



Coulomb interactions in hadronic femtoscopy: applications to the pp and $\alpha\alpha$ systems

Miguel Albaladejo (IFIC, CSIC-UV, Valencia)

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Acknowledged projects:

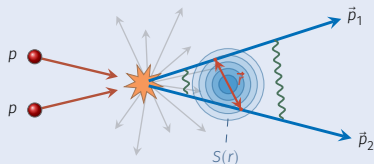
PID2023-147458NBC21 · CEX2023-001292-S

RYC2022-038524-I · PIE 20245AT019

- 1 Introduction
- 2 Theoretical approach
- 3 Applications
 - pp : comparison with Koonin (theory)
 - pp : comparison with ALICE (experiment)
 - Application to $\alpha\alpha$
- 4 Conclusions

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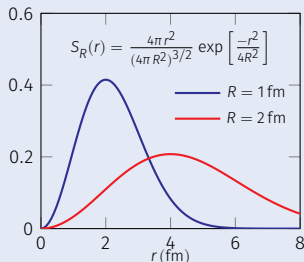
(Very basic introduction to) femtoscopy



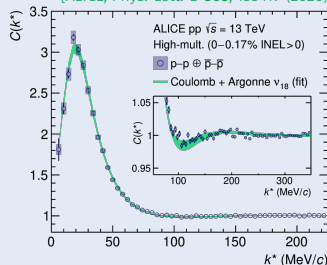
$$C_{\text{exp}}(k) = \xi(k) \frac{N_{\text{same}}(k)}{N_{\text{mixed}}(k)}$$

$$C_{\text{th}}(k) = \sum_{\ell=0} (2\ell+1) \int dr S_R(r) |\psi_{\ell}(r, k)|^2$$

$$\psi_{\ell}^{[\text{asy}, \text{nr}]}(k, r) = j_{\ell}(kr) + f_{\ell}(k) \frac{e^{i(kr - \pi\ell/2)}}{r}$$



[ALICE, Phys. Lett. B 805, 135419 (2020)]



- Known the **source $S(r)$** , explore **interactions** (encoded in the wave function)
- A new method to explore **hadron interactions**

[L. Fabbietti, V. Mantovani Sarti, and O. Vazquez Doce, Ann. Rev. Nucl. Part. Sci. 71, 377 (2021)]

- Many more details in several talks:

- | | |
|--------------------------------|-------------------------------|
| ▶ W.-H. Liang [Mon 15:30] | ▶ L.-S. Geng [Tue 15:50] |
| ▶ P.-P. Shi [Mon 16:50] | ▶ K. Khemchandani [Tue 16:10] |
| ▶ D. Fujii [Mon 19:15] | ▶ A. Feijoo [Tue 16:30] |
| ▶ J. M. Nieves [Tue 15:00] | ▶ O. Vázquez Doce [Tue 16:50] |
| ▶ J. Torres-Rincón [Tue 15:30] | ▶ P. Encarnación [Tue 19:15] |
| | ▶ M. F. Barbat [Tue 19:55] |

Coulomb interactions: why they are inevitable

[M. Albaladejo, A. Garcia-Lorenzo, and J. Nieves, PTEP 2025, 093D02 (2025)]

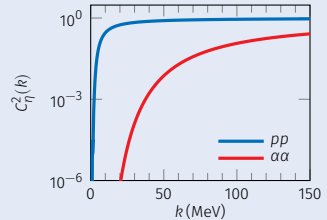
- Where Most measured pairs are **charged**: pp , K^-p , $p\Xi^-$, $\alpha\alpha$, ...

