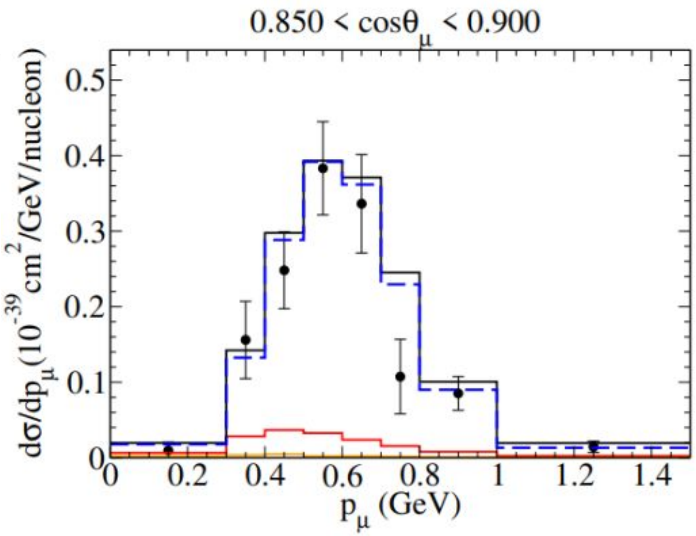
Fig. 2 | Quasi-elastic reconstructed energy. The 1.159 GeV $C(e,e')_{0\pi}$ cross-section plotted as a function of the reconstructed energy E_{QE} for data (black points), GENIE SuSAv2 (solid black curve) and GENIE G2018 (dotted black curve). The coloured lines show the contributions of different processes to the

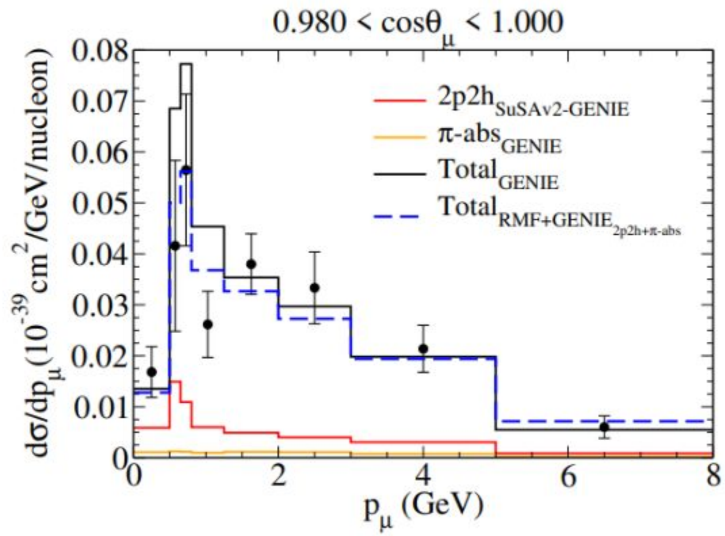


SuSAv2 can **overestimate** data at very forward angles in the **QE peak and below** (low-energy nuclear effects at low q and ω), but **QE+2p2h** in the *dip region* should <u>not overestimate</u> the data. **EDRMF** would improve data agreement at very low kinematics.

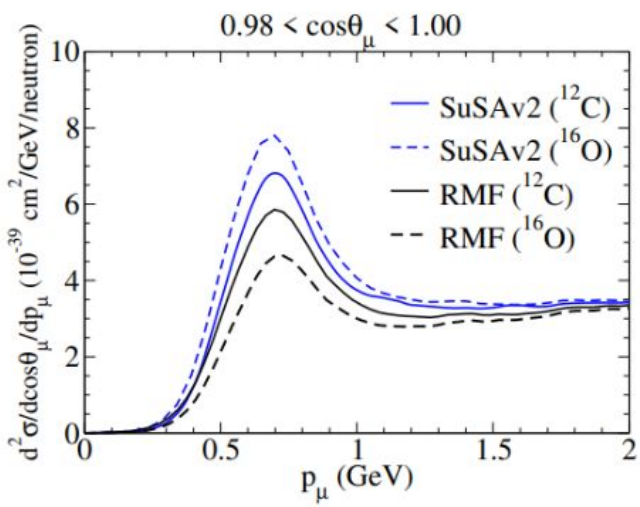
Low-energy effects at T2K CC0π 0p >500 MeV/c   *arXiv:1905.08556*

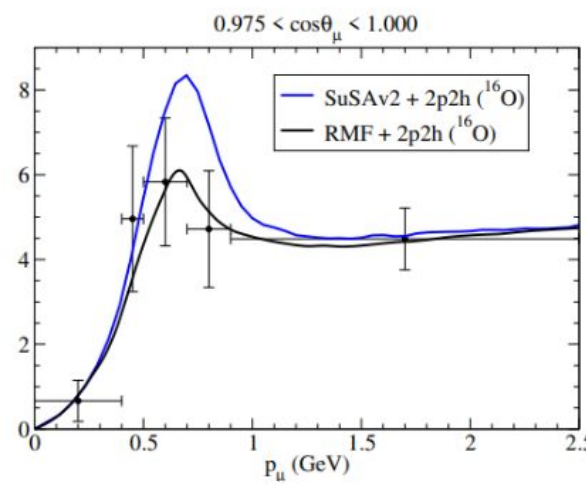
Low-energy effects and scaling violations are only appreciable at very forward angles (low q_3, q_0 values). RMF is more accurate than SuSAv2 at these kinematics.





Low-energy nuclear effects and its proper description can have an important effect in the C to O extrapolation, which is essential for T2K and HK.

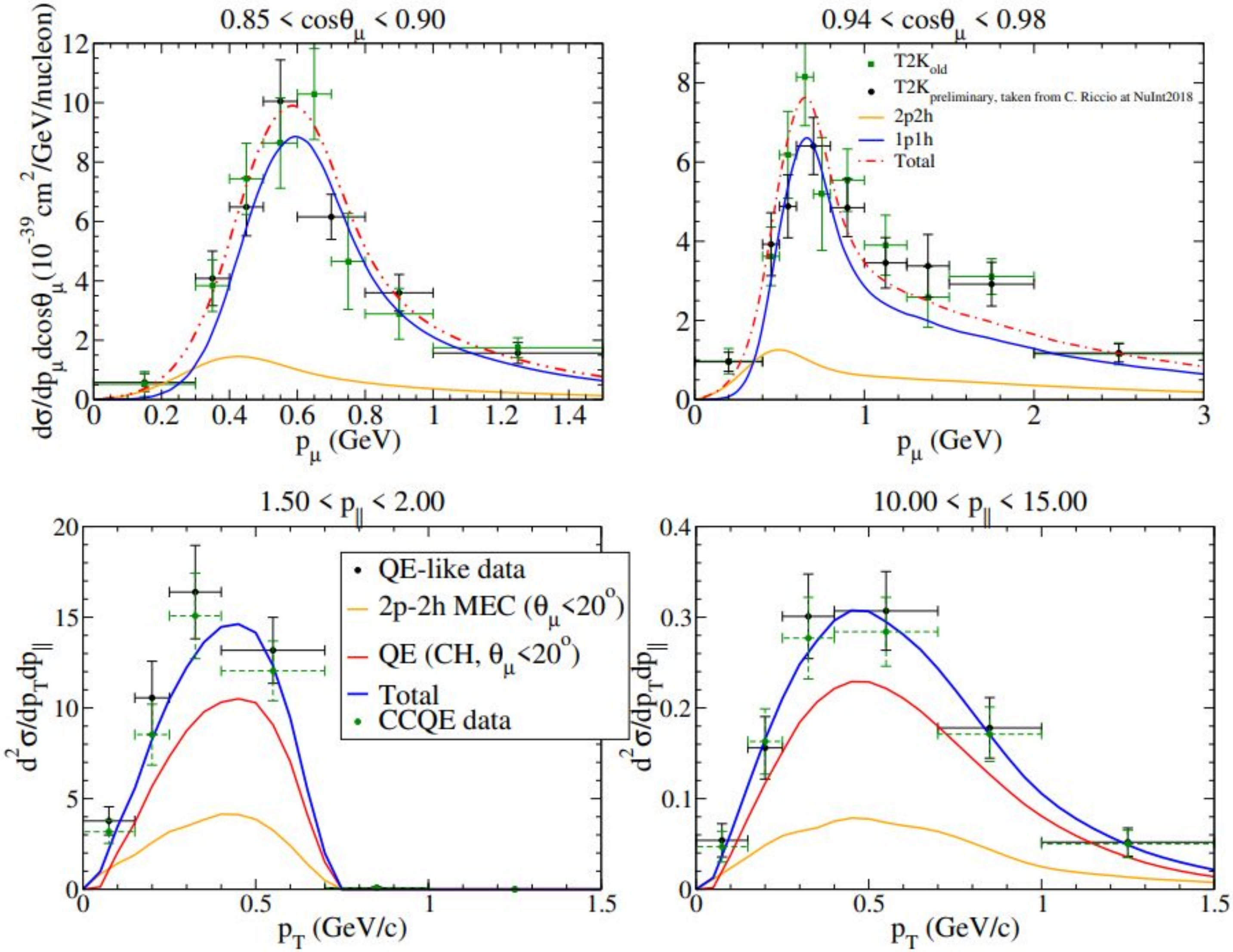




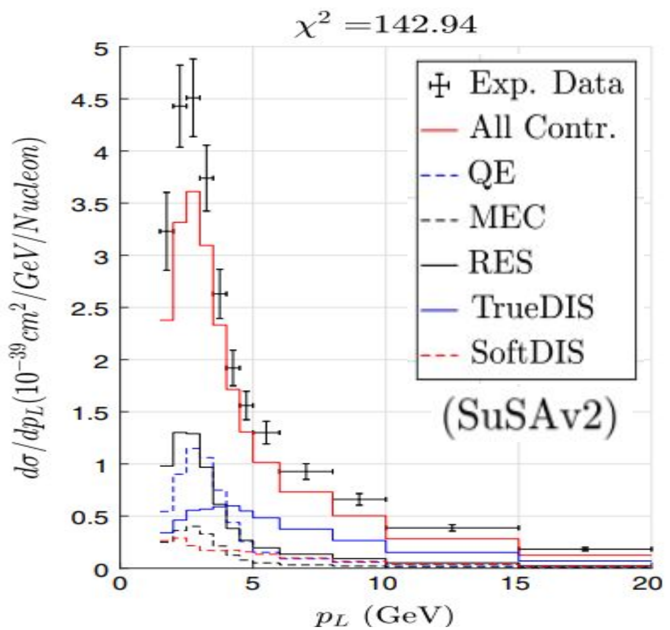
Comparison with CC0π ν_μ-nucleus data

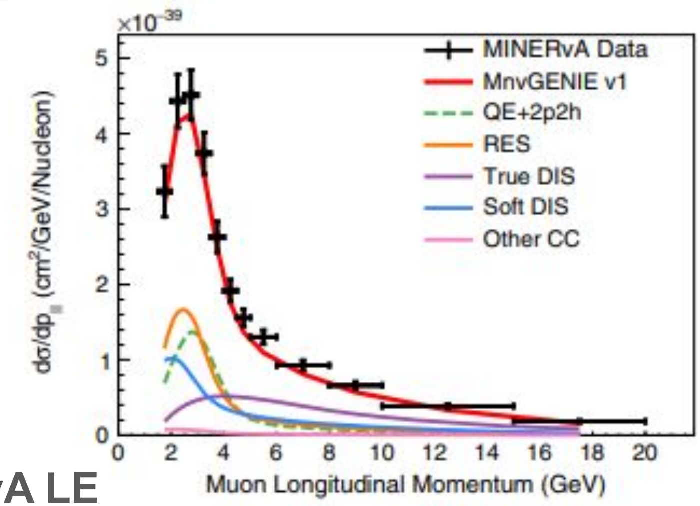
T2K CC0π $\nu_\mu - C_8H_8$

MINERνA $\bar{\nu}_\mu$-CH



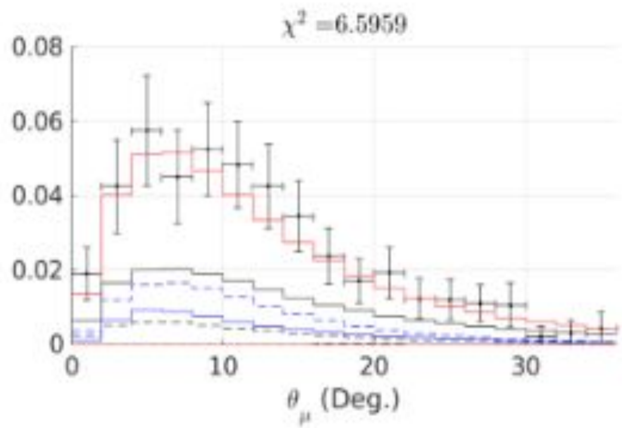
Model uncertainties in RES and DIS models are the largest ones