- Scales

$$\eta(k) = \frac{k_c}{k}, \quad k_c = Z_1 Z_2 \alpha_{\text{em}} \mu = \frac{1}{a_B},$$

$$C_\eta^2 = |\psi_{\vec{k}}(0)|^2 = \frac{2\pi\eta}{e^{2\pi\eta} - 1} \quad (\text{Sommerfeld factor})$$

	k_c	$a_B = 1/k_c$	R
pp	3.4 MeV	57.6 fm	1.25 fm
$\alpha\alpha$	54.4 MeV	3.6 fm	5.4 – 6.5 fm



- Cross sections vs. CFs

- Scattering: $dN/dk \sim k^2 |\psi_k|^2$, the **phase space suppresses** the region $k \lesssim k_c$, precisely where $C_\eta^2 \rightarrow 0 \Rightarrow$ Coulomb is a **correction**, undone through the Coulomb-modified ERE, with δ_ℓ extracted where $\eta \ll 1$
- Femtoscscopy: in $C(k) = \xi(k) N_{\text{same}}(k) / N_{\text{mixed}}(k)$ the phase space **cancels**, leaving $\langle |\psi_k|^2 \rangle_R$ itself, measured at $k \lesssim 100$ MeV ($\eta \gtrsim 1$) $\Rightarrow$ Coulomb is **very relevant**

This work

!

- Analytic** Coulomb: no Schrödinger equation to be solved numerically
- Beyond LL [R. Lednicky and V. L. Lyuboshits, Yad. Fiz. 35, 1316 (1981)]:
(1) exact unitarity, (2) EFT amplitudes, (3) off-shell effects

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Coulomb interactions: formalism

[M. Albaladejo, A. Garcia-Lorenzo, and J. Nieves, PTEP 2025, 093D02 (2025)]

- Notation**

• Free (plain) waves	$ \vec{k}\rangle$	$[\hat{V} = 0]$
• Only Coulomb	$ \psi_{\vec{k}}^{(\pm)}\rangle$	$[\hat{V} = \hat{V}_C]$
• Full (Coulomb and strong)	$ \phi_{\vec{k}}^{(\pm)}\rangle$	$[\hat{V} = \hat{V}_C + \hat{V}_S]$

$$\hat{H} = \hat{H}_C + \hat{V}_S$$

$$\hat{H}_C = \hat{H}_0 + \hat{V}_C$$
- Amplitudes** Gell-Mann-Goldberger decomposition: (two-potential trick; distorted wave theory)

[J. Nieves, Phys. Lett. B **568**, 109 (2003)]

[X. Kong and F. Ravndal, Nucl. Phys. A **665**, 137 (2000)]

$$T(\vec{k}', \vec{k}) = \langle \vec{k}' | \hat{T}_C | \vec{k} \rangle + \langle \psi_{\vec{k}'}^{(-)} | \hat{T}_{SC} | \psi_{\vec{k}}^{(+)} \rangle \equiv T_C(\vec{k}', \vec{k}) + T_{SC}(\vec{k}', \vec{k})$$

- ▶ T_{SC} : strong amplitude **in the presence of Coulomb**, taken between **Coulomb** states

- ▶ **Origin of T_{SC}** two exact relations:

$$[\hat{T}_{SC} = \hat{V}_S + \hat{V}_S \hat{G}_C^{(+)} \hat{T}_{SC}; \hat{G}_{SC}^{(\pm)} = \hat{G}_C^{(\pm)} + \hat{G}_C^{(\pm)} \hat{V}_S \hat{G}_{SC}^{(\pm)}]$$

$$(a) \hat{T}_{SC} |\psi_{\vec{k}}^{(+)}\rangle = \hat{V}_S |\phi_{\vec{k}}^{(+)}\rangle \quad (b) |\phi_{\vec{k}}^{(\pm)}\rangle = \sum_{n=0}^{\infty} (\hat{G}_C^{(\pm)}(E_k) \hat{V}_S)^n |\psi_{\vec{k}}^{(\pm)}\rangle$$

$$\Rightarrow T_{SC}(\vec{k}', \vec{k}) = \langle \psi_{\vec{k}'}^{(-)} | \hat{V}_S |\phi_{\vec{k}}^{(+)}\rangle = \sum_{n=0}^{\infty} \langle \psi_{\vec{k}'}^{(-)} | \hat{V}_S (\hat{G}_C^{(+)} \hat{V}_S)^n | \psi_{\vec{k}}^{(+)}\rangle$$