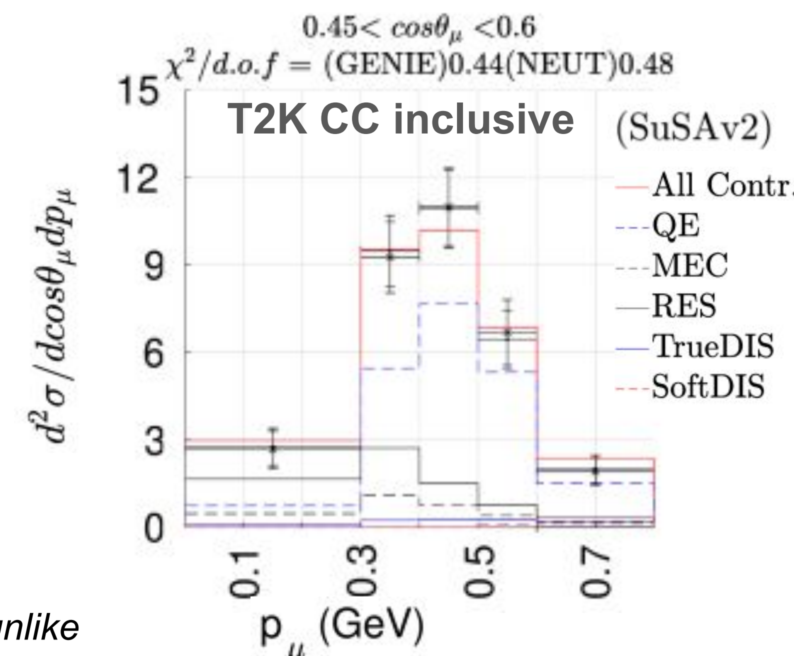


MINERvA LE CC inclusive

MINERvA data: PRD101, 112007 (2020)
T2K data: PRD98, 012004 (2018)
SuSAv2 results: PRD108, 113008 (2023)



ArgoNEUT $CC\bar{\nu}_\mu$, $< E_{\nu_\mu} > \sim 3.6$ GeV

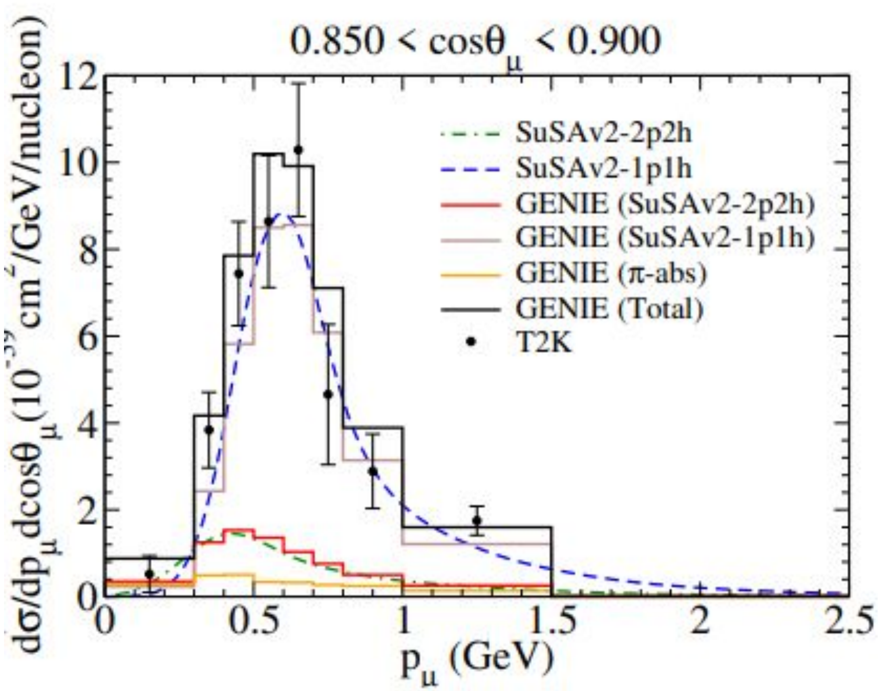


SuSAv2-RES is similar to RMF-1pi and Ghent Hybrid Model, unlike Berger-Sehgal GENIE. Comparison with NEUT-DCC is on the way.

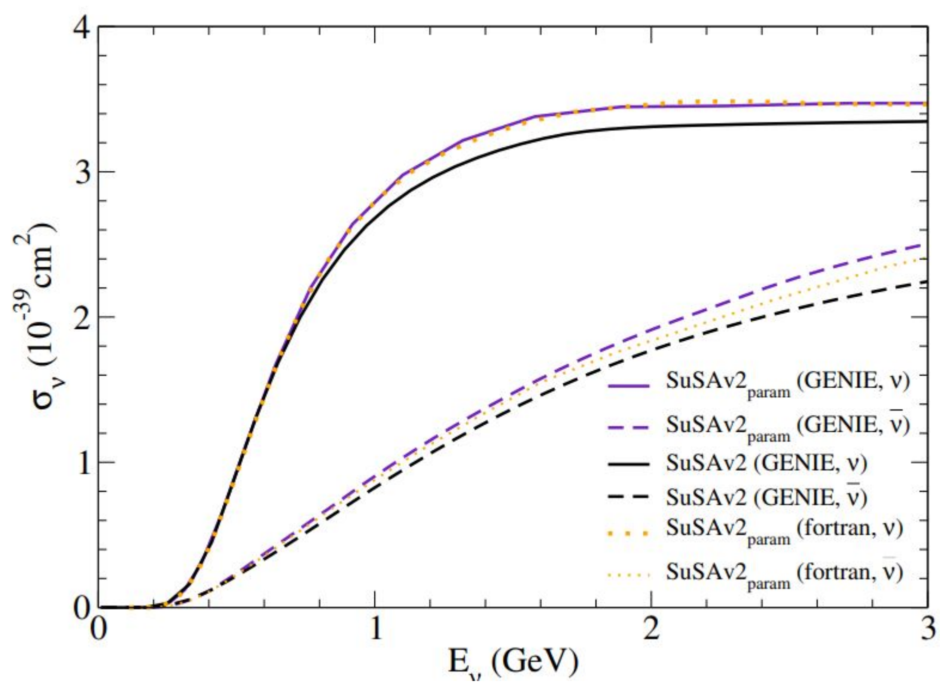
Neutrinos: SuSAv2-QE and 2p2h implementation in GENIE vs. original codes

GENIE interpolation methods for 2p2h hadron tensor tables works better that SuSAv2 2p2h parameterization formula but results are very similar for 2p2h. Same is true for QE where GENIE interpolates hadron tensor tables from original SuSAv2 code.