- ▶ Contact strong interaction (S-wave): $\langle \vec{q}' | \hat{V}_S | \vec{q} \rangle = C_0 \Rightarrow$ **it factorizes**

$$\langle \psi_{\vec{k}'}^{(-)} | \hat{V}_S | \psi_{\vec{k}}^{(+)} \rangle \stackrel{\text{on-shell}}{=} C_0 [\psi_{\vec{k}}^{(+)}(\vec{0})]^2 = C_0 C_\eta^2 e^{2i\sigma_0}, \quad \text{each loop: } \langle \vec{0} | \hat{G}_C^{(+)}(E_k) | \vec{0} \rangle = J_0(E_k)$$

$$T_{SC}(E_k) = C_\eta^2 e^{2i\sigma_0} \left(C_0 + C_0 J_0(E_k) C_0 + \dots \right) = \frac{C_\eta^2 e^{2i\sigma_0}}{C_0^{-1} - J_0(E_k)}$$

Coulomb interactions: formalism (II)

[M. Albaladejo, A. Garcia-Lorenzo, and J. Nieves, PTEP 2025, 093D02 (2025)]

- Coulomb propagator** $\hat{G}_C^{(+)}(E_k) = \frac{1}{E_k - \hat{H}_C + i\epsilon}$ resums **all** Coulomb photons
 - ▶ All we need is $\langle \vec{r} | \hat{G}_C^{(+)}(E_k) | \vec{0} \rangle$: $\vec{r} = \vec{0} \rightarrow$ **amplitude**; $r \neq 0 \rightarrow$ **wave function**

- (Modified) loop function** $J_0(E_k) = \langle \vec{r} | \hat{G}_C^{(+)}(E_k) | \vec{0} \rangle \Big|_{\vec{r}=\vec{0}} = \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{|\psi_{\vec{q}}^{(+)}(0)|^2}{E_k - \frac{\vec{q}^2}{2\mu} + i\epsilon}$

$$J_0(E_k) = \text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \dots$$

The diagrams represent the expansion of the loop function $J_0(E_k)$ in terms of Coulomb interactions. Each diagram consists of a horizontal oval with two external lines (represented by circles with an 'x') at the ends. The first diagram is a simple oval. The second diagram has a wavy line inside the oval. The third diagram has two wavy lines inside the oval. The ellipsis indicates higher-order terms in the expansion.

- ▶ UV divergent $\rightarrow$ **one subtraction** [$\lambda = +1(-1)$ for repulsive (attractive) Coulomb; $\eta = \lambda\alpha\mu/k$]

$$J_0(E_k) = J_0(0) - \lambda \frac{\mu^2 \alpha}{\pi} H^\lambda(\eta), \quad H^\lambda(\eta) = h^\lambda(\eta) + i \frac{C_\eta^2}{2\eta}$$

- Coulomb ERE** $T_{SC}(E_k) = -\frac{2\pi}{\mu} C_\eta^2 e^{2i\sigma_0} f_{SC}(k), \quad f_{SC}^{-1}(k) = C_\eta^2 k (\cot \delta_0 - i)$

$$f_{SC}^{-1}(k) + 2\lambda\alpha\mu H^\lambda(\eta) = -\frac{1}{a_0} + \frac{1}{2}r_0 k^2 + v_2 k^4 + \dots = -\frac{2\pi}{\mu} [C_0^{-1}(k^2) - J_0(0)]$$

- ▶ Energy-dependent $C_0(k^2)$: the **ERE** (or **any EFT**) fixes the input

- Wave function** same resummation as in T_{SC} , now at $r \neq 0$:

$$\phi_0^*(r, k) = \psi_0^*(r, k) + \frac{T_{SC}(E_k) G_C^{(+)}(E_k; r)}{e^{-\pi\eta/2} \Gamma(1+i\eta)}, \quad G_C^{(+)}(E_k; r) = \langle \vec{r} | \hat{G}_C^{(+)}(E_k) | \vec{0} \rangle = -\frac{\mu}{2\pi r} \Gamma(1+i\eta) W_{-i\eta, \frac{1}{2}}(-2ikr)$$

Coulomb interactions: formalism (III)

[M. Albaladejo, A. Garcia-Lorenzo, and J. Nieves, PTEP 2025, 093D02 (2025)]