T2K CC0pi inclusive cross-section



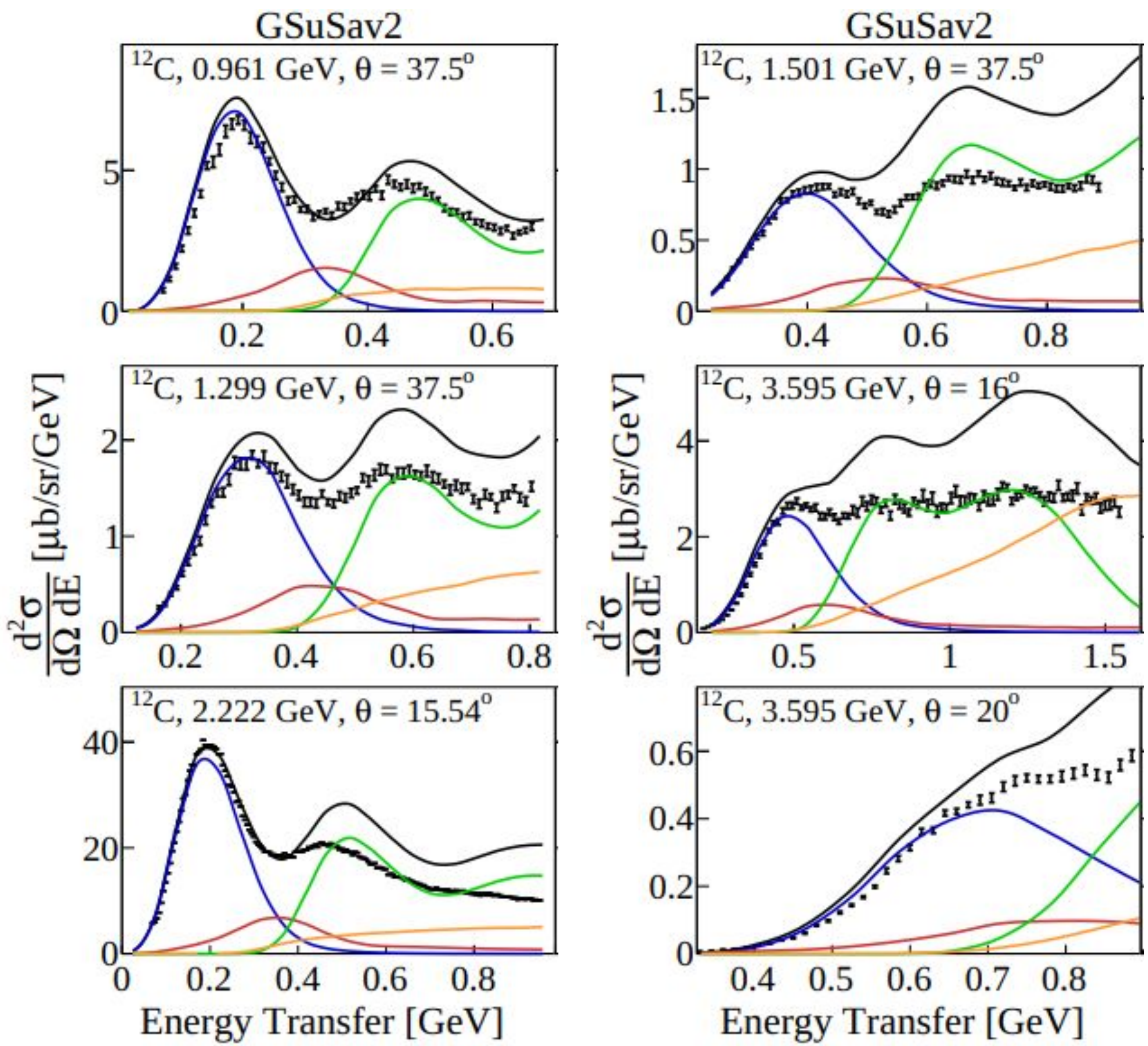
SuSAv2 RFG-2p2h total cross section

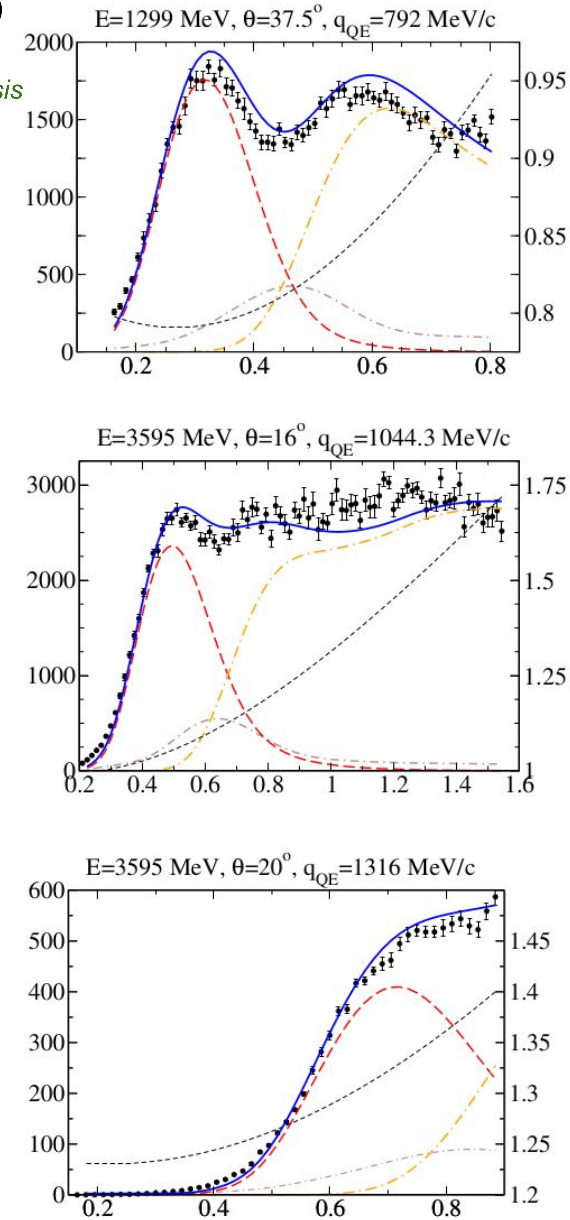


Electrons: SuSAv2-QE and 2p2h in GENIE (left and mid panels) vs. original codes (right panels)

GENIE-Inelastic is not SuSAv2-inelastic in left and mid panels.

Right panels: Original SuSAv2 (QE+2p2h+Inelastic): *PRD94 (2016) 013012 and G.D. Megias' PhD thesis*





Inelastic Nuclear Responses within the SuSAv2 Approach

➲ Extension of the SuSAv2 formalism to the complete inelastic spectrum ⇒ resonant (Δ), nonresonant and deep inelastic scattering (DIS).

$$R_{QE}^{L,T} = \frac{\mathcal{N}\xi_F}{\eta_F^3 \kappa m_N} R_{s.n.}^{L,T} f_{SuSAv2}^{L,T}(q_0^{QE}, \psi') \quad ; \quad R_{inel}^{L,T} = \frac{\mathcal{N}\xi_F}{\eta_F^3 \kappa} \int d\mu_X \mu_X R_{inel(s.n.)}^{L,T} f_{SuSAv2}^{L,T}(q_0^{inel}, \psi'_X),$$

$\mu_X = \frac{W_X}{m_N}$: dimensionless invariant mass, ψ'_X: inelastic scaling variable, and $U_{inel}^{L,T}$ depends on the single-nucleon inelastic structure functions W_1, W_2, W_3 ($\equiv F_1, F_2, F_3$), obtained by using:

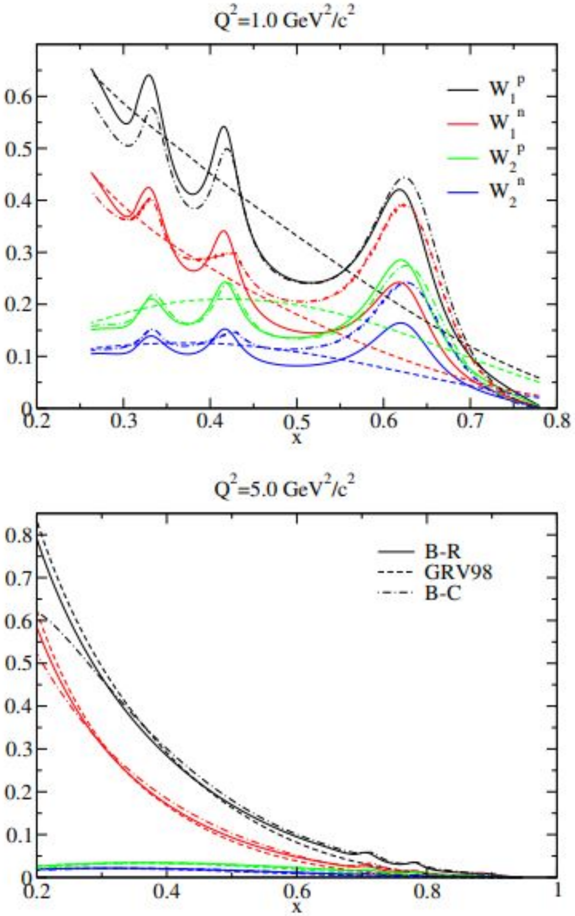
- Fits of the inelastic structure functions (Bodek-Ritchie, **Bosted-Christy**, ...)
- *PDFs* (GRV98 model, ...)

$$F_2^{eN} = \frac{1}{2}\left(F_2^{ep} + F_2^{en}\right) = \frac{5x}{18}\Big(u(x) + \bar{u}(x) + d(x) + \bar{d}(x)\Big).$$

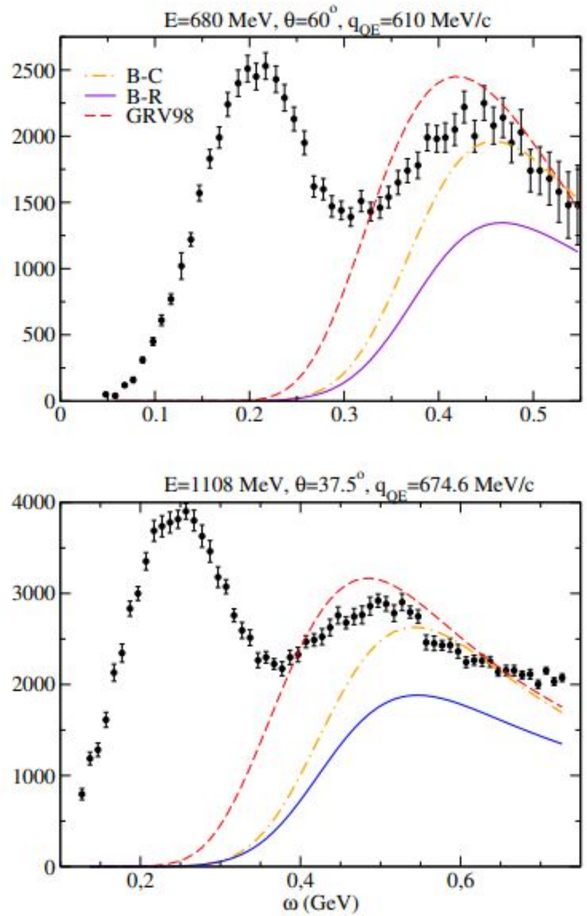
$$F_2^{\nu N} \approx \frac{18}{5} F_2^{eN}. \qquad xF_3^{\nu N} = F_2^{\nu N} - 2\bar{Q}(x)$$

J. Gonzalez-Rosa et al. https://arxiv.org/abs/2203.12308

Inelastic structure functions



Inclusive $^{12}C(e,e')$ double differential cross section

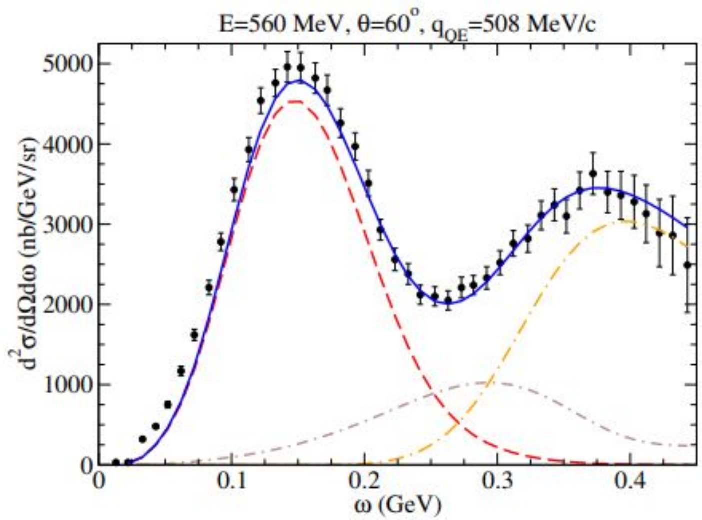


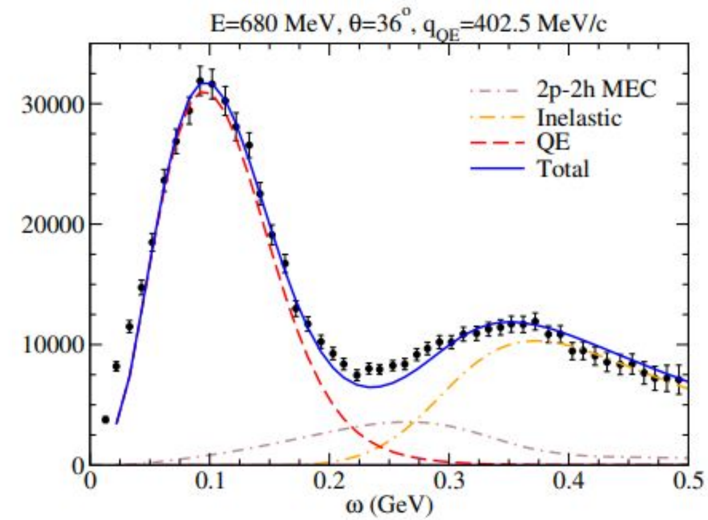
Bodek-Ritchie: poor description of the resonance region.

Bosted-Christy: Good description of the resonant structures observed in (e,e') reactions.

GRV98: No resonant structures (average) and poor description at $Q^2 \lesssim 1$ GeV^2.

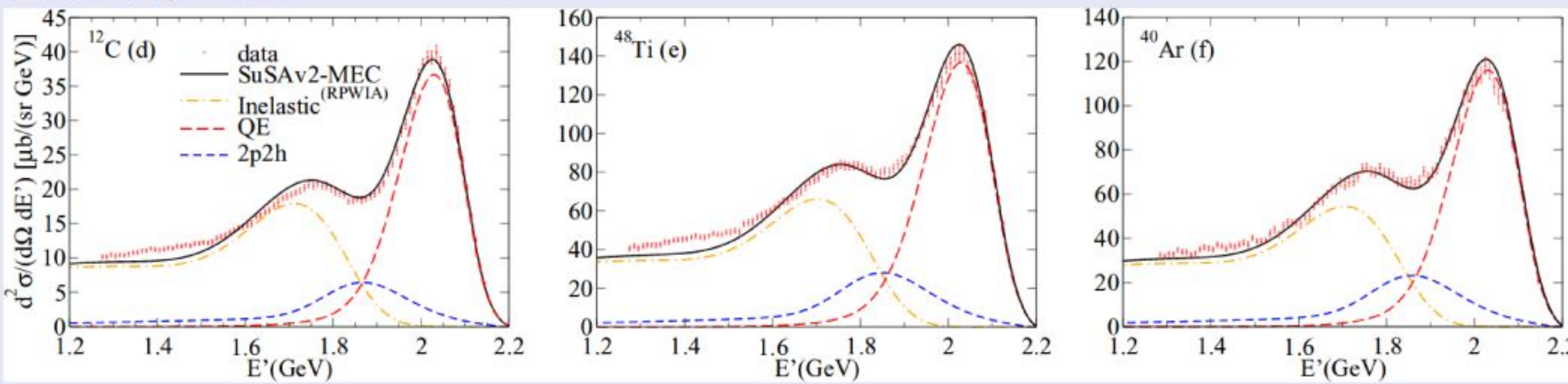
Inclusive $^{12}C(e, e')$ cross sections *PRD 94, 013012 (2016)*





(e, e') JLab data on ^{12}C, ^{40}Ar, ^{48}Ti vs. SuSAv2-MEC. *PRC99, 042501(R), 2019*

$E_i = 2.2$ GeV, $\theta_e = 15.5^o$



Summary of main ideas and tasks

- **Validation against (*e,e'*) data is a solid benchmark** for nuclear models used in **ν oscillation experiments** where nuclear-medium uncertainties are one of the major source of systematics.

- **Satisfactory comparison** of SuSAv2 and RMF models with (*e, e'*) and (*ν, l*) data makes them promising candidates to reduce these uncertainties.

- Analysis of **more exclusive neutrino-nucleus reactions** (joint detection of leptons and hadrons), more sensitive to nuclear model details, can improve **model selection** in **MC experimental generators**.

- **RES** and **DIS** are the channels where model uncertainties are largest. Having electron and neutrino scattering data with detailed final hadron detection ($1\pi^+$, $1\pi^-$, $1\pi^0$, 2π, etc.) can help to develop nuclear models in the inelastic regime.

- **Model tuning** in MC generators must be realistic and connected with nuclear model uncertainties.

- Application of MC **neutrino energy reconstruction** methods using electron experiments (well-known energy beam) can help to improve neutrino oscillation analyses as well as to improve nuclear model selection in these studies.

- **Ongoing tasks related to JENNIFER3**:

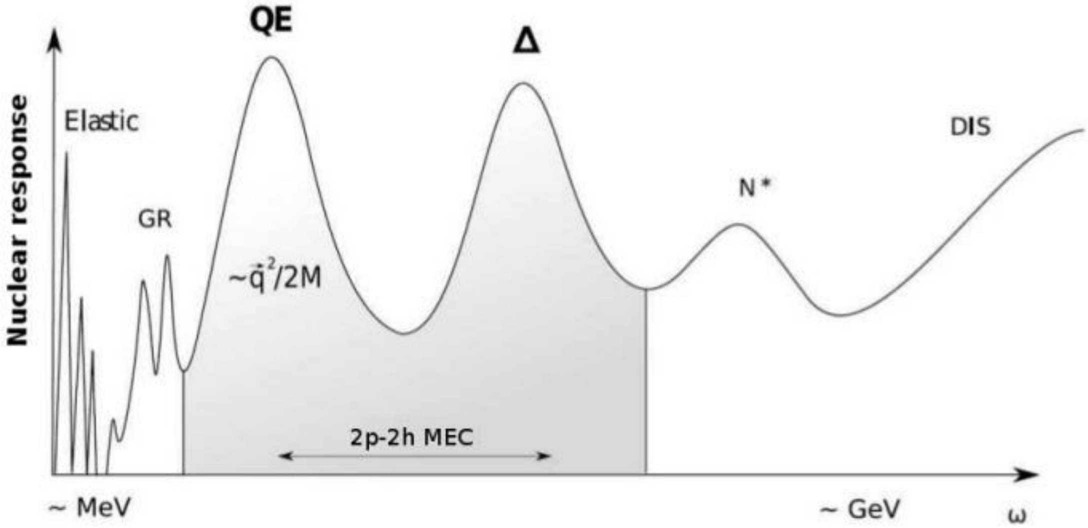
  - **Implementation** of SuSAv2 QE, 2p2h, RES, inelastic and RMF QE + RES models in NEUT and GENIE.

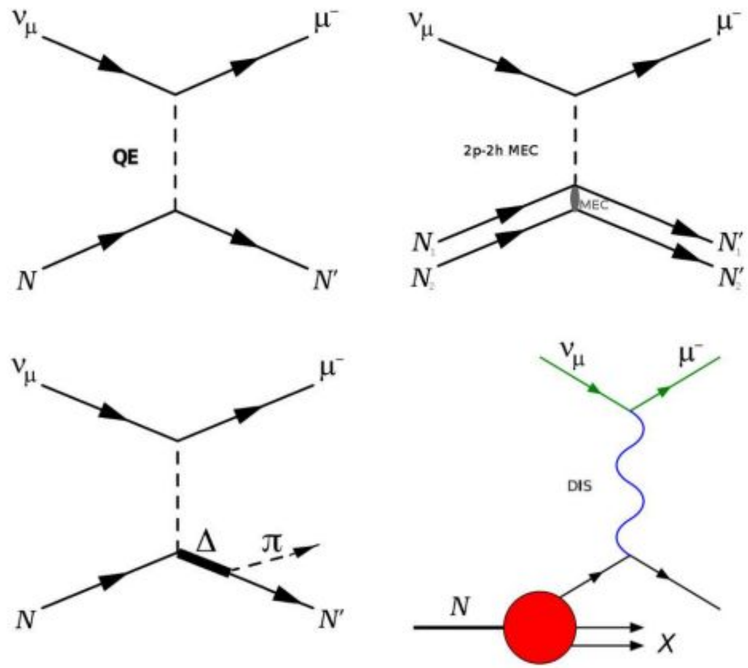
  - **Benchmarking cascade models** with predictions from widely-tested nuclear potentials (ROP, RMF) and analysis of FSI effects.

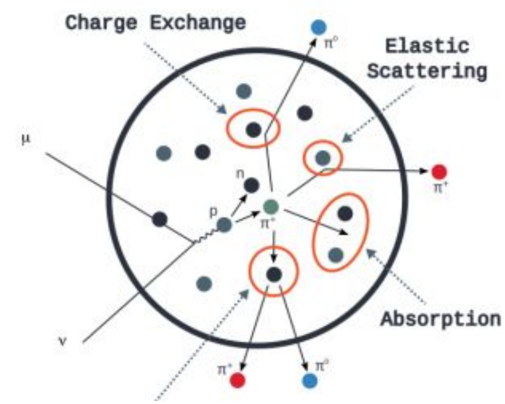
  - **Test implementations** and new features of nuclear models with electron and neutrino scattering data. Add ***model uncertainties*** and ***reweighting***.

Nuclear response in terms of the energy transfer

Neutrino-nucleus interactions over the whole experimental range (of the order of 100s of MeV up to 10s of GeV) implies a large variety of nuclear effects: quasielastic (QE), multinucleon excitations (2p-2h MEC), meson production via nucleon resonances (Δ) and deep inelastic processes. At these kinematics, charged-current quasielastic (CCQE) ν-A scattering is the dominant interaction in oscillation experiments ($\gtrsim 40\%$).







Challenges for nuclear models

Significant improvements of nuclear models by theorists are essential and should include:

1. The development of a unified model of nuclear structure giving the initial kinematics and dynamics of nucleons bound in the nucleus.
2. Providing consistent descriptions for all particles in the FS (leptons, hadrons, mesons, etc.) and nuclear reaction mechanisms.
3. Improving our understanding of FSI (rescattering of produced hadrons in the nucleus), initial nucleon-nucleon correlations, low-energy nuclear effects (binding energies, shell effects, Pauli blocking, etc.).
4. Expressing these improvements of the nuclear model in terms that can be successfully incorporated in the simulation of neutrino events by neutrino event generators.

Challenge for nuclear models and cross section analysis: How to improve understanding of $\nu - A$ interactions for oscillation analysis?

- Improve theory: FSI, ground states, PB and shell effects (E_b), nuclear potentials, etc.
- Use external constraints (e, e'), $(e, e'p)$ to characterize nuclear effects and improve model selection in ν event generators
- Compare several nuclear models with e- and $nu - A$ inclusive (*lepton* detection), semi-inclusive ($l + N$, $l + \pi$) and more exclusive ($l + p + \pi$) cross section measurements

Neutrino- and electron-nucleus scattering are connected (CVC) to each other and a **reliable model** must be able to describe both processes. Neutrinos can probe both V and A nuclear responses, unlike electrons which are only sensitive to the V response.

Differential $\nu - A$ cross section $\chi = +(-) \equiv \nu_\mu(\bar{\nu}_\mu)$

$$\left[\frac{d\sigma}{dk_\mu d\Omega_\mu}\right]_\chi = \sigma_0 \left[V_L R_L + V_T R_T + \chi(2 V_{T'} R_{T'})\right]$$

$$V_L R_L = V_{CC} R_{CC} + 2 V_{CL} R_{CL} + V_{LL} R_{LL}$$

$$R_L = R_L^{VV} + R_L^{AA} \; ; \; R_T = R_T^{VV} + R_T^{AA} \; ; \; R_{T'} = R_{T'}^{VA}$$

See M. Barbaro's talk for more details.

$$\begin{aligned} R_{CC} &= W^{00} \\ R_{CL} &= -\frac{1}{2}\left(W^{03} + W^{30}\right) \\ R_{LL} &= W^{33} \\ R_T &= W^{11} + W^{22} \\ R_{T'} &= -\frac{i}{2}\left(W^{12} - W^{21}\right) \end{aligned}$$

Comparison with (e, e') reactions

$$\left[\frac{d\sigma}{dk_\mu d\Omega}\right] = \sigma_{Mott}\left(v_L R_L^{VV} + v_T R_T^{VV}\right) \quad ; \quad \sigma_{Mott} = \frac{\alpha^2 \cos^2\theta/2}{4 E_i \sin^4\theta/2}$$

Theoretical description: ν-A inclusive cross section

Double differential cross section $\chi = +(-) \equiv \nu_\mu(\bar{\nu}_\mu)$

$$\left[\frac{d\sigma}{dk_\mu d\Omega_\mu}\right] = \frac{|\vec{k}_l|}{|\vec{k}_{\nu_l}|}\frac{G_F^2}{4\pi^2}\widetilde{\eta}_{\mu\nu}\widetilde{W}^{\mu\nu}$$

$$= \sigma_0(V_{CC}R_{CC}^{AA} + 2V_{CL}R_{CL}^{AA} + V_{LL}R_{LL}^{AA} + V_L R_L^{VV} + V_T R_T^{VV}$$

$$+ \quad V_T R_T^{AA} + \chi\,[2V_{T'}R_{T'}])$$

$$\sigma_0 = \frac{\left(G_F^2\cos\theta_c\right)^2}{2\pi^2}\left(k_\mu \cos\frac{\tilde{\theta}}{2}\right)^2$$

$\widetilde{\eta}_{\mu\nu}(\widetilde{W}^{\mu\nu})$: leptonic (hadronic) tensor

Comparison with (e, e') reactions

$$\left[\frac{d\sigma}{dk_e d\Omega_e}\right] = \sigma_{Mott}\left(v_L R_L^{VV} + v_T R_T^{VV}\right)$$

$$\sigma_{Mott} = \frac{\alpha^2\cos^2\theta/2}{4E_i\sin^4\theta/2}$$

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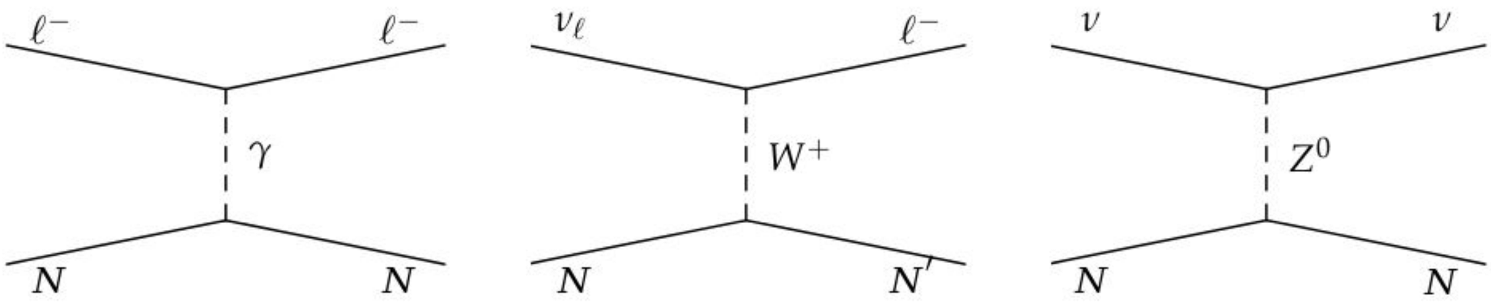
Connection between ν-A and e-A reactions