- **Correlation functions** Koonin-Pratt: $C(E_k) = \int d^3\vec{r} \tilde{S}(r) |\phi(\vec{r}, \vec{k})|^2$ $[S(r) = 4\pi r^2 \tilde{S}(r)]$
- Adding and subtracting $|\psi_0|^2$ (Coulomb) and $|\phi_0|^2$ (asymptotic):

$$C(E_k) = C_C(E_k) + \Delta C_{LL}(E_k) + \delta C(E_k)$$

▶ **Purely Coulomb CF:** $C_C(E_k) = \int d^3\vec{r} \tilde{S}(r) |\psi(\vec{r}, \vec{k})|^2$ (all partial waves; dominated by C_η^2)

▶ **LL term:** $\Delta C_{LL}(E_k) = \int dr S(r) (|\phi_0(r, k)|^2 - |\psi_0(r, k)|^2)$ (exact on-shell T_{SC} , asymptotic w.f.)

▶ **FRP term:** $\delta C(E_k) = \int dr S(r) (|\phi_0^{\text{ex}}(r, k)|^2 - |\phi_0(r, k)|^2) \simeq -\beta \frac{d_0(k) C_\eta^2 |f_{SC}(k)|^2}{4\sqrt{\pi} R^3}$

(Famous)
off-shell effects

$$d_0(k) = r_0 + 4v_2 k^2 + \dots = 2 \frac{d}{dk^2} (\text{Re } f_{SC}^{-1}(k) + 2\lambda\alpha\mu h^\lambda(\eta))$$

[E. Epelbaum et al., Phys. Rev. Lett. 136, 212301 (2026)]

Summary of our approach

- No numerical sol. of Schrödinger equation
- Connection with EFTs, contact interactions
- Off-shell effects partially included (β)
- Paper: all derivations, connection to other works

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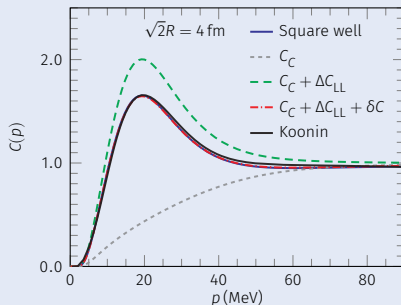
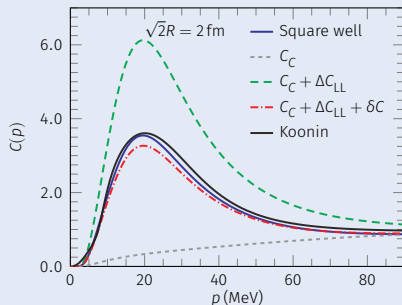
Coulomb interactions: comparison with Koonin

[M. Albaladejo, A. Garcia-Lorenzo, and J. Nieves, PTEP 2025, 093D02 (2025)]

- Koonin: using the Coulomb and RSC central, spin-orbit, and diagonal tensor potentials in all pp channels ($\ell \leq 3$) [S. E. Koonin, Phys. Lett. B 70, 43 (1977)] [R. V. Reid Jr., Annals Phys. 50, 411 (1968)]
- $C(E_p) \neq 1$ (appreciably) only for a small range of p : 1S_0 wave (our case)
- Our input: ERE for $f_{sc}^{-1}(p) = -\frac{1}{a_0} + \frac{1}{2}r_0p^2 + v_2p^4 + \dots$, same parameters as obtained with RSC
- We compare for several values of R ($\sqrt{2}R = 2 \dots 6$ fm):

$C(E_p) = C_C(E_p) + \Delta C_{LL}(E_p) + \delta C(E_p)$

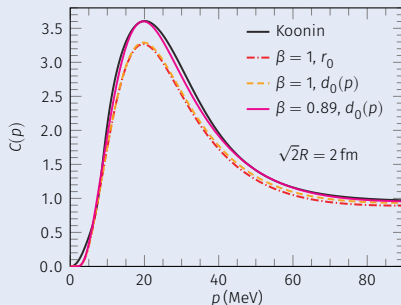
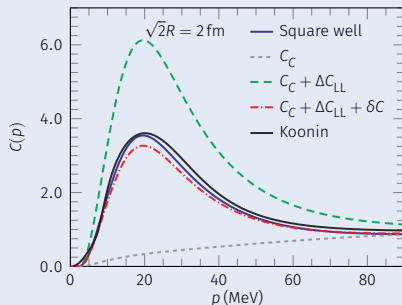
- ▶ ΔC_{LL} **always overestimates** the true CF.
- ▶ For $\sqrt{2}R \gtrsim 4$ fm, all curves (not LL or C_C) are very similar
- ▶ For $\sqrt{2}R = 2$ fm, the FRP can be improved by considering β



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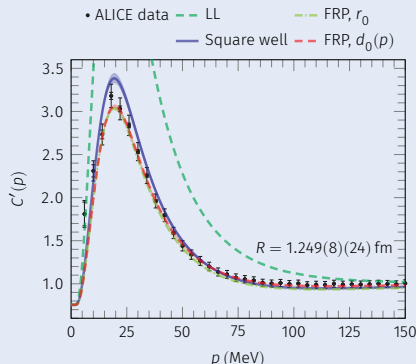


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Coulomb interactions: comparison with ALICE pp data

[M. Albaladejo, A. Garcia-Lorenzo, and J. Nieves, PTEP 2025, 093D02 (2025)]

- Analysis of pp CF: [ALICE, Phys. Lett. B 805, 135419 (2020)]
- Data contain feed-down contributions: $C'(p) = 1 + \lambda_{pp}(C_{pp}(p) - 1) + \lambda_{pp_\Lambda}(C_{pp_\Lambda}(p) - 1)$
- $C_{pp_\Lambda}(p)$: protons from the weak Λ decay
- Same formulae for $C_{pp_\Lambda}(p)$ as ALICE (LL formula, same values $\lambda_{pp} = 0.67$, $\lambda_{pp_\Lambda} = 0.223$)
- Genuine pp CF [$C_{pp}(p)$]: again, our input is the ERE for $f_{SC}(p)$ (RSC paramaters)
- Results:
 - ▶ ΔC_{LL} overestimates the data
 - ▶ Square well near the data

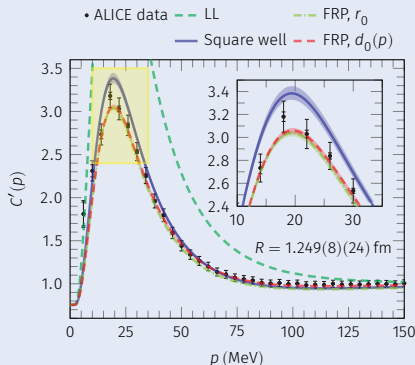


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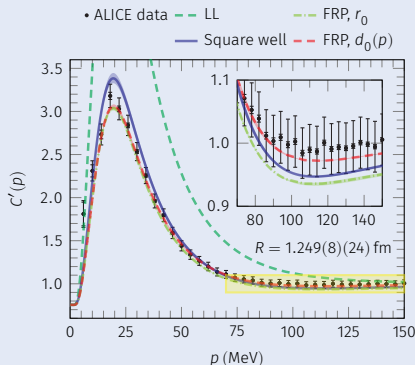


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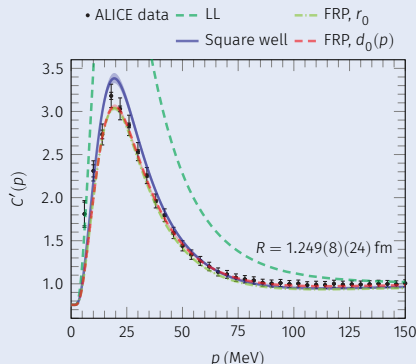
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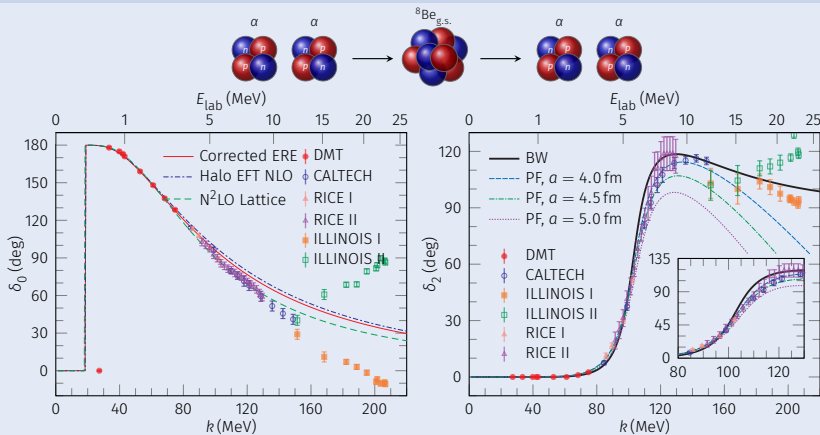
- No fit is performed, parameters taken from ALICE or other theoretical works
- This confirms the validity of the scheme presented here