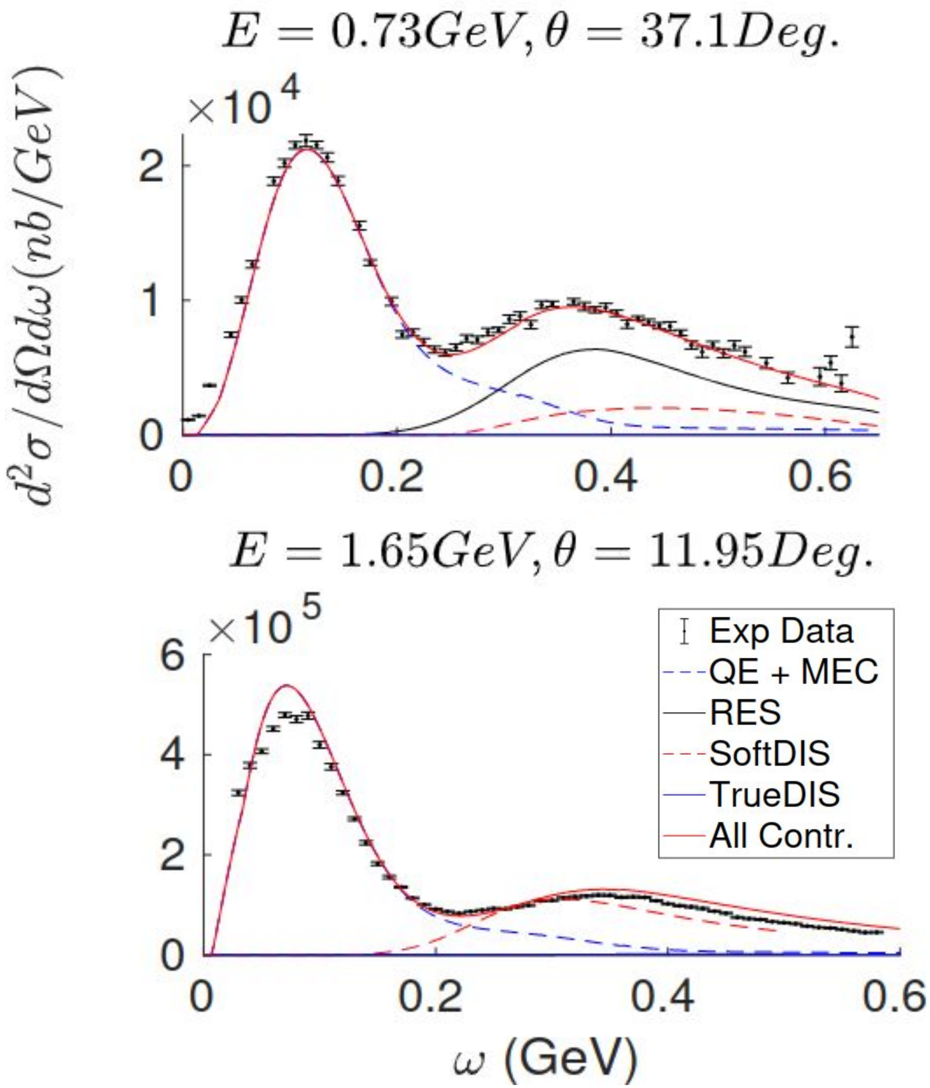
(a) Electromagnetic scattering (b) Charged-current scattering (c) Neutral-current scattering

$l = e, \mu, \tau$

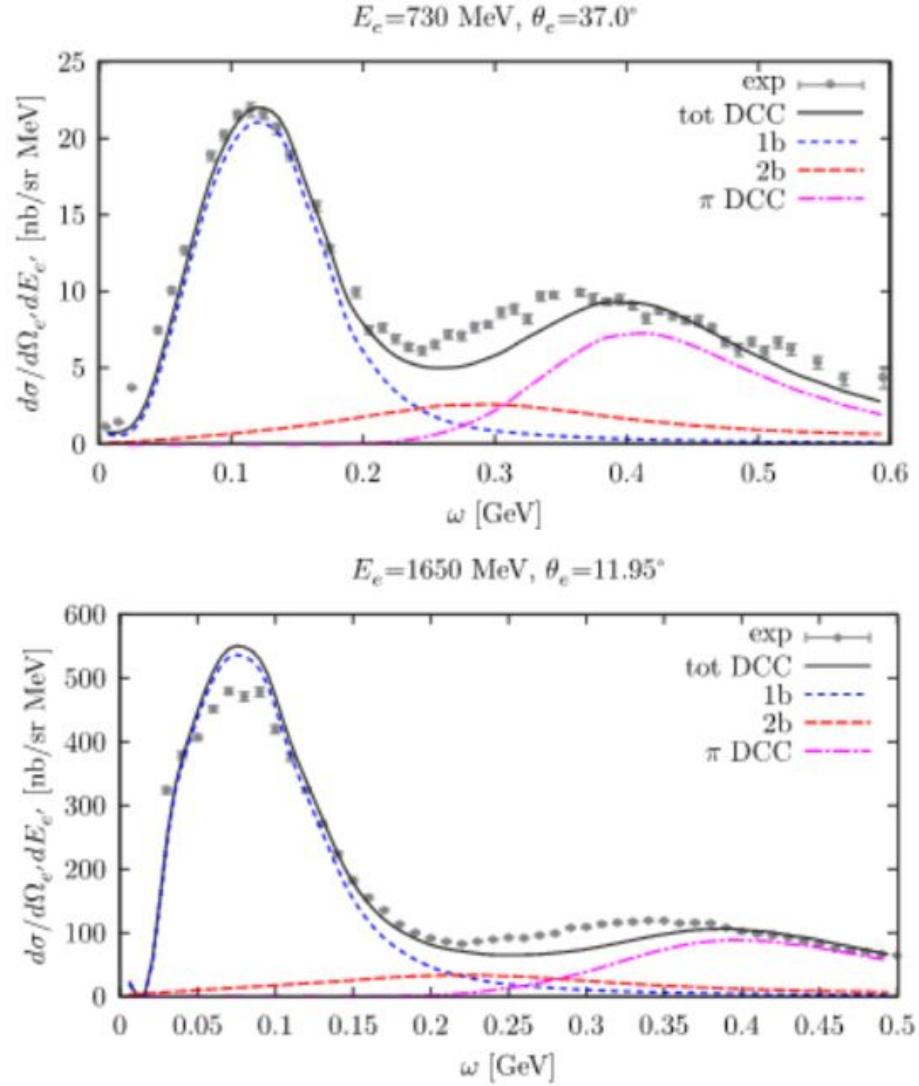
- Experimental conditions are different:
  ➠ (e, e'): E_e is well determined and different channels can be clearly identified by knowing the energy and momentum transfer
  ➠ $CC(\nu_l, l)$: E_ν is broadly distributed in the neutrino beam and different channels and nuclear effects can contribute to the same kinematics of the outgoing lepton
- From a theoretical framework, neutrino- and electron-nucleus scattering are obviously connected (CVC) to each other and a **reliable model** must be able to describe both processes.
- Neutrinos can probe both the vector and axial nuclear responses, unlike electrons which are only sensitive to the vector response.

$\Longrightarrow$ Although not sufficient to fully constrain neutrino cross sections, electron scattering constitutes a necessary test and a solid benchmark for nuclear models.

- ***Electron scattering data*** provide **better** study of reaction channels (monochromatic well known flux).
- **Nuclear models** must describe properly ***electron scattering data*** **before** being implemented or tested against ν data.



SuSAv2 Phys. Rev. D 108, 113008 (2023)

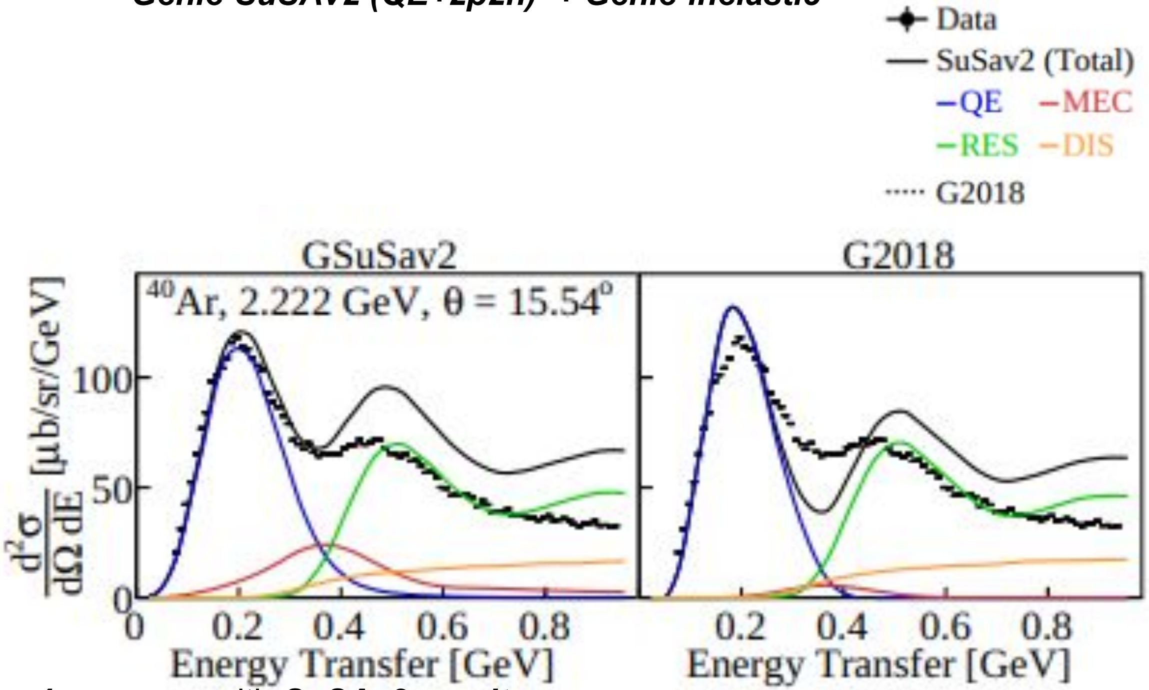


Extended Fact. Scheme. PRC 100 (2019) 4, 045503

Which data are interesting for theorists? Mostly electrons but also neutrinos

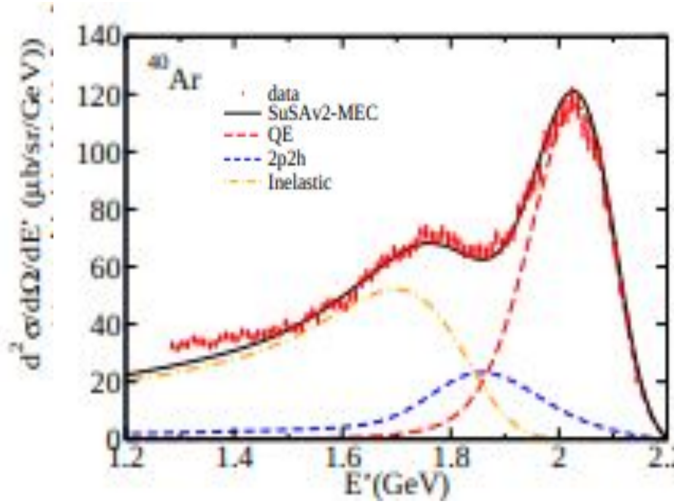
- More **exclusive measurements** with information from hadrons in FS to go deeper into nuclear-medium effects.
- Analysis of **MC simulations with electrons** is essential to validate neutrino simulations (*Nature 599, 565 (2021)*).
- Example: SuSAv2-implementation in e-GENIE and nu-GENIE. *Partially implemented (QE and 2p2h)*. Work in progress for ***inelastic*** (RES, DIS, others). Limitations: only predicts lepton kinematics, q0 and q3. Some approaches to predict hadron kinematics: factorization with RMF (~~LFG~~) mom. dist.. ED-RMF would be better but more computationally demanding.
- Need of proper implementation and use of models in generators. If not complete → ***know what/how is being used***.
- Know the **limitations of the models** before and/or after being implemented in generators.