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Femtoscscopy for nuclear physics: $\alpha\alpha$ scattering input

[A. Garcia-Lorenzo, M. Albaladejo, A. Feijoo, and J. Nieves, *in preparation* (2026)]



- $^8\text{Be}(0^+)$: $E_R = 92(6)$ keV, $\Gamma_R = 5.7(4.0)$ eV, i.e. $a_0 \simeq -1.9 \times 10^3$ fm vs. $r_0 \simeq 1.1$ fm
- 2^+ : one-level Coulomb R -matrix, $a_c = 4.5$ fm $\simeq r_c(A_1^{1/3} + A_2^{1/3})$
- Theoretical amplitudes (S-wave) fixed **before** any fit to the CF data:

— ERE [G. Rasche, Nucl. Phys. A **94**, 301 (1967)]
 --- Halo EFT NLO [R. Higa, H.-W. Hammer, and U. van Kolck, Nucl. Phys. A **809**, 171 (2008)]
 --- $N^2\text{LO}$ Lattice [S. Elhatisari, T. A. Lähde, D. Lee, U.-G. Meißner, and T. Vonk, JHEP **02**, 001 (2022)]

Scattering
input

Femtoscscopy for nuclear physics: $\alpha\alpha$ CFs

[A. Garcia-Lorenzo, M. Albaladejo, A. Feijoo, and J. Nieves, *in preparation* (2026)]

Genuine CF Two identical spin-0 bosons $\Rightarrow$ exchange-**even** projection. Only $\ell = 0, 2$ kept.

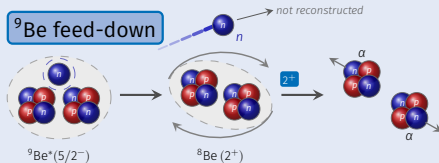
$$C_{\alpha\alpha}(k; R) = C_C^{(E)} + 8\pi \sum_{\ell=0,2,\dots} (2\ell+1) \langle |\phi_\ell|^2 - |\psi_\ell|^2 \rangle_R + \Delta C_{\text{FRP}}^{(E)}$$

$$\Delta C_{\text{FRP},0}^{(E)} \simeq -4\pi d_0(k) C_\eta^2 |f_0(k)|^2 S_R(0) \quad \text{factor 2 from Bose symmetry}$$

What is fitted **not** the genuine CF:

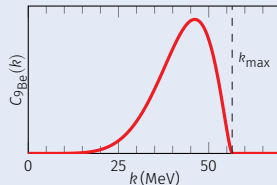
$$C_{\text{raw}}(k; R) = N [1 + \lambda (C_{\text{ext}} - 1) + \beta C_{\text{FRP}} + \gamma C_{9\text{Be}}(k)]$$

- λ : core-halo dilution
- β : strength of the FRP term ($\beta = 1$ extended source, $\beta = 0$ no FRP correction)
- γ : ^9Be feed-down
- N : normalization
- Folded with the momentum **resolution**,
 $C_{\text{fit}} = C_{\text{raw}} \otimes G_{\sigma_p}$: it is what smears the $^8\text{Be}_{\text{g.s.}}$ peak
- Fit: $R, \lambda, \beta, \gamma, N, \sigma_p$
- **The amplitude is fixed beforehand**