Genie-SuSAv2 (QE+2p2h) + Genie-inelastic



e4nu papers with ***SuSAv2 results***:
Phys. Rev. D 103, 113003 (2021), Nature 599, 565 (2021)

Original SuSAv2 calculations (QE+2p2h+inelastic)



Phys. Rev. C 99, 042501 (2019)

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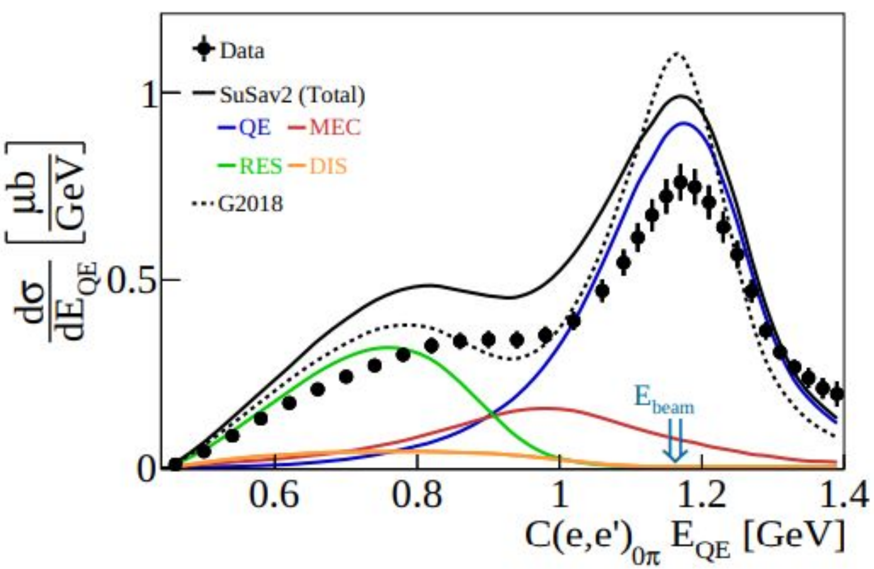

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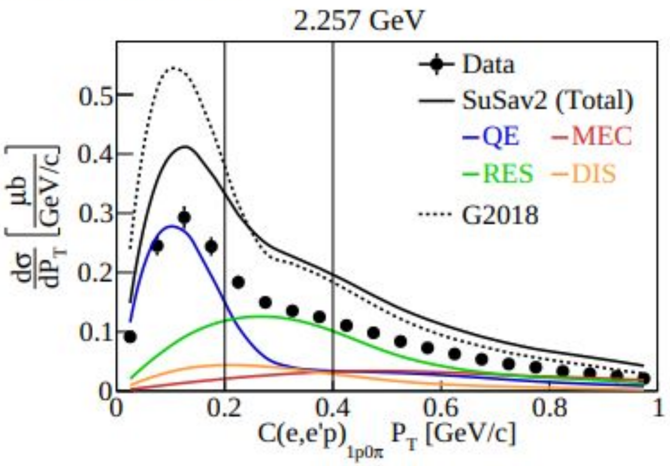
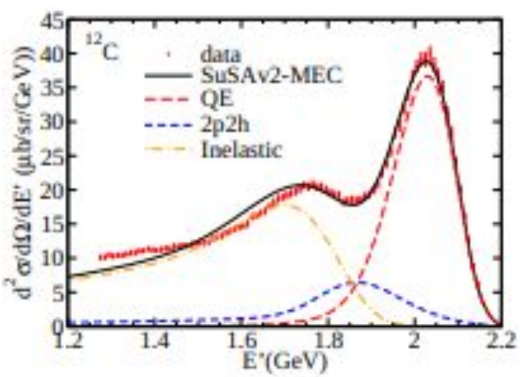




SuSAv2 comparison with electron reactions on 12C: Phys. Rev. D 94, 013012 (2016), Phys. Rev. C 99, 042501 (2019)

Final state interactions: Alternative approaches: **RMF and ROP models**

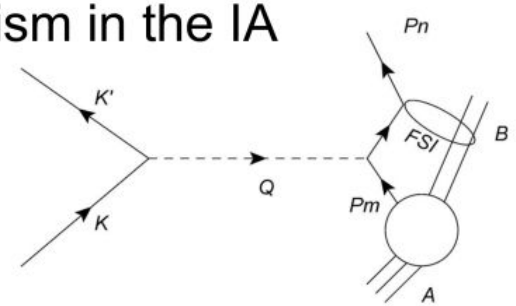
- **RMF-based models (ED-RMF, ROP, SuSAv2) assume DWIA** to predict lepton and hadron kinematics for the 1p1h channel and also 2p2h (SuSAv2 only). No factorization approach (SF).

Can be implemented in **generators**. Partially implemented in GENIE. Work in progress for NEUT.

- **Semi-inclusive CS:** final lepton + nucleon events.

Semi-inclusive neutrino-nucleus formalism in the IA

$$\left\langle \frac{d^6\sigma}{dk_l d\Omega_l dp_N d\Omega_N} \right\rangle = \int dk\, \phi(k) \times K \times L_{\mu\nu} H^{\mu\nu}$$



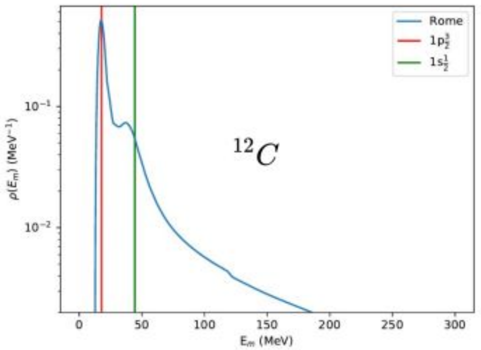
$$H^{\mu\nu}_{\kappa} = \rho_\kappa(E_m) \times \sum_{m_j,\, s_N} \left[J_{\kappa,\, m_j,\, s_N}(Q, P_N)\right]^{\star} J_{k,\, m_j,\, s_N}(Q,\, P_N) \Longrightarrow$$ No factorization!!

$$J^{\mu}_{\kappa,\, m_j,\, s_N} = \int d\mathbf{r}\, e^{i\mathbf{r}\cdot\mathbf{q}} \overline{\Psi}_{s_N}(\mathbf{p}_N,\, \mathbf{r}) \left(F_1\gamma^\mu + \frac{iF_2}{2m_N}\sigma^{\mu\nu}Q_\nu + G_A\gamma^\mu\gamma^5 + \frac{G_P}{2m_N}Q^\mu\gamma^5\right) \Psi^{m_j}_{\kappa}(\mathbf{r}) \Longrightarrow$$

- W.F. scattered nucleon (red)
- CC2 operator (blue)
- W.F. bound nucleon (green)

Description of the initial state:

- Pure shell model (first approximation): missing energy profile is given by a Dirac delta per shell
- Realistic model, i.e. Rome (Benhar spectral function) used in electron exclusive processes: short- and long-range correlations included

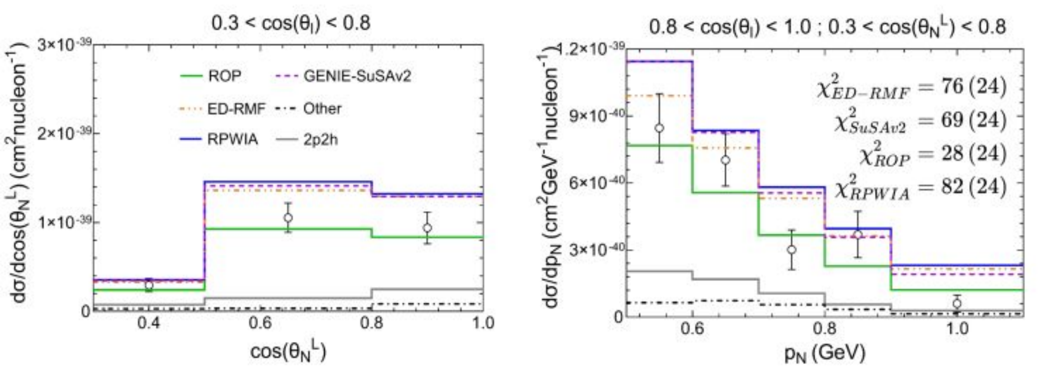


Scattered Nucleon Description

Regarding the scattered nucleon, we can consider several situations:

- Relativistic Plane-Wave Impulse Approximation (RPWIA): the ejected nucleon is considered a plane-wave (i.e, there are not final state interactions)
- Energy-Dependent Relativistic Mean Field (ED-RMF): W.F. solution of the Dirac equation in the continuum using the same RMF potential that describes the initial state times a phenomenological function that weakens the potentials at high energies.
- Relativistic Optical Potential (ROP): The scattered nucleon travels under the influence of a phenomenological relativistic optical potential fitted to elastic proton-nucleus scattering data.

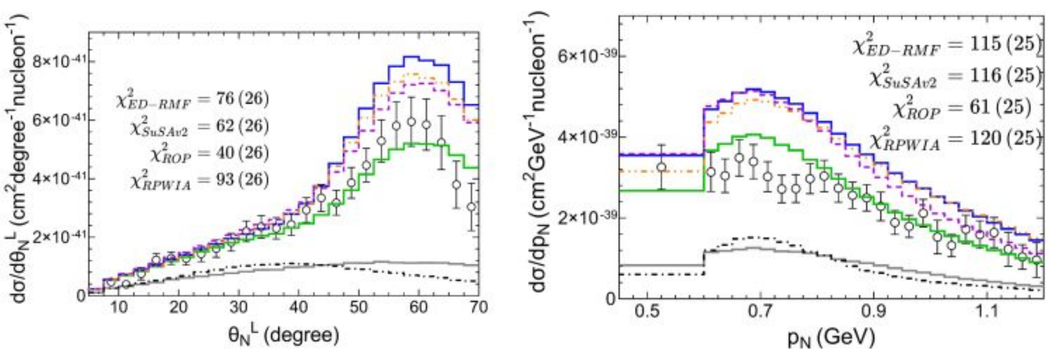
Cross sections vs proton kinematics: T2K and MINERvA



Phys. Rev. D 98, 032003(2018)

$T2K\ \nu_\mu - CC0\pi Np$

$p_N > 500\ MeV/c$

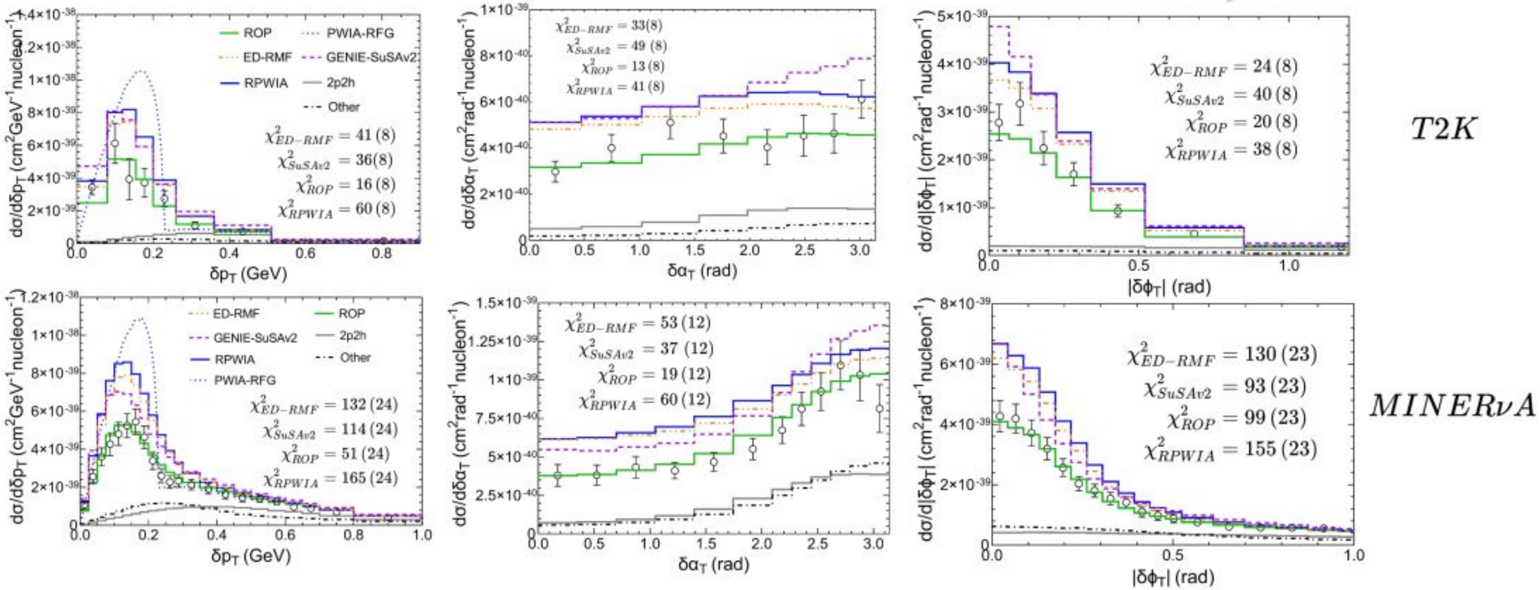


Phys. Rev. D 101, 092001(2020)

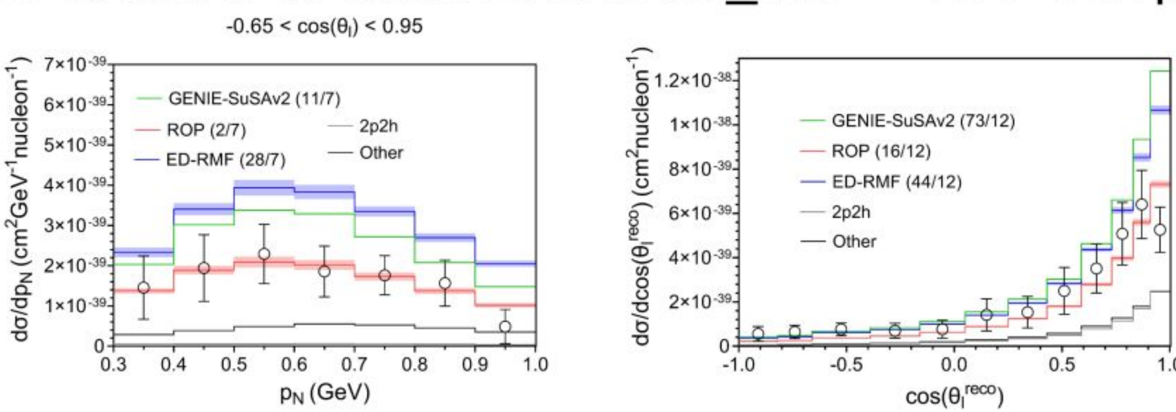
$MINER\nu A\ \nu_\mu - CC0\pi Np$

k'	$\cos\theta_l$	p_N	$\cos\theta_N^L$	ϕ_N^L
1.5-10 GeV	> 0.939	0.45-1.2 GeV	> 0.342	-

Cross sections vs transverse kinematic imbalances



Cross sections vs MicroBooNE nu_mu -> 40Ar CC0piNp



We can also add uncertainties in the nuclear potential parameters, SF profile, binding energies, occupancy, transparency, etc. Error bands included in MicroBooNE plots for reference.

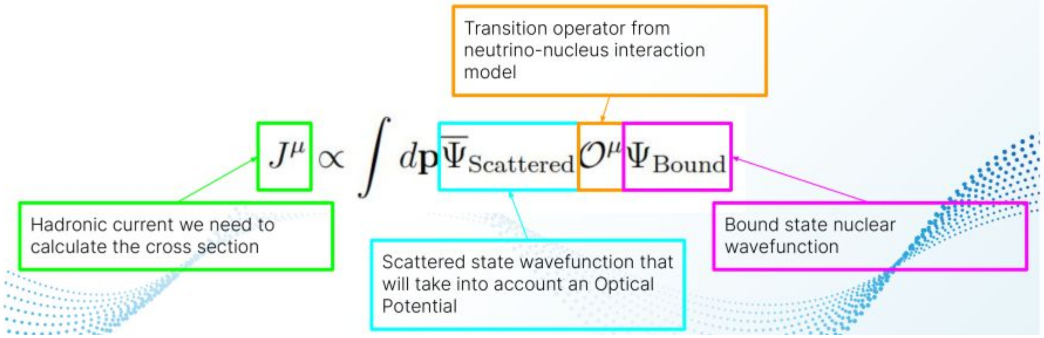
See Phys. Rev. D 106, 113005 (2022) and Phys. Rev. D 109, 013004 (2024) for details.

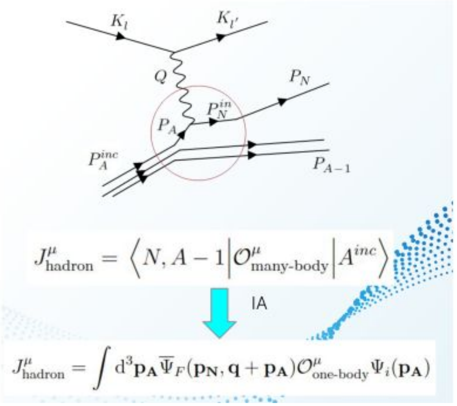
Final state interactions: Alternative approaches. Optical potentials

- **PWIA**: outgoing w.f. is a plane wave ⇒ No FSI but SF formalism.
- **T2K uses SF-PWIA** for CCQE and RFG for other channels. This approach **fails** at low kinematics. No nuclear potential for scattered nucleons.
- **FSI** can be treated alternatively as a distortion of the outgoing nucleon w.f. wave functions by a nuclear potential ⇒ **DWIA:** distorted wave impulse approximation.
- **How to address FSI? Optical potentials (OP)**. OP represents $e-A$ and $\nu-A$ reactions by using phenom. OP adjusted to $e-A$ data.

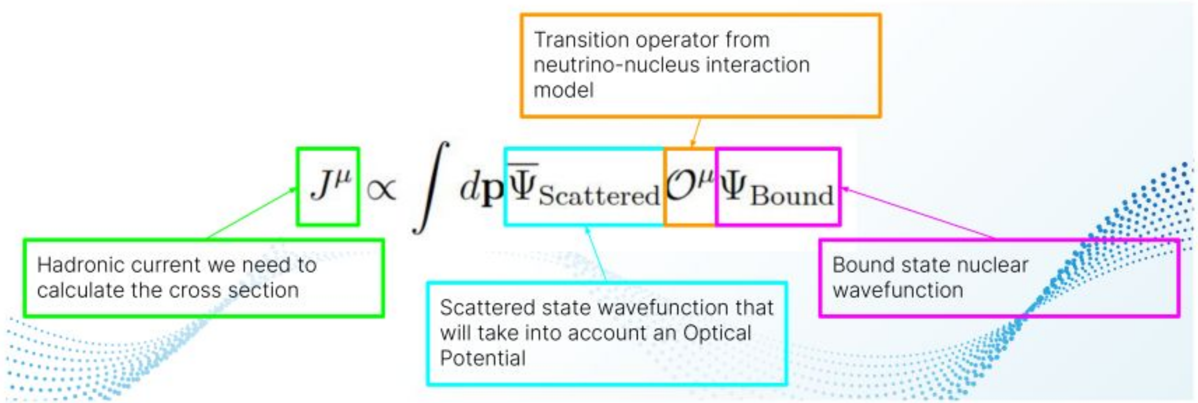
  - **OP real part** distorts kinematics of the outgoing nucleon.

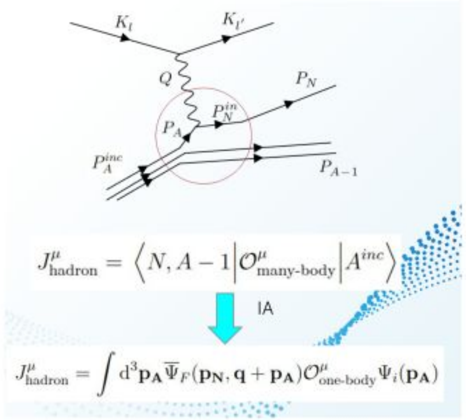
  - **OP imaginary part** accounts for absorption of reaction flux from the elastic channel (*1 scattered N: 1p1h*) to inelastic reaction channels thus assuring 1p1h FS for QE reactions. Similar as for cascade models but differences appears at low kinematics (*PRC105, 054603*).





- **RPA and FSI effects** can introduce important differences in the experimental analyses.
- These differences affect OA: E dependence of the CS, ν vs. $\bar{\nu}$, ν_e/ν_μ, C/O.
- Largest uncertainty in semi-inclusive neutrino CS (lepton+hadron kinematics).

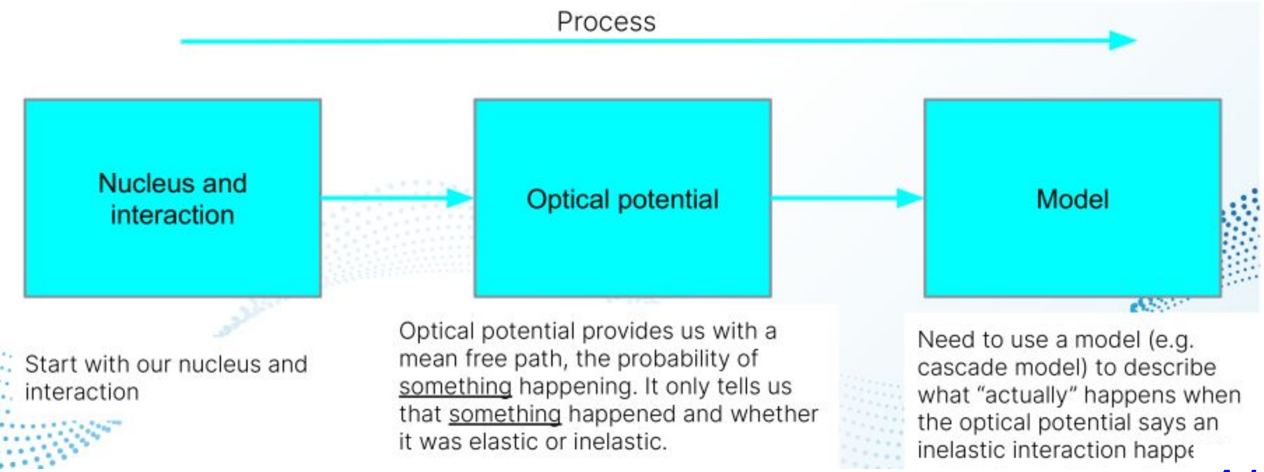




Hadronic tensor from $\nu - N$ operator, convoluted with nuclear initial state and the scattered nucleon. Scattered state w.f. uses OP ($\sim$ effect of NEUT FSI cascade)