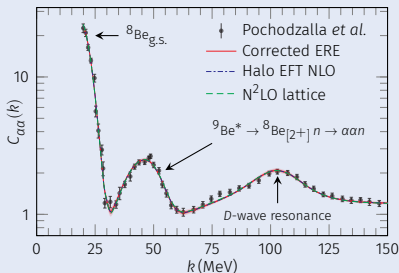
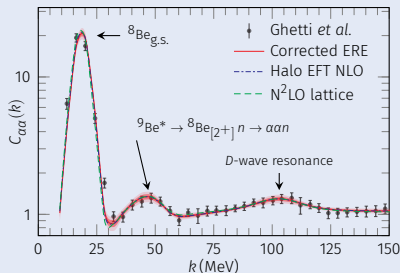
- $N \propto k^{2\ell_{\alpha\alpha}+2} (k_{\text{max}}^2 - k^2)^{\frac{2\ell_n+1}{2}}, k_{\text{max}} = 56.5 \text{ MeV}$
- This mechanism: $(\ell_{\alpha\alpha}, \ell_n) = (2, 1)$

$$C_{9\text{Be}} = \mathcal{C}_0 x^6 (1-x^2)^{3/2} \theta(1-x), \quad x = k/k_{\text{max}}$$



Femtoscscopy for nuclear physics: fits to $\alpha\alpha$ CFs

[A. Garcia-Lorenzo, M. Albaladejo, A. Feijoo, and J. Nieves, *in preparation* (2026)]



Data	Input	R (fm)	σ_p (MeV)	β	χ^2_{red}
Ghetti	Corr. ERE	6.47(35)	3.40(10)	0.40(10)	1.99
	N ² LO latt.	6.42(34)	3.29(9)	-0.03(17)	2.14
Pochodzalla	Corr. ERE	5.43(13)	4.09(7)	0.61(3)	1.29
	N ² LO latt.	5.58(11)	4.25(7)	0.33(5)	1.24

Halo EFT NLO indistinguishable from the corrected ERE

Data: [R. Ghetti et al., *Phys. Rev. C* **70**, 027601 (2004)]

[J. Pochodzalla et al., *Phys. Rev. C* **35**, 1695 (1987)]

- R **stable** against th. inputs, β is not
- Errors rescaled by $\simeq 3.5$
- $\lambda \simeq 0.5$ absorbs more than core-halo
- Not included: higher waves, other resonances, other feed-downs, ...

Outlook for $\alpha\alpha$ /nuclear

- ! We are able to obtain the **source** (in agreement with exp. expectations)
- With **better experiments** (statistics, resolution, finer bins) we might be able to discriminate/improve theoretical models
- Could this help in pinning down **other nuclear states**?

- 1 Introduction
- 2 Theoretical approach
- 3 Applications
 - pp : comparison with Koonin (theory)
 - pp : comparison with ALICE (experiment)
 - Application to $\alpha\alpha$
- 4 Conclusions

General conclusions

- **Femtoscscopy** gives new experimental access to hadron and nuclear interactions
- Many measured pairs are **charged**. Coulomb is not just a correction, but the **dominant** effect at low energies, and must be treated together with the strong interaction

Coulomb interactions: analytical methods

[PTEP 2025, 093D02 (2025)]

- No need of numerical solution of Sch. eq., possibility to connect with EFTs
- Exact elastic unitarity and full Coulomb left-hand discontinuity
- **Finite-range** (off-shell) corrections, controlled by $d_0(p) = r_0 + 4v_2p^2 + \dots$
- Good agreement with Koonin theoretical calculations and with pp ALICE CFs data, **with no fit**

$\alpha\alpha$: femtoscopy for nuclear physics

[A. Garcia-Lorenzo et al., *in preparation* (2026)]

- First fully theoretical description of the $\alpha\alpha$ CF
- Coulomb-distorted waves, Bose symmetry, 0^+ and 2^+ channels of ^8Be , $^9\text{Be}^*$ feed-down, momentum resolution
- Source radii **stable** against the three S-wave inputs

Outlook

- Decades-old $\alpha\alpha$ data are **already sensitive** to the interaction model and to the finite-range term
- **Precise** measurements would turn $\alpha\alpha$ CFs into an independent benchmark of nuclear-cluster dynamics
- Extension to 3α systems, and to any charged pair

Thank you!



Coulomb interactions in hadronic femtoscopy: applications to the pp and $\alpha\alpha$ systems

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Backup slides