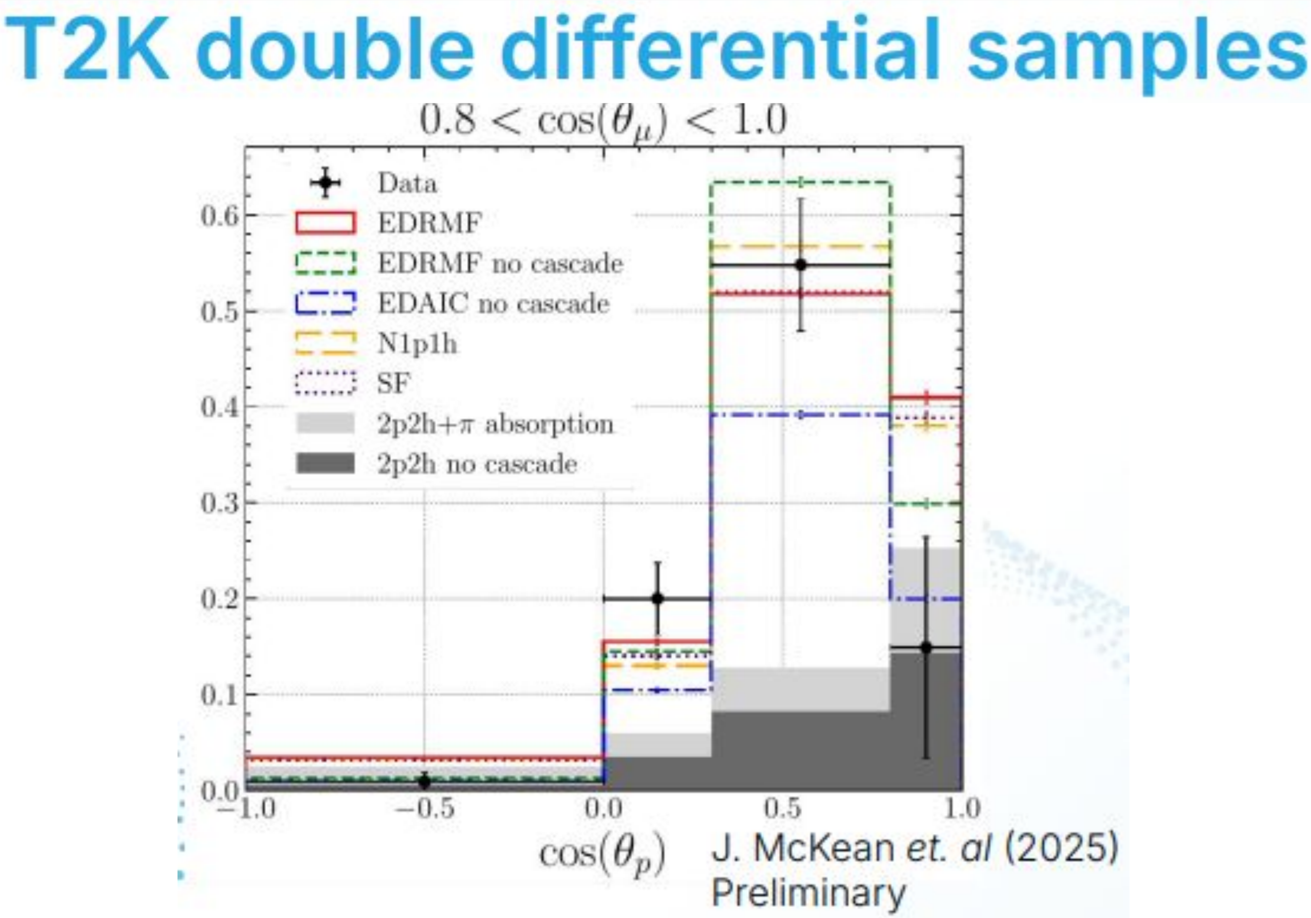
Findings: Validity of OP employed against $p-{}^{28}$Si data. Some flexibility in the optical potentials $\Rightarrow$ can be tuned for $e - A$ and $\nu - A$ scattering.

Plans: Applying the previous scheme using **MK SPP operator** as $\nu - N$ operator, scattered state w.f. provided by **RMF codes** with Woods-Saxon **nuclear potentials** from UCM Madrid group, and bound state using **TIMORA** software which solves Dirac-Hartree-Fock equations.



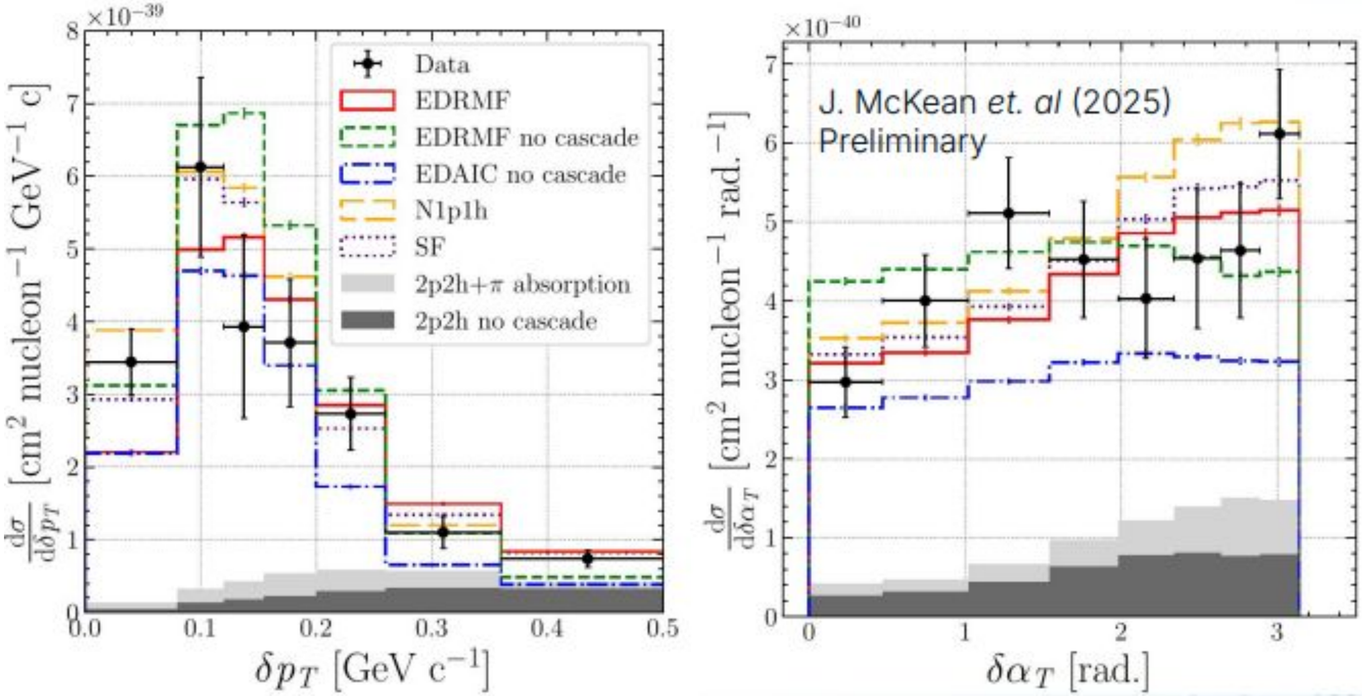
Need to check results into this formalism.

Adapted from Jake McKean's talks at T2K CM



- **ED-RMF** is being implemented in **NEUT** for 12C, 16O and 40Ar via hadron <u>*tensor tables*</u> containing full lepton and nucleon kinematics.
- Different choice of nuclear potentials (real part) to analyze **elastic FSI**.
- <u>*Absorption effects related to **inelastic FSI***</u> can be generated by **NEUT cascade** or via **imaginary part** of phenomenological optical potentials.

T2K TKI



Model	δp_T	$\delta\phi_T$	$\delta\alpha_T$
EDMRF cas	42.1/8	27.1/8	17.6/8
EDMRF no cas	60.4/8	51.2/8	27.5/8
N1p1h cas	5.62/8	12.6/8	31.3/8
SF cas	11.3/8	8.49/8	19.4/8
EDAIC no cas	24.6/8	19.3/8	18.8/8

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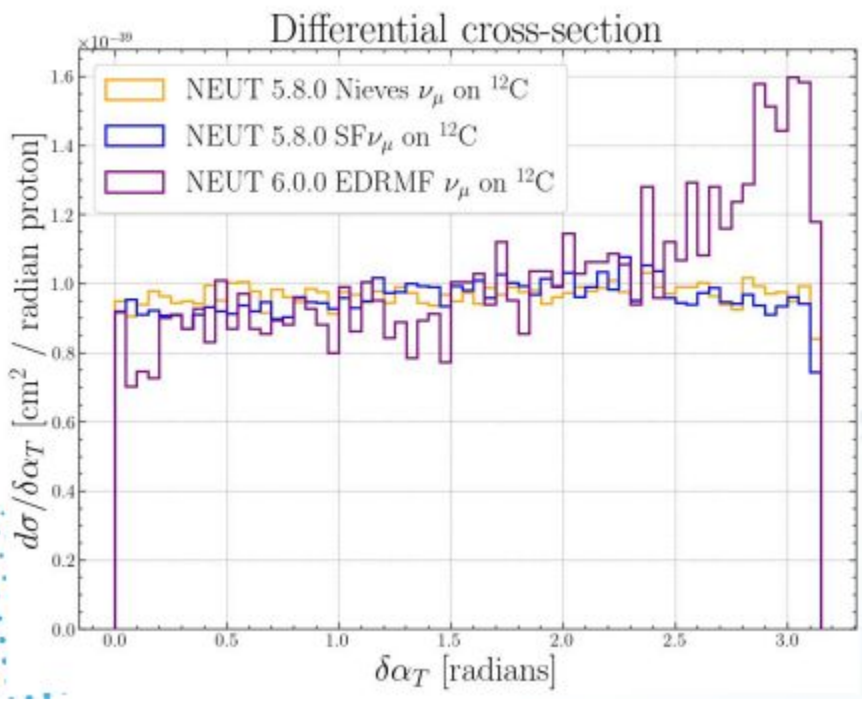

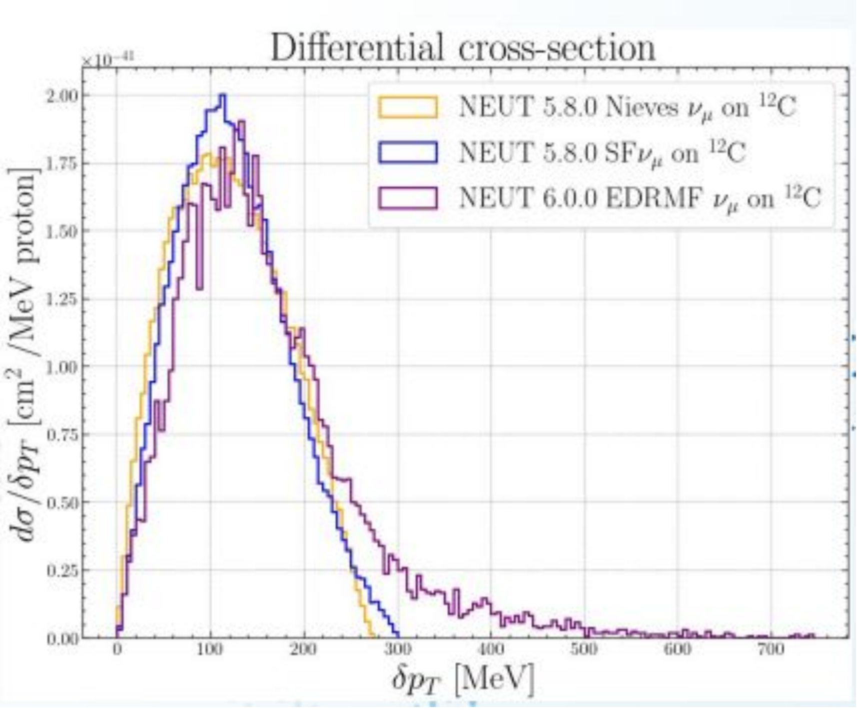

- **ED-RMF** is being implemented in **NEUT** for 12C, 16O and 40Ar via hadron *tensor tables* containing full lepton and nucleon kinematics.
- Different choice of nuclear potentials (real part) to analyze **elastic FSI**.
- *Inelastic FSI* can be generated by **NEUT cascade** or via **imaginary part** of nuclear potentials.

- *MC tuning, reweighting or model approaches in the RES and SoftDIS (Background) channels for neutrinos should be validated against electron scattering data.*
- *GENIE Mnv RES approach is based on a single-nucleon Rein-Sehgal pion production model with lepton mass corrections and other modifications to include nuclear effects. But, it looks similar to a low-pF RFG model.